

Neutrino Physics

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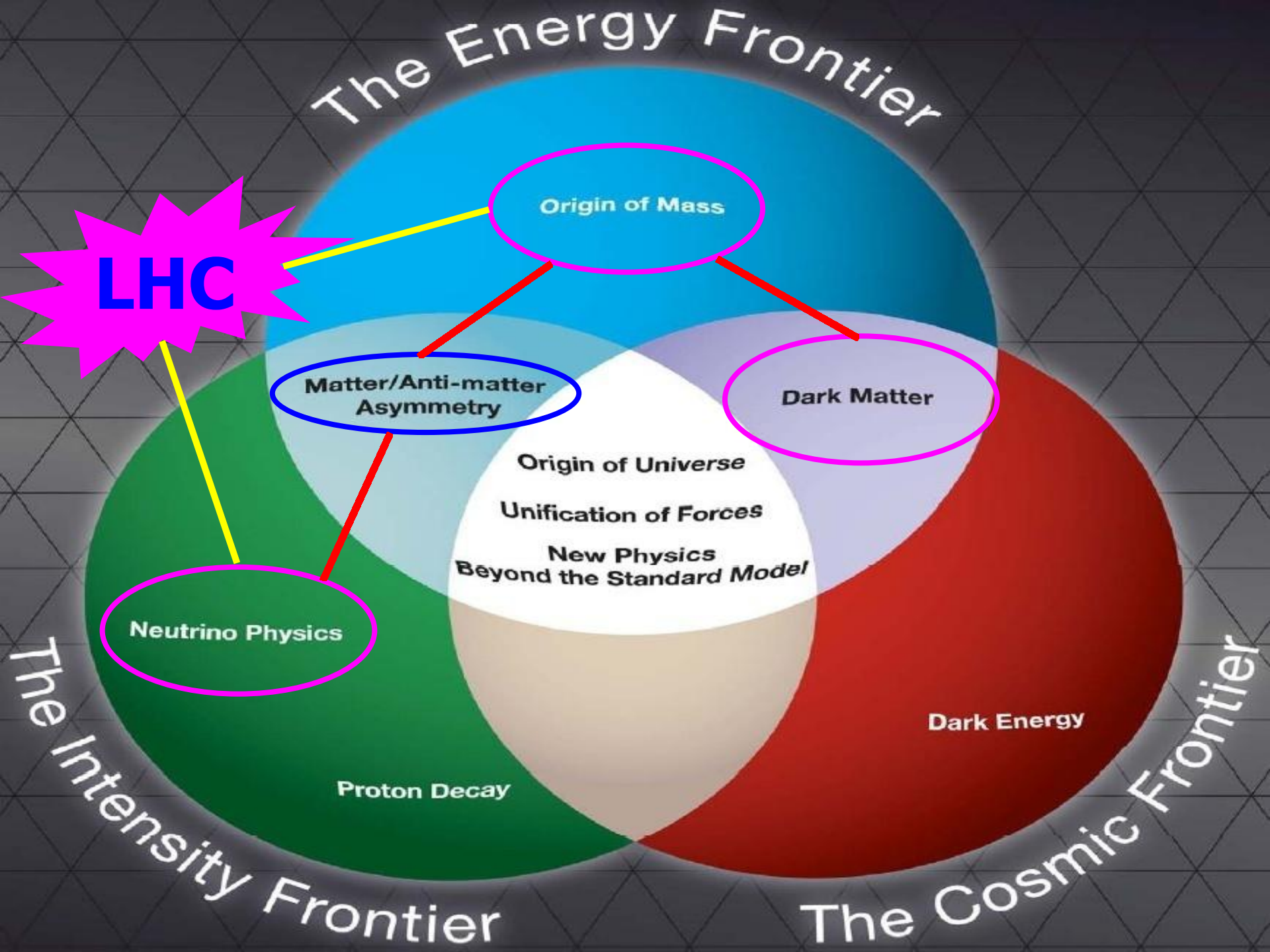
- A1:** Neutrino's history & lepton families
- A2:** Dirac & Majorana neutrino masses
- B1:** Lepton flavor mixing & CP violation
- B2:** Neutrino oscillation phenomenology
- C1:** Seesaw & leptogenesis mechanisms
- C2:** Extreme corners in the neutrino sky



@ The 1st Asia-Europe-Pacific School of HEP, 10/2012, Fukuoka

Lecture C1

- ★ **Ways to Generate Neutrino Mass**
- ★ **TeV Seesaws: Natural/Testable?**
- ★ **Collider Signals of TeV Seesaws?**



Within the SM

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All ν 's are **massless** in the SM, a result of the model's simple structure:

---- $SU(2)_L \times U(1)_Y$ **gauge symmetry** and **Lorentz invariance**;

Fundamentals of the model, mandatory for consistency of a QFT.

---- Economical **particle content**:

No right-handed neutrinos --- a **Dirac** mass term is not allowed.

Only one Higgs doublet --- a **Majorana** mass term is not allowed.

---- Mandatory **renormalizability**:

No dimension ≥ 5 operators: a **Majorana** mass term is forbidden.

To generate ν -masses, one or more of the constraints must be relaxed.

--- The **gauge symmetry** and **Lorentz invariance** cannot be abandoned;

--- The **particle content** can be modified;

--- The **renormalizability** can be abandoned.

How many ways?

Beyond the SM (1)

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Way 1: to relax the requirement of **renormalizability** (S. Weinberg **79**)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{\mathcal{L}_{\text{d}=5}}{\Lambda} + \frac{\mathcal{L}_{\text{d}=6}}{\Lambda^2} + \dots$$

In the SM, the **lowest-dimension operator** that violates **lepton/baryon** number is

$$\frac{1}{M} H H L L$$

neutrino mass

$$\text{Seesaw: } m_{1,2,3} \sim \langle H \rangle^2 / M$$

$$m_{1,2,3} < 1 \text{ eV} \Rightarrow M > 10^{13} \text{ GeV}$$

$$\frac{1}{M^2} Q Q Q L$$

proton decay

$$\text{Example : } p \rightarrow \pi^0 + e^+$$

$$\tau_p > 10^{33} \text{ years} \Rightarrow M > 10^{15} \text{ GeV}$$

Neutrino masses/proton decays: windows onto physics at high scales

Beyond the SM (2)

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Way 2: to add **3 right-handed** neutrinos & demand a **(B - L)** symmetry

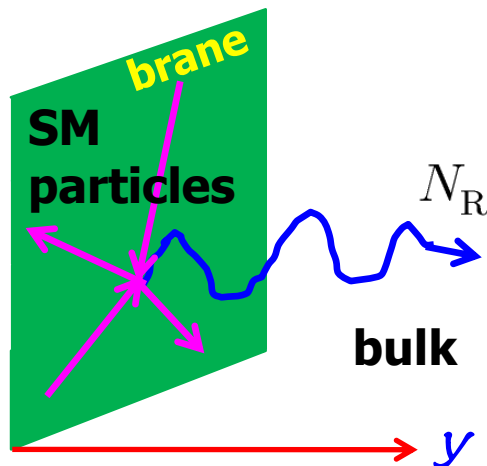
A pure **Dirac** mass term

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_\nu \tilde{H} N_R + \text{h.c.}$$

$$\begin{aligned} M_l &= \frac{v}{\sqrt{2}} Y_l \\ M_\nu &= \frac{v}{\sqrt{2}} Y_\nu \end{aligned}$$

The hierarchy problem: $y_i/y_e = m_i/m_e \lesssim 0.5 \text{ eV}/0.5 \text{ MeV} \sim 10^{-6}$

A very speculative way out: the smallness of **Dirac** masses is ascribed to the assumption that **N_R** have access to an extra spatial dimension (Dienes, Dudas, Gherghetta **98**; Arkani-Hamed, Dimopoulos, Dvali, March-Russell **98**) :



The wavefunction of **N_R** spreads out over the extra dimension **y** , giving rise to a suppressed Yukawa interaction at **$y = 0$** .

$$\left[\bar{l}_L Y_\nu \tilde{H} N_R \right]_{y=0} \sim \frac{1}{\sqrt{L}} \left[\bar{l}_L Y_\nu \tilde{H} N_R \right]_{y=L}$$

(e.g., King **08**)

$$\Lambda_{\text{String}}/\Lambda_{\text{Planck}} \sim 10^{-12}$$

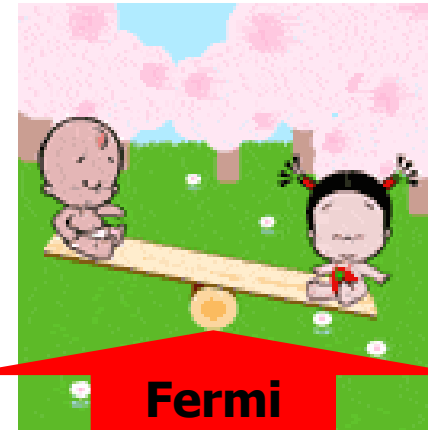
Beyond the SM (3)

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Seesaw: add new heavy degrees of freedom and allow (B-L) violation:



Seesaw — A Footnote Idea:
H. Fritzsch, M. Gell-Mann,
P. Minkowski, PLB 59 (1975) 256



**Fermi
scale**

T-1: SM + 3 right-handed neutrinos (Minkowski 77;
 Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79;
 Mohapatra, Senjanovic 79)

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.}$$

T-2: SM + 1 Higgs triplet (Konetschny, Kummer 77; Magg, Wetterich 80;
 Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80; Mohapatra, Senjanovic 80)

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \frac{1}{2} \bar{l}_L Y_\Delta \Delta i \sigma_2 l_L^c - \lambda_\Delta M_\Delta H^T i \sigma_2 \Delta H + \text{h.c.}$$

variations

T-3: SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} \text{Tr} (\bar{\Sigma} M_\Sigma \Sigma^c) + \text{h.c.}$$

combinations

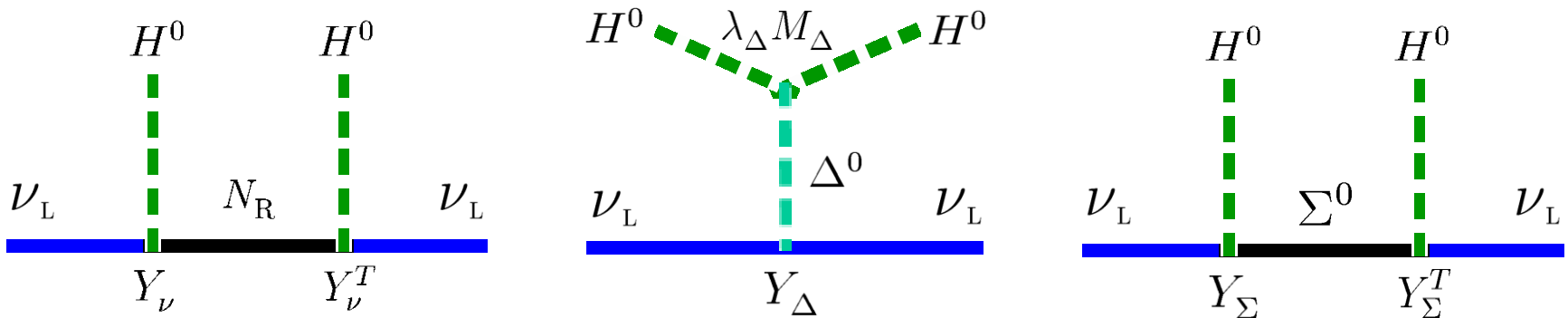
Seesaws

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Weinberg operator: the unique **dimension-five** operator of **ν -masses** after integrating out the heavy degrees of freedom.

$$\frac{\mathcal{L}_{d=5}}{\Lambda} = \left\{ \begin{array}{l} \frac{1}{2} \left(Y_\nu M_R^{-1} Y_\nu^T \right)_{\alpha\beta} \bar{l}_{\alpha L} \tilde{H} \tilde{H}^T l_{\beta L}^c + \text{h.c.} \\ -\frac{\lambda_\Delta}{M_\Delta} (Y_\Delta)_{\alpha\beta} \bar{l}_{\alpha L} \tilde{H} \tilde{H}^T l_{\beta L}^c + \text{h.c.} \\ \frac{1}{2} \left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T \right)_{\alpha\beta} \bar{l}_{\alpha L} \tilde{H} \tilde{H}^T l_{\beta L}^c + \text{h.c.} \end{array} \right. \quad M_\nu = \left\{ \begin{array}{ll} -\frac{1}{2} Y_\nu \frac{v^2}{M_R} Y_\nu^T & \text{(Type 1)} \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & \text{(Type 2)} \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & \text{(Type 3)} \end{array} \right.$$

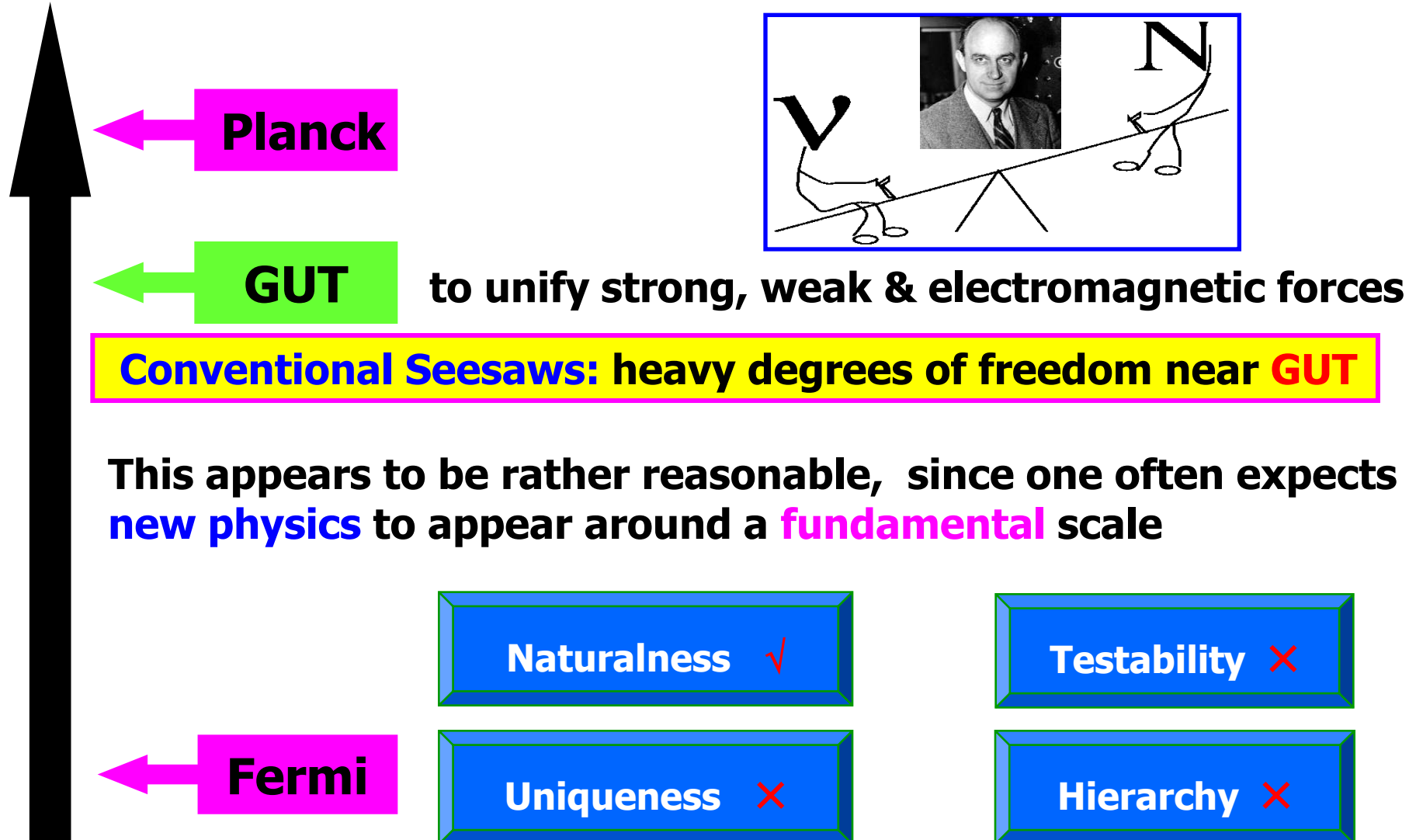
After SSB, a Majorana mass term is $-\mathcal{L}_{\text{mass}} = \frac{1}{2} \bar{\nu}_L M_\nu \nu_L^c + \text{h.c.}$ $\langle \tilde{H} \rangle = v/\sqrt{2}$



Seesaw Scale?

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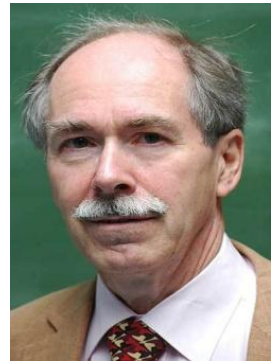
What is the energy scale at which the **seesaw** mechanism works?



Lower Scale?

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There is no direct evidence for a large or extremely large seesaw scale. So **eV-**, **keV-**, **MeV-** or **GeV-**scale seesaws are all possible, at least in principle; they are **technically natural** according to 't Hooft's naturalness criterion.



't Hooft's naturalness criterion (80):

At any energy scale μ , a set of parameters, $\alpha_i(\mu)$ describing a system can be small, if and only if, in the limit $\alpha_i(\mu) \rightarrow 0$ for each of these parameters, the system exhibits an enhanced symmetry.

Potential problems of low-scale seesaws:

- No obvious connection to a theoretically well-justified fundamental scale (for example, Fermi scale, TeV scale, GUT or Planck scale).
- The neutrino Yukawa couplings are simply tiny, no actual explanation of why the masses of three known neutrinos are so small.
- A very low seesaw scale doesn't allow canonical thermal leptogenesis to work, though there might be a very *contrived* way out.

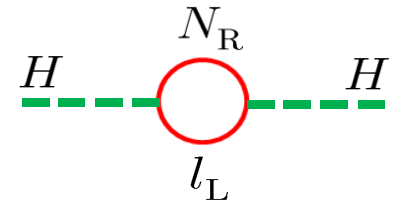
Hierarchy Problem

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Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom in seesaw (Vissani 98; Casas et al 04; Abada et al 07)

Type 1:

$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2} \left(\Lambda^2 + M_i^2 \ln \frac{M_i^2}{\Lambda^2} \right)$$

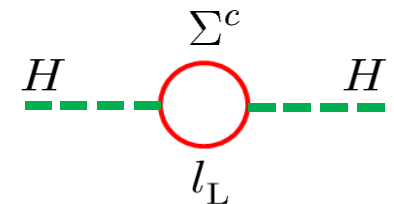


Type 2:

$$\delta m_H^2 = \frac{3}{16\pi^2} \left[\lambda_3 \left(\Lambda^2 + M_\Delta^2 \ln \frac{M_\Delta^2}{\Lambda^2} \right) + 4\lambda_\Delta^2 M_\Delta^2 \ln \frac{M_\Delta^2}{\Lambda^2} \right]$$

Type 3:

$$\delta m_H^2 = -\frac{3y_i^2}{8\pi^2} \left(\Lambda^2 + M_i^2 \ln \frac{M_i^2}{\Lambda^2} \right)$$



here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning

$$M_i \sim \left[\frac{(2\pi v)^2 |\delta m_H^2|}{m_i} \right]^{1/3} \sim 10^7 \text{ GeV} \left[\frac{0.2 \text{ eV}}{m_i} \right]^{1/3} \left[\frac{|\delta m_H^2|}{0.1 \text{ TeV}^2} \right]^{1/3}$$

Possible way out: (1) **Supersymmetric** seesaw? (2) **TeV-scale** seesaw?

The Seesaw Scale?

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Planck scale

$\Lambda \sim 10^{19} \text{ GeV}$ **The SM vacuum stability for a light Higgs**

GUT scale?

$\Lambda \sim 10^{16} \text{ GeV}$

Seesaw scale?

$\Lambda \sim 10^{12} \text{ GeV}$

TeV / SUSY?

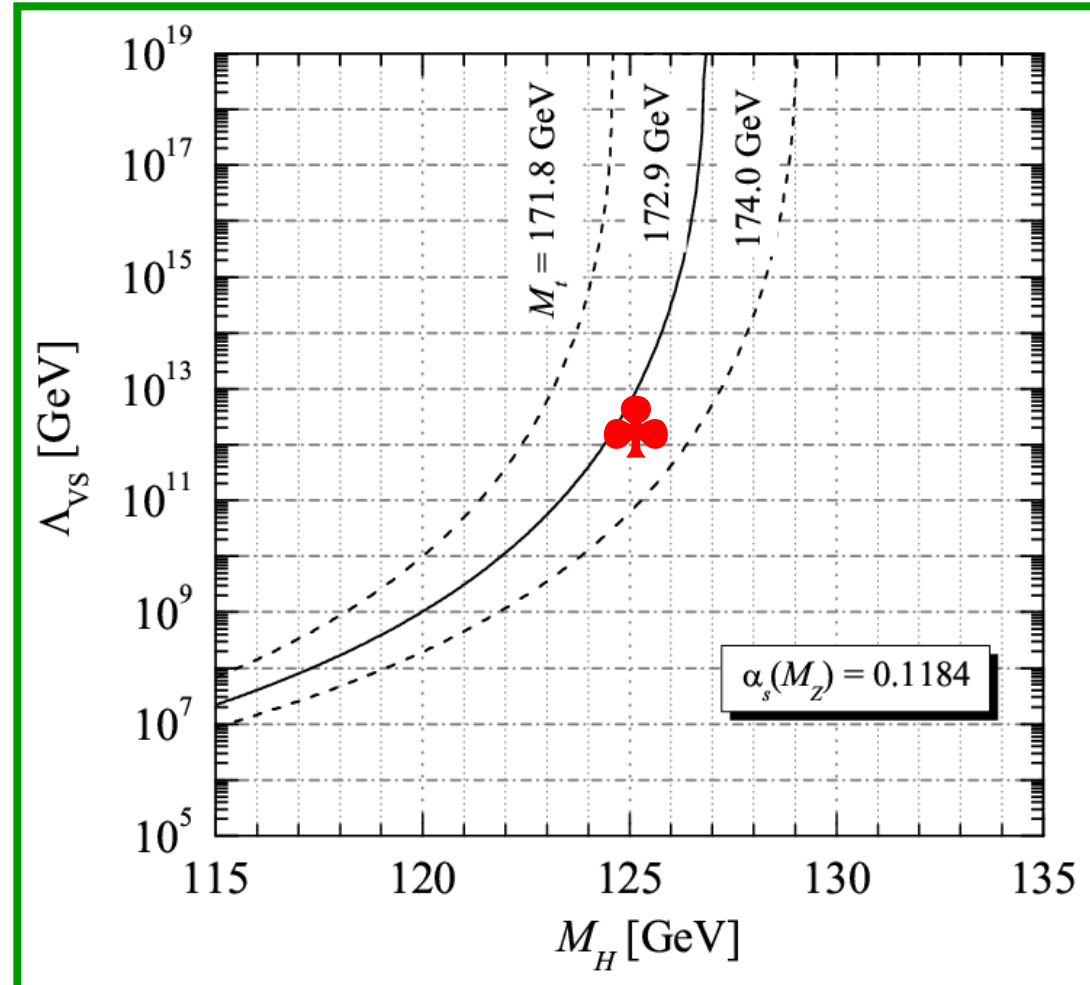
$\Lambda \sim 10^3 \text{ GeV}$

Fermi scale

$\Lambda \sim 10^2 \text{ GeV}$

QCD scale

$\Lambda \sim 10^2 \text{ MeV}$



Elias-Miro et al., arXiv:1112.3022;
Xing, Zhang, Zhou, arXiv:1112.3112; ...

TeV Neutrino Physics?

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to discover the SM Higgs boson

to verify Yukawa interactions

to pin down heavy seesaw particles

to test seesaw mechanism(s)

to measure low-energy effects

Why

Not

Try



Type-1 Seesaw

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Type-1 Seesaw: add **3 right-handed** Majorana neutrinos into the SM.

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \overline{N_R^c} M_R N_R + \text{h.c.}$$

or

$$-\mathcal{L}_{\text{mass}} = \bar{e}_L M_l E_R + \frac{1}{2} \overline{(\nu_L \quad N_R^c)} \begin{pmatrix} \mathbf{0} & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

Diagonalization (flavor basis \Rightarrow mass basis):

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}$$

$$V^\dagger V + S^\dagger S = VV^\dagger + RR^\dagger = 1$$

Hence **V** is not unitary

Seesaw:

$$M_\nu \equiv V \widehat{M}_\nu V^T \approx -M_D M_R^{-1} M_D^T$$

$$R \sim S \sim M_D / M_R$$

Strength of Unitarity Violation

$$V \approx \left(1 - \frac{1}{2} R R^\dagger \right) V_{\text{unitary}}$$

Natural or Unnatural?

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Natural case: no large cancellation in the leading seesaw term.

$$M_\nu \approx M_D M_R^{-1} M_D^T$$

0.01 eV

10^{15} GeV

100 GeV

$$R \sim S \sim M_D / M_R \sim 10^{-13}$$

$$\text{Unitarity Violation} \sim 10^{-26}$$

Unnatural case: large cancellation in the leading seesaw term.

$$M_\nu \approx M_D M_R^{-1} M_D^T$$

0.01 eV

1 TeV

100 GeV

$$R \sim S \sim M_D / M_R \sim 10^{-1}$$

$$\text{Unitarity Violation} \sim 10^{-2}$$

TeV-scale (right-handed) Majorana neutrinos: small masses of **3** light **Majorana** neutrinos come from **sub-leading perturbations**.

Structural Cancellation

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Given diagonal M_R with 3 mass eigenvalues M_1 , M_2 and M_3 , the leading (i.e., **type-I seesaw**) term of the active neutrino mass matrix vanishes, if and only if M_D has rank 1,

and if

$$M_D = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}$$

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0$$

$$M_\nu \approx M_D M_R^{-1} M_D^T = 0$$

(Buchmueller, Greub 91; Ingelman, Rathsman 93; Heusch, Minkowski 94;; Kersten, Smirnov 07).

Tiny ν -masses can be generated from tiny corrections to this complete “**structural cancellation**”, by deforming M_D or M_R .

Simple example:

$$M'_D = M_D + \epsilon X_D$$

$$\begin{aligned} M'_\nu &= M'_D M_R^{-1} M'^T_D \\ &\approx \epsilon \left(M_D M_R^{-1} X_D^T + X_D M_R^{-1} M_D^T \right) + \mathcal{O}(\epsilon^2) \end{aligned}$$

Fast Lessons

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Lesson 1: two necessary conditions to test a seesaw model with heavy right-handed Majorana neutrinos at the **LHC**:

---Masses of heavy Majorana neutrinos must be of $O(1)$ **TeV** or below

---Light-heavy neutrino mixing (i.e. M_D/M_R) must be large enough

Lesson 2: A collider signature of the heavy Majorana ν 's is essentially decoupled from masses and mixing parameters of light ν 's.

Lesson 3: **non-unitarity** of the light ν flavor mixing matrix might lead to observable effects in ν oscillations and rare processes.

Lesson 4: nontrivial limits on heavy Majorana ν 's could be derived at the **LHC**, if the SM backgrounds are small for a specific final state.

$\Delta L = 2$ like-sign dilepton events

$$pp \rightarrow W^\pm W^\pm \rightarrow \mu^\pm \mu^\pm jj \text{ and } pp \rightarrow W^\pm \rightarrow \mu^\pm N \rightarrow \mu^\pm \mu^\pm jj$$

Collider Signature

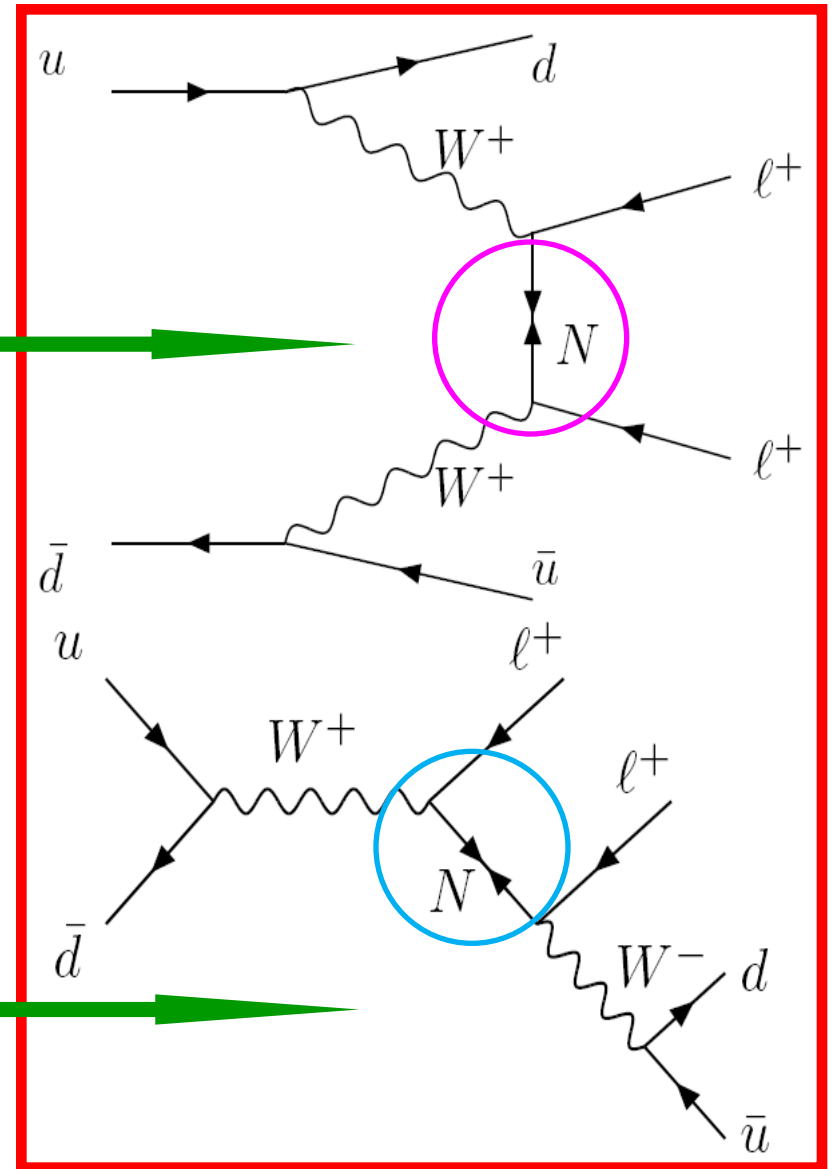
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Lepton number violation: like-sign dilepton events at hadron colliders, such as Tevatron ($\sim 2 \text{ TeV}$) and LHC ($\sim 14 \text{ TeV}$).

collider analogue to $0\nu\beta\beta$ decay

dominant channel

N can be produced on resonance



Testability at the LHC

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Distinguishing seesaw models at LHC
with multi-lepton signals

F. del Aguila, J. A. Aguilar-Saavedra

2 recent comprehensive works:

arXiv:0808.2468v2 [hep-ph] 12 Sep 2008

The Search for Heavy Majorana Neutrinos

arXiv:0901.3589v1 [hep-ph] 23 Jan 2009

Anupama Atre^{1,2}, Tao Han^{2,3,4}, Silvia Pascoli⁵, Bin Zhang^{4*}

We also extend the search to hadron collider experiments. We find that, at the Tevatron with 8 fb^{-1} integrated luminosity, there could be 2σ (5σ) sensitivity for resonant production of a Majorana neutrino in the $\mu^\pm \mu^\pm$ modes in the mass range of $\sim 10 - 180 \text{ GeV}$ ($10 - 120 \text{ GeV}$). This reach can be extended to $\sim 10 - 375 \text{ GeV}$ ($10 - 250 \text{ GeV}$) at the LHC of 14 TeV with 100 fb^{-1} . The production cross section at the LHC of 10 TeV is also presented for comparison. We study the $\mu^\pm e^\pm$ modes as well and find that the signal could be large enough even taking into account the current bound from neutrinoless double-beta decay. The signal from the gauge boson fusion channel $W^+ W^+ \rightarrow \ell_1^+ \ell_2^+$ at the LHC is found to be very weak given the rather small mixing parameters. We comment on the search strategy when a τ lepton is involved in the final state.

Non-unitarity

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Type-1 seesaw: a typical signature would be the **unitarity violation** of the 3×3 neutrino mixing matrix **V** in the charged-current interactions

Current experimental constraints at the 90% C.L. (Antusch *et al* 07):

$$|VV^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

$\mu \rightarrow e + \gamma$ etc,
W / Z decays,
universality,
 ν -oscillation.

$$|V^\dagger V| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

**accuracy
of a few
percent!**

Extra CP-violating phases exist in a non-unitary ν mixing matrix may lead to observable **CP-violating effects** in **short- or medium-baseline** ν oscillations (Fernandez-Martinez *et al* 07; Xing 08).

Typical example: non-unitary CP violation in the $\nu_\mu \rightarrow \nu_\tau$ oscillation, an effect probably **at the percent level**.

Type-2 Seesaw

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Type-2 (Triplet) Seesaw: add **one** **SU(2)_L** Higgs triplet into the SM.

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \frac{1}{2} \bar{l}_L Y_\Delta \Delta i \sigma_2 l_L^c + \text{h.c.}$$

$$\Delta \equiv \begin{pmatrix} H^- & -\sqrt{2} H^0 \\ \sqrt{2} H^{--} & -H^- \end{pmatrix}$$

or

$$-\mathcal{L}_{\text{mass}} = \bar{e}_L M_l E_R + \frac{1}{2} \bar{\nu}_L M_L \nu_L^c + \text{h.c.}$$

$$M_L \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Potential:

$$V(H, \Delta) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \frac{1}{2} M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) - [\lambda_\Delta M_\Delta H^T i \sigma_2 \Delta H + \text{h.c.}]$$

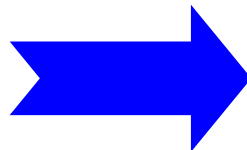
L and **B-L** violation

Naturalness? (t' Hooft **79**, ..., Giudice **08**)

- (1) **M_Δ** is **O(1) TeV** or close to the scale of gauge symmetry breaking.
- (2) **λ_Δ** must be tiny, and **$\lambda_\Delta = 0$** enhances the symmetry of the model.

$$M_L \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

0.01 eV
1 TeV



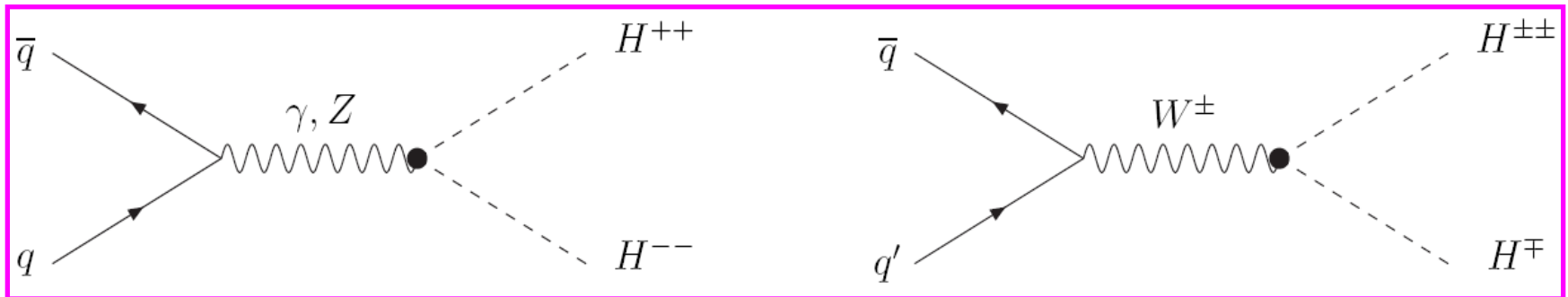
$$\lambda_\Delta Y_\Delta \sim 10^{-12} \Rightarrow \begin{cases} Y_\Delta \sim 1, \lambda_\Delta \sim 10^{-12} \\ \lambda_\Delta \sim Y_\Delta \sim 10^{-6} \\ \dots \end{cases}$$

Collider Signature

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From a viewpoint of **direct tests**, the triplet seesaw has an advantage:

The **SU(2)_L** Higgs triplet contains a **doubly-charged scalar** which can be produced at colliders: it is dependent on its mass but independent of the (small) Yukawa coupling.



Typical **LNV** signatures: $H^{\pm\pm} \rightarrow l_\alpha^\pm l_\beta^\pm$ $H^+ \rightarrow l_\alpha^+ \bar{\nu}_\beta$ $H^- \rightarrow l_\alpha^- \nu$

$$\mathcal{B}(H^{\pm\pm} \rightarrow l_\alpha^\pm l_\beta^\pm) = \frac{(2 - \delta_{\alpha\beta}) |(M_L)_{\alpha\beta}|^2}{\sum_{\rho, \sigma} |(M_L)_{\rho\sigma}|^2}, \quad \mathcal{B}(H^+ \rightarrow l_\alpha^+ \bar{\nu}) = \frac{\sum_{\beta} |(M_L)_{\alpha\beta}|^2}{\sum_{\rho, \sigma} |(M_L)_{\rho\sigma}|^2}$$

Testability at the LHC

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Lesson one: the above branching ratios **purely** depend on 3 neutrino masses, 3 flavor mixing angles and the CP-violating phases.

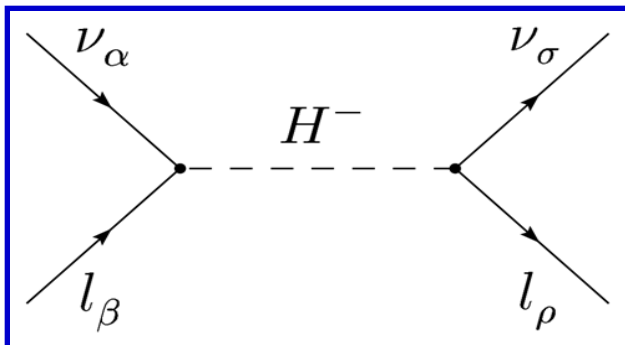
Lesson two: the **Majorana phases** may affect LNV $H^{\pm\pm} \rightarrow l_{\alpha}^{\pm} l_{\beta}^{\pm}$ decay modes, but they do not enter $H^+ \rightarrow l_{\alpha}^+ \bar{\nu}_{\beta}$ and $H^- \rightarrow l_{\alpha}^- \nu$ processes.

$$|(M_L)_{\alpha\beta}|^2 = \left| \sum_{i=1}^3 (m_i V_{\alpha i} V_{\beta i}) \right|^2, \quad \sum_{\beta} |(M_L)_{\alpha\beta}|^2 = \sum_{i=1}^3 (m_i^2 |V_{\alpha i}|^2)$$

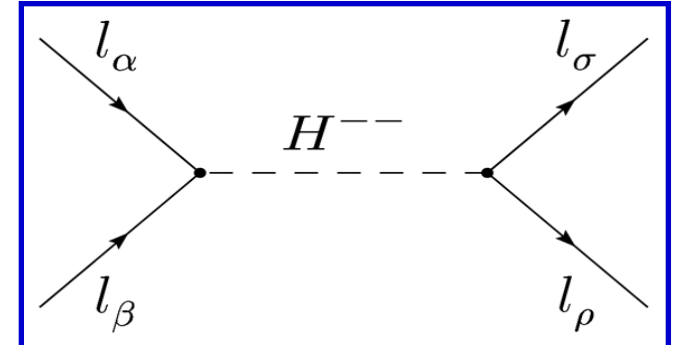
Dimension-6 operator:
(2 low-energy effects)

$$\frac{\mathcal{L}_{d=6}}{\Lambda^2} = -\frac{(Y_{\Delta})_{\alpha\beta} (Y_{\Delta})_{\rho\sigma}^{\dagger}}{4M_{\Delta}^2} (\bar{l}_{\alpha L} \gamma^{\mu} l_{\sigma L}) (\bar{l}_{\beta L} \gamma_{\mu} l_{\rho L})$$

1) **NSIs** of 3 neutrinos



2) **LFV** of 4 charged leptons



Type-3 Seesaw

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Type-3 Seesaw: add **3 SU(2)_L** triplet fermions (**Y = 0**) into the SM.

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} \text{Tr} (\bar{\Sigma} M_\Sigma \Sigma^c) + \text{h.c.}$$

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$$

or

$$-\mathcal{L}_{\text{mass}} = \overline{(e_L \ \Psi_L)} \begin{pmatrix} M_l & \sqrt{2} M_D \\ 0 & M_\Sigma \end{pmatrix} \begin{pmatrix} E_R \\ \Psi_R \end{pmatrix} + \frac{1}{2} \overline{(\nu_L \ \Sigma^0)} \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \Sigma^{0c} \end{pmatrix} + \text{h.c.}$$

$$M_l = Y_l v / \sqrt{2}, \quad M_D = Y_\Sigma v / \sqrt{2}, \quad \Psi = \Sigma^- + \Sigma^{+c}$$

Diagonalization of the neutrino mass matrix:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & 0 \\ 0 & \widehat{M}_\Sigma \end{pmatrix}$$

Seesaw formula:

$$M_\nu \equiv V \widehat{M}_\nu V^T \approx -M_D M_\Sigma^{-1} M_D^T$$

Comparison between type-1 and type-3 seesaws (Abada et al 07):

- a) The 3×3 flavor mixing matrix **V** is **non-unitary** in both cases (**CC**);
- b) The modified couplings between **Z** & neutrinos are different (**NC**);
- c) **Non-unitary** flavor mixing is also present in the coupling between **Z** and charged leptons in the **type-3** seesaw (**NC**).

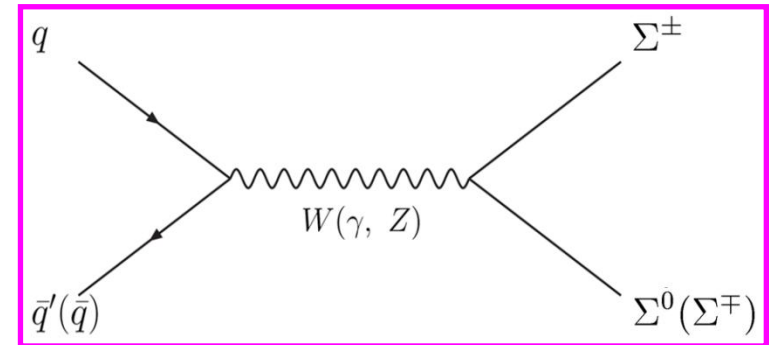
Testability at the LHC

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LVN signatures at the LHC:

$$pp \rightarrow \Sigma^+ \Sigma^0 \rightarrow l_\alpha^+ l_\beta^+ + Z^0 W^- (\rightarrow 4j)$$

$$pp \rightarrow \Sigma^- \Sigma^0 \rightarrow l_\alpha^- l_\beta^- + Z^0 W^+ (\rightarrow 4j)$$



PHYSICAL REVIEW D **78**, 033002 (2008)

Type-III seesaw mechanism at CERN LHC

Roberto Franceschini,¹ Thomas Hambye,² and Alessandro Strumia³

Neutrino masses can be generated by fermion triplets with TeV-scale mass, that would manifest at LHC as production of two leptons together with two heavy standard model (SM) vectors or Higgs, giving rise to final states such as $2\ell + 4j$ (that can violate lepton number and/or lepton flavor) or $\ell + 4j + \cancel{E}_T$. We devise cuts to suppress the SM backgrounds to these signatures. Furthermore, for most of the mass range suggested by neutrino data, triplet decays are detectably displaced from the production point, allowing to infer the neutrino mass parameters. We compare with LHC signals of type-I and type-II seesaw.

Distinguishing seesaw models at LHC
with multi-lepton signals

F. del Aguila, J. A. Aguilar-Saavedra

2 latest comprehensive works.

arXiv:0808.2468v2 [hep-ph] 12 Sep 2008

Low-energy Tests

26

Type-3 seesaw: a typical signature would be the **non-unitary effects** of the 3×3 lepton flavor mixing matrix **N** in both **CC** and **NC** interactions.

Current experimental constraints at the 90% C.L. (Abada *et al* 07):

$$|NN^\dagger| \approx \begin{pmatrix} 1.001 \pm 0.002 & < 1.1 \cdot 10^{-6} & < 1.2 \cdot 10^{-3} \\ < 1.1 \cdot 10^{-6} & 1.002 \pm 0.002 & < 1.2 \cdot 10^{-3} \\ < 1.2 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} & 1.002 \pm 0.002 \end{pmatrix}$$

**accuracy
at 0.1%.**

These bounds are **stronger** than those obtained in the **type-1 seesaw**, as the flavor-changing processes with charged leptons are allowed at the tree level in the **type-3 seesaw**.

Two types of LFV processes:

Radiative decays of charged leptons: $\mu \rightarrow e + \gamma$, $\tau \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$.

Tree-level rare decays of charged leptons: $\mu \rightarrow 3e$, $\tau \rightarrow 3e$, $\tau \rightarrow 3\mu$, $\tau \rightarrow e + 2\mu$, $\tau \rightarrow 2e + \mu$ (Abada et al 07, 08; He, Oh 09)

TeV leptogenesis or muon g-2 problems? (Strumia 08, Blanchet, Chacko, Mohapatra 08, Fischler, Flauger 08; Chao 08, Biggio 08;)

Seesaw Trivialization

27

Linear trivialization: use three types of seesaws to make a family tree.

Type 1 + Type 2

Type 1 + Type 3

Type 2 + Type 3

Type 1 + Type 2 + Type 3

Weinberg's 3rd law of progress in theoretical physics (83):

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry What could be better?



Linearly trivialized seesaws usually work at super-high energies.

Multiple trivialization: well motivated to lower the seesaw scale.

Illustration

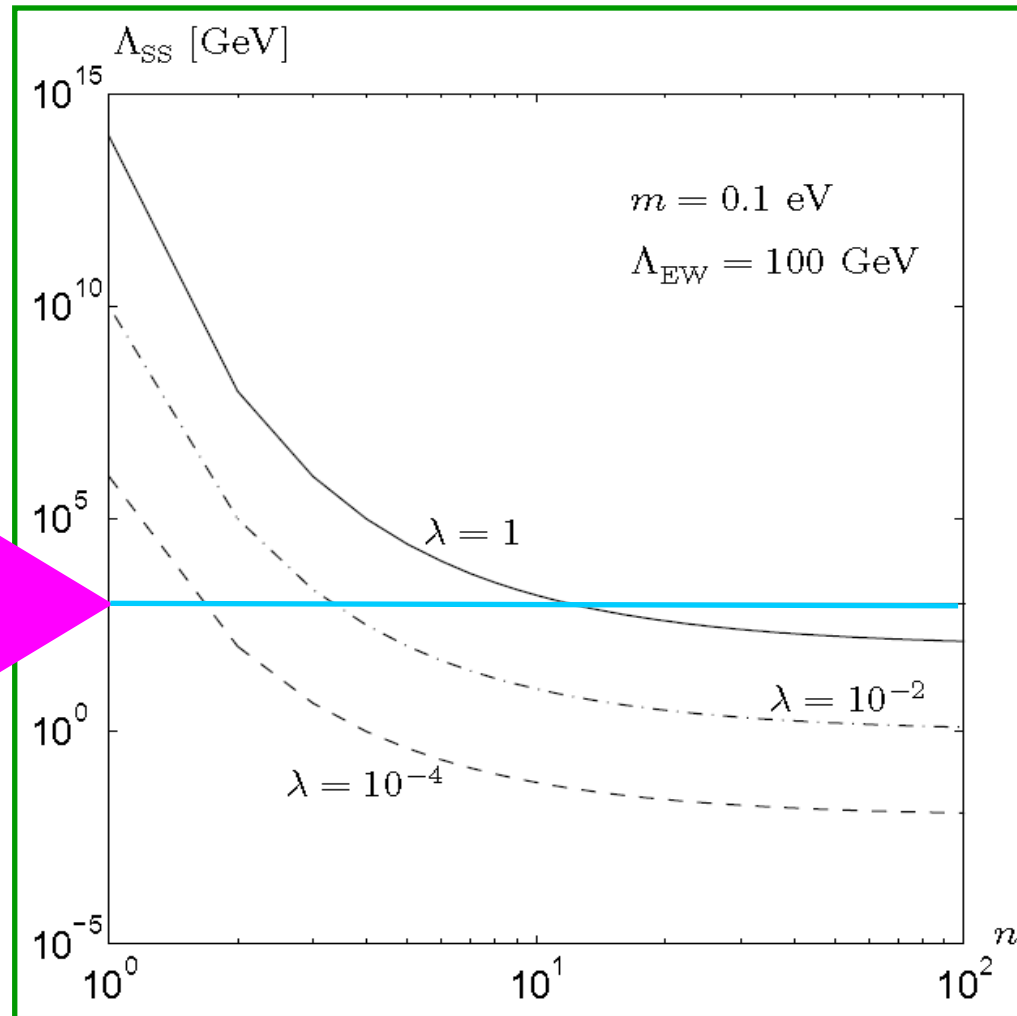
28

Neutrino mass:

$$m \sim (\lambda \Lambda_{\text{EW}})^{n+1} / \Lambda_{\text{SS}}^n$$

$$\Lambda_{\text{SS}} \sim \lambda^{\frac{n+1}{n}} \left[\frac{\Lambda_{\text{EW}}}{100 \text{ GeV}} \right]^{\frac{n+1}{n}} \left[\frac{0.1 \text{ eV}}{m} \right]^{\frac{1}{n}} 10^{\frac{2(n+6)}{n}} \text{ GeV}$$

TeV scale →



Example: Inverse Seesaw 29

The Inverse Seesaw: SM + 3 heavy right-handed neutrinos + 3 gauge singlet neutrinos + one Higgs singlet (Wyler, Wolfenstein 83; Mohapatra, Valle 86; Ma 87).

$$-\mathcal{L}_{\text{lepton}} = \overline{l}_L Y_l H E_R + \overline{l}_L Y_\nu \tilde{H} N_R + \overline{N}_R^c Y_S \Phi S_R + \frac{1}{2} \overline{S}_R^c \mu S_R + \text{h.c.}$$

↑ **LNV: tiny**

v-mass matrix:

$$\begin{pmatrix} \overline{\nu}_L & \overline{N}_R^c & \overline{S}_R^c \end{pmatrix} \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_S \\ 0 & M_S^T & \mu \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \\ S_R \end{pmatrix} \quad \begin{cases} M_D = Y_\nu \langle H \rangle \\ M_S = Y_S \langle \Phi \rangle \end{cases}$$

Effective light v-mass matrix

$$M_\nu \approx M_D \frac{1}{M_S^T} \mu \frac{1}{M_S} M_D^T \longleftrightarrow -\mathcal{L}_{\text{mass}} = \frac{1}{2} \overline{\nu}_L M_\nu \nu_L^c + \text{h.c.}$$

Merit: more natural tiny v-masses and appreciable collider signatures;
Fault: some new degrees of freedom. **Is Weinberg's 3rd law applicable?**

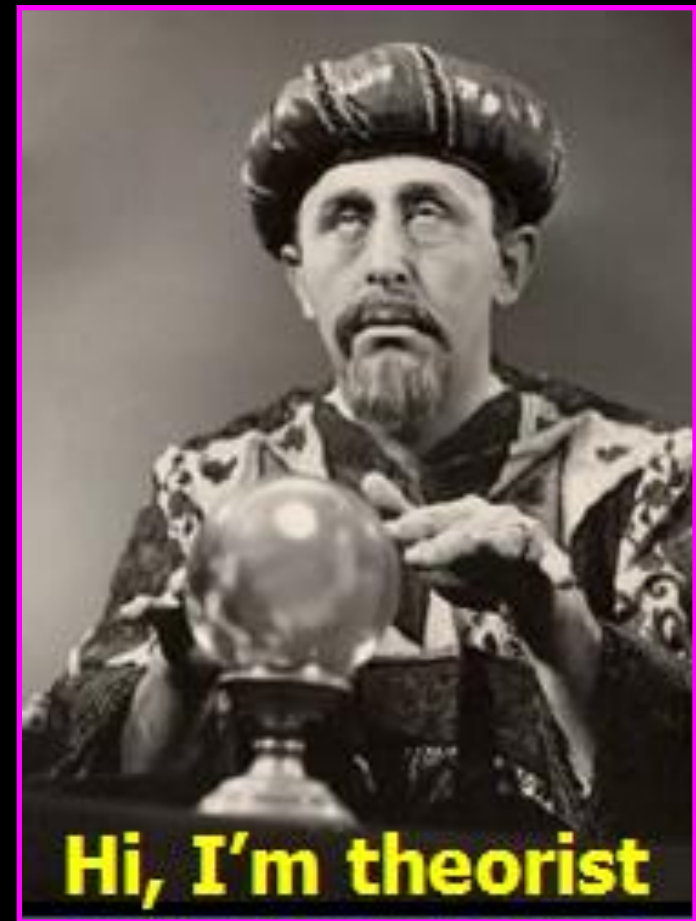
Multiple seesaw mechanisms: to *naturally* lower seesaw scales to TeV (Babu et al 09; Xing, Zhou 09; Bonnet et al 09, etc).

Appendix

30

Misguiding principles for a **theorist** to go **beyond the SM**
(Schellekens 08: "The Emperor's Last Clothes?")

- **Agreement with observation**
- **Consistency**
- **Uniqueness**
- **Naturalness**
- **Simplicity**
- **Elegance**
- **Beauty**
- **.....**



Lecture C2

- ★ **Baryogenesis via Leptogenesis**
- ★ **The Cosmic Neutrino Background**
- ★ **UHE Cosmic Neutrino Telescopes**

Dirac's Expectation

32

PAUL A. M. DIRAC

Theory of electrons and positrons

Nobel Lecture, December 12, 1933



If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.

The Puzzle

33

Why is there **not** an anti-
Universe as expected by Dirac?

$$t = 10^{16} \text{ sec}$$

$$r = 10^{29} \text{ cm}$$

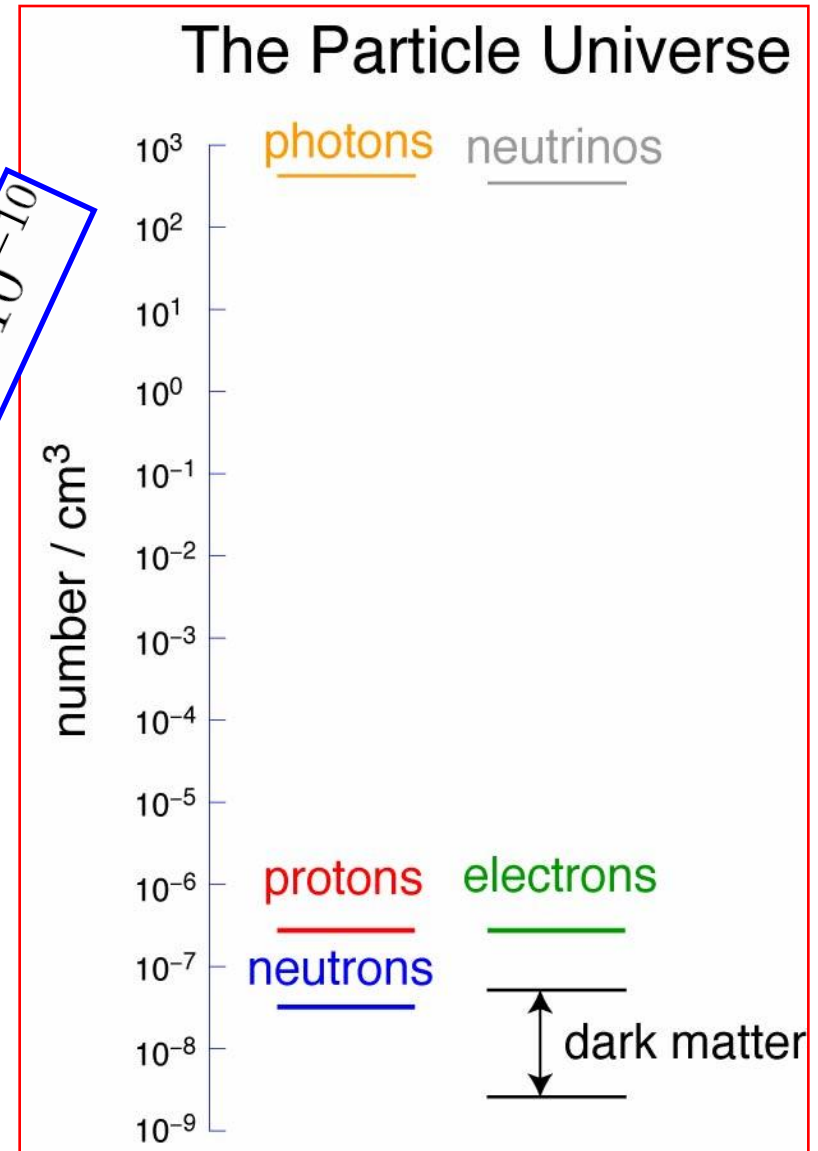
$$T = 2.7 \text{ K}$$

$$400 \gamma / \text{cm}^3$$

$$10^{80} p, n$$

$$0 \quad \bar{p}, \bar{n}$$

$$\eta_B \equiv n_B / n_\gamma = (6.1 \pm 0.2) \times 10^{-10}$$

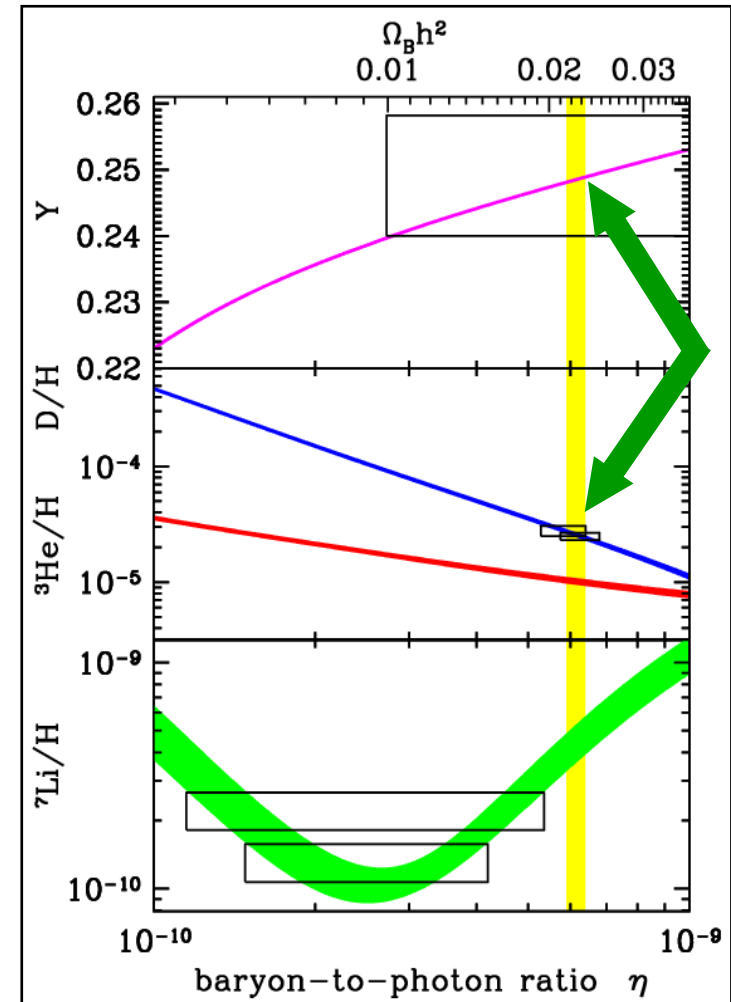
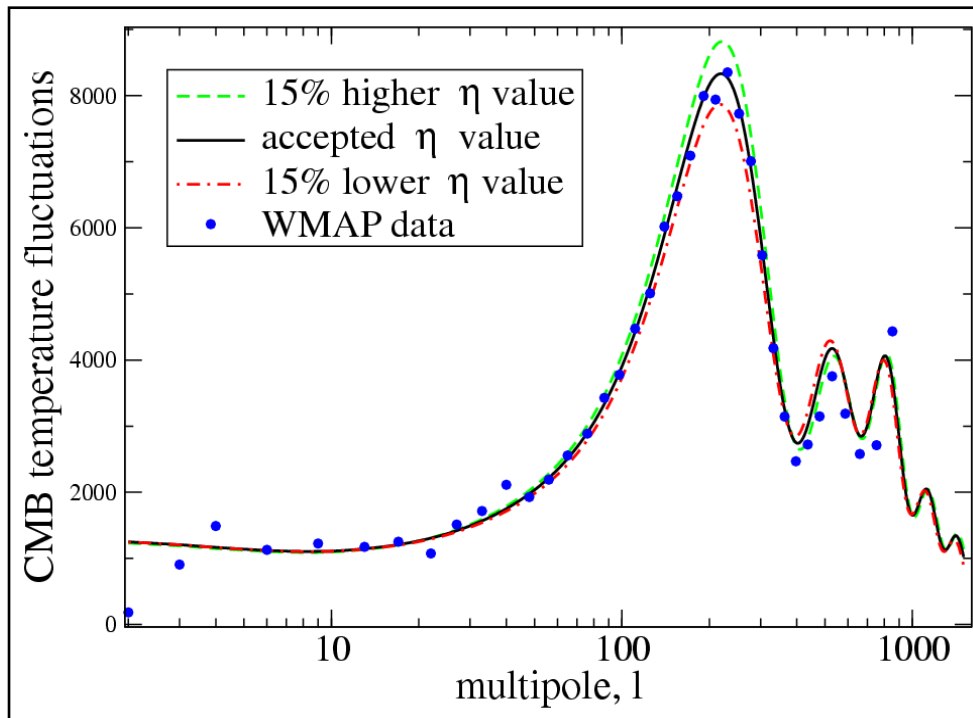


Evidence

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η_B was historically determined from the **Big Bang Nucleosynthesis**: Primordial abundances of BBN light elements are sensitive to it.

η_B can now be measured from **Cosmic Microwave Background**: Relative sizes of those Doppler peaks of CMB temperature anisotropy are sensitive to it.



Sakharov Conditions

35

Baryogenesis: ★ Just-So: $B > 0$ from the very beginning up to now;
★ ★ Dynamical picture: $B > 0$ evolved from $B = 0$ after inflation.

Condition 1: baryon number (B) violation.

[GUT, SUSY & even SM allow it, but no direct experimental evidence]

Condition 2: breaking of C and CP symmetries.

[C & CP asymmetries are both needed to keep B violation survivable]

Condition 3: departure from thermal equilibrium.

[Thermal equilibrium might erase B asymmetry due to CPT symmetry]



Baryogenesis Mechanisms

- ◆ Planck/GUT Baryogenesis;
- ◆ Electroweak Baryogenesis;
- ◆ Leptogenesis;
- ◆ Affleck-Dine Mechanism; ...

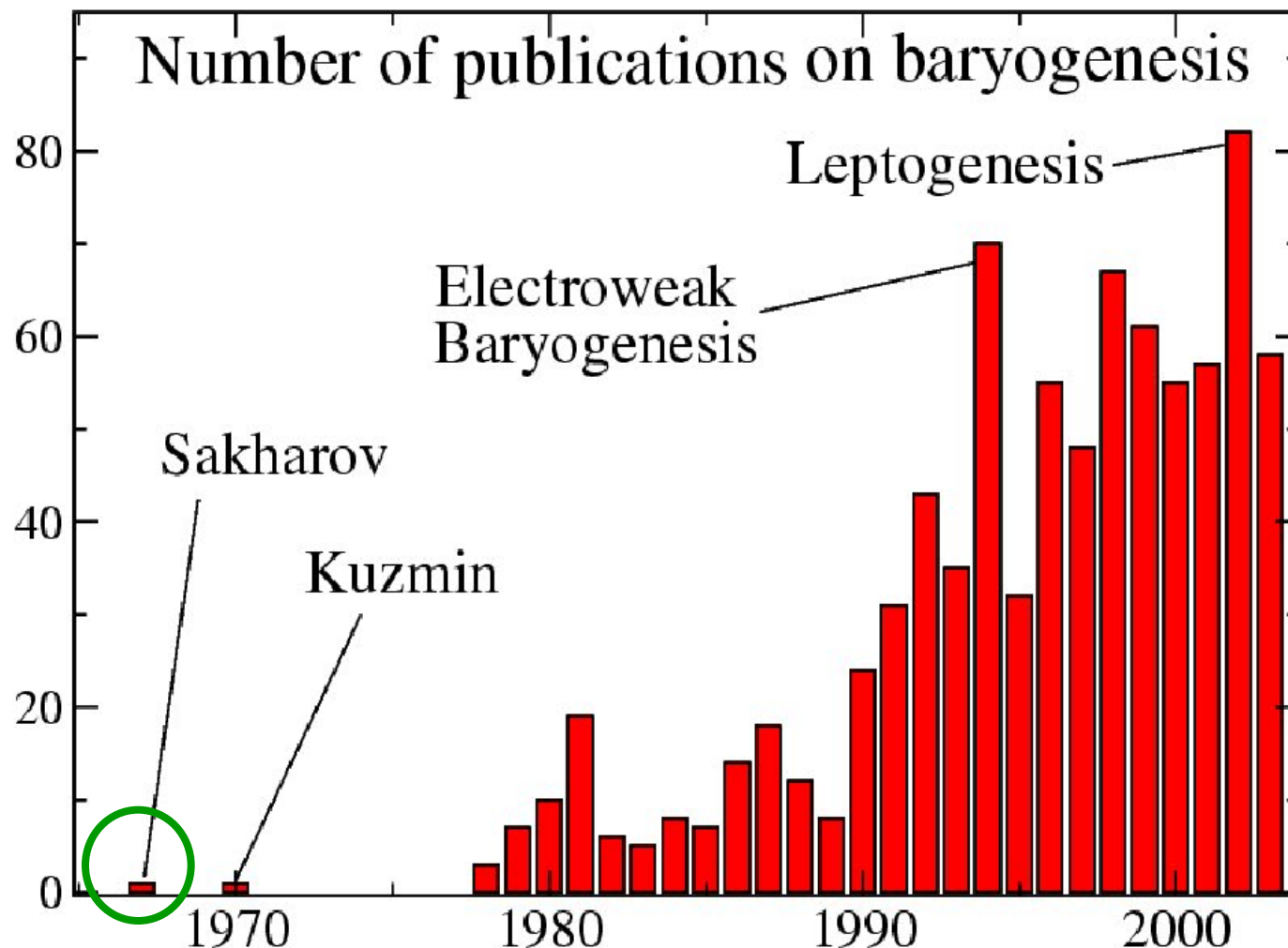
Sakharov's paper:
almost no citation
during 1967-1979

Now >1300 times

Neutrino
Physics

Hot Topic

36



Lesson: if you publish a paper that nobody cares today, **don't worry**, as it might actually be a seminal work and become popular tomorrow.

Remarks on CP Violation

37

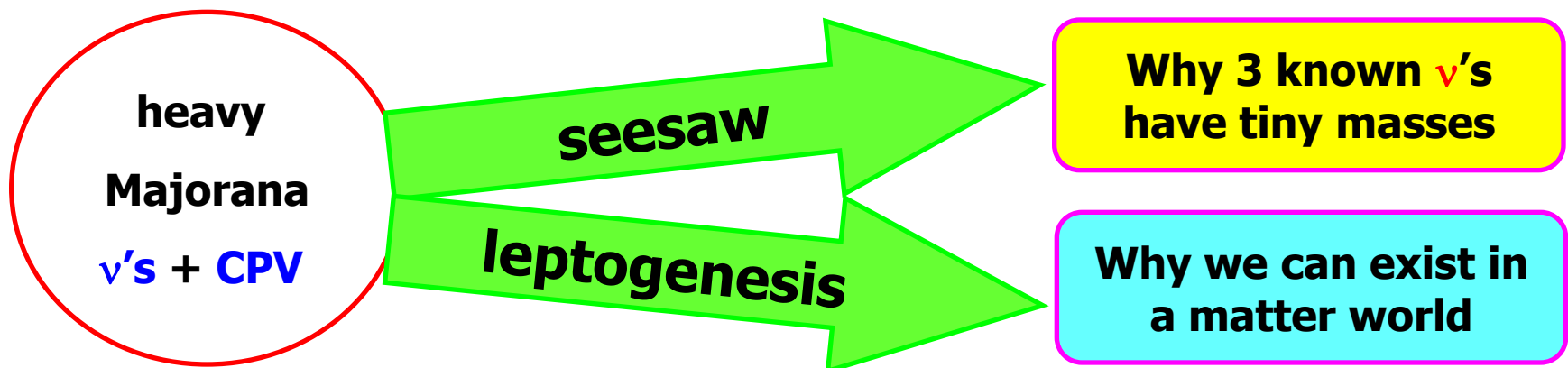
CP violation from the **CKM** quark mixing matrix is not the whole story to explain the **matter-antimatter asymmetry** of the visible Universe.



Two reasons for this in the **SM**:

- **CP** violation from the **SM**'s quark sector is highly suppressed;
- The electroweak phase transition is not strongly first order.

New sources of CP violation are necessarily required.



Thermal Leptogenesis

38

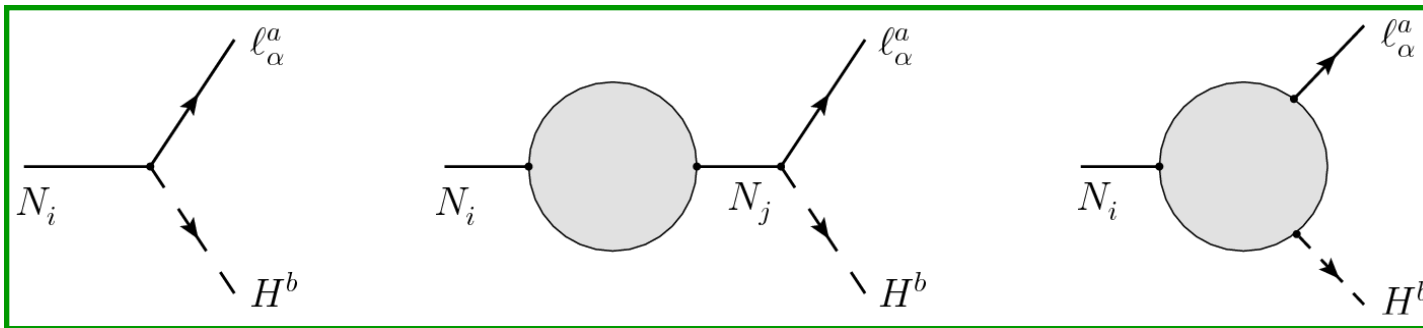
- ◆ add 3 **heavy right-handed Majorana neutrinos** into SM & keep its $SU(2) \times U(1)$ gauge symmetry:

$$-\mathcal{L}_{\text{lepton}} = \bar{\ell}_L Y_l H E_R + \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.}$$



Fukugita, Yanagida 86

- ◆ **lepton-number-violating & CP-violating** decays of heavy neutrinos:



$$\begin{aligned} \varepsilon_i &\equiv \frac{\sum_{\alpha} \left[\Gamma(N_i \rightarrow \ell_{\alpha} + H) - \Gamma(N_i \rightarrow \bar{\ell}_{\alpha} + \bar{H}) \right]}{\sum_{\alpha} \left[\Gamma(N_i \rightarrow \ell_{\alpha} + H) + \Gamma(N_i \rightarrow \bar{\ell}_{\alpha} + \bar{H}) \right]} \\ &\approx \frac{1}{8\pi(Y_{\nu}^{\dagger} Y_{\nu})_{ii}} \sum_j \text{Im} \left[(Y_{\nu}^{\dagger} Y_{\nu})_{ij} \right]^2 \left[f_V \left(\frac{M_j^2}{M_i^2} \right) + f_S \left(\frac{M_j^2}{M_i^2} \right) \right] \end{aligned}$$

$$\begin{aligned} f_V(x) &= \begin{cases} \sqrt{x} \left[1 - (1+x) \ln \frac{1+x}{x} \right] & (\text{SM}) , \\ -\sqrt{x} \ln \frac{1+x}{x} & (\text{SUSY}) ; \end{cases} \\ f_S(x) &= \begin{cases} \frac{\sqrt{x}}{1-x} & (\text{SM}) , \\ \frac{2\sqrt{x}}{1-x} & (\text{SUSY}) . \end{cases} \end{aligned}$$

Thermal Leptogenesis

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◆ to prevent **CP** asymmetries from being washed out by the inverse decays and scattering processes, the decays of heavy neutrinos must be **out of thermal equilibrium** (their decay rates must be smaller than the expansion rate of the Universe.

$$\Gamma(N_i \rightarrow \ell_\alpha + H) < H(T = M_i)$$

The **net** lepton number asymmetry:

$$Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \frac{1}{g_*} \sum_i \kappa_i \varepsilon_i$$

κ_i : efficiency factors

g_* : number of relativistic d.o.f

s : entropy density

(Boltzmann equations for time evolution of particle number densities)

◆ non-perturbative but **(B-L)-conserving** weak **sphaleron** reactions convert a lepton number asymmetry to a baryon number asymmetry.

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_f}{32\pi^2} \left(-g^2 W_{\mu\nu}^i \tilde{W}^{i\mu\nu} + g'^2 B_{\mu\nu} \tilde{B}^{\mu\nu} \right)$$

at the quantum level
via triangle anomaly.

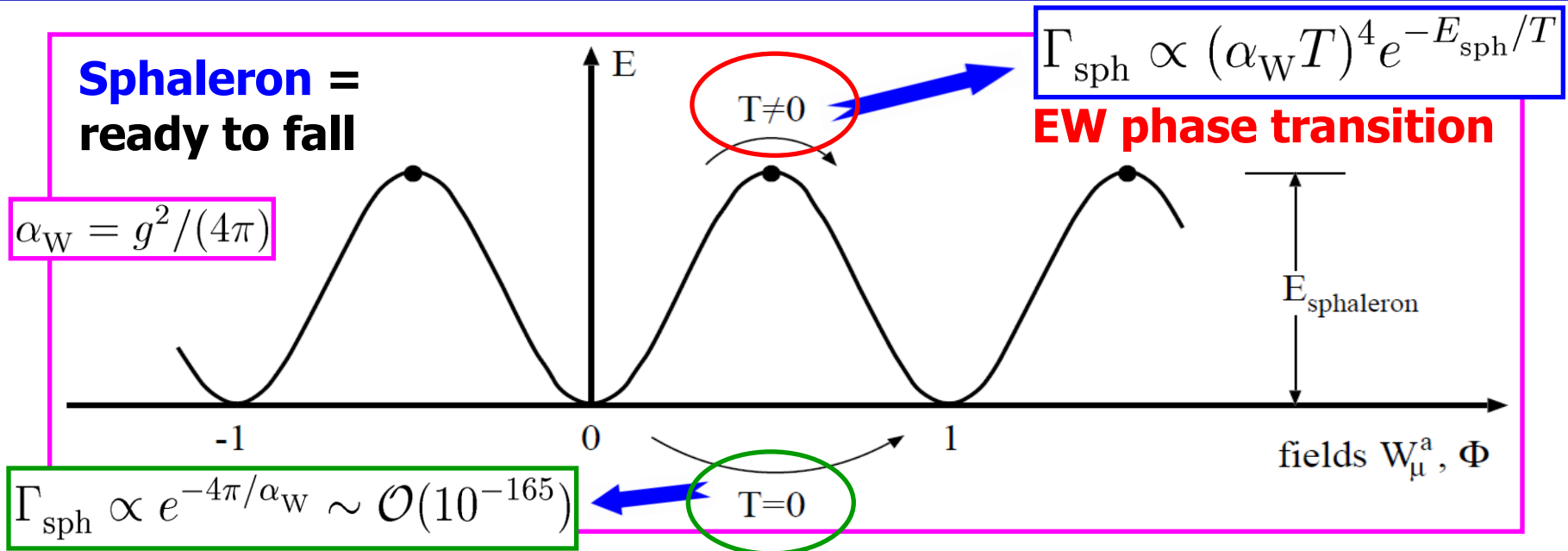
$$B - L = \int d^3x (J_B^0 - J_L^0) = 0 \quad \text{(B-L) is conserved in the SM ('t Hooft, 76)}$$

Chern-Simons (CS) numbers = $\pm 1, \pm 2, \dots$

$$\Delta B = \Delta L = N_f \Delta N_{\text{CS}}$$

Thermal Leptogenesis

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Sphaleron-induced ($B+L$)-violating process is in thermal equilibrium when the temperature:

$$10^2 \text{ GeV} < T < 10^{12} \text{ GeV}$$

Baryogenesis via leptogenesis is realized:

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = -C Y_L$$

$$\left. \frac{n_B}{s} \right|_{\text{equilibrium}} = C \left. \frac{n_B - n_L}{s} \right|_{\text{equilibrium}} = -C \left. \frac{n_L}{s} \right|_{\text{initial}}$$

$$\left. \frac{n_{\bar{B}}}{s} \right|_{\text{equilibrium}} = C \left. \frac{n_{\bar{B}} - n_{\bar{L}}}{s} \right|_{\text{equilibrium}} = -C \left. \frac{n_{\bar{L}}}{s} \right|_{\text{initial}}$$

$$C = \frac{8N_f + 4N_\Phi}{22N_f + 13N_\Phi}$$

$$= \begin{cases} 28/79 & (\text{SM}) \\ 8/23 & (\text{MSSM}) \end{cases}$$

History of the Universe

something occurred
over there
13.7 billion years ago

BIG
BANG

Inflation

10^{-44}
 10^{-32}
 10^{-19}

L

↕

B

possible dark matter relics

cosmic microwave radiation visible

so

we

are

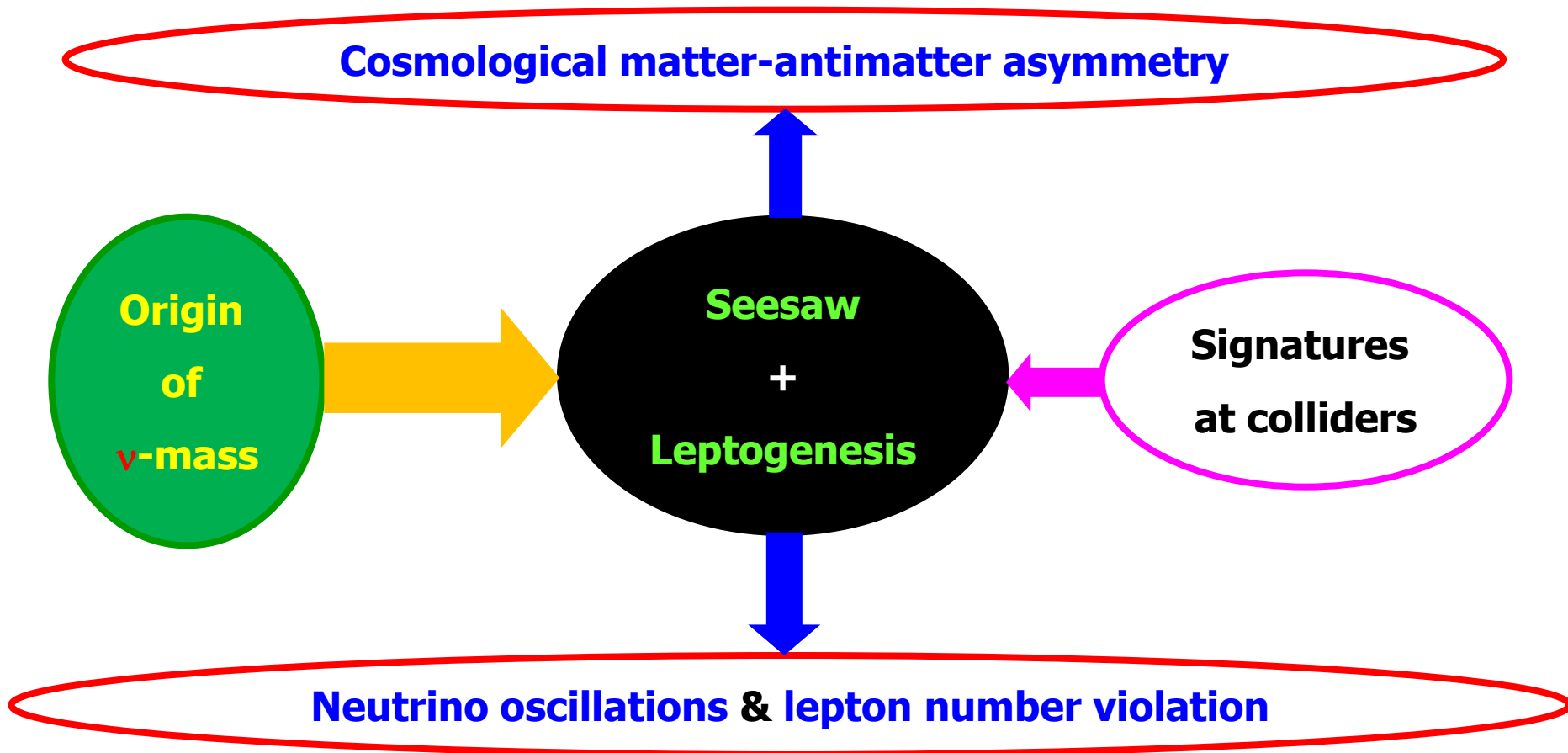
here

today

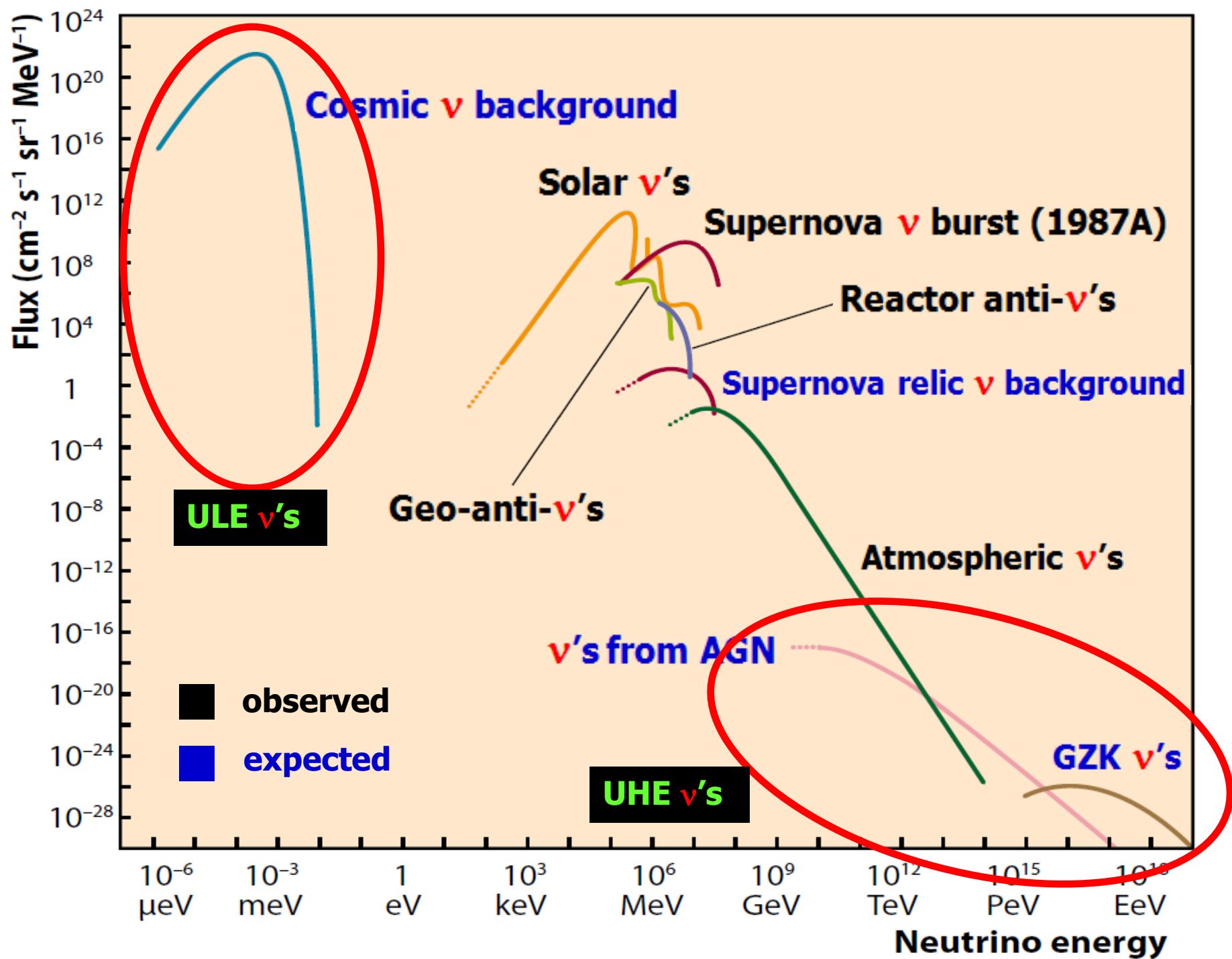
Key:		
W, Z bosons	photon	
q quark	meson	star
g gluon	baryon	galaxy
e electron	ion	black hole
m muon	atom	
t tau		
n neutrino		

A Grand Picture?

42



Cosmic messenger: neutrino astronomy and neutrino cosmology.
Surprise maker: history of neutrino physics was full of surprises.



Formation of CνB

As $T \sim$ a few MeV in the Universe, the survival relativistic particles were photons, electrons, positrons, neutrinos and antineutrinos.

Electroweak reactions:

$$\gamma + \gamma \rightleftharpoons e^+ + e^- \rightleftharpoons \nu_\alpha + \bar{\nu}_\alpha \quad (\text{for } \alpha = e, \mu, \tau)$$

$$\nu_e + n \rightleftharpoons e^- + p, \quad \bar{\nu}_e + p \rightleftharpoons e^+ + n \quad \bar{\nu}_e + e^- + p \rightleftharpoons n$$

Neutrinos decoupled from matter:

Weak interactions

Hubble expansion

$$\Gamma \sim G_F^2 T^5$$

$$H \sim \frac{\sqrt{g_*} T^2}{M_{\text{Pl}}}$$

$$\Gamma > H$$

$$\Gamma \sim H$$

$$\Gamma < H$$

Number density of 6 relic ν's:

$$n_\nu = \frac{9}{11} n_\gamma \approx 336 \left(\frac{T_\gamma}{2.725 \text{ K}} \right)^3 \text{ cm}^{-3}$$

ν's in thermal contact with cosmic plasma

ν's not in thermal contact with matter

arrow of time

neutrino and photon temperatures (blue)

neutrino decoupling

$$T < m_e \quad e^+ + e^- \rightarrow \gamma + \gamma$$

$$T_\nu = T_\gamma$$

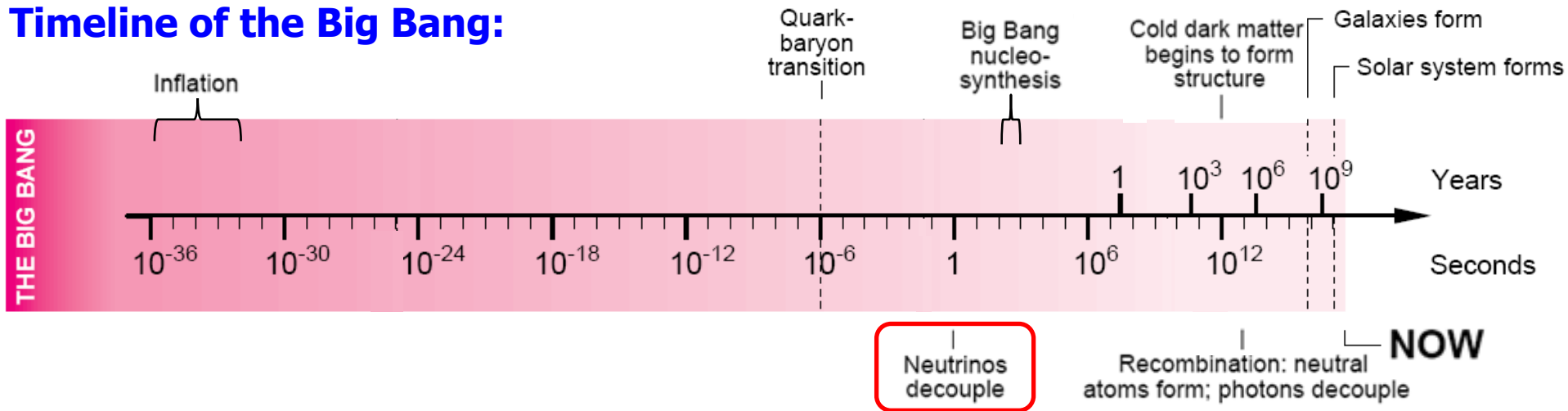
$$T_{\text{fr}} \sim \left(\frac{\sqrt{g_*}}{G_F^2 M_{\text{Pl}}} \right)^{1/3} \sim 1 \text{ MeV}$$

$$T_\nu = \left(\frac{4}{11} \right)^{1/3} T_\gamma$$

Witness / Participant

CMB and **LSS**: the existence of **relic neutrinos** had an impact on the epoch of **matter-radiation equality**, their **species** and **masses** could affect the CMB anisotropies and large scale structures.

Timeline of the Big Bang:



At the time of **recombination** ($t \sim 380\,000$ yrs):

$$\rho_\gamma + \rho_\nu = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_\nu^{\text{CMB}} \right]$$

The **C_vB** contribution to the total energy density of the Universe today

relativistic



$$\Omega_\nu = \frac{21}{8} \left(\frac{4}{11} \right)^{4/3} \Omega_\gamma \approx 1.68 \times 10^{-5} h^{-2}$$

non-relativistic

$$\Omega_\nu = \frac{8\pi G_N}{3H^2} \sum_i m_i (n_{\nu_i} + n_{\bar{\nu}_i}) \approx \frac{1}{94 h^2 \text{ eV}} \sum_i m_i$$

Is C_vB Detectable?

Today's **matter** & **energy** densities in the Universe (Dunkley et al **09**; Komatsu et al **09**; Nakamura et al **10**): **5-year WMAP** + **Λ CDM** model

Parameter	Value
Hubble parameter h	0.72 ± 0.03
Total matter density Ω_m	$\Omega_m h^2 = 0.133 \pm 0.006$
Baryon density Ω_B	$\Omega_B h^2 = 0.0227 \pm 0.0006$
Vacuum energy density Ω_v	$\Omega_v = 0.74 \pm 0.03$
Radiation density Ω_r 	$\Omega_r h^2 = 2.47 \times 10^{-5}$
Neutrino density Ω_ν 	$\Omega_\nu h^2 = \sum m_i / (94 \text{ eV})$
Cold dark matter density Ω_{CDM}	$\Omega_{\text{CDM}} h^2 = 0.110 \pm 0.006$

The **CMB** (**t ~ 380 000 years**) is already measured today

Is it likely to detect the **C_vB** (**t ~ 1 s**) in the foreseeable future? ---- Here we'll look at a **Gedankenexperiment**.

Detection of $C\nu B$

Way 1: $C\nu B$ -induced **mechanical effects** on Cavendish-type torsion balance;

Way 2: **Capture** of relic ν 's on radioactive β -decaying nuclei (Weinberg 62);

Way 3: **Z-resonance annihilation** of UHE cosmic ν 's and relic ν 's (Weiler 82).

Temperature today

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.945 \text{ K}$$

Mean momentum today

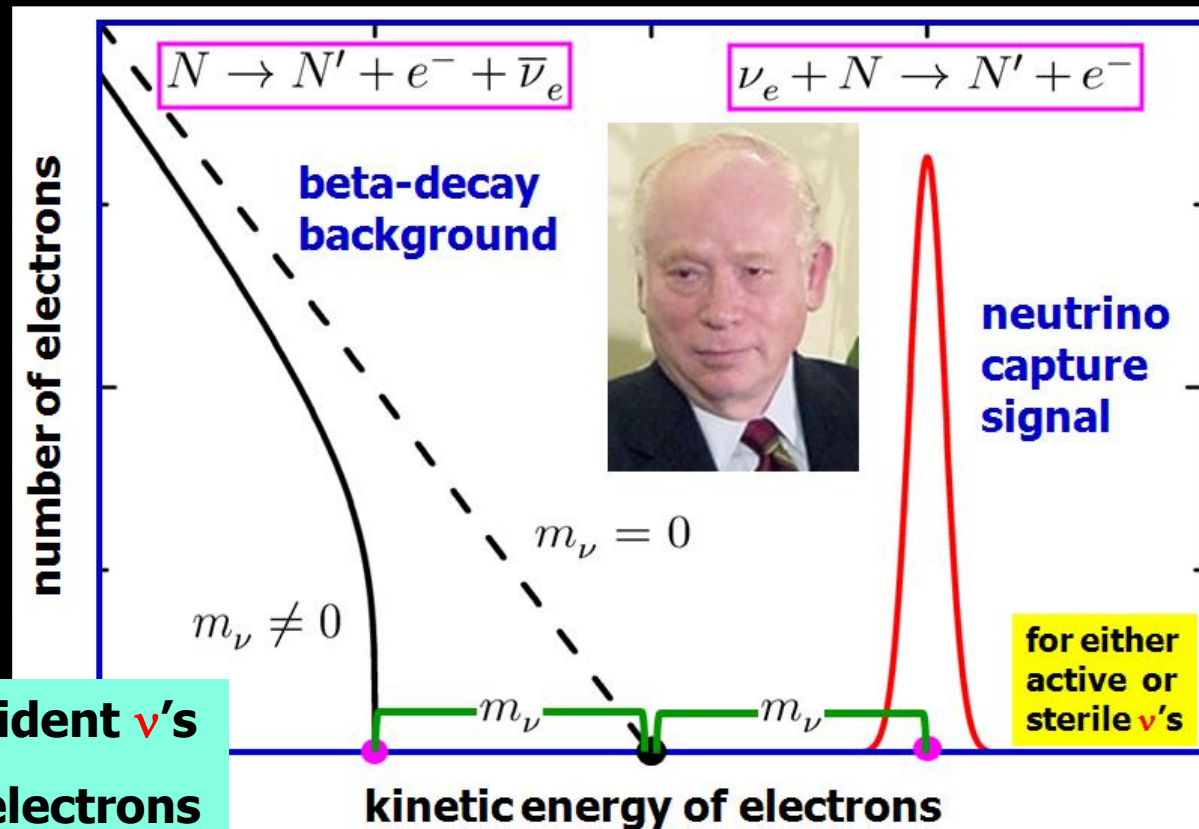
$$\begin{aligned}\langle p_\nu \rangle &\simeq 3.151 T_\nu \\ &\simeq 5.281 \times 10^{-4} \text{ eV}\end{aligned}$$

At least **2 ν 's** **cold** today

How to detect ULE ν 's ?

(Irvine & Humphreys, 83)

Relic neutrino capture on β -decaying nuclei



- no energy threshold on incident ν 's
- **mono**-energetic outgoing electrons

Example

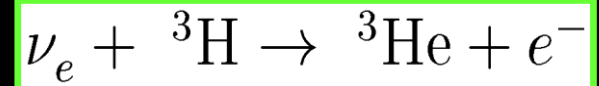
Salient feature: the cross section of a capture reaction scales with $\frac{c}{v_\nu}$ so that the number of events converges to a constant for $v_\nu \rightarrow 0$:

$$\sigma(\nu_e N) \cdot \frac{v_\nu}{c} \Big|_{v_\nu \rightarrow 0} = \text{const.}$$

e.g.

$$\sigma(\nu_e {}^3\text{H}) \cdot \frac{v_\nu}{c} \Big|_{v_\nu \rightarrow 0} \simeq (7.84 \pm 0.03) \times 10^{-45} \text{cm}^2$$

(Cocco et al **07**, Lazauskas et al **08**).



Capture rate: (1 MCi = 100 g = $N_T \approx 2.1 \times 10^{25}$ tritium atoms)

$$\frac{d\mathcal{N}_{\text{C}\nu\text{B}}}{dT_e} \approx 6.5 \sum_i |V_{ei}|^2 \frac{n_{\nu_i}}{\langle n_{\nu_i} \rangle} \cdot \frac{1}{\sqrt{2\pi} \sigma} \exp \left[-\frac{(T_e - T_e^i)^2}{2\sigma^2} \right] \text{yr}^{-1} \text{MCi}^{-1}$$

$$T_e^i = Q_\beta + E_{\nu_i}$$

Background: (the tritium β -decay)

$$E_e = T'_e + m_e$$

$$\langle n_{\nu_i} \rangle \approx \langle n_{\bar{\nu}_i} \rangle \approx 56 \text{ cm}^{-3}$$

$$\begin{aligned} \frac{d\mathcal{N}_\beta}{dT_e} \approx & 5.55 \int_0^{Q_\beta - \min(m_i)} dT'_e \left\{ N_T \frac{G_F^2 \cos^2 \theta_C}{2\pi^3} F(Z, E_e) \sqrt{E_e^2 - m_e^2} E_e (Q_\beta - T'_e) \right. \\ & \times \sum_i \left[|V_{ei}|^2 \sqrt{(Q_\beta - T'_e)^2 - m_i^2} \Theta(Q_\beta - T'_e - m_i) \right] \frac{1}{\sqrt{2\pi} \sigma} \exp \left[-\frac{(T_e - T'_e)^2}{2\sigma^2} \right] \Big\} \end{aligned}$$

Energy resolution (Gaussian function) :

$$\Delta = 2\sqrt{2 \ln 2} \sigma \approx 2.35482 \sigma$$

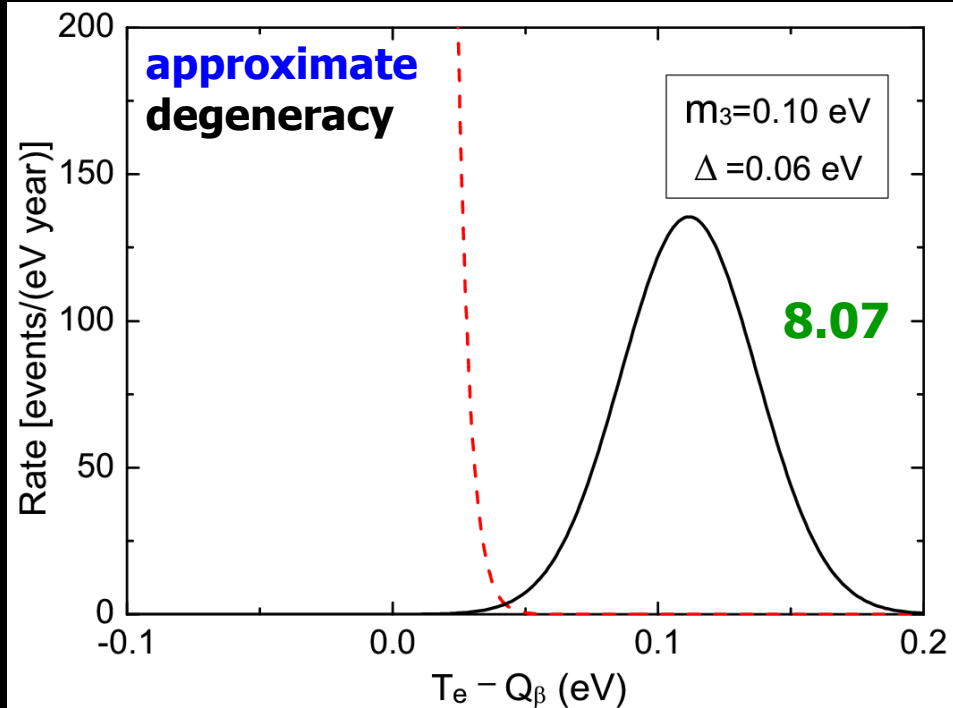
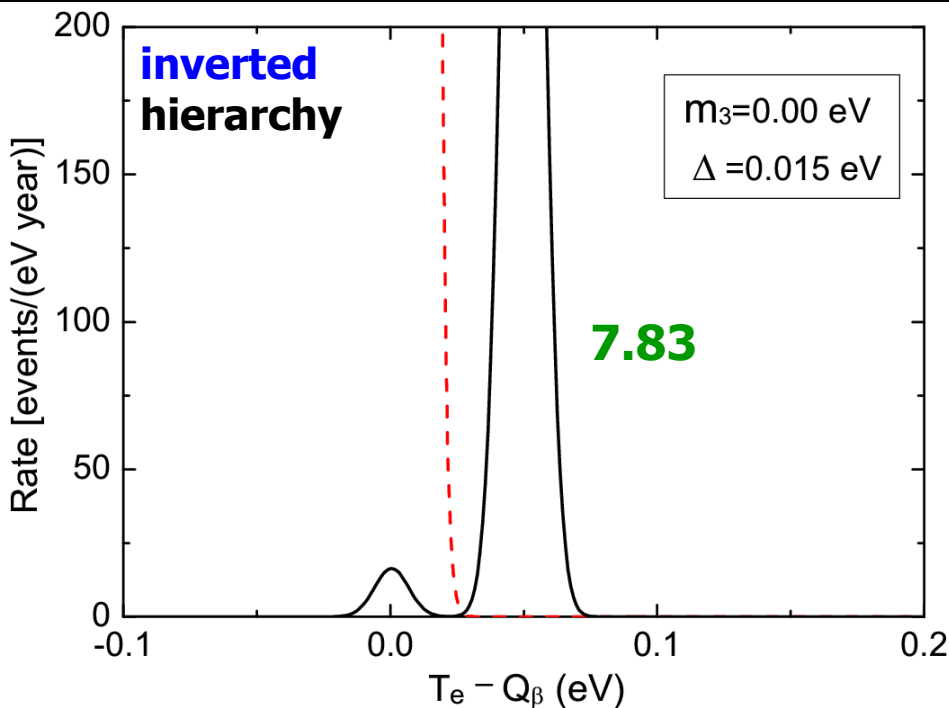
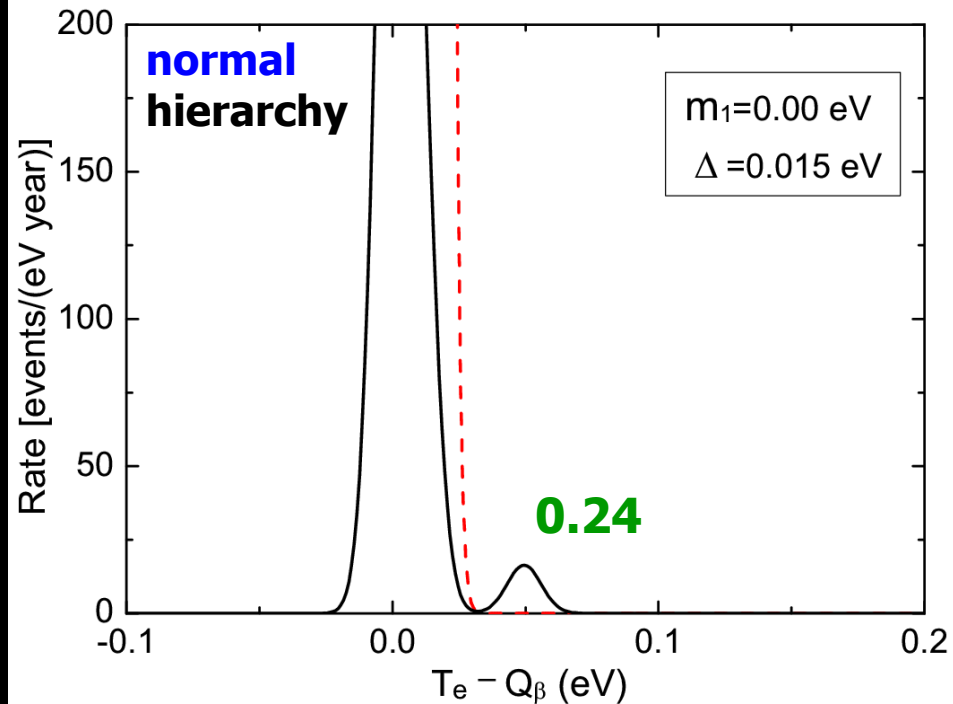
Illustration

Target mass: 100 g tritium atoms

Input $\theta(13)$: 10 degrees

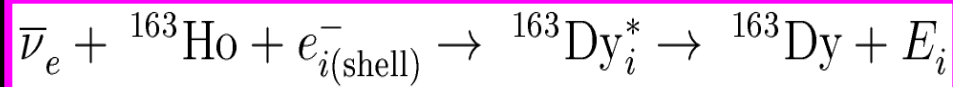
Number of events per year: ~ 8

The **gravitational clustering** effect may help enhance the signal rates (Ringwald & Wong, **04**).

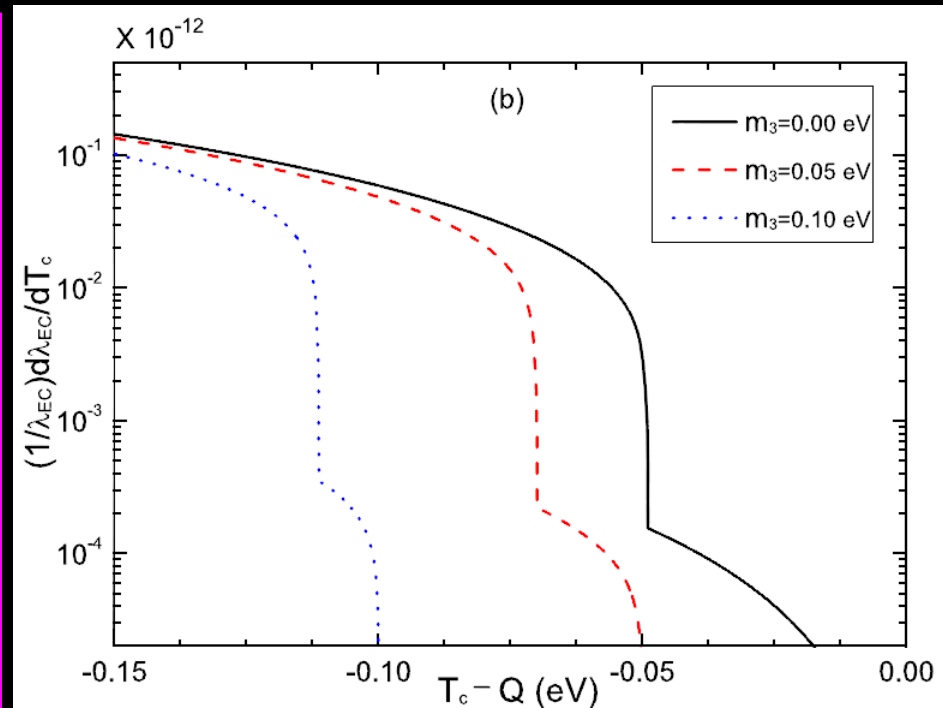
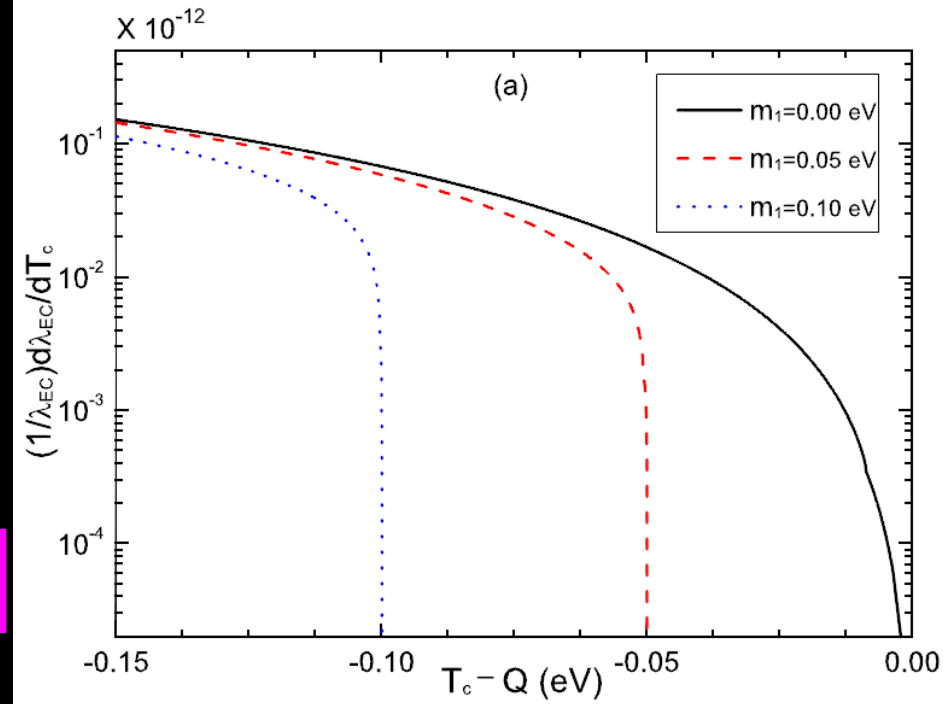
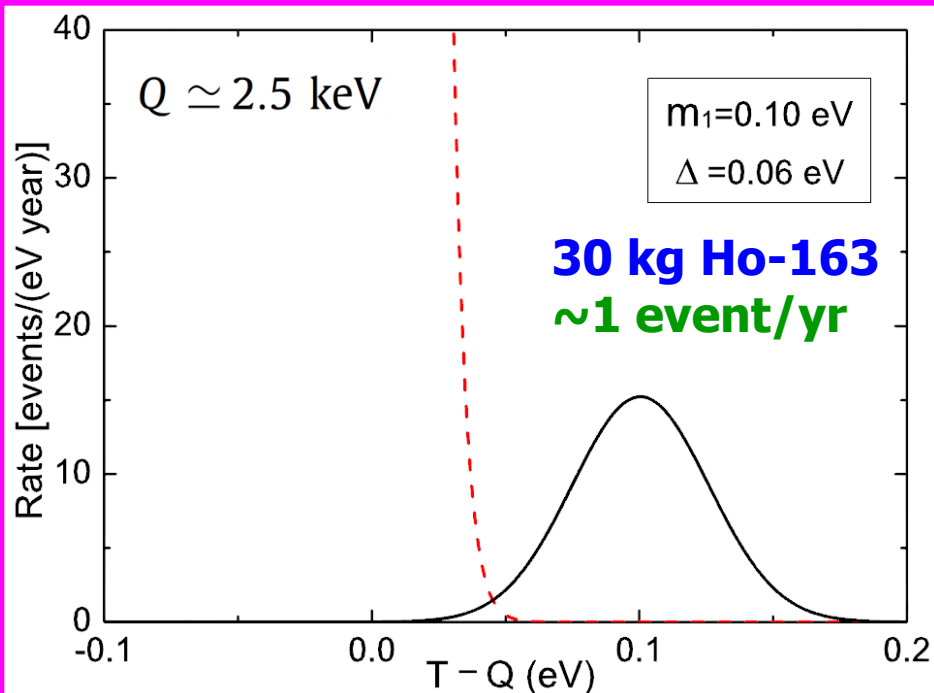


Cosmic anti- ν Background?

Relic antineutrino capture on
EC-decaying Ho-163 nuclei.



(Lusignoli, Vignati, **11**; Li, Xing, **11**)



A Naïve (Why Not) Picture



Hot dark matter: $C_{\nu B}$ is guaranteed but not significant.

Cold dark matter: most likely? At present most popular.

Warm dark matter: suppress the small-scale structures.

If you think so,

**Do not put all your
eggs in one basket**



**hot
dark
matter**

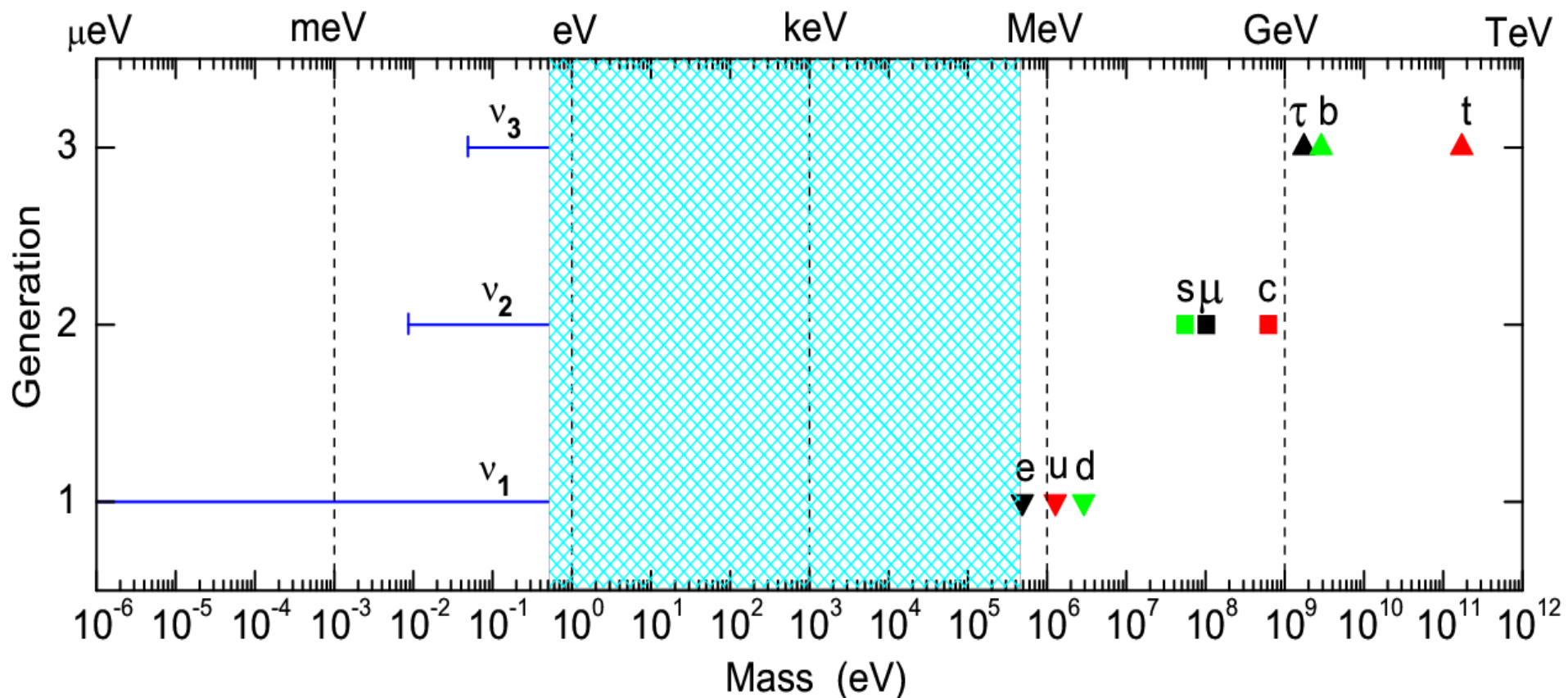
**warm
dark
matter**



keV sterile ν Dark Matter

NO strong prior theoretical motivation for the existence of keV sterile ν 's. **Typical models:** Asaka et al, 05; Kusenko et al, 10; Lindner et al, 11....

A purely phenomenological argument to support keV sterile ν 's in the **FLAVOR DESERT** of the standard model (Xing, 09).



keV sterile ν Dark Matter

NO strong prior theoretical ν 's. **Typical models:** Asa

keV sterile ν 's

the existence of keV sterile ν 's. **Typical models:** Asa et al, **10**; Lindner et al, **11**....

support keV sterile ν 's in the

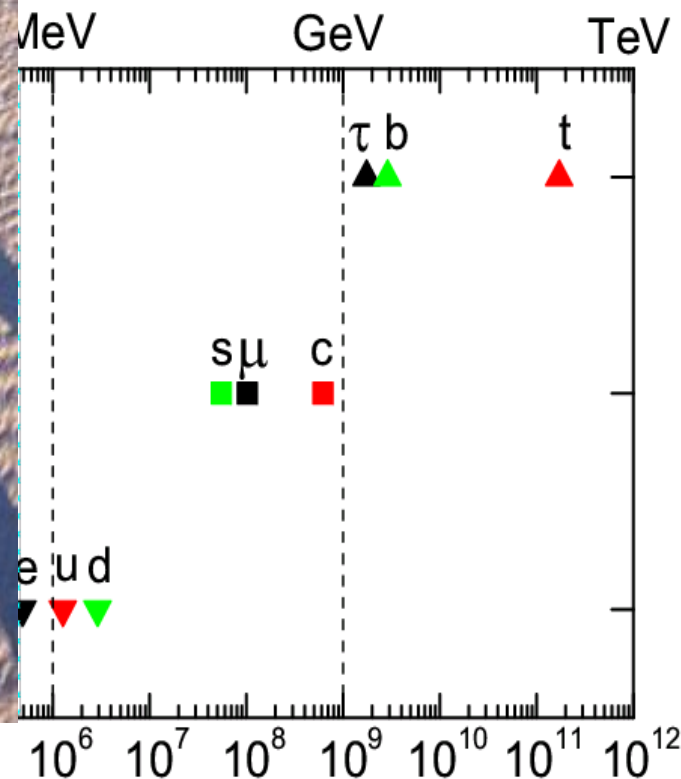


Planck

gauge **hierarchy**
+
desert problems



Fermi



keV sterile ν Dark Matter

Production: via active-sterile ν oscillations in the early Universe, etc;
Salient feature: warm DM in the form of keV sterile ν 's can suppress the formation of dwarf galaxies and other small-scale structures.

Bounds on 2-flavor parameters:
 (Abazajian, Koushiappas, 2006)

For simplicity, we assume only one type of keV sterile neutrinos:

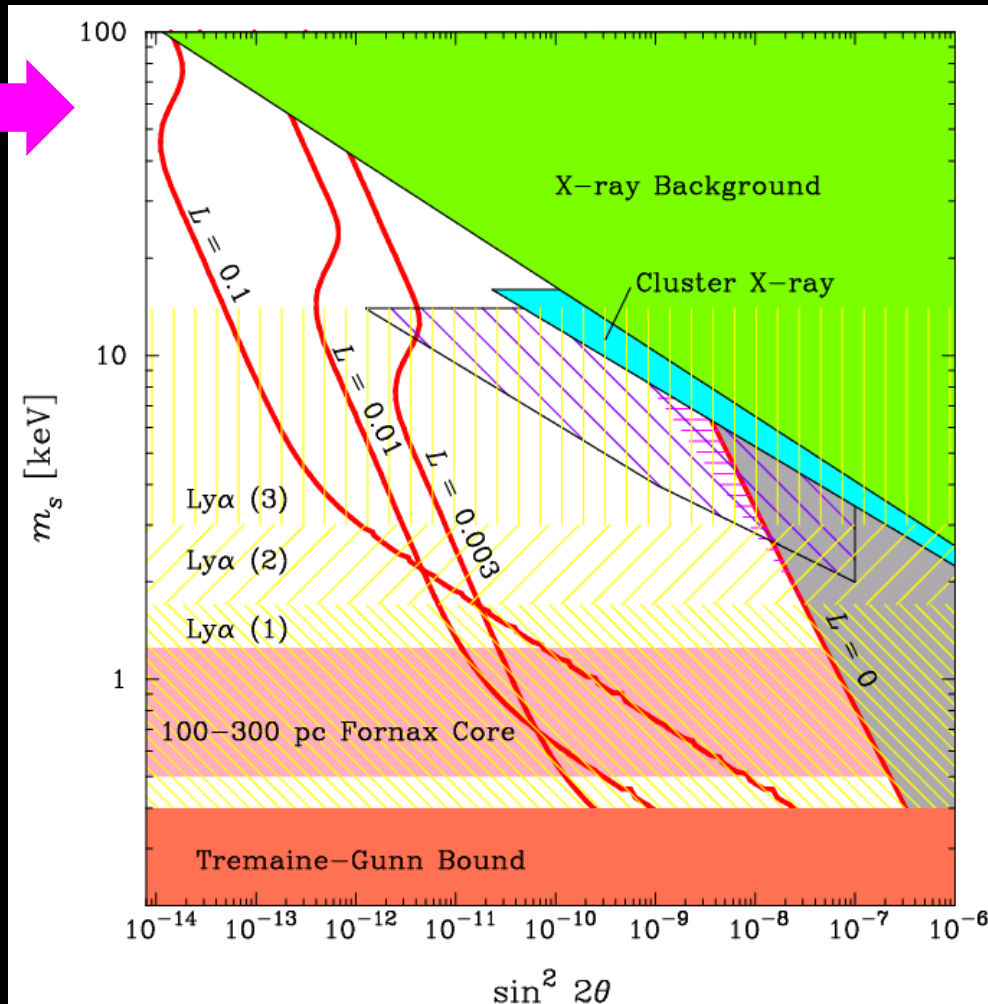
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} & V_{e4} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} & V_{\mu4} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} & V_{\tau4} \\ V_{s1} & V_{s2} & V_{s3} & V_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

Standard parameterization of V :
 6 mixing angles & 3 (Dirac) or 6 (Majorana) CP-violating phases.

$$V_{s1} \simeq s_{14} e^{-i\delta_{14}}, \quad V_{s2} \simeq s_{24} e^{-i\delta_{24}}$$

$$V_{s3} \simeq s_{34} e^{-i\delta_{34}}, \quad V_{s4} \simeq 1$$

$$V_{e4} \simeq -c_{12}c_{13}s_{14}e^{i\delta_{14}} - s_{12}c_{13}s_{24}e^{i(\delta_{24}-\delta_{12})}$$



Decay Rates

Dominant decay mode [$C_\nu = 1$ (Dirac) or 2 (Majorana)]:

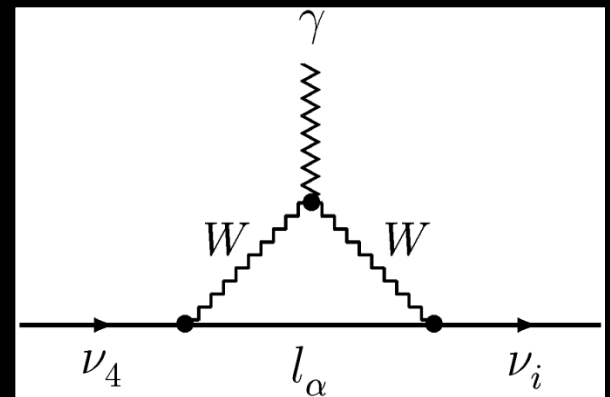
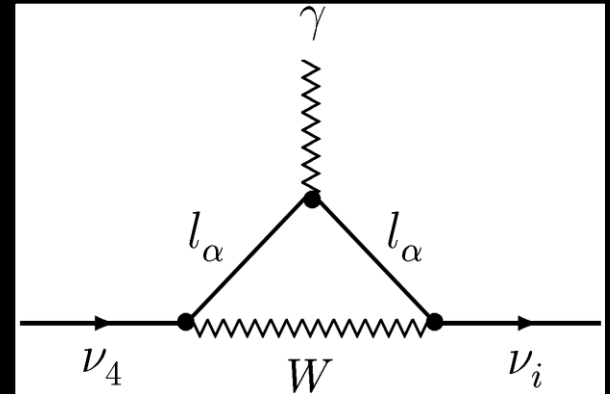
$$\sum_{\alpha=e}^{\tau} \sum_{\beta=e}^{\tau} \Gamma(\nu_4 \rightarrow \nu_\alpha + \nu_\beta + \bar{\nu}_\beta) = \frac{C_\nu G_F^2 m_4^5}{192\pi^3} \sum_{\alpha=e}^{\tau} |V_{\alpha 4}|^2 = \frac{C_\nu G_F^2 m_4^5}{192\pi^3} \sum_{i=1}^3 |V_{si}|^2$$

Lifetime (the Universe's age $\sim 10^{17}$ s):

$$\tau_{\nu_4} \simeq \frac{2.88 \times 10^{27}}{C_\nu} \left(\frac{m_4}{1 \text{ keV}} \right)^{-5} \left(\frac{s_{14}^2 + s_{24}^2 + s_{34}^2}{10^{-8}} \right)^{-1} \text{ s}$$

Radiative decay: X-ray and Lyman-alpha forest observations.

$$\begin{aligned} \sum_{i=1}^3 \Gamma(\nu_4 \rightarrow \nu_i + \gamma) &\simeq \frac{9\alpha_{\text{em}} C_\nu G_F^2 m_4^5}{512\pi^4} \sum_{i=1}^3 \left| \sum_{\alpha=e}^{\tau} V_{\alpha 4} V_{\alpha i}^* \right|^2 \\ &= \frac{9\alpha_{\text{em}} C_\nu G_F^2 m_4^5}{512\pi^4} \sum_{i=1}^3 |V_{s4} V_{si}^*|^2 \\ &\simeq \frac{9\alpha_{\text{em}} C_\nu G_F^2 m_4^5}{512\pi^4} (s_{14}^2 + s_{24}^2 + s_{34}^2) \end{aligned}$$



Detection in the Lab

The same method as the detection of the **CvB** in the lab.

$$\nu_e + N \rightarrow N' + e^-$$

Capture rate with a Gaussian energy resolution:

$$Q_\beta = m_N - m_{N'} - m_e$$

$$\frac{d\mathcal{N}_\nu}{dT_e} = \sum_{i=1}^4 N_T |V_{ei}|^2 \sigma_{\nu_i} v_{\nu_i} n_{\nu_i} \frac{1}{\sqrt{2\pi} \sigma} \exp \left[-\frac{(T_e - T_e^i)^2}{2\sigma^2} \right]$$

$$N \rightarrow N' + e^- + \bar{\nu}_e$$

Assumption: the number density of **sterile ν 's** is equivalent to the total amount of DM in our galactic neighborhood.

$$\rho_{\text{DM}}^{\text{local}} \simeq 0.3 \text{ GeV cm}^{-3}$$

$$n_{\nu_4} \simeq 10^5 (3 \text{ keV}/m_4) \text{ cm}^{-3}$$

Half-life effect of target nuclei (Li, Xing, 11)

$$N_T = \frac{N(0)}{\lambda t} (1 - e^{-\lambda t}), \quad \lambda = \frac{\ln 2}{t_{1/2}}$$

Two sources (Liao, 10; Li, Xing, 11):

$${}^3\text{H} : Q_\beta = 18.6 \text{ keV}, \quad t_{1/2} = 3.888 \times 10^8 \text{ s}, \quad \sigma_{\nu_i} v_{\nu_i}/c = 7.84 \times 10^{-45} \text{ cm}^2$$

$${}^{106}\text{Ru} : Q_\beta = 39.4 \text{ keV}, \quad t_{1/2} = 3.228 \times 10^7 \text{ s}, \quad \sigma_{\nu_i} v_{\nu_i}/c = 5.88 \times 10^{-45} \text{ cm}^2$$

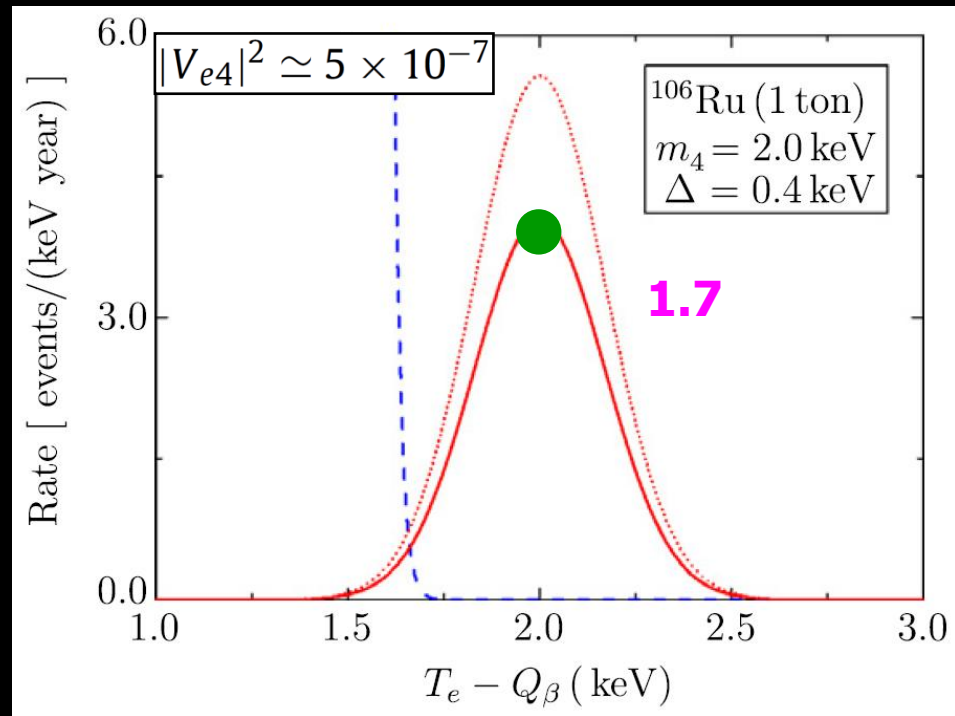
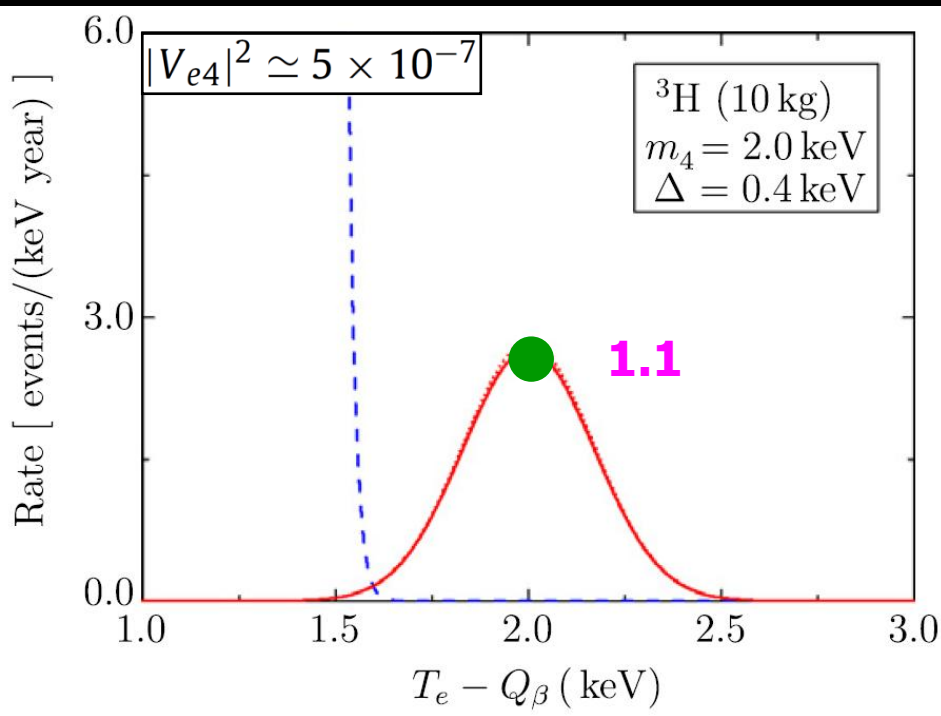
This method & the X-ray detection probe different parameter space.

$$|V_{e4}|^2 \simeq c_{12}^2 s_{14}^2 + s_{12}^2 s_{24}^2 + 2c_{12} s_{12} s_{14} s_{24} \cos(\delta_{24} - \delta_{12} - \delta_{14})$$

Illustration

For illustration: **solid** (**dotted**) curves **with** (**without**) half-life effects.

Number of events per year: **pink**



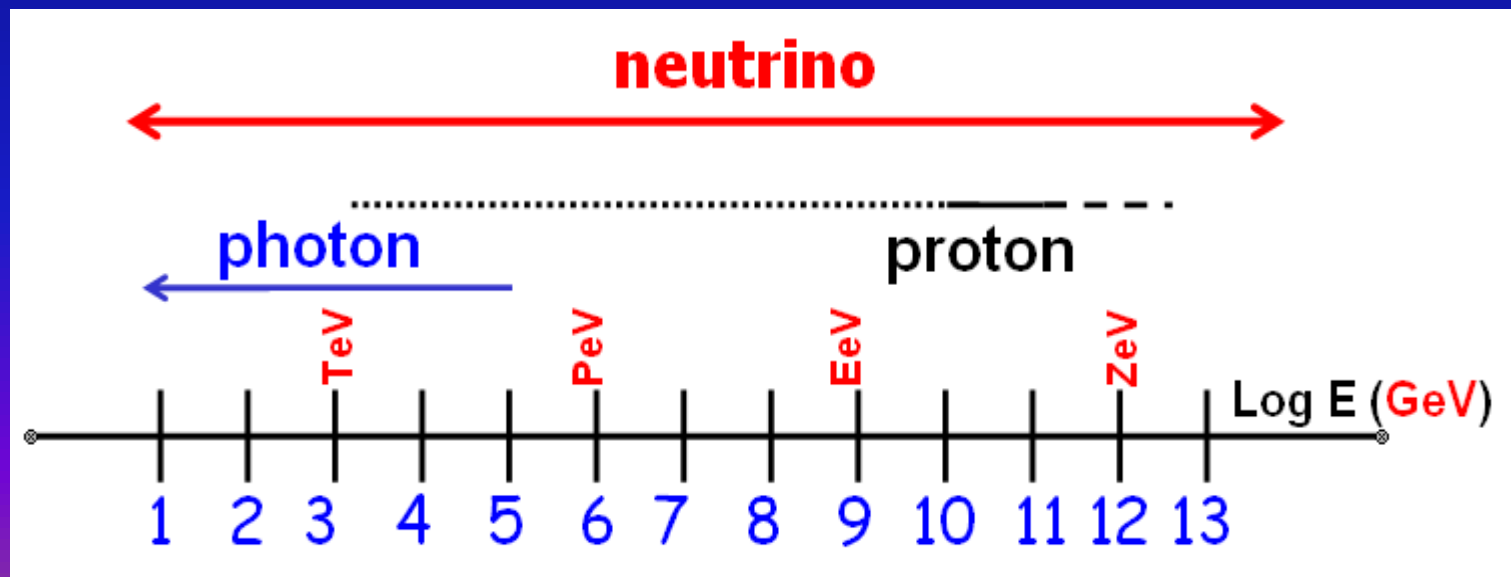
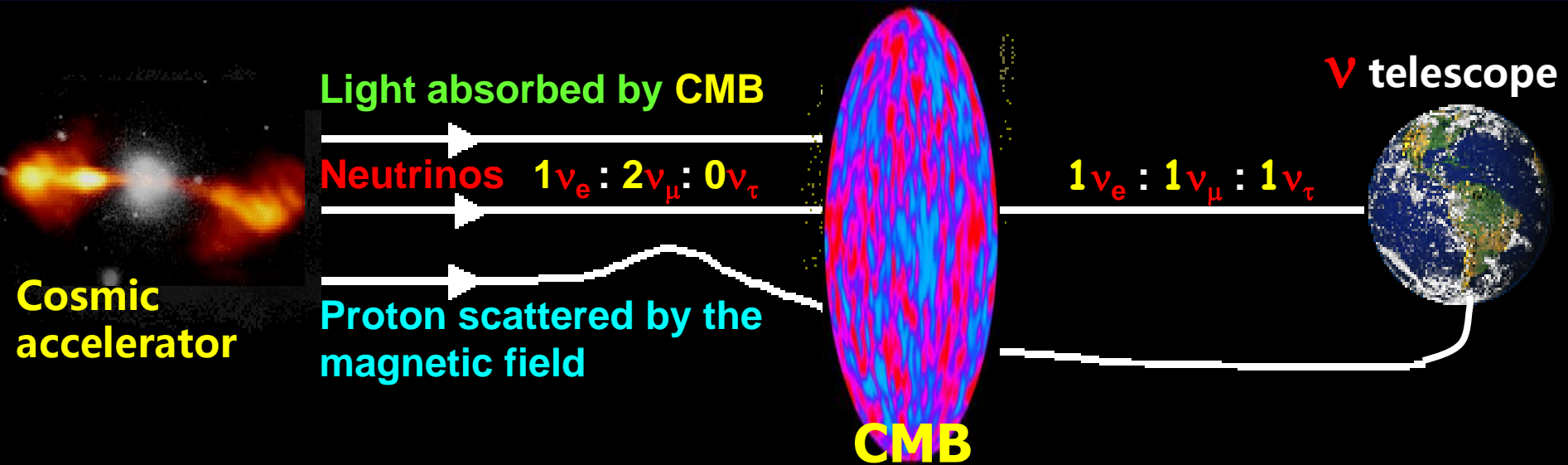
Dim and remote observability of **keV** sterile neutrino DM in this way:

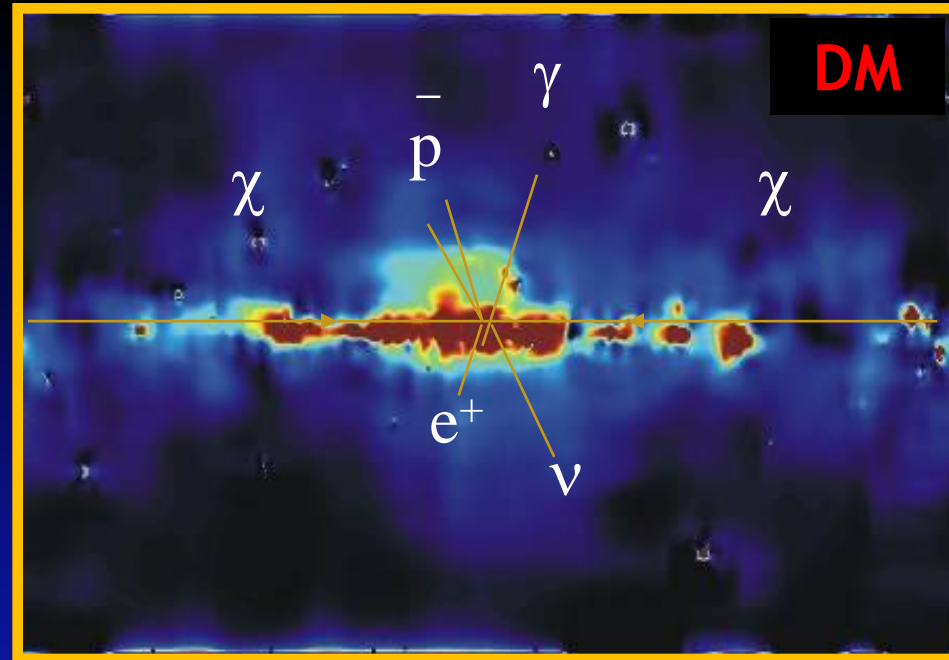
--- tiny active-sterile neutrino mixing angles (**main problem**)

--- background: keV solar neutrinos or $\nu_4 + e^- \rightarrow \nu_i + e^-$ scattering.

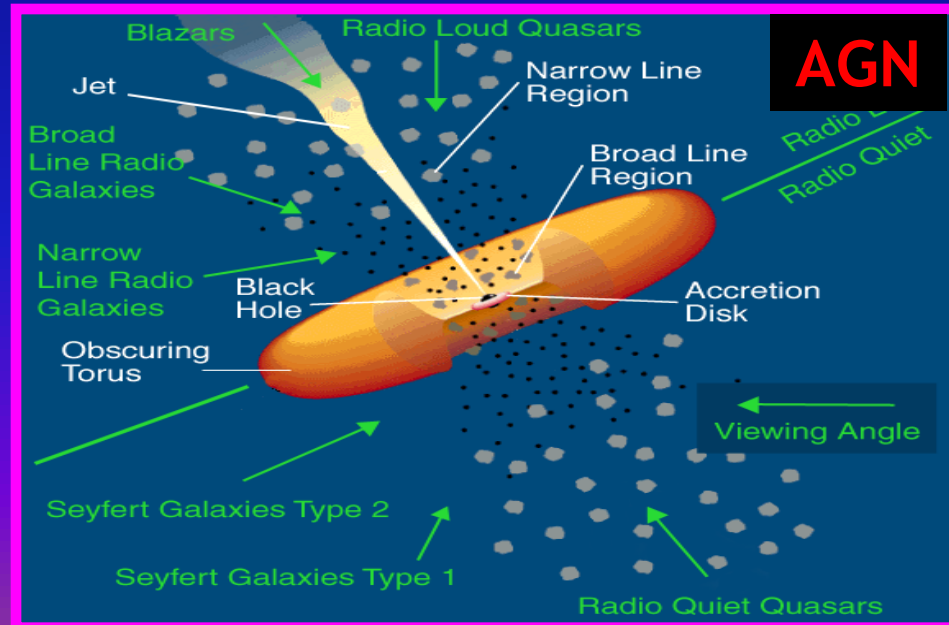
UHE Cosmic Messenger

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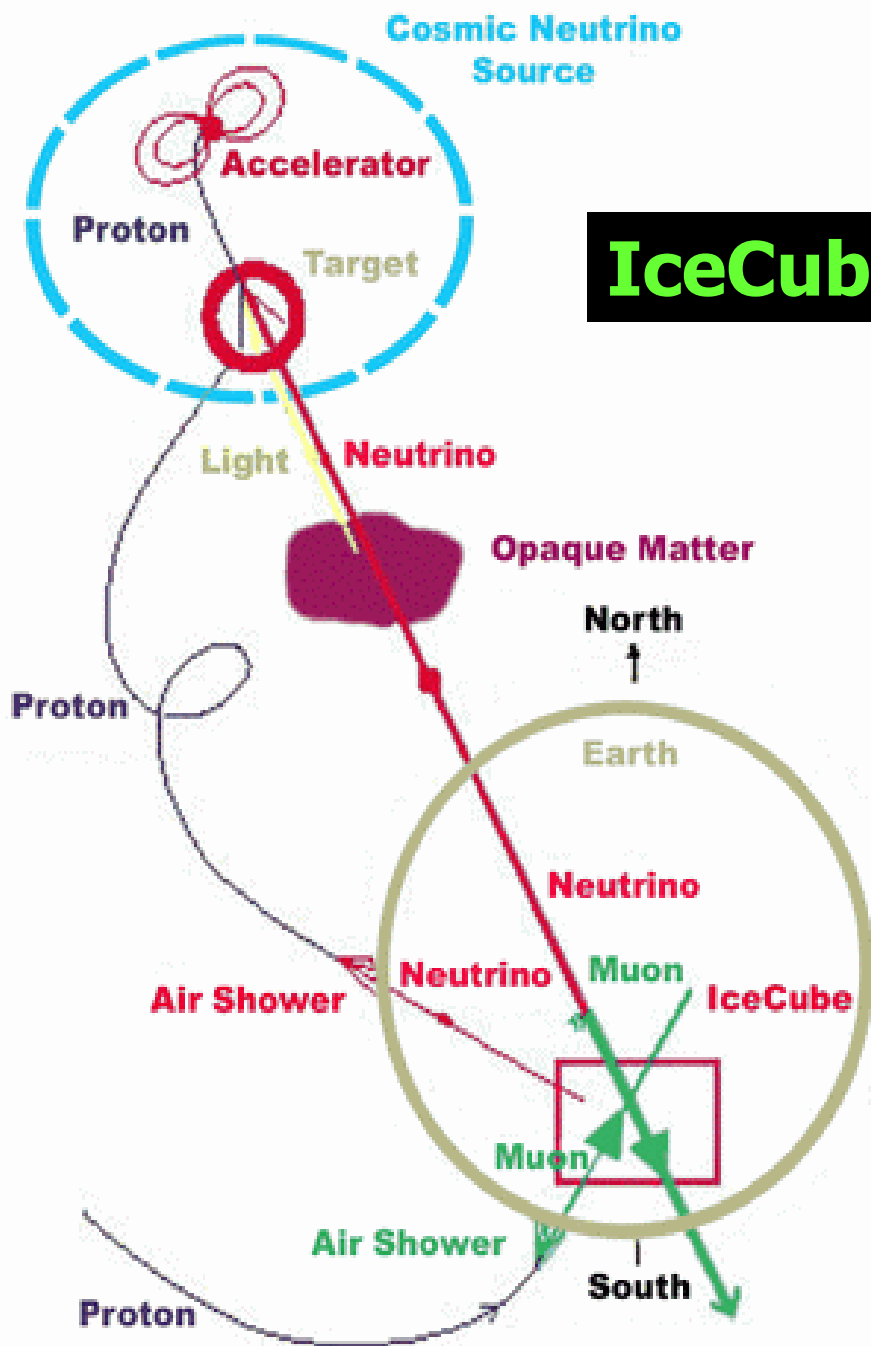
Possible astrophysical sources of UHE cosmic neutrinos ...



Optical Cherenkov NTs

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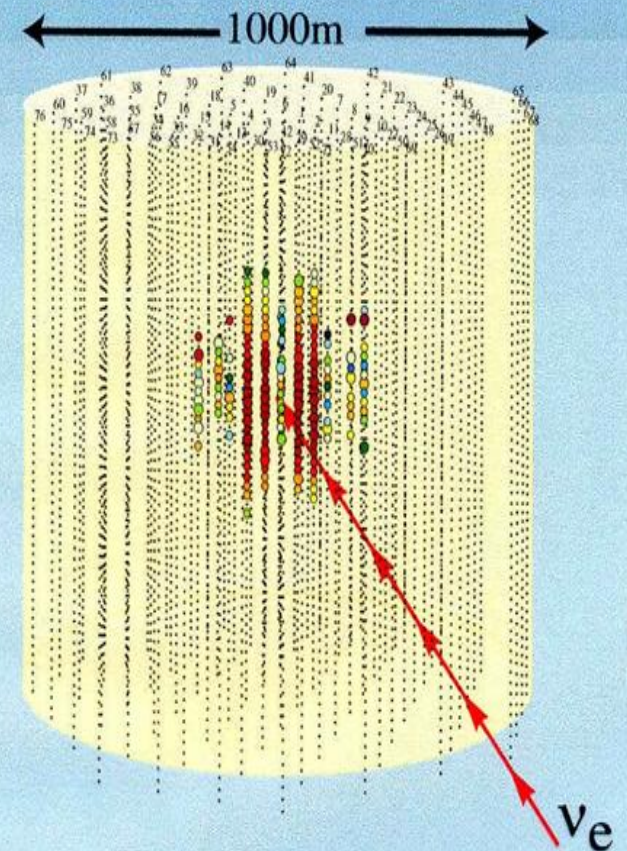




IceCube is working



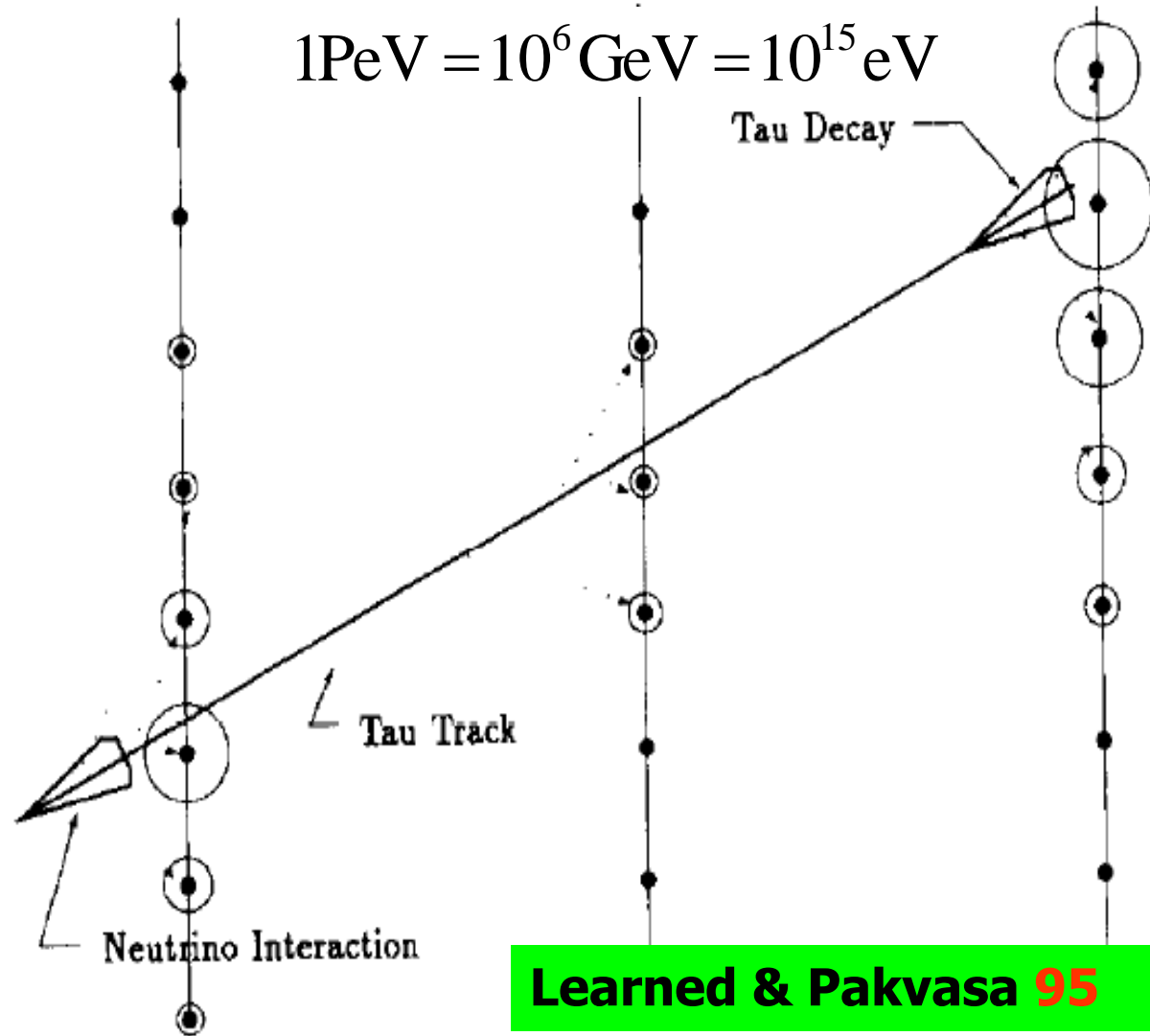
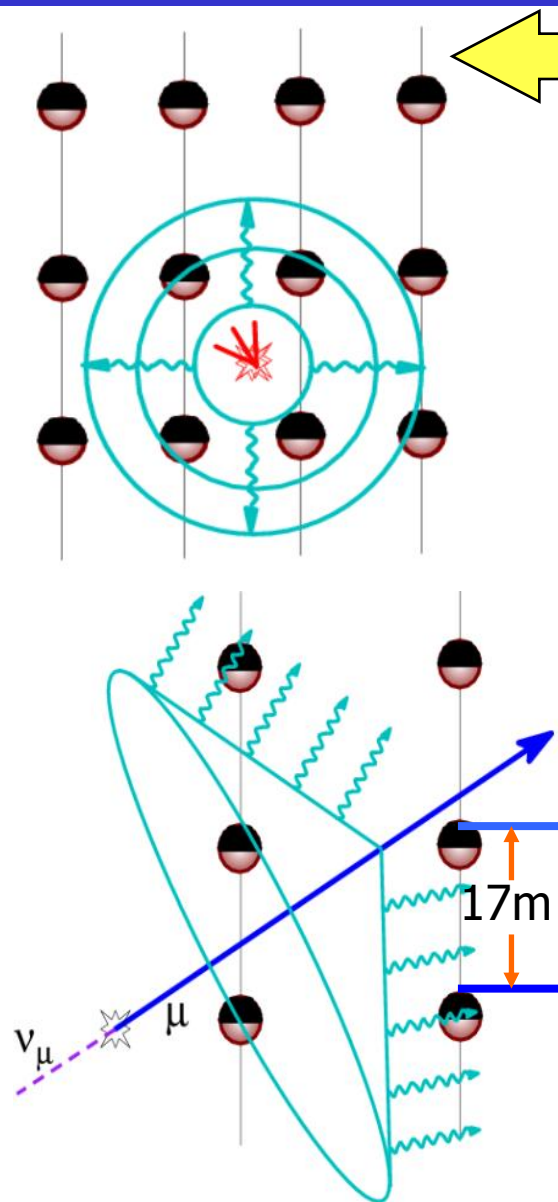
IceCube



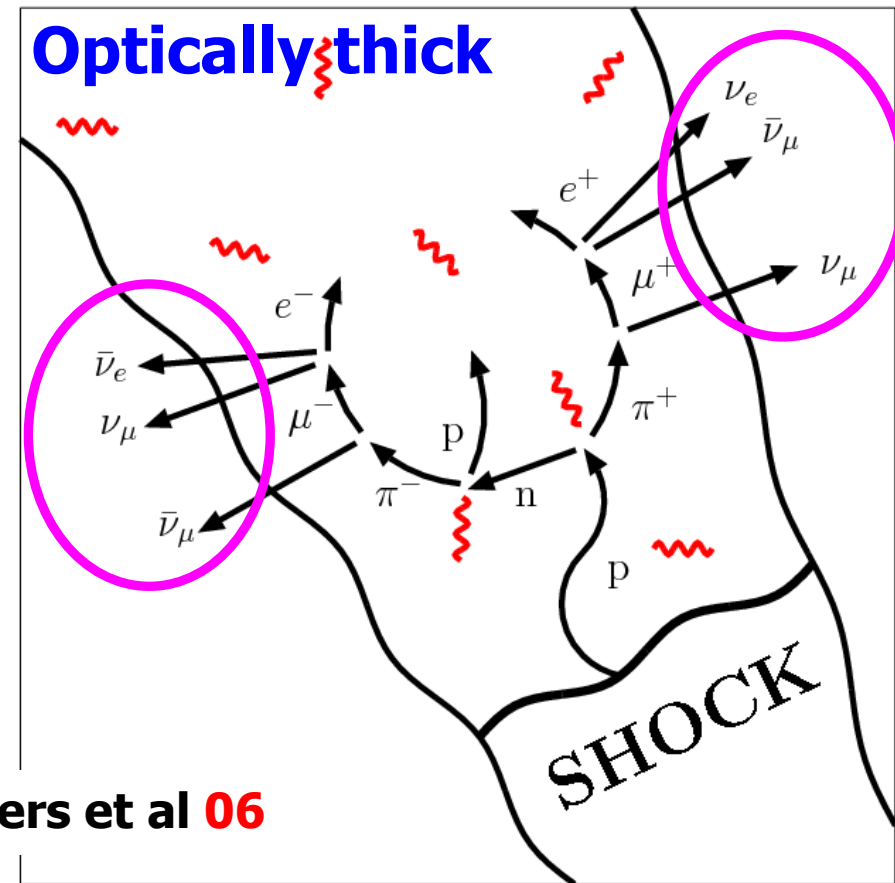
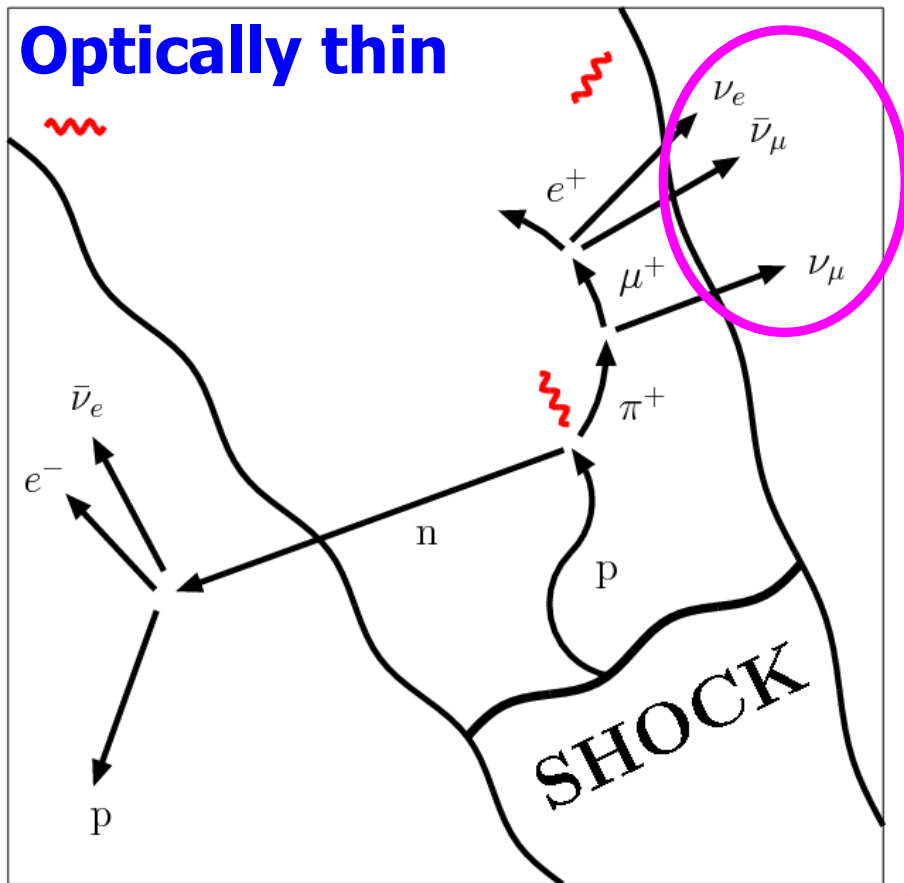
Flavor Identification

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Halzen, astro-ph/0602132



Learned & Pakvasa 95



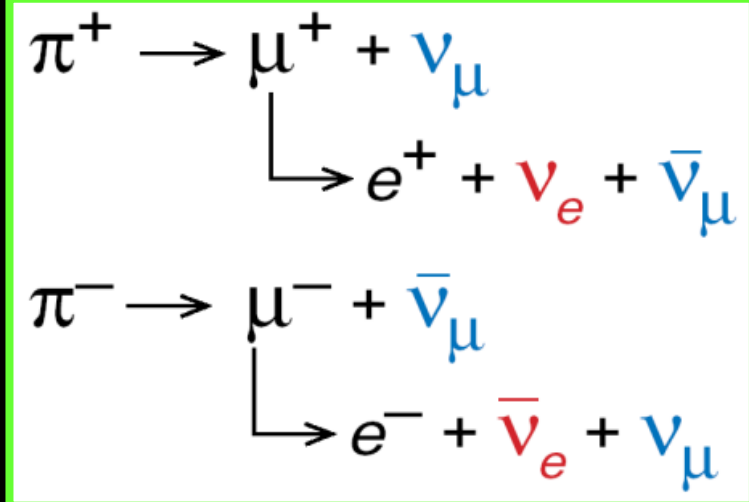
Ahlers et al 06

Conventional mechanism:

$$p + \gamma \rightarrow \Delta^+ \rightarrow \pi^+ + n$$

$$p + p \rightarrow \pi^\pm + X$$

$$\Phi_e^S : \Phi_\mu^S : \Phi_\tau^S = 1 : 2 : 0$$



Oscillations

The transition probability:

$$\alpha, \beta = e, \mu, \tau$$

$$j, k = 1, 2, 3$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j=1}^3 |V_{\alpha j}|^2 |V_{\beta j}|^2 + 2\text{Re} \sum_{j < k} V_{\alpha j} V_{\beta k} V_{\alpha k}^* V_{\beta j}^* \exp \left\{ -i \frac{\Delta m_{kj}^2 L}{2E} \right\}$$

Expected sources (AGN) at a typical distance: ~ 100 Mpc.

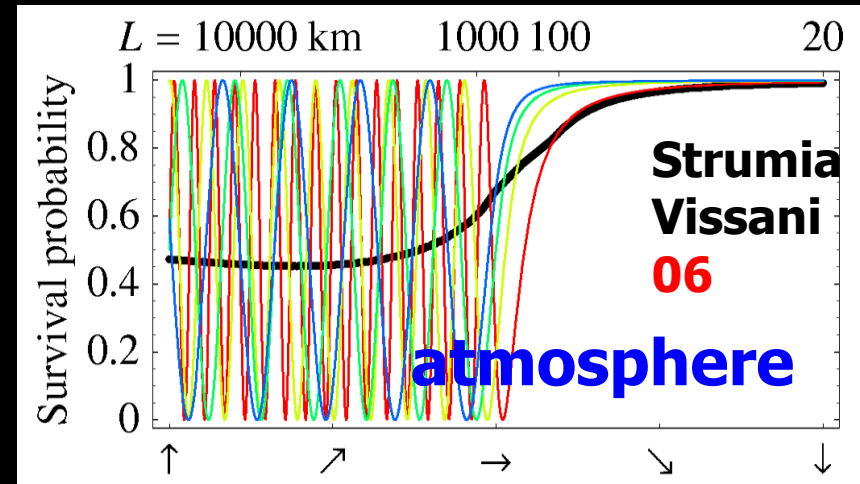
For $|\Delta m^2| \sim 10^{-4} \text{ eV}^2$, the oscillation length in vacuum:

$$L_{\text{OSC}} \equiv \frac{4\pi E_\nu}{|\Delta m^2|} \sim 8 \times 10^{-25} \text{ Mpc} \left(\frac{E_\nu}{1 \text{ eV}} \right)$$

$$1 \text{ Mpc} \approx 3.1 \times 10^{22} \text{ m}$$

After many oscillations, the averaged probability of UHE cosmic neutrinos is

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j=1}^3 |V_{\alpha j}|^2 |V_{\beta j}|^2$$



Flavor Democracy

At an astrophysical source:

$$\Phi_e^S : \Phi_\mu^S : \Phi_\tau^S = 1 : 2 : 0$$

At a ν -telescope:

$$\Phi_\beta^T = \sum_\alpha \Phi_\alpha^S P(\nu_\alpha \rightarrow \nu_\beta) = \sum_\alpha \sum_{i=1}^3 \Phi_\alpha^S |V_{\alpha i}|^2 |V_{\beta i}|^2$$

If there is a μ - τ symmetry for V :

$$|V_{\mu i}| = |V_{\tau i}|$$

$$(i = 1, 2, 3)$$

Then the unitarity of V leads to:

$$\Phi_e^T : \Phi_\mu^T : \Phi_\tau^T = 1 : 1 : 1$$

In the PDG parametrization (Xing, Zhou, 08):

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & +c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

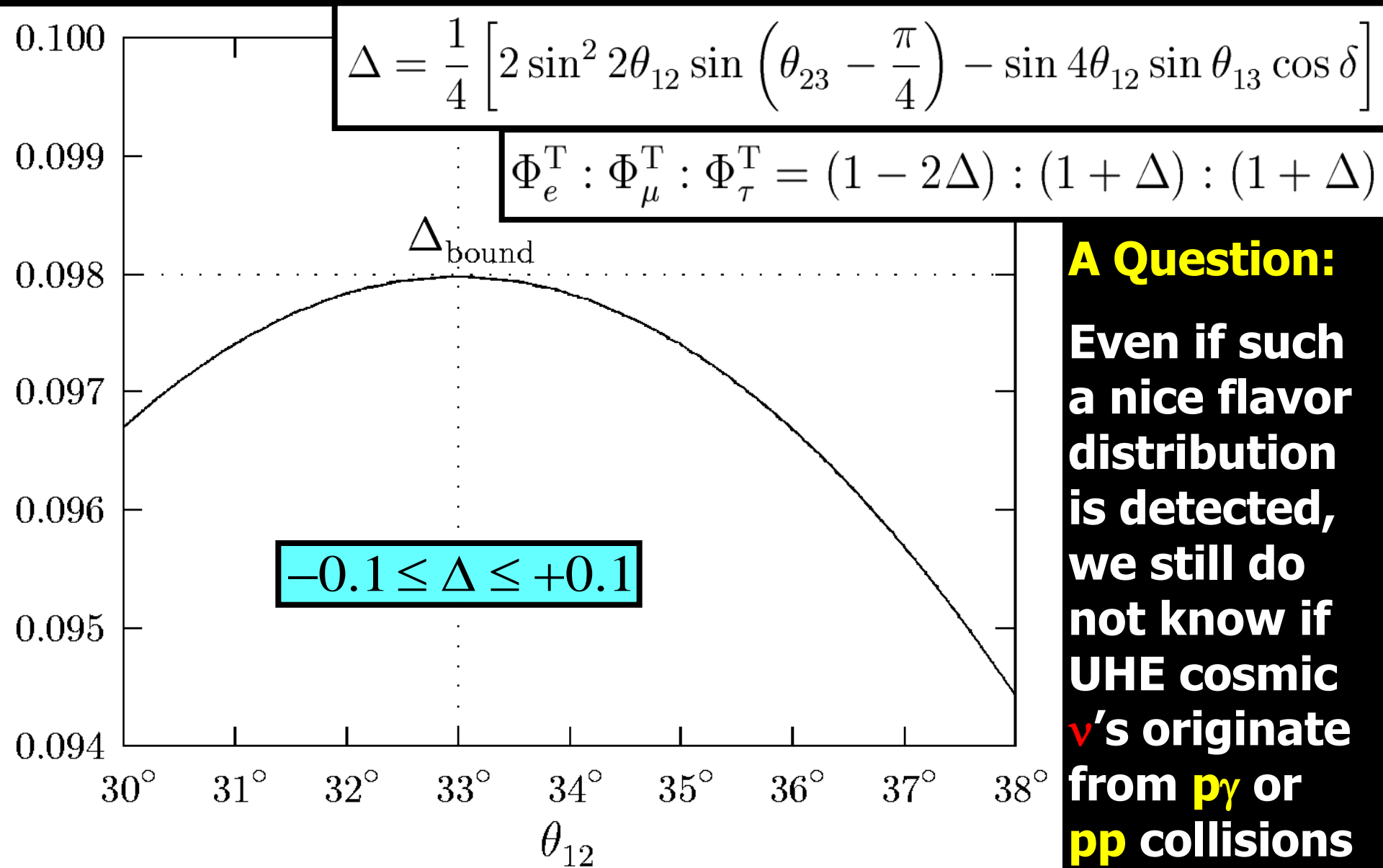
$$\begin{array}{l} \text{CPC:} \left\{ \begin{array}{l} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{array} \right. \\ \text{or} \\ \text{CPV:} \left\{ \begin{array}{l} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right. \end{array}$$

Near flavor democracy (Learned, Pakvasa, 95)

**μ - τ symmetry breaking
(Xing, 06, 12)**

$$\Phi_e^T : \Phi_\mu^T : \Phi_\tau^T = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)$$

μ - τ Symmetry Breaking



The Glashow Resonance

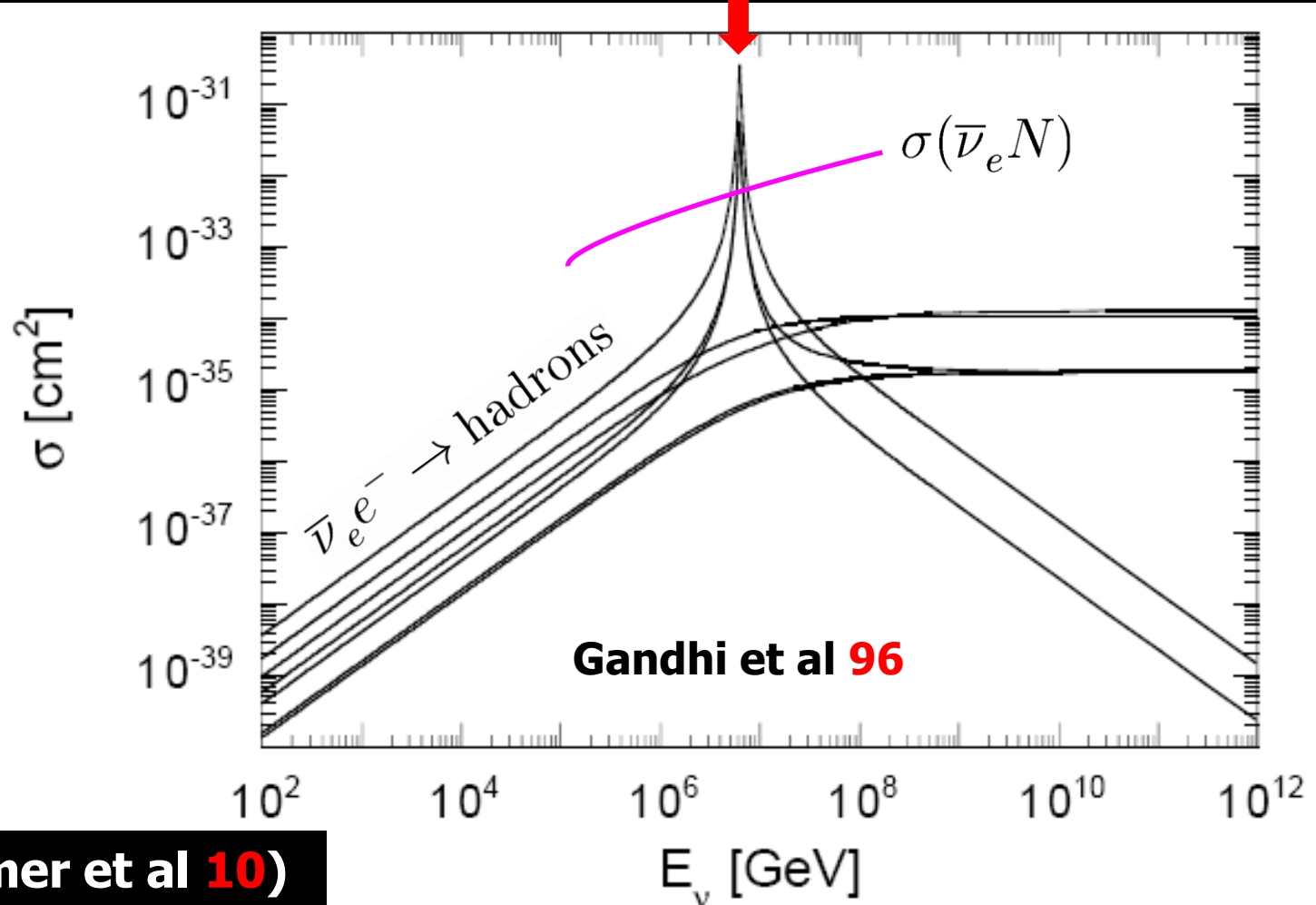
$$\bar{\nu}_e + e^- \rightarrow W^- \rightarrow \text{anything}$$

Unique for electron anti- ν 's!



(Glashow 60)

$$E_{\bar{\nu}_e} \simeq M_W^2 / (2m_e) \simeq 6.3 \text{ PeV}$$



An interesting **discriminator** between **$p\gamma$** & **pp** collisions at an optically thin source of cosmic rays. (Anchordoqui et al 05, Hummer et al 10)

Cosmic Flavor Physics

C ν B

Hot DM

keV ν 's
Warm DM

Baryogenesis
Leptogenesis

Energetic ν 's
from cold DM

UHE
Cosmic ν 's

Supernova ν 's
(relic background)

.....

A New Road Ahead?