Beyond the Standard Model

Lecture 4

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Outline:

- *Electroweak symmetry breaking (Lecture 1)*
- *Quark and lepton masses; vectorlike quarks (Lecture 2)*
- *New gauge bosons (Lecture 3)*
- **Extra dimensions; MSSM; WIMPs and cascade decays (Lectures 4 & 5)**

September 2011 - European School of HEP

"New physics" at the TeV scale could change the basic hypotheses of the Standard Model:

local quantum field theory
in 3 spatial + 1 time dimensions,
invariant under $SO(3,1)$ Lorentz transformations.

... "terra incognita" ... "uncharted waters" ...

Theories with extra spatial dimensions:

- **graviton only propagates in $n \geq 2$ flat extra dimensions (ADD)**
  (rephrases the hierarchy problem)
- **bosons only propagate in some flat extra dimensions (DDG)**
  (allows gauge coupling unification at lower scales)
- **all particles propagate in some flat extra dimensions (UED)**
  (has dark matter candidate; explains proton stability for $n = 2$)
- **graviton only propagates in a warped extra dimension (RS)**
  (solves hierarchy problem)
- **all particles propagate in a warped extra dimension**
  (makes fermion masses exponentially sensitive to inputs)
- ...

Bosons in extra spatial dimensions

4D flat spacetime $\perp$ one dimension of size $L = \pi R$:

A scalar field in the bulk, $\phi(x^\alpha)$, $\alpha = 0, 1, ..., 4$:

$$\mathcal{L}_{5D} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - \left(\partial^4 \phi\right)^\dagger \partial_4 \phi - m_0^2 \phi^\dagger \phi \ , \qquad \mu = 0,1,2,3$$

$\Rightarrow$ Equation of motion: $\left(\partial^\mu \partial_\mu - \partial^4 \partial_4\right) \phi = m_0^2 \phi$

m_0 is the 5D mass of ϕ.

Neumann boundary conditions for “even” fields:

$$\frac{\partial}{\partial x^4}\phi(x^\mu,0)=\frac{\partial}{\partial x^4}\phi(x^\mu,\pi R)=0$$

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Solution to the equation of motion:

$$\phi(x^\mu,x^4)=\frac{1}{\sqrt{\pi R}}\left[\phi^{(0)}(x^\mu)+\sqrt{2}\sum_{j\geq 1}\phi^{(j)}(x^\mu)\cos\left(\frac{jx^4}{R}\right)\right]$$

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Kaluza-Klein decomposition

Zero-mode (wave function is constant along x^4)

Kaluza-Klein modes: particles of definite momentum along x^4

4D point of view: a tower of massive particles:

$$\mathcal{L}_{4D}=\int_0^{\pi R}dx^4\,\mathcal{L}_{5D}\qquad\Rightarrow\qquad m_j^2=m_0^2+\frac{j^2}{R^2}$$

Dirichlet boundary conditions for “odd” fields:

$$\phi(x,0)=\phi(x,\pi R)=0$$

KK decomposition: $$\phi(x^\mu,x^4)=\frac{\sqrt{2}}{\sqrt{\pi R}}\sum_{j\geq 1}\phi^{(j)}(x^\mu)\sin\left(\frac{jx^4}{R}\right)$$

There is no zero-mode.
The lightest KK mode is $\phi^{(1)}$, of mass $\sqrt{1/R^2+m_0^2}$

Homework: *Check that the normalization condition for KK functions requires the factor of* $\sqrt{2}$.
Why $j<0$ *is not allowed?*

Gauge bosons in 5D:

$A_\mu(x^\nu, x^4)$, $\mu, \nu = 0, 1, 2, 3$, and
$A_4(x^\nu, x^4)$ – polarization along the extra dimension.

From the point of view of the 4D theory:
$A_4(x^\nu, x^4)$ is a tower of spinless KK modes.

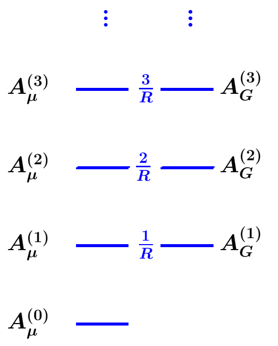
Gauge invariance requires A_μ to have a zero-mode:

$$\partial_4 A_\mu(x^\nu, 0) = \partial_4 A_\mu(x^\nu, \pi R) = 0$$

$$A_\mu(x^\nu, x^4) = \frac{1}{\sqrt{\pi R}}\left[A_\mu^{(0)}(x^\nu) + \sqrt{2}\sum_{j\geq 1} A_\mu^{(j)}(x^\nu)\cos\left(\frac{jx^4}{R}\right)\right]$$

Kaluza-Klein spectrum of gauge bosons

$A_G^{(j)}(x^\nu)$ becomes the longitudinal degree of freedom of the spin-1 KK mode $A_\mu^{(j)}(x^\nu)$.



$$\text{Dirichlet B.C}: \quad A_4(x^\nu, 0) = A_4(x^\nu, \pi R) = 0$$

$$\text{KK decomposition}: \quad A_4(x^\nu, x^4) = \sqrt{\frac{2}{L}}\sum_{j\geq 1} A_G^{(j)}(x)\sin\left(\frac{jx^4}{R}\right)$$

$\rightarrow$ $A_4(x^\nu, x^4)$ does not have a 0-mode! *(Odd field)*

Fermions in a compact dimension

Gamma matrices – require 5 anti-commuting matrices:

γ^μ, $\mu = 0, 1, 2, 3$, and $\gamma^4 = i\gamma_5$

These are 4×4 matrices $\rightarrow$ 5D fermions have 4 components.

$\Rightarrow$ 5D fermions are vector-like:

$$\chi(x^\mu, x^4) = \chi_L(x^\mu, x^4) + \chi_R(x^\mu, x^4)$$

Chiral boundary conditions:

$$\begin{aligned} \chi_L(x^\mu, 0) &= \chi_L(x^\mu, \pi R) = 0 \\ \frac{\partial}{\partial x^4}\chi_R(x^\mu, 0) &= \frac{\partial}{\partial x^4}\chi_R(x^\mu, \pi R) = 0 \end{aligned}$$

Dirac equation in 5D:

$$i\gamma^\mu \partial_\mu \chi_R = (\partial_4 + m_0)\chi_L$$

$$i\gamma^\mu \partial_\mu \chi_L = (-\partial_4 + m_0)\chi_R$$

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Kaluza-Klein decomposition:

$$\chi = \frac{1}{\sqrt{\pi R}} \left\{ \chi_R^0(x^\mu) + \sqrt{2} \sum_{j \geq 1} \left[\chi_R^j(x^\mu) \cos\left(\frac{\pi j x^4}{L}\right) + \chi_L^j(x^\mu) \sin\left(\frac{\pi j x^4}{L}\right) \right] \right\}$$

0-mode is a chiral fermion!

KK modes are vectorlike fermions.

Homework: solve 5D Dirac equation when χ_R is odd and χ_L is even.

Kaluza-Klein spectrum of quarks and leptons

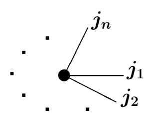
$(t_L^{(3)}, b_L^{(3)})$	$\frac{3}{R}$	$(T_R^{(3)}, B_R^{(3)})$	$T_L^{(3)}$	$\frac{3}{R}$	$t_R^{(3)}$
$(t_L^{(2)}, b_L^{(2)})$	$\frac{2}{R}$	$(T_R^{(2)}, B_R^{(2)})$	$T_L^{(2)}$	$\frac{2}{R}$	$t_R^{(2)}$
$(t_L^{(1)}, b_L^{(1)})$	$\frac{1}{R}$	$(T_R^{(1)}, B_R^{(1)})$	$T_L^{(1)}$	$\frac{1}{R}$	$t_R^{(1)}$
(t_L, b_L)					t_R

Universal Extra Dimensions

All Standard Model particles propagate in $D \geq 5$ dimensions.

Momentum conservation $\longrightarrow$ KK-number conservation

$$\mathcal{L}_{4D} = \int_0^{\pi R} dx^4 \, \mathcal{L}_{5D}$$



At each interaction vertex:

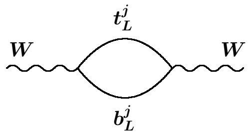
$j_1 \pm j_2 \pm ... \pm j_n = 0$ **for a certain choice of $\pm$**

In particular: $0 \pm \cdots \pm 0 \neq 1$

$\Rightarrow$ tree-level exchange of KK modes does not contribute to currently measurable quantities

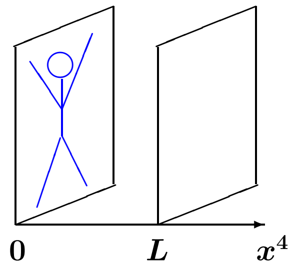
$\Rightarrow$ no single KK 1-mode production at colliders

Bounds from one-loop shifts in W and Z masses, and other observables:

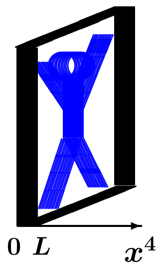


$$\frac{1}{R} \gtrsim 300 - 500\ \mathrm{GeV}$$

Bosons in 5D,
localized fermions:

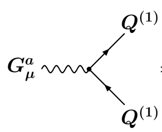


A 4-th universal spatial dimension:

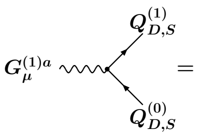


We all have a constant thickness in the 4th spatial dimension.

Feynman rules relevant for QCD production of KK particles at hadron colliders:



$= -ig_s\gamma^\mu T^a$



$= -ig_s\gamma^\mu P_{L,R} T^a$

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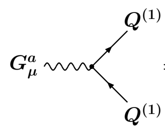

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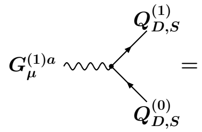

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Interaction of a level-1 quark with a level-1 gluon is chiral

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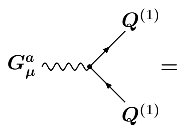

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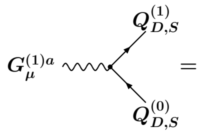
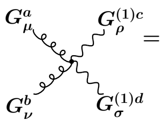


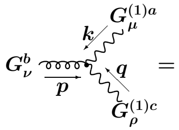
$= -ig_s\gamma^\mu P_{L,R} T^a$

Interaction of a level-1 quark with a level-1 gluon is chiral

Feynman rules for interactions of standard-model gluons with KK modes are fixed by gauge invariance:



$$= -ig_s^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$$

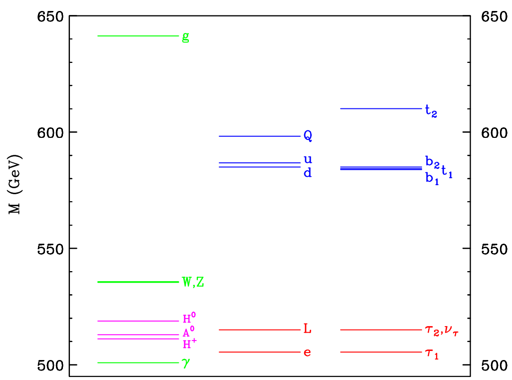


$$= g_s f^{abc}[(k-p)_\lambda g_{\mu\nu} + (p-q)_\mu g_{\nu\rho} + (q-k)_\nu g_{\mu\rho}]$$

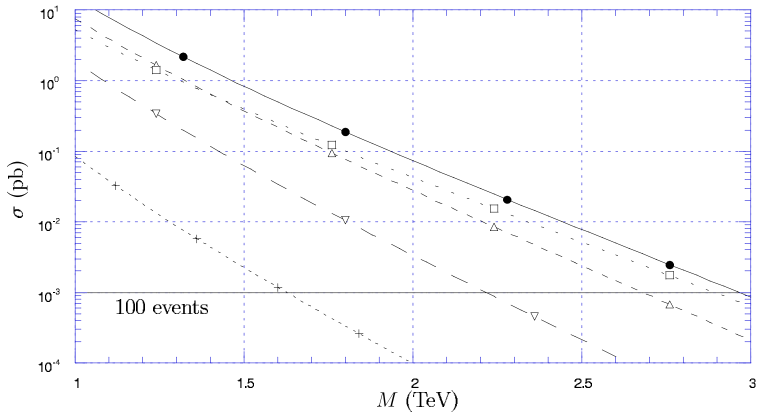
(1) modes have a tree-level mass of $1/R$, and KK parity $-$.

One-loop contributions (and electroweak symmetry breaking) split the spectrum *(Cheng, Matchev, Schmaltz, hep-ph/0204342)*

Mass spectrum of the (1) level:

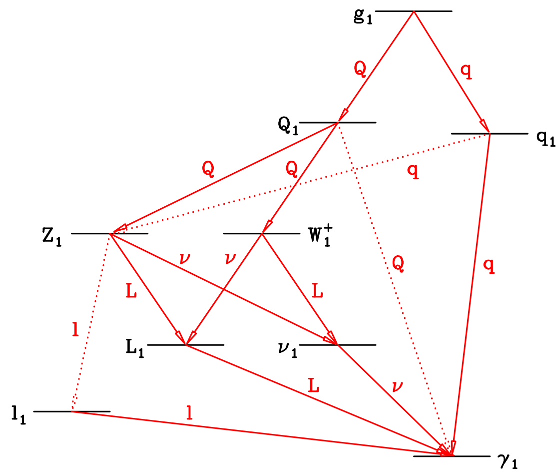


Cross section for production of a pair of level-1 particles at the LHC14, as a function of the compactification scale $1/R$:



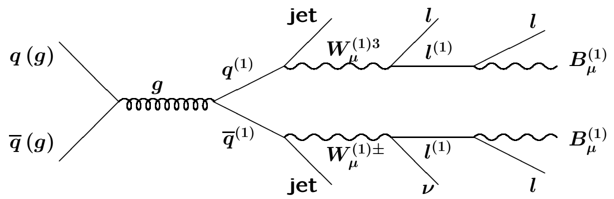
(Macesanu, McMullen, Nandi, hep-ph/0201300)

Decay modes of the KK particles:



Homework: compute the branching fractions of the level-1 particles.

Pair production of (1) modes at hadron colliders:



Look for: 2 hard leptons (~ 100 GeV)
$+$ 1 soft lepton (~ 10 GeV)
$+$ 2 jets (~ 50 GeV)
$+$ $\not{E}_T$

(Cheng, Matchev, Schmaltz, hep-ph/0205314; ...)

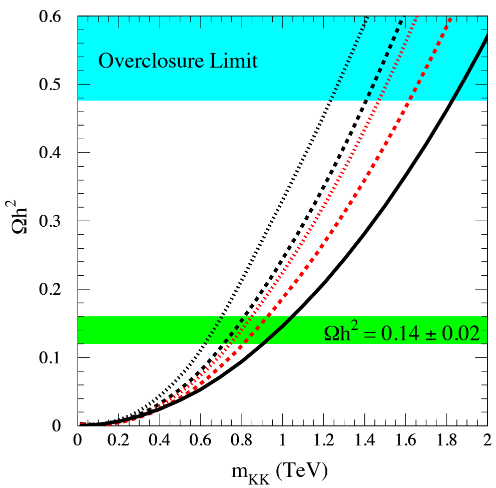
Homework: draw other diagrams which contribute to this signal.

CDF analysis of $3l + \not{E}_T$: $1/R > 280$ GeV (Run I)

Lightest KK particle is stable in UED:

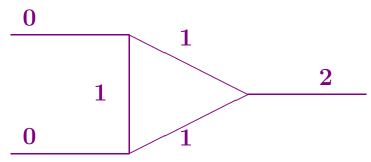
$\gamma^{(1)}$ is a viable dark matter candidate

(from Servant, Tait, hep-ph/0206071)



At one-loop level: $j_1 \pm j_2 \pm ... \pm j_n =$ even

At colliders: s-channel production of the 2-modes

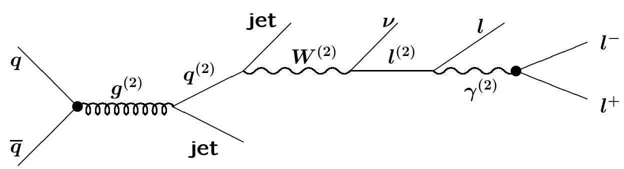


Kaluza-Klein parity: invariance under reflections with respect to the center of the compact dimension.

KK parity $(-1)^j$ is conserved $\Rightarrow$ lightest KK-odd particle is stable.

(only KK modes with odd j are odd under KK parity)

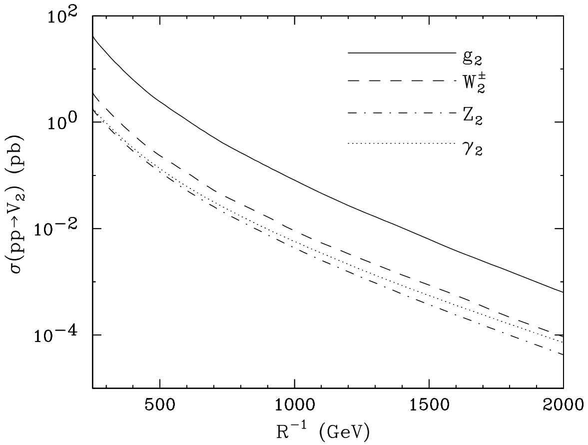
Second-level masses: $\sim 2/R$.
Cascade decay of the 2-mode is followed by $\gamma^{(2)}$ decay into hard leptons:



Allows discrimination of UED & MSSM

(A. Datta, K. Kong, K. Matchev, hep-ph/0509246)

Cross section for s-channel production of a level-2 boson (of mass $2/R$ + corrections) at the LHC14:



(A. Datta, K. Kong, K. Matchev, hep-ph/0509246)

Conclusions to Lecture 4

- Any particle that propagates in $D \geq 5$ would appear in experiments as a tower of heavy 4-dimensional particles.

- Kaluza-Klein modes of the quarks and leptons are vectorlike fermions. Chirality of 0-modes arises due to boundary conditions.

- One Universal Extra Dimension

– compactification scale can be as low as ~ 500 GeV.

– lightest KK mode is a dark matter candidate

- Look for Kaluza-Klein modes at the Tevatron and the LHC:

– 3 soft leptons + jets + $\not{E}_T$

– series of narrow $\ell^+\ell^-$ resonances due to level-2 particles.