Temperature dependent EoS in hydro

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Equations of hydrodynamics

- Basic eqs: continuity and energy-mom. conservation

$$\partial_\mu(nu^\mu) = 0, \tag{1}$$
$$\partial_\nu T^{\mu\nu} = 0. \tag{2}$$

- n is some kind of number density (nonzero chemical potential)
- Energy-momentum tensor $T^{\mu\nu}$ in a perfect fluid:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}, \tag{3}$$

- Equation of State additionally (κ may depend on time):

$$\epsilon = \kappa p = \kappa(T)nT \tag{4}$$

Separation of energy and momentum conservation

- Projection with u^μ: energy conservation

$$(\epsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0. \tag{5}$$

- Multiply with u^ν and substract from the original: Euler equation

$$(\epsilon + p)u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu p, \tag{6}$$

- Substitute $\epsilon = \kappa(T)p$ and $p = nT$.

Ellipsoidal scaling

- Introduce an ellipsoidal scaling variable:

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2} \tag{7}$$

- $s = 1$ is an ellipsoid with principal axes X, Y and Z.
- X, Y and Z are time dependent $\rightarrow$ expansion!
- Look for u^μ such as

$$u^\mu \partial_\mu s = 0 \tag{8}$$

- Then for any function $f(s)$ $u^\mu \partial_\mu f(s) = 0$ also

Continuity

- We choose a 3D Hubble flow

$$u^\mu = \gamma \left(1, \frac{\dot{X}}{X} r_x, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z\right) \tag{9}$$

- Fulfills $u^\mu \partial_\mu s = 0$
- Acceleration possible if X, Y, Z are not constant
- Let the density be:

$$n = n_0 \frac{V_0}{V} \nu(s) \tag{10}$$

- $\nu(s)$ arbitrary, V has to fulfill:

$$u^\mu \partial_\mu V = V \partial_\mu u^\mu \tag{11}$$

- Example: $V = \tau^3$ or $V = XYZ$

Energy conservation and temperature

- Continuity + energy conservation + $\kappa(T)$:

$$\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T \partial_\mu u^\mu = 0. \tag{12}$$

- Generally true (if κ depends only on temperature)!
- Means $d(\kappa T)/dT \neq 0$ ($\partial_\mu u^\mu$ not realistic)
- Also, this is ϵ/n, expected to be monotonic in T
- Can be solved implicitly:

$$\int_{T_0}^{T} \left(\frac{d\kappa(T')T'}{dT'} \frac{1}{T'} \right) dT' = \ln \left(\nu(s) \frac{V_0}{V} \right) \tag{13}$$

- with arbitrary $\mu(s)$ (cancels anyway)
- If κ=constant:

$$T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \mu(s)^{1/\kappa} \tag{14}$$

Euler-equation

- Putting everything together:

$$(\kappa+1)u^\nu\partial_\nu u^\mu - \frac{k+1}{k}u^\mu\partial_\nu u^\nu = \tag{15}$$
$$=\frac{k+1}{k}\partial^\mu \ln\frac{V_0}{V} + \left(\frac{\nu'(s)}{\nu(s)} + \frac{\mu'(s)}{k\mu(s)}\right)\partial_\mu s$$

- with $k = d\kappa T/dT$
- If κ =constant, then $\kappa = k$, thus:

$$\kappa u^\nu\partial_\nu u^\mu - u^\mu\partial_\nu u^\nu = \partial^\mu \ln\left(\frac{V_0}{V}\nu(s)\mu(s)^{1/\kappa}\right) \tag{16}$$

- Easier to solve if $\nu(s) = \mu(s)^{-1/\kappa}$.

A known solution

- A known solution

$$s = \frac{r_x^2}{\dot{X}^2 t^2} + \frac{r_y^2}{\dot{Y}^2 t^2} + \frac{r_z^2}{\dot{Z}^2 t^2}, \tag{17}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{18}$$

$$n = n_0 \frac{V_0}{V} \nu(s), \text{ with } \nu(s) = \text{arbitrary}, \tag{19}$$

$$V = \tau^3, \tag{20}$$

$$T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \frac{1}{\nu(s)}, \text{ with } \kappa = \text{constant} \tag{21}$$

- T. Csörgő, L. P. Csernai, Y. Hama *et al.*, Heavy Ion Phys. **A21** (2004) 73-84.
- No acceleration: $\dot{X}, \dot{Y}, \dot{Z}$ constant.
- Here κ=constant

A new solution

- A partly implicit non-acceleration new solotion:

$$n = n_0 \frac{V_0}{V}, \tag{22}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{23}$$

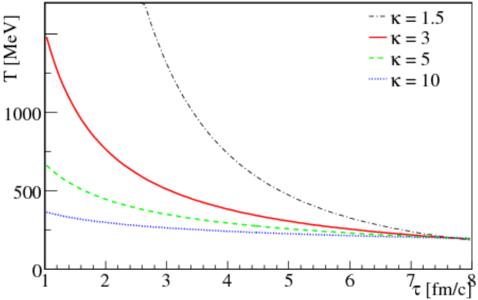
$$V = \tau^3, \tag{24}$$

$$T \text{ from } \int_{T_0}^{T} \left(\frac{d\kappa(T')T'}{dT'} \frac{1}{T'} \right) dT' = \ln \frac{V_0}{V} \tag{25}$$

- No acceleration, but $\kappa(T)$ possible
- Time dependence of the temperature to calculate more exactly

An example, κ =constant

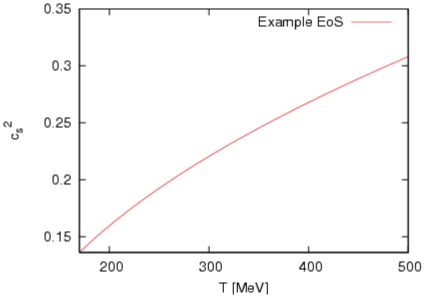
- $T(t)$ can be calculated directly:



- How about $\kappa(T)$?

An example $\kappa(T)$

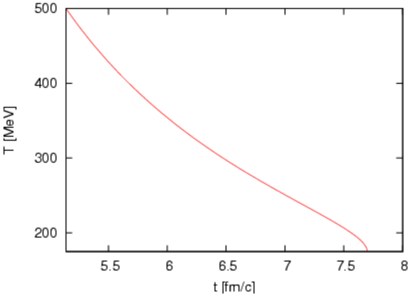
- Take an example $\kappa(T)$, displayed as $c_s^2(T)$.



- In the right regime, just took an example dependence
- Not physical, just an example!
- This can be plugged into the solution

Temperature evolution

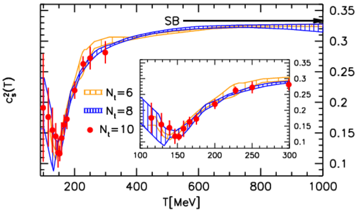
- $T(t)$ can be calculated from the implicit equation for the temperature:



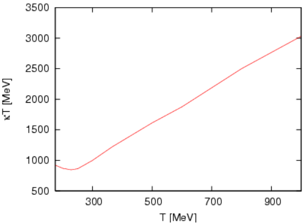
- How about realistic $c_s^2(T)$?

A realistic EoS

- Take EoS from lattice QCD:



$\Rightarrow$



- S. Borsányi, G. Endrődi, Z. Fodor *et al.* arXiv:1007.2580
- Problem: $\frac{d\kappa T}{dT}$ =0 at $T = 200 - 250$ MeV
- Recall: $\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T \partial_\mu u^\mu = 0$
- Thus $\partial_\mu u^\mu = 0$ here: unrealistic!

Summary

- A general ellipsoidally symmetric class of solutions
- A new solution with arbitrary $\kappa(T)$
- Lattice QCD EoS versus $\frac{d\kappa T}{dT} u^\mu \partial_\mu T + T\partial_\mu u^\mu$?