Introduction to RooFit

1. Introduction and overview
2. Creation and basic use of models
3. Composing models
4. Working with (profile) likelihood
5. Simultaneous fits and combined models

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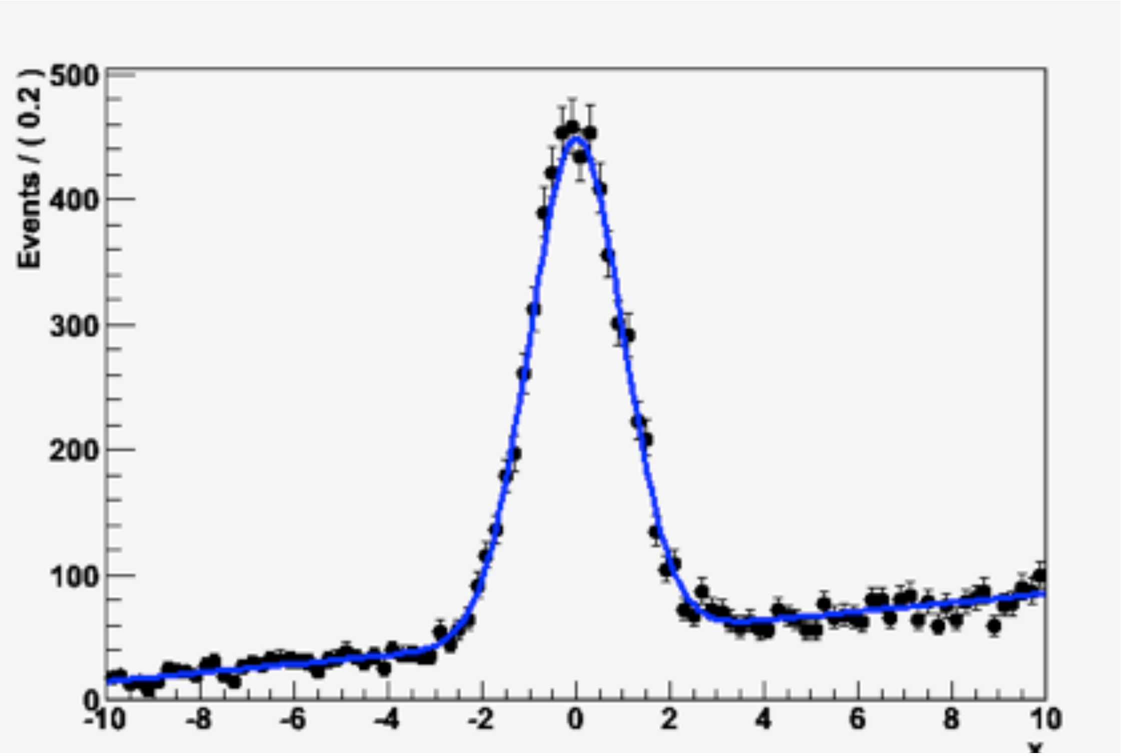
1 Introduction & Overview

Introduction -- Focus: coding a probability density function

- Focus on one practical aspect of many data analysis in HEP: How do you formulate your p.d.f. in ROOT
  - For ‘simple’ problems (gauss, polynomial) this is easy



  - But if you want to do unbinned ML fits, use non-trivial functions, or work with multidimensional functions you quickly find that you need some tools to help you

Introduction – Why RooFit was developed

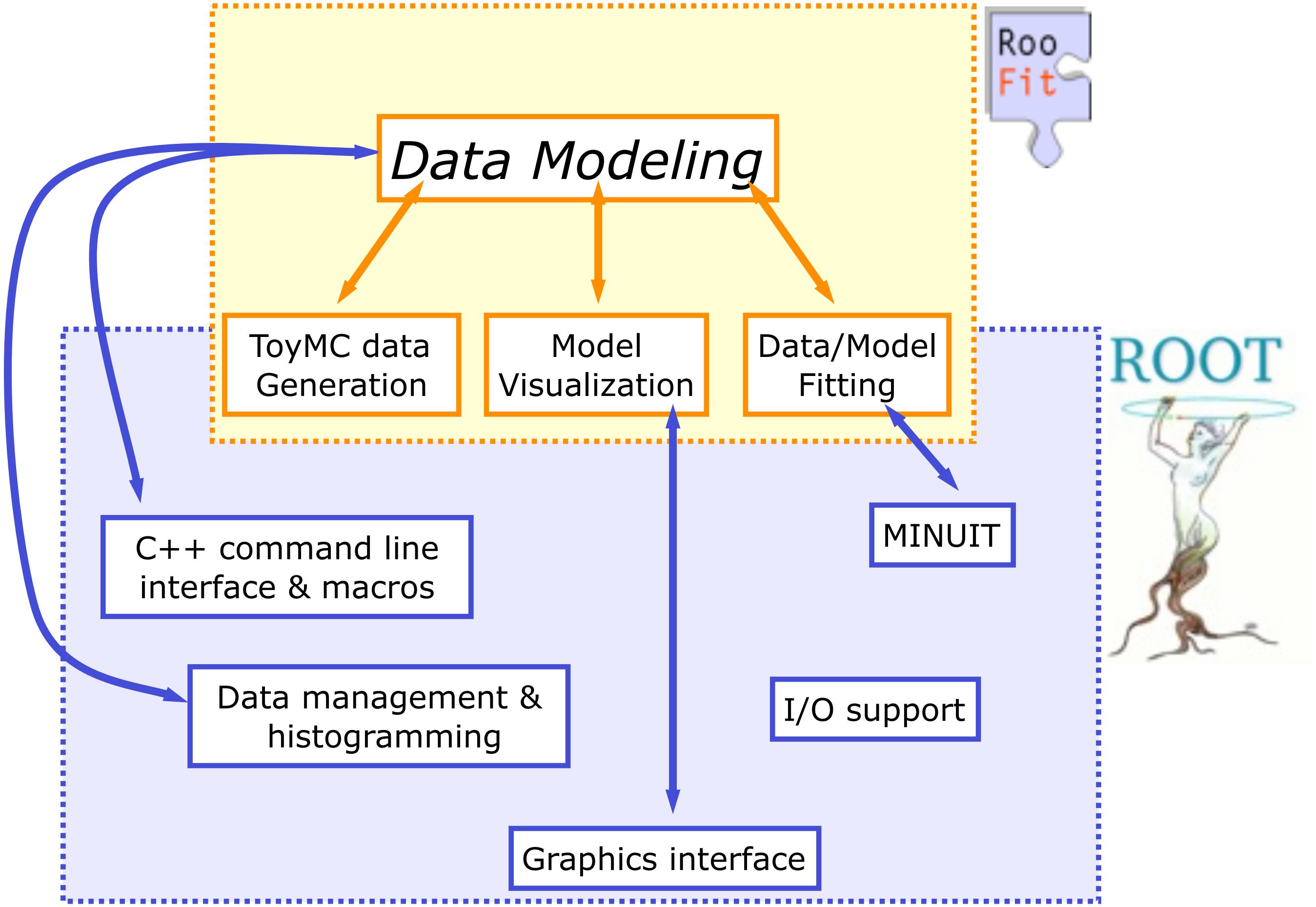
- BaBar experiment at SLAC: Extract sin(2β) from time_ dependent CP violation of B decay: e^+e^- → Y(4s) → BB
  - Reconstruct both Bs, measure decay time difference
  - Physics of interest is in decay time dependent oscillation

$$
\begin{aligned}
& f_{sig} \times \Big[\mathrm{SigSel}(m;\bar{p}_{sig}) \times \Big(\mathrm{SigDecay}(t;\vec{q}_{sig},\sin(2\beta)) \otimes \mathrm{SigResol}(t\,|\,dt;\vec{r}_{sig})\Big)\Big] + \\
& (1-f_{sig})\Big[\mathrm{BkgSel}(m;\bar{p}_{bkg}) \times \Big(\mathrm{BkgDecay}(t;\vec{q}_{bkg}) \otimes \mathrm{BkgResol}(t\,|\,dt;\vec{r}_{bkg})\Big)\Big]
\end{aligned}
$$

- Many issues arise
  - Standard ROOT function framework clearly insufficient to handle such complicated functions → must develop new framework
  - Normalization of p.d.f. not always trivial to calculate → may need numeric integration techniques
  - Unbinned fit, >2 dimensions, many events → computation performance important → must try optimize code for acceptable performance
  - Simultaneous fit to control samples to account for detector performance

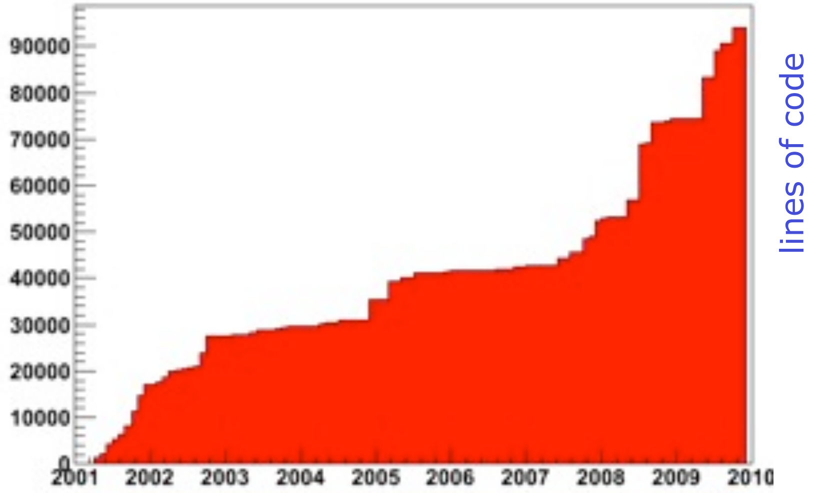
Introduction – Relation to ROOT

Extension to ROOT – (Almost) no overlap with existing functionality



Project timeline

- 1999 : Project started
  - First application: ‘sin2b’ measurement of BaBar (model with 5 observables, 37 floating parameters, simultaneous fit to multiple CP and control channels)
- 2000 : Complete overhaul of design based on experience with sin2b fit
  - Very useful exercise: new design is still current design
- 2003 : Public release of RooFit with ROOT
- 2004 : Over 50 BaBar physics publications using RooFit
- 2007 : Integration of RooFit in ROOT CVS source
- 2008 : Upgrade in functionality as part of RooStats project
  - Improved analytical and numeric integration handling, improved toy MC generation, addition of workspace
- 2009 : Now ~100K lines of code
  - (For comparison RooStats proper is ~5000 lines of code)



RooFit core design philosophy

- Mathematical objects are represented as C++ objects

Mathematical concept		RooFit class
variable	x	**RooRealVar**
function	$f(x)$	**RooAbsReal**
PDF	$f(x)$	**RooAbsPdf**
space point	$\vec{x}$	**RooArgSet**
integral	$\int_{x_{\min}}^{x_{\max}} f(x)dx$	**RooRealIntegral**
list of space points		**RooAbsData**

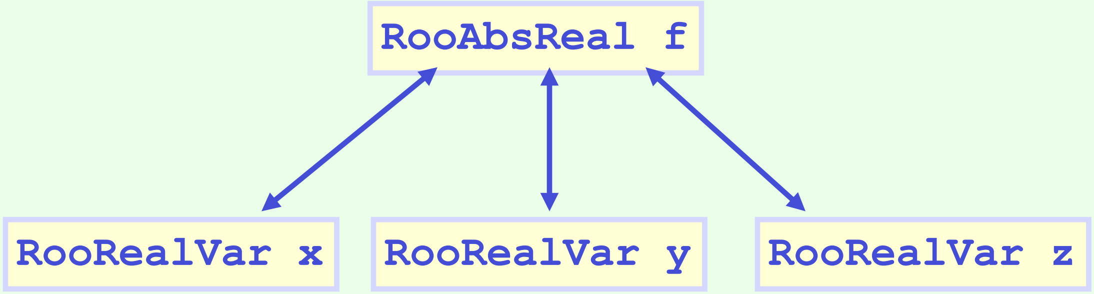
RooFit core design philosophy

- Represent relations between variables and functions as client/server links between objects

Math

$f(x,y,z)$

RooFit diagram



RooAbsReal f

RooRealVar x

RooRealVar y

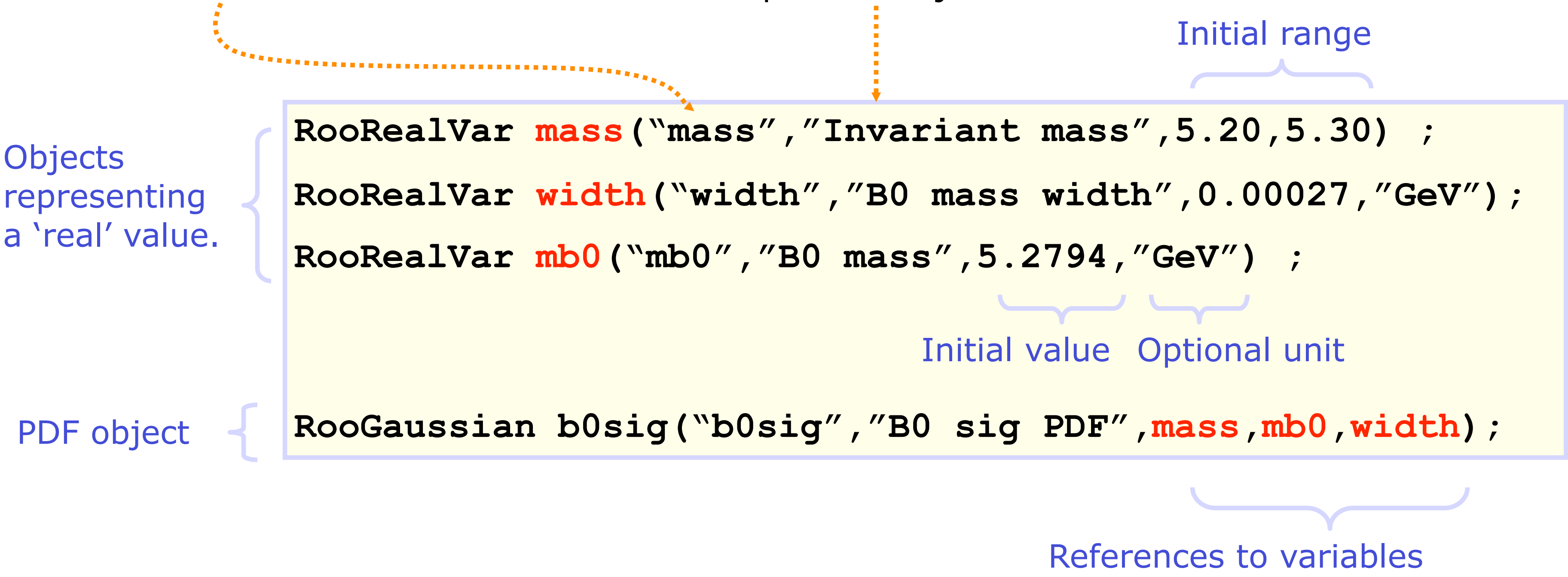
RooRealVar z

RooFit code

```
RooRealVar x(“x”,”x”,5) ;
RooRealVar y(“y”,”y”,5) ;
RooRealVar z(“z”,”z”,5) ;
RooBogusFunction f(“f”,”f”,x,y,z) ;
```

Object-oriented data modeling

- All objects are **self documenting**
  - **Name** - Unique identifier of object
  - **Title** – More elaborate description of object



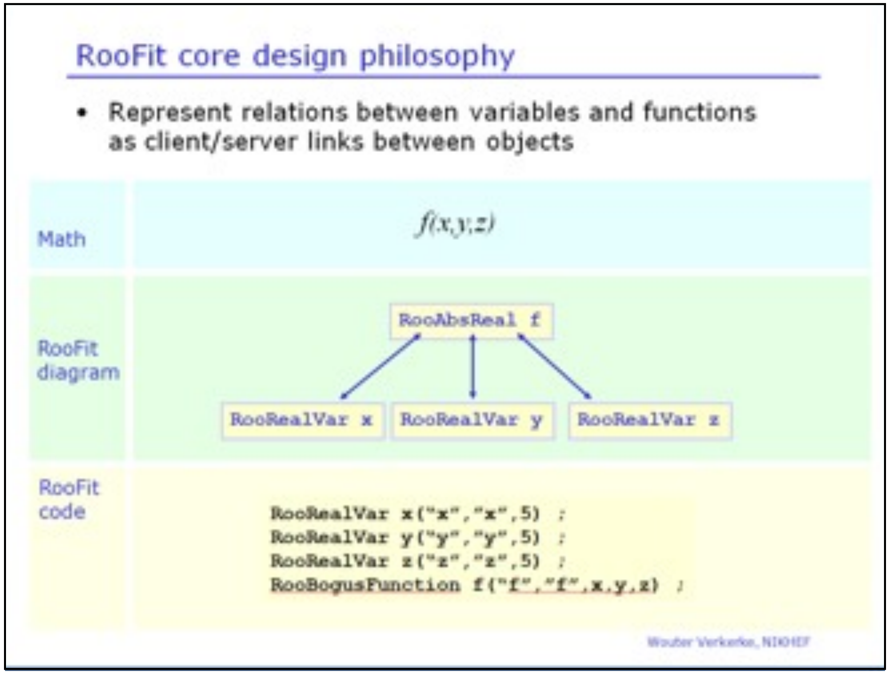
```
RooRealVar mass(“mass”,”Invariant mass”,5.20,5.30) ;
RooRealVar width(“width”,”B0 mass width”,0.00027,”GeV”);
RooRealVar mb0(“mb0”,”B0 mass”,5.2794,”GeV”) ;

RooGaussian b0sig(“b0sig”,”B0 sig PDF”,mass,mb0,width);
```

2 Basic use

Factory and Workspace

- *One C++ object per math symbol* provides ultimate level of control over each objects functionality, but results in lengthy user code for even simple macros
- Solution: add factory that auto-generates objects from a math-like language



RooFit core design philosophy

- Represent relations between variables and functions as client/server links between objects

```
RooRealVar x("x","x",5) ;
RooRealVar y("y","y",5) ;
RooRealVar z("z","z",5) ;
RooBogusFunction f("f","f",x,y,z) ;
```

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```
Gaussian::f(x[-10,10],mean[5],sigma[3])
```

```
RooRealVar x(“x”,”x”,-10,10) ;
RooRealVar mean(“mean”,”mean”,5) ;
RooRealVar sigma(“sigma”,”sigma”,3)  ;
RooGaussian f(“f”,”f”,x,mean,sigma) ;
```

Factory and Workspace

- This is *not* the same as reinventing Mathematica!
  String **constructs** an expression in terms of C++ objects, rather than being the expression
  - Objects can be tailored after construction through object pointers
  - For example: tune parameters and algorithms of numeric integration to be used with a given object
- Implementation in RooFit:

  Factory makes objects, **Workspace** owns them

```
RooWorkspace w(“w”,kTRUE) ;
w.factory(“Gaussian::f(x[-10,10],mean[5],sigma[3])”) ;

w.Print(“t”) ;
variables
---------
(mean,sigma,x)

p.d.f.s
-------
RooGaussian::f[ x=x mean=mean sigma=sigma ] = 0.249352
```

Accessing the workspace contents

- Contents can be accessing in two ways

- Through C++ namespace corresponding through w’space
  - Super easy
  - But works in ROOT interpreted macros only

```
RooWorkspace w(“w”,kTRUE) ;
w.factory(“Gaussian::g(x[-10,10],0,3)”) ;

w::g.Print() ;
```

- Through accessor methods
  - A bit more clutter, but 100% ISO compliant C++ (and compilable)

```
RooAbsPdf* g = w.pdf(“g”) ;
RooRealVar* x = w.var(“x”) ;
```

Factory language

- The factory language has a 1-to-1 mapping to the constructor syntax of RooFit classes
  - With a few handy shortcuts for variables

- Creating variables

```
x[-10,10]    // Create variable with given range, init val is midpoint
x[5,-10,10]  // Create variable with initial value and range
x[5]         // Create initially constant variable
```

- Creating pdfs (and functions)

```
Gaussian::g(x,mean,sigma) → RooGaussian(“g”,”g”,x,mean,sigma)
Polynomial::p(x,{a0,a1}) → RooPolynomial(“p”,”p”,x”,RooArgList(a0,a1));
```

  - Can always omit leading ‘Roo’
  - Curly brackets translate to set or list argument (depending on context)

Factory language

- Composite expression are created by nesting statements
  - No limit to recursive nesting

```
Gaussian::g(x[-10,10],mean[-10,10],sigma[3])
    →    x[-10,10]
         mean[-10,10]
         sigma[3]
         Gaussian::g(x,mean,sigma)
```

- You can also use numeric constants whenever an unnamed constant is needed

```
Gaussian::g(x[-10,10],0,3)
```

- Names of nested function objects are optional
    - SUM syntax explained later

```
SUM::model(0.5*Gaussian(x[-10,10],0,3),Uniform(x)) ;
```

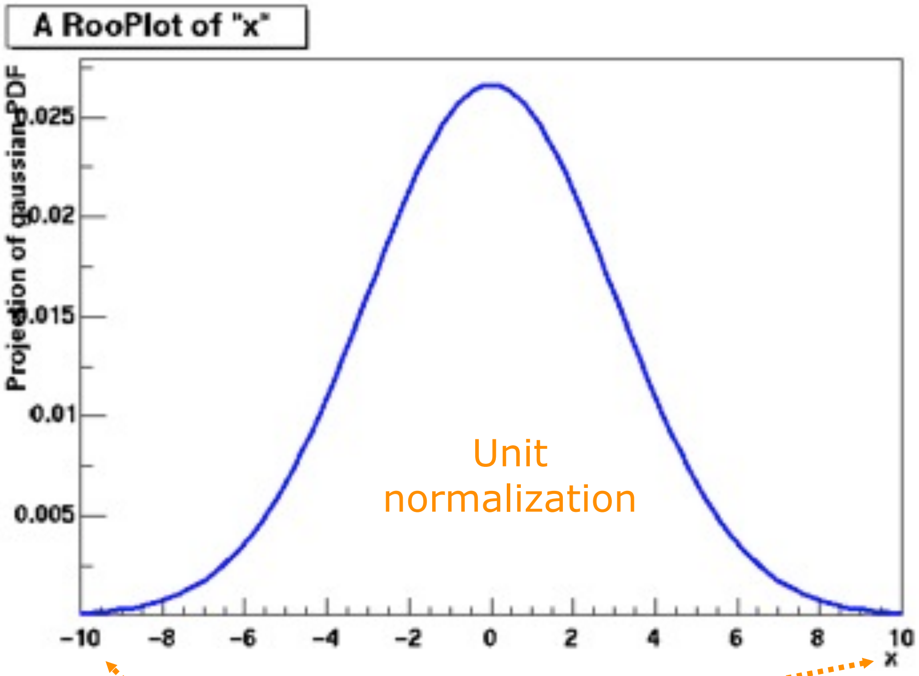
Basics – Creating and plotting a Gaussian p.d.f

Setup gaussian PDF and plot

```
// Build Gaussian PDF
w.factory(“Gaussian::gauss(x[-10,10],mean[-10,10],sigma[3,1,10]”)

// Plot PDF
RooPlot* xframe = w::x.frame() ;
w::gauss.plotOn(xframe) ;
xframe->Draw() ;
```

Axis label from `gauss` title

A `RooPlot` is an empty frame capable of holding anything plotted versus it variable



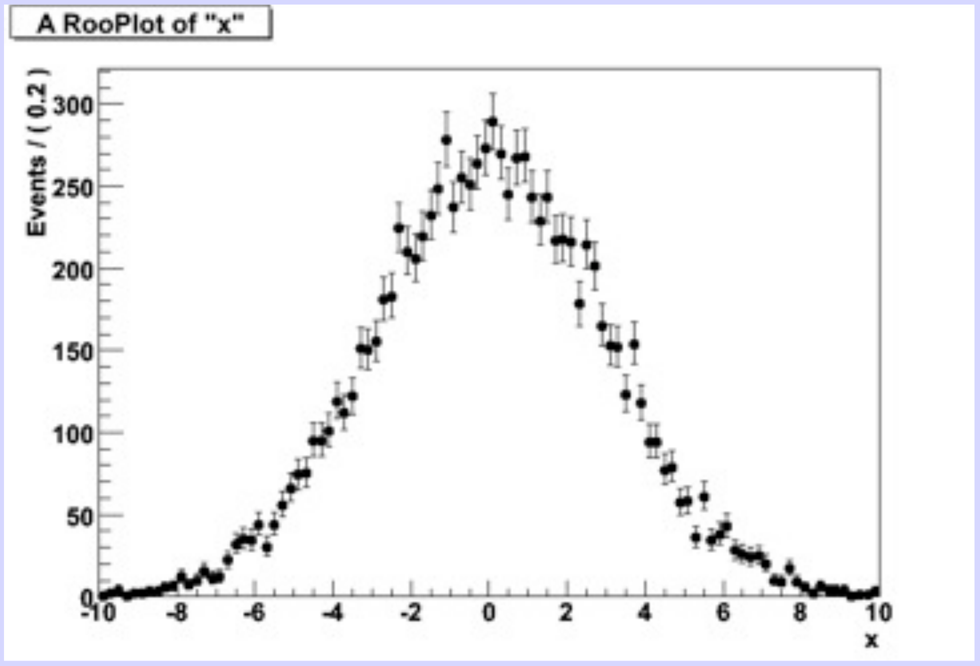
Unit normalization

Plot range taken from limits of `x`

Basics – Generating toy MC events

Generate 10000 events from Gaussian p.d.f and show distribution

```
// Generate an unbinned toy MC set
RooDataSet* data = w::gauss.generate(w::x,10000) ;

// Generate an binned toy MC set
RooDataHist* data = w::gauss.generateBinned(w::x,10000) ;

// Plot PDF
RooPlot* xframe = w::x.frame()
data->plotOn(xframe) ;
xframe->Draw() ;
```



Can generate both binned and unbinned datasets

Basics – Importing data

- Unbinned data can also be imported from ROOT **TTrees**

```
// Import unbinned data
RooDataSet data(“data”,”data”,w::x,Import(*myTree)) ;
```

  - Imports **TTree** branch named “x”.
  - Can be of type **Double_t**, **Float_t**, **Int_t** or **UInt_t**. All data is converted to Double_t internally
  - Specify a **RooArgSet** of multiple observables to import multiple observables

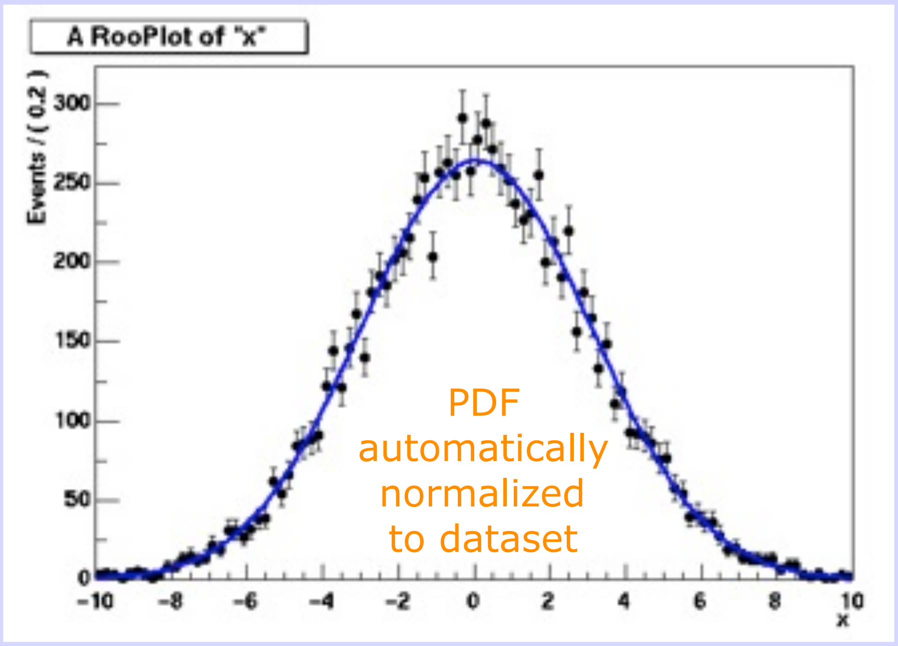
- Binned data can be imported from ROOT **THx** histograms

```
// Import unbinned data
RooDataHist data(“data”,”data”,w::x,Import(*myTH1)) ;
```

  - Imports values, binning definition *and* SumW2 errors (if defined)
  - Specify a **RooArgList** of observables when importing a TH2/3.

Basics – ML fit of p.d.f to *unbinned* data

```
// ML fit of gauss to data
w::gauss.fitTo(*data) ;
(MINUIT printout omitted)

// Parameters if gauss now
// reflect fitted values
w::mean.Print()
RooRealVar::mean = 0.0172335 +/- 0.0299542
w::sigma.Print()
RooRealVar::sigma = 2.98094  +/- 0.0217306

// Plot fitted PDF and toy data overlaid
RooPlot* xframe = w::x.frame() ;
data->plotOn(xframe) ;
w::gauss.plotOn(xframe) ;
```



PDF automatically normalized to dataset

Basics – ML fit of p.d.f to *unbinned* data

- Can also choose to save full detail of fit

```
RooFitResult* r = w::gauss.fitTo(*data,Save()) ;

r->Print() ;
  RooFitResult: minimized FCN value: 25055.6,
                estimated distance to minimum: 7.27598e-08
                coviarance matrix quality:
                Full, accurate covariance matrix

    Floating Parameter    FinalValue +/-  Error
  --------------------  --------------------------
                  mean    1.7233e-02 +/-  3.00e-02
                 sigma    2.9809e+00 +/-  2.17e-02


r->correlationMatrix().Print() ;

2x2 matrix is as follows

     |      0    |      1    |
-------------------------------
   0 |          1    0.0005869
   1 |  0.0005869            1
```

Basics – Observables and parameters of Gauss

- Class **RooGaussian** has *no intrinsic notion* of distinction between observables and parameters

- Distinction always implicit in use context with dataset
  - **x** = observable (as it is a variable in the dataset)
  - **mean,sigma** = parameters

- Choice of observables (for unit normalization) must be passed to **gauss.getVal()** when called directly

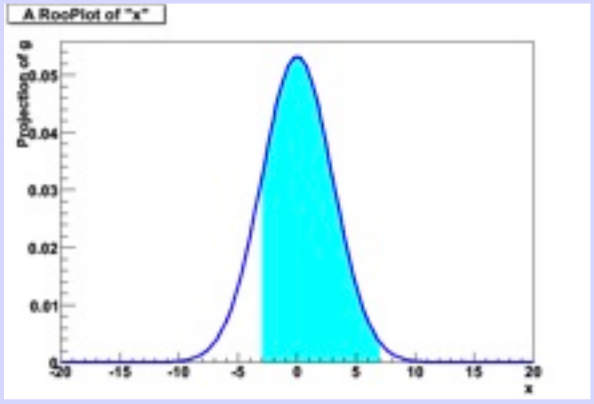
```
gauss.getVal() ;  // Not normalized (i.e. this is _not_ a pdf)
gauss.getVal(x) ; // Guarantees Int[xmin,xmax] Gauss(x,m,s)dx==1
gauss.getVal(s) ; // Guarantees Int[smin,smax] Gauss(x,m,s)ds==1
```

  - This flexibility is useful in e.g. Bayesian context

Basics – Integrals over p.d.f.s

- It is easy to create an object *representing integral* over a normalized p.d.f in a sub-range

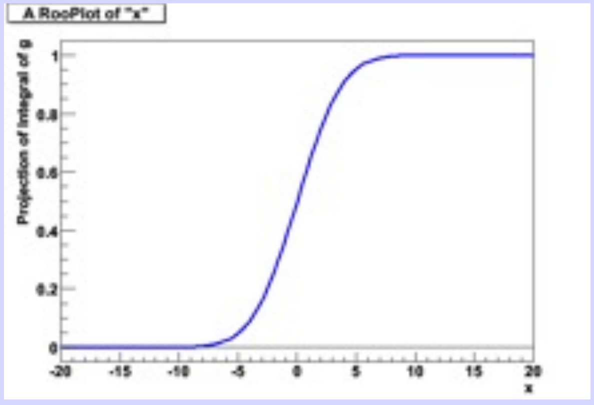
```
w::x.setRange(“sig”,-3,7) ;
RooAbsReal* ig = w::g.createIntegral(x,NormSet(x),Range(“sig”)) ;
cout << ig.getVal() ;
0.832519
mean=-1 ;
cout << ig.getVal() ;
0.743677
```



- Similarly, one can also request the *cumulative distribution function*

$$C(x) = \int_{x_{\min}}^{x} F(x')dx'$$

```
RooAbsReal* cdf = gauss.createCdf(x) ;
```



Model building – (Re)using standard components

- RooFit provides a collection of compiled standard PDF classes

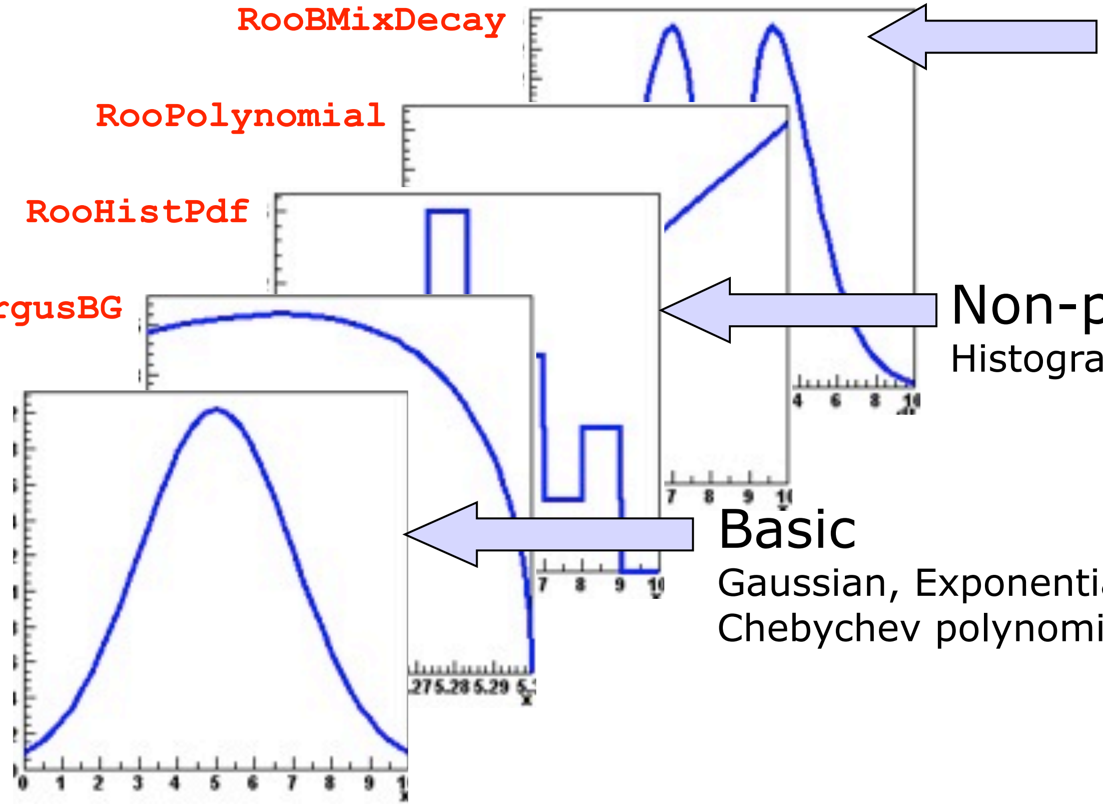
RooBMixDecay

RooPolynomial

RooHistPdf

RooArgusBG

RooGaussian



Physics inspired
ARGUS,Crystal Ball,
Breit-Wigner, Voigtian,
B/D-Decay,….

Non-parametric
Histogram, **KEYS**

Basic
Gaussian, Exponential, Polynomial,…
Chebychev polynomial

Easy to extend the library: each p.d.f. is a separate C++ class

Model building – (Re)using standard components

- List of most frequently used pdfs and their factory spec

Gaussian	`Gaussian::g(x,mean,sigma)`
Breit-Wigner	`BreitWigner::bw(x,mean,gamma)`
Landau	`Landau::l(x,mean,sigma)`
Exponential	`Exponental::e(x,alpha)`
Polynomial	`Polynomial::p(x,{a0,a1,a2})`
Chebychev	`Chebychev::p(x,{a0,a1,a2})`
Kernel Estimation	`KeysPdf::k(x,dataSet)`
Poisson	`Poisson::p(x,mu)`
Voigtian (=BW⊗G)	`Voigtian::v(x,mean,gamma,sigma)`

Model building – Making your own

- Interpreted expressions

```
w.factory(“EXPR::mypdf(‘sqrt(a*x)+b’,x,a,b)”) ;
```

- Customized class, compiled and linked on the fly

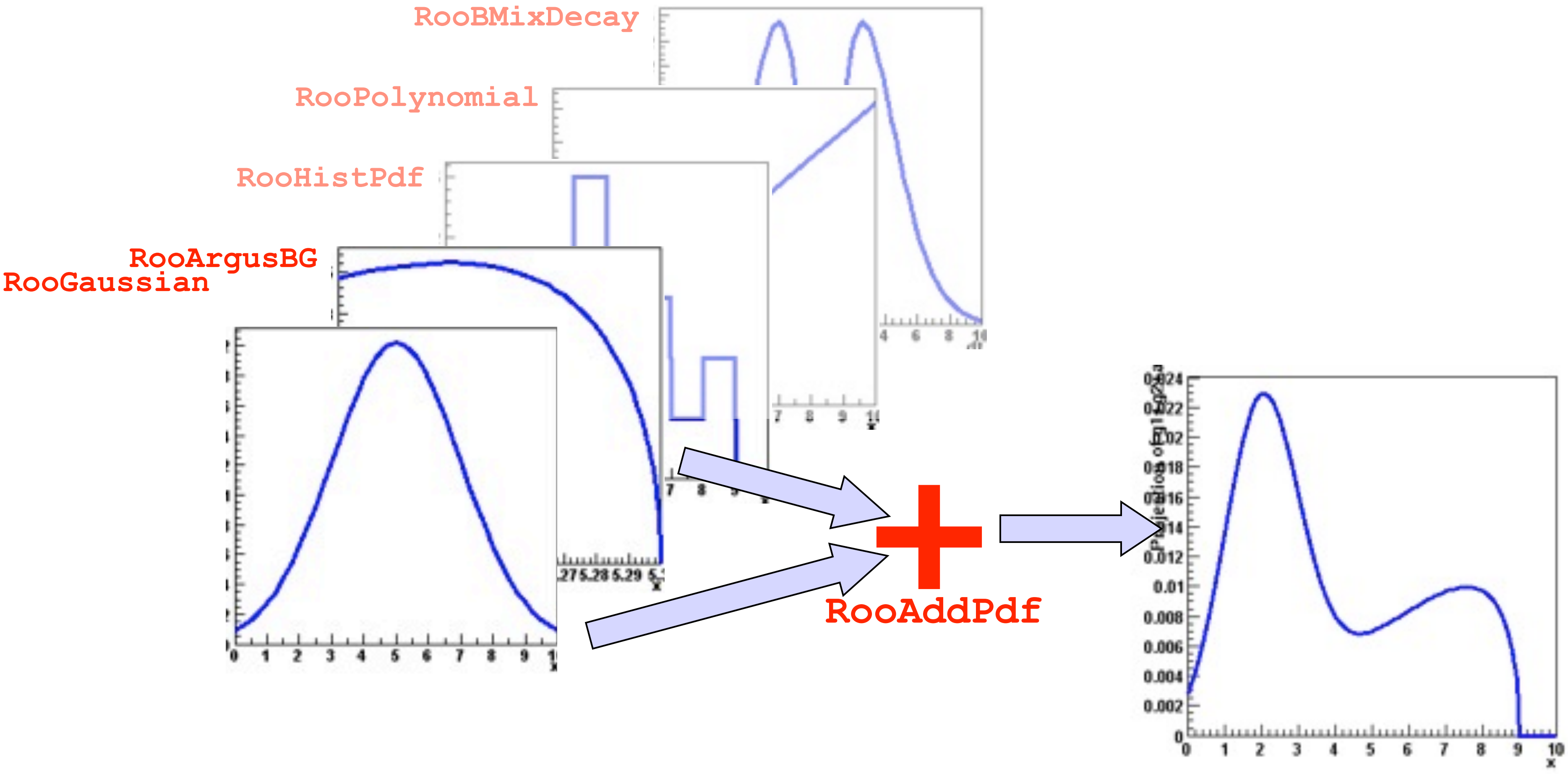
```
w.factory(“CEXPR::mypdf(‘sqrt(a*x)+b’,x,a,b)”) ;
```

- Custom class written by you
  - Offer option of providing analytical integrals, custom handling of toy MC generation (details in RooFit Manual)
- Compiled classes are faster in use, but require O(1-2) seconds startup overhead
  - Best choice depends on use context

3 Composite models

Model building – (Re)using standard components

- Most realistic models are constructed as the sum of one or more p.d.f.s (e.g. signal and background)
- Facilitated through operator p.d.f `RooAddPdf`



RooBMixDecay

RooPolynomial

RooHistPdf

RooArgusBG

RooGaussian

RooAddPdf

Adding p.d.f.s – Mathematical side

- From math point of view adding p.d.f is simple
  - Two components *F, G*

$$S(x) = fF(x) + (1-f)G(x)$$

  - Generically for *N* components P_0-P_N

$$S(x) = c_0P_0(x) + c_1P_1(x) + \ldots + c_{n-1}P_{n-1}(x) + \left(1 - \sum_{i=0,n-1} c_i\right)P_n(x)$$

- For *N* p.d.f.s, there are *N-1* fraction coefficients that should sum to less 1
  - The remainder is by construction 1 minus the sum of all other coefficients

Adding p.d.f.s – Factory syntax

- Additions created through a SUM expression

```
SUM::name(frac1*PDF1,frac2*PDF2,...,PDFN)
```

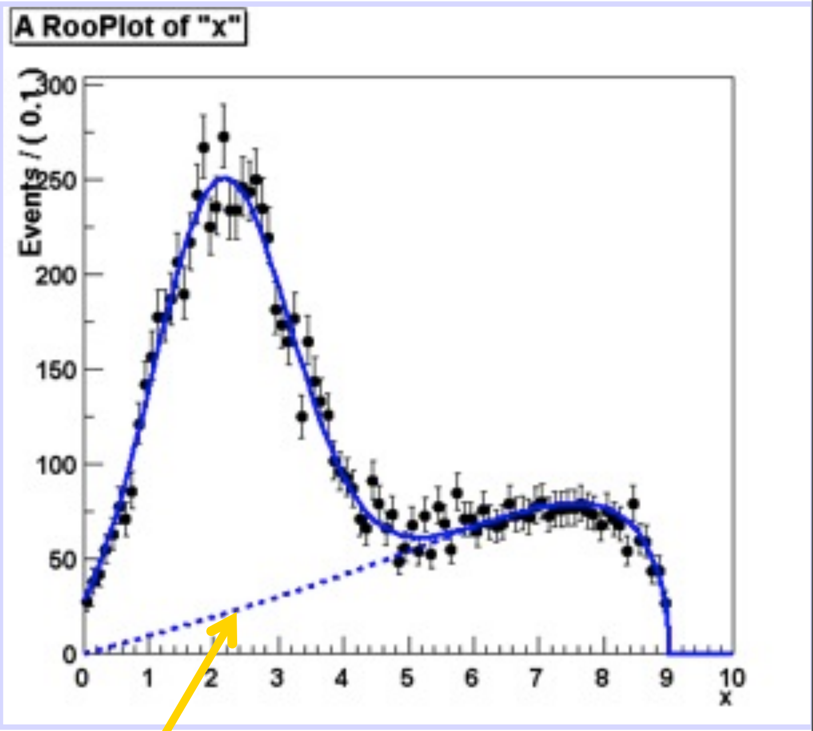
  - Note that last PDF does not have an associated fraction

- Complete example

```
w.factory(“Gaussian::gauss1(x[0,10],mean1[2],sigma[1]”) ;
w.factory(“Gaussian::gauss2(x,mean2[3],sigma)”) ;
w.factory(“ArgusBG::argus(x,k[-1],9.0)”) ;

w.factory(“SUM::sum(g1frac[0.5]*gauss1, g2frac[0.1]*gauss2, argus)”)
```

Component plotting - Introduction

- Plotting, toy event generation and fitting works identically for composite p.d.f.s
  - Several optimizations applied behind the scenes that are specific to composite models (e.g. delegate event generation to components)
- Extra plotting functionality specific to composite pdfs
  - Component plotting



```
// Plot only argus components
w::sum.plotOn(frame,Components(“argus”),LineStyle(kDashed)) ;

// Wildcards allowed
w::sum.plotOn(frame,Components(“gauss*”),LineStyle(kDashed)) ;
```

Extended ML fits

- In an extended ML fit, an extra term is added to the likelihood

Poisson(N_{obs},N_{exp})

- This is most useful in combination with a composite pdf

shape *normalization*

$$F(x) = f \times S(x) + (1-f)B(x) \quad ; \quad N_{\mathrm{exp}} = N$$

$$f, N \Rightarrow N_S, N_B$$

$$F(x) = \frac{N_S}{N_S + N_B} \times S(x) + \frac{N_B}{N_S + N_B} B(x) \quad ; \quad N_{\mathrm{exp}} = N_S + N_B$$

Write like this,
extended term automatically included in –log(L)

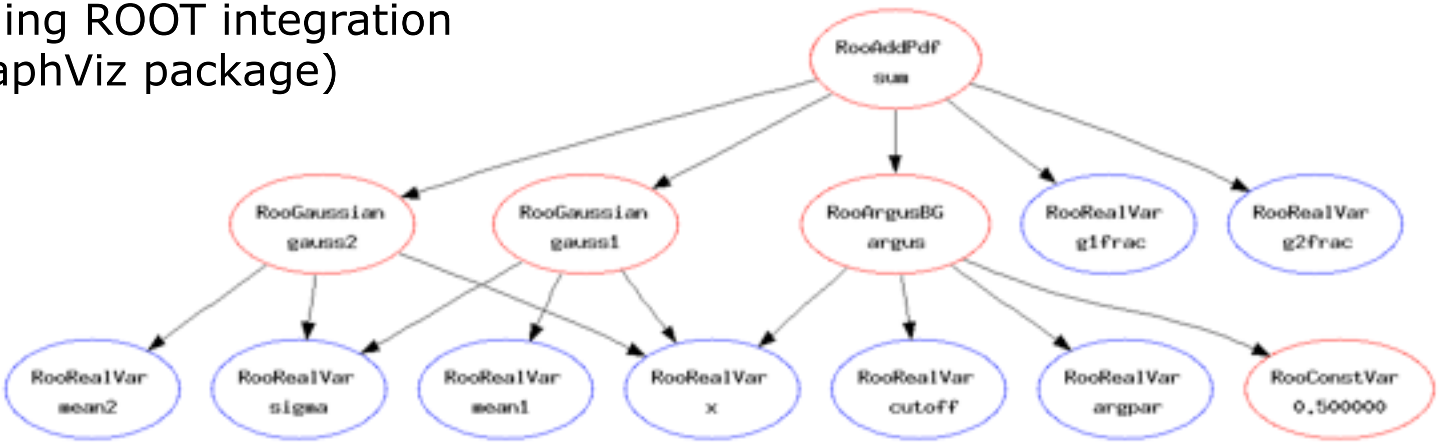
```
SUM::name(Nsig*S,Nbkg*B)
```

Operations on specific to composite pdfs

- Tree printing mode of workspace reveals component structure – **w.Print("t")**

```
RooAddPdf::sum[ g1frac * g1 + g2frac * g2 + [%] * argus ] = 0.0687785
    RooGaussian::g1[ x=x mean=mean1 sigma=sigma ] = 0.135335
    RooGaussian::g2[ x=x mean=mean2 sigma=sigma ] = 0.011109
    RooArgusBG::argus[ m=x m0=k c=9 p=0.5 ] = 0
```

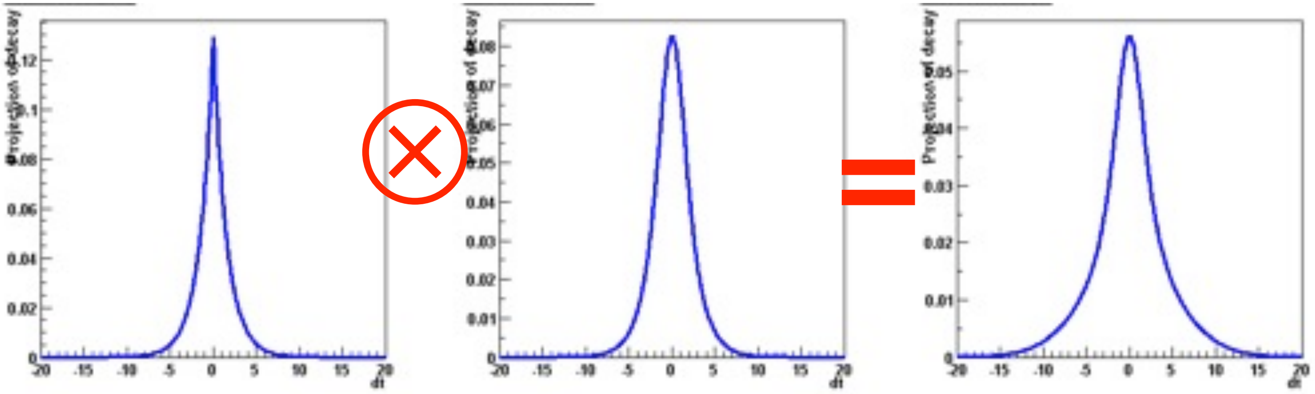
  - Can also make input files for GraphViz visualization (**w::sum.graphVizTree("myfile.dot")**)
  - Graph output on ROOT Canvas in near future (pending ROOT integration of GraphViz package)



Convolution

- Model representing a convolution of a theory model and a resolution model often useful

$$f(x)\otimes g(x)=\int_{-\infty}^{+\infty} f(x)g(x-x')dx'$$



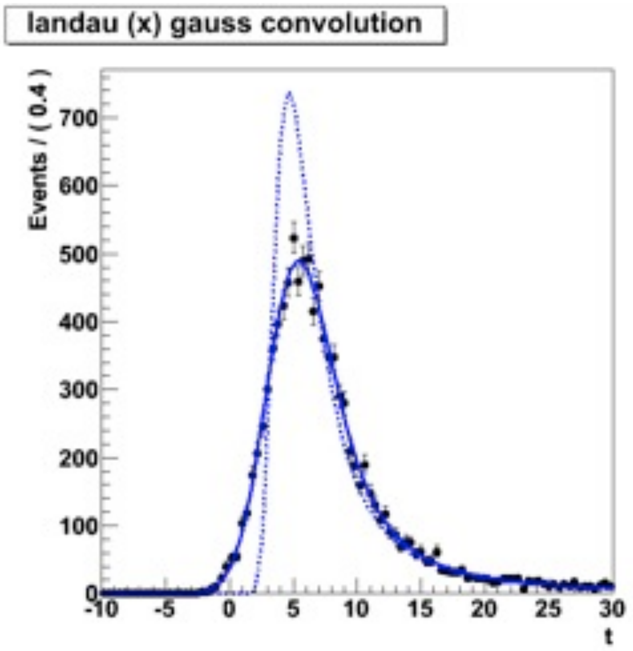
- But numeric calculation of convolution integral can be challenging. No one-size-fits-all solution, but 3 options available
  - Analytical convolution (BW⊗Gauss, various B physics decays)
  - Brute-force numeric calculation (slow)

Convolution

- Example

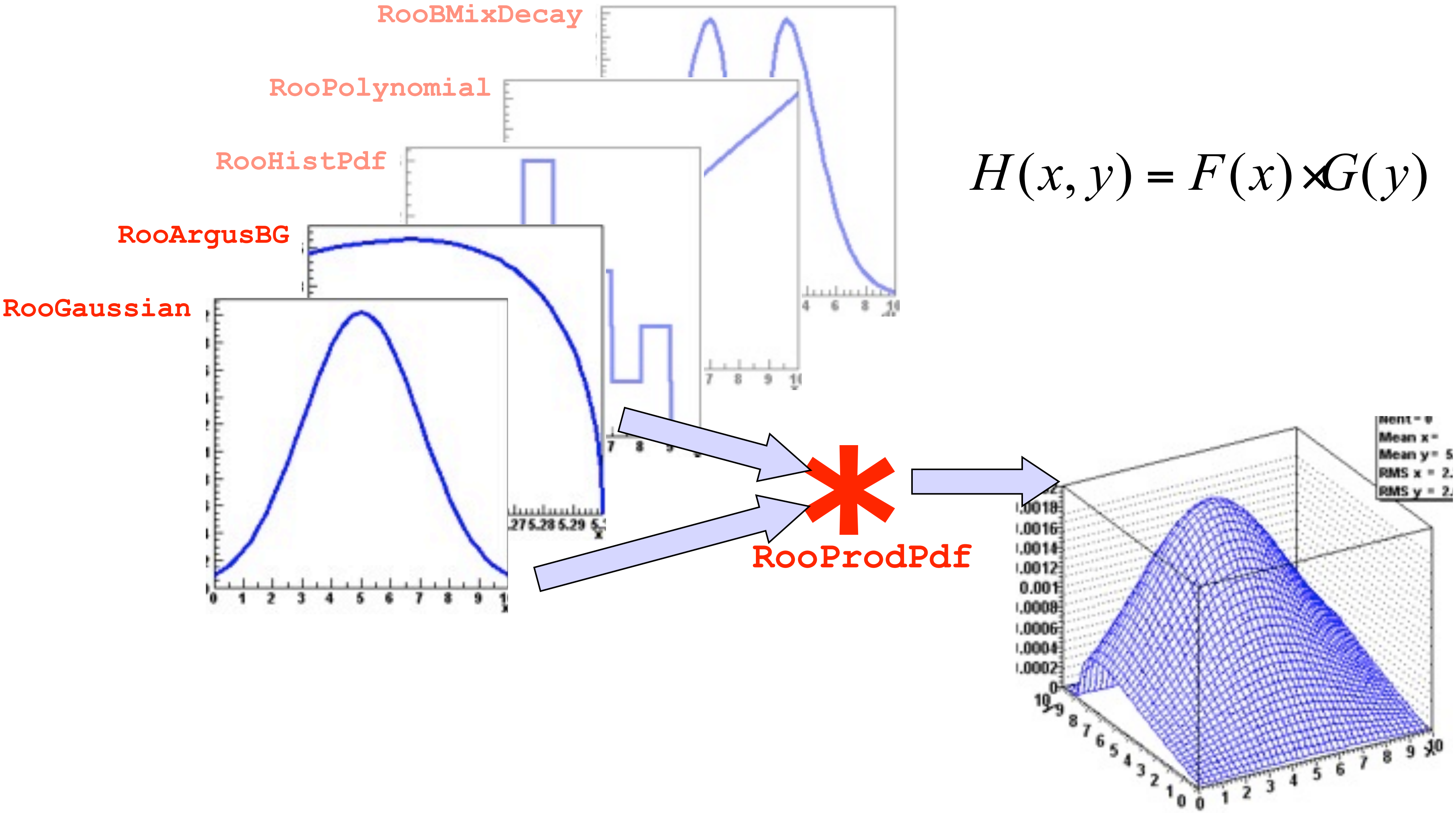
```
  w.factory(“Landau::L(x[-10,30],5,1)”) :
  w.factory(“Gaussian::G(x,0,2)”) ;

  w::x.setBins(“cache”,10000) ; // FFT sampling density
  w.factory(“FCONV::LGf(x,L,G)”) ; // FFT convolution

  w.factory(“NCONV::LGb(x,L,G)”) ; // Numeric convolution
```

- FFT usually best
  - Fast: unbinned ML fit to 10K events take ~5 seconds
  - NB: Requires installation of FFTW package (free, but not default)
  - Beware of cyclical effects (some tools available to mitigate)



Model building – Products of uncorrelated p.d.f.s

$$H(x,y) = F(x) \times G(y)$$



RooBMixDecay

RooPolynomial

RooHistPdf

RooArgusBG

RooGaussian

RooProdPdf

Uncorrelated products – Mathematics and constructors

- Mathematical construction of products of uncorrelated p.d.f.s is straightforward

2D **nD**

$$H(x,y) = F(x) \times G(y) \qquad H(x^{\{i\}}) = \prod_i F^{\{i\}}(x^{\{i\}})$$

  - No explicit normalization required → If input p.d.f.s are unit normalized, product is also unit normalized
  - (Partial) integration and toy MC generation automatically uses factorizing properties of product, e.g. $\int H(x,y)dx \equiv G(y)$ is deduced from structure.

- Corresponding factory operator is PROD

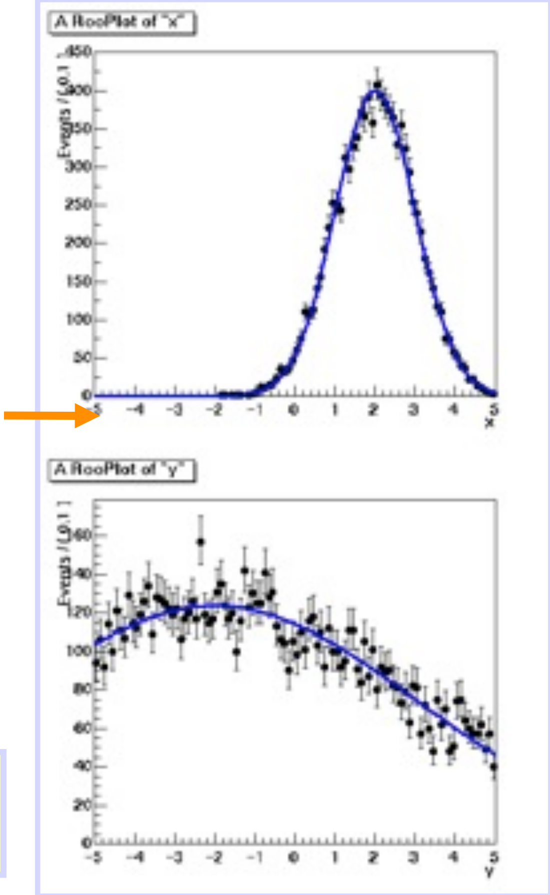
```
w.factory(“Gaussian::gx(x[-5,5],mx[2],sx[1])”) ;
w.factory(“Gaussian::gy(y[-5,5],my[-2],sy[3])”) ;

w.factory(“PROD::gxy(gx,gy)”) ;
```

Plotting multi-dimensional models

- N-D models usually projected on 1-D for visualization
  - Happens automatically.
    RooPlots tracks observables of plotted data,
    subsequent models automatically integrated

```
RooDataSet* dxy =
w::gxy.generate(RooArgSet(w::x,w::y,10000));

RooPlot* frame = w::x.frame() ;
dxy->plotOn(frame) ;
w::gxy.plotOn(frame) ;
```

$$P_{gxy}(x) = \int gxy(x,y)dy$$



  - Projection integrals analytically reduced whenever possible
    (e.g. in case of factorizing pdf)

- To make 2,3D histogram of pdf

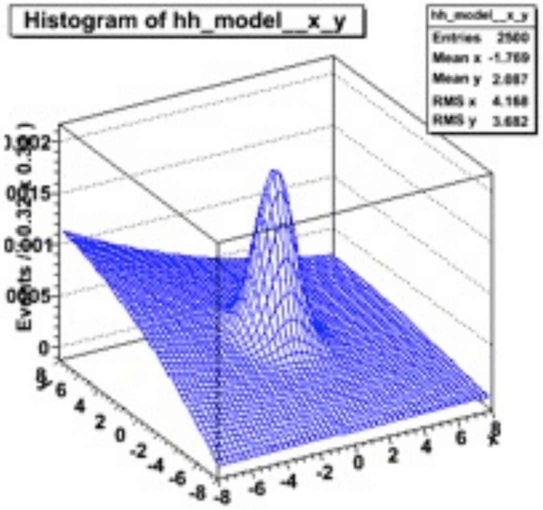
```
TH2* hh = w::gxy.createHistogram(“x,y”,50,50);
```

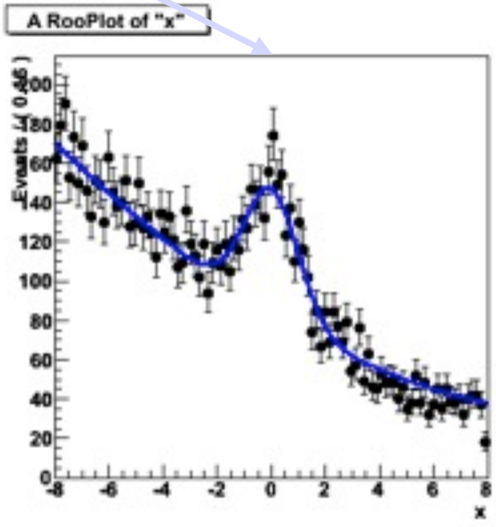
Can also project slices of a multi-dimensional pdf

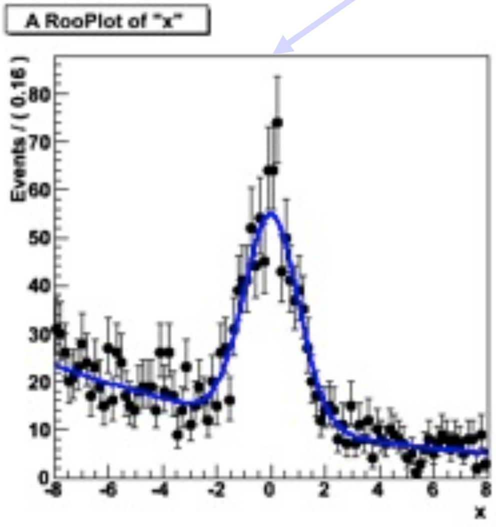
model(x,y) = gauss(x)*gauss(y) + poly(x)*poly(y)

```
RooPlot* xframe = x.frame() ;
data->plotOn(xframe) ;
model.plotOn(xframe) ;
```

```
y.setRange(“sig”,-1,1) ;
RooPlot* xframe2 = x.frame() ;
data->plotOn(xframe2,CutRange("sig")) ;
model.plotOn(xframe2,ProjectionRange("sig")) ;
```







→ *Works also with >2D projections (just specify projection range on all projected observables)*

→ *Works also with multidimensional p.d.fs that have correlations*

Introducing correlations through composition

- RooFit pdf building blocks do not require variables as input, just real-valued functions
  - Can substitute any variable with a function expression in parameters and/or observables

$$f(x;p) \Rightarrow f(x,p(y,q)) = f(x,y;q)$$

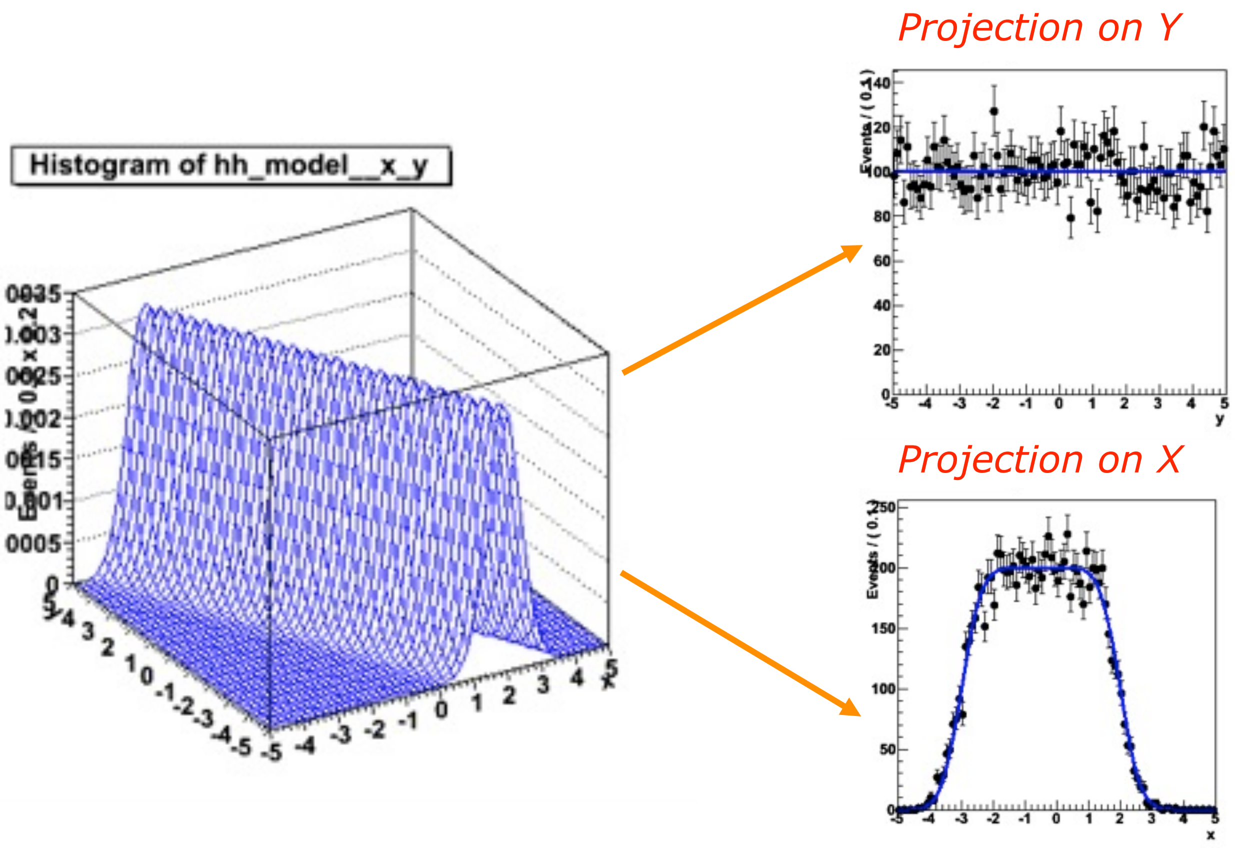
  - Example: Gaussian with shifting mean

```
w.factory(“expr::mean(‘a*y+b’,y[-10,10],a[0.7],b[0.3])”) ;
w.factory(“Gaussian::g(x[-10,10],mean,sigma[3])”) ;
```

  - No assumption made in function on a,b,x,y being observables or parameters, any combination will work

What does the example p.d.f look like?

- Use example model with x,y as observables

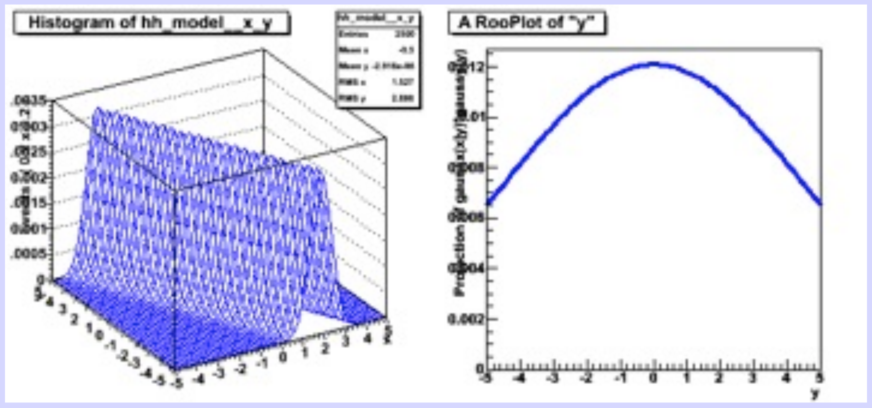


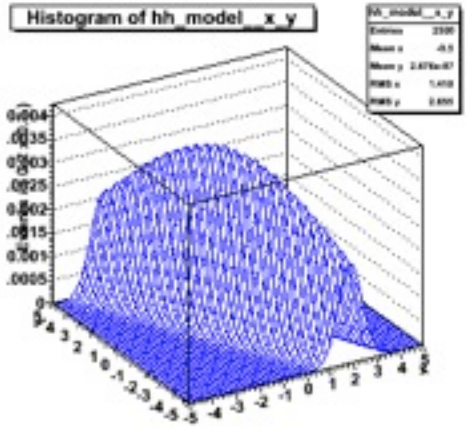
Projection on Y

Projection on X

- Note flat distribution in y. Unlikely to describe data, solutions:
  1. Use as conditional p.d.f g(x|y,a,b)
  2. Use in conditional form multiplied by another pdf in y: g(x|y)*h(y)

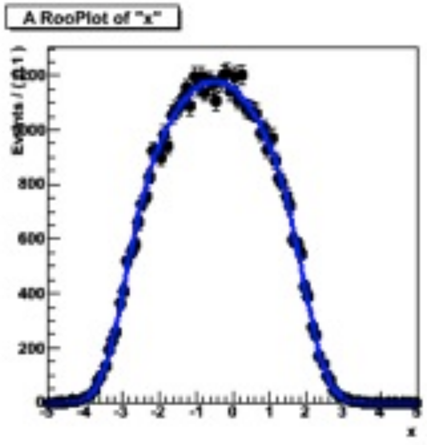
Example with product of conditional and plain p.d.f.





gx(x|y) * gy(y) = model(x,y)

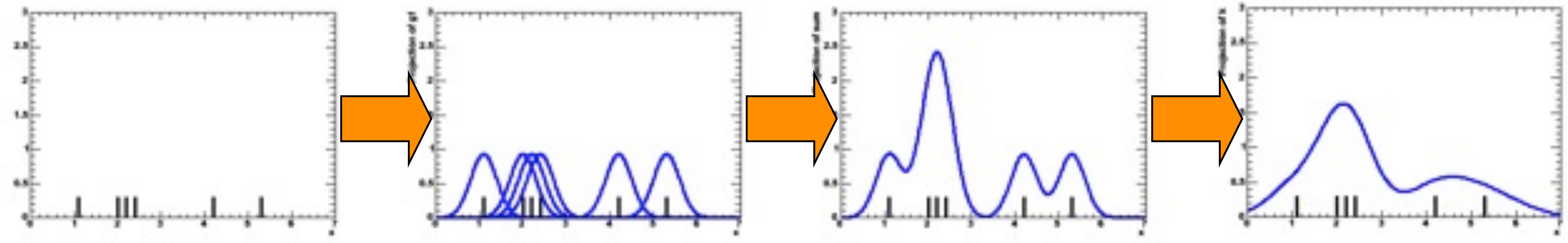
```
// I - Use g as conditional pdf g(x|y)
w::g.fitTo(data,ConditionalObservables(w::y)) ;

// II - Construct product with another pdf in y
w.factory("Gaussian::h(y,0,2)") ;
w.factory("PROD::gxy(g|y,h)") ;
```



$$\int gx(x|y)g(y)dy$$

Special pdfs – Kernel estimation model

- Kernel estimation model
  - Construct smooth pdf from unbinned data, using kernel estimation technique



Sample of events

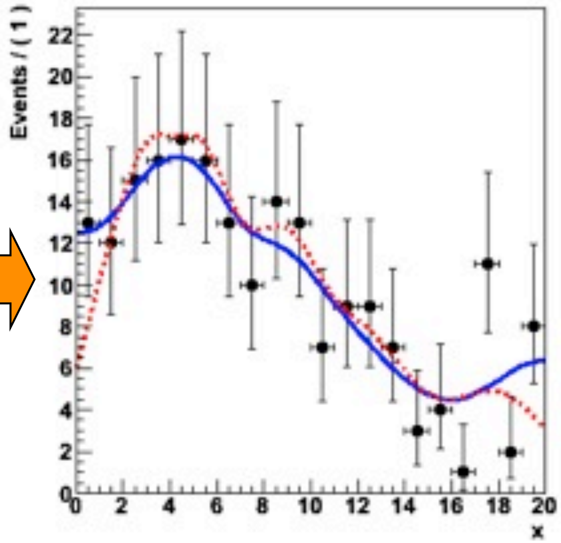
Gaussian pdf for each event

Summed pdf for all events

Adaptive Kernel: width of Gaussian depends on local event density

- Example

```
w.import(myData,Rename(“myData”)) ;
w.factory(“KeysPdf::k(x,myData)”) ;
```

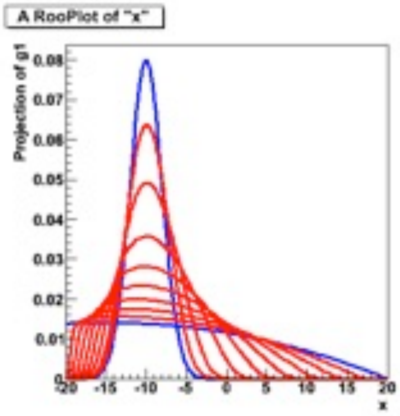


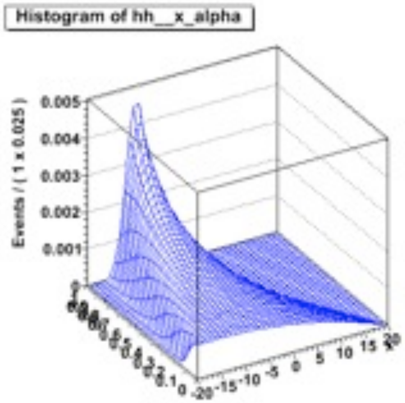
- Also available for n-D data

Special pdfs – Morphing interpolation

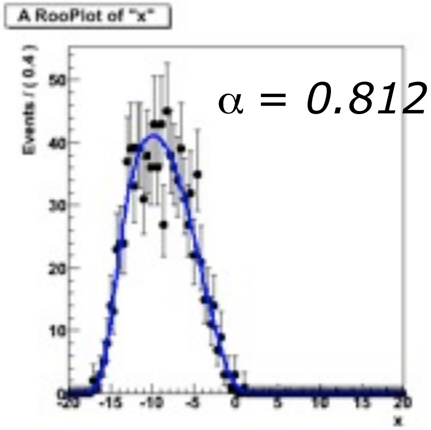
- Special operator pdfs can interpolate existing pdf shapes
  - Ex: interpolation between Gaussian and Polynomial

```
w.factory(“Gaussian::g(x[-20,20],-10,2)”) ;
w.factory(“Polynomial::p(x,{-0.03,-0.001})”) ;
w.factory(“IntegralMorph::gp(g,p,x,alpha[0,1])”) ;
```





Fit to data



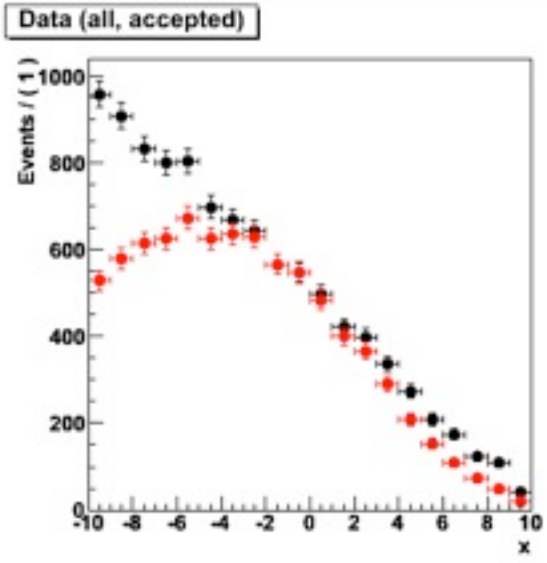
α = *0.812 ± 0.008*

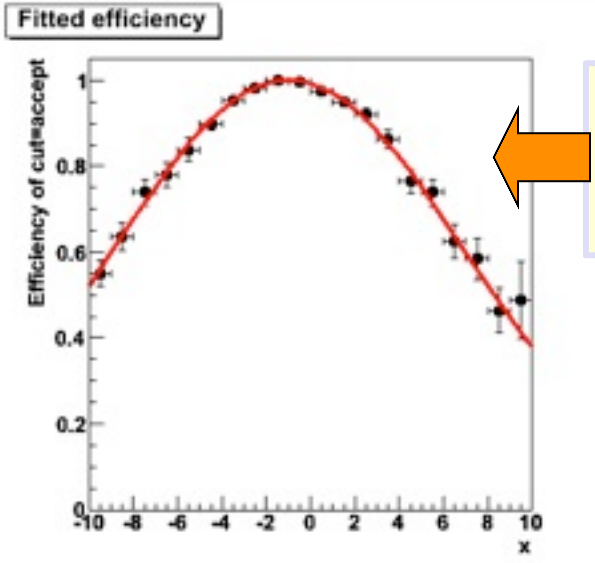
- Two morphing algorithms available
  - IntegralMorph (Alex Read algorithm).
    CPU intensive, but good with discontinuities
  - MomentMorph (Max Baak).
    Fast, can handling multiple observables (and soon multiple interpolation

Special pdfs – Unbinned ML fit for efficiency function

- Binomial pdf
  - Constructs pdf that can estimate *efficiency function e(x)* in from dataset D(x,c) where ‘c’ distinguishes accepted and rejected events

```
w.factory(“expr::e(‘(1-a)+a*cos((x-c)/b)’,x,a,b,c);
w.factory(“Efficiency::model(e,cut[acc,rej],"acc")”) ;

w::model.fitTo(data,ConditionalObservables(w::x)) ;
```


Data (all, accepted)


Fitted efficiency

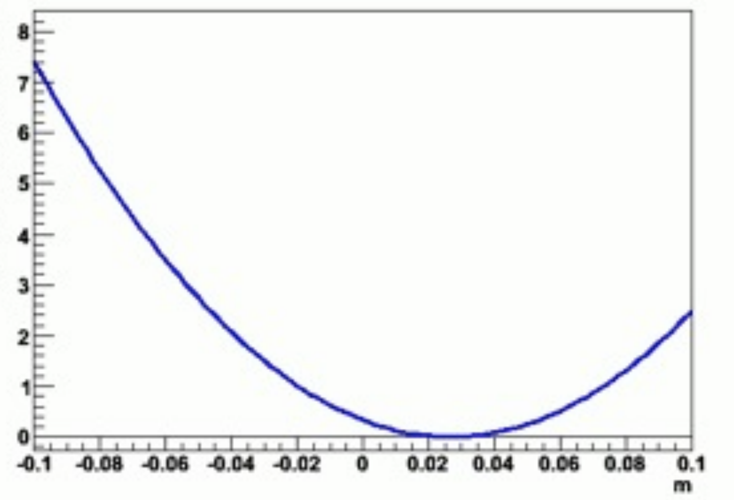
```
RooPlot* frame = w::x.frame() ;
data->plotOn(frame,
             Efficiency(cut)) ;
e.plotOn(frame) ;
```

4 Likelihood & Profile Likelihood

Constructing the likelihood

- So far focus on construction of pdfs, and basic use for fitting and toy event generation
- Can also explicitly construct the likelihood function of and pdf/data combination
  - Can use (plot, integrate) likelihood like any RooFit function object

```
RooAbsReal* nll = w::model.createNLL(data) ;

RooPlot* frame = w::param.frame() ;
nll->plotOn(frame,ShiftToZero()) ;
```



Constructing the likelihood

- Example – Manual MINUIT invocation
  - After each MINUIT command, result of operation are immediately propagated to RooFit variable objects (values and errors)
  - NB: Also other minimizers (Minuit2, GSL etc) supported since 5.24

```
// Create likelihood (calculation parallelized on 8 cores)
RooAbsReal* nll = w::model.createNLL(data,NumCPU(8)) ;

RooMinuit m(*nll) ;   // Create MINUIT session
m.migrad() ;          // Call MIGRAD
m.hesse() ;           // Call HESSE
m.minos(w::param) ;   // Call MINOS for 'param'

RooFitResult* r = m.save() ; // Save status (cov matrix etc)
```

- Can also create χ^2 functions objects

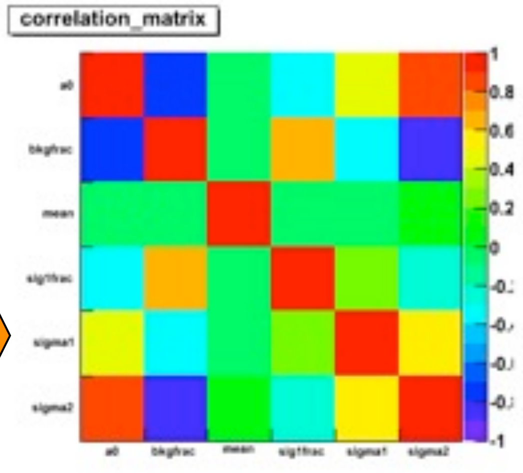
```
RooAbsReal* chi2 = w::model.createChi2(binnedData) ;
RooAbsReal* chi2 = w::model.createXYChi2(xyData) ;
```

Using the fit result output

- The fit result class contains the full MINUIT output

- Easy visualization of correlation matrix

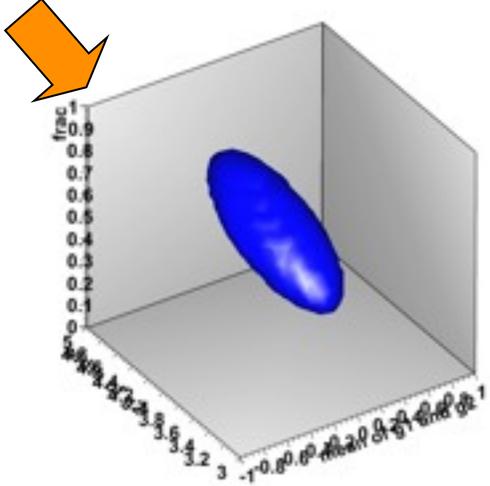
```
fitresult->correlationHist->Draw(“colz”) ;
```



- Construct multi-variate Gaussian pdf representing pdf on parameters

```
RooAbsPdf* paramPdf = fr->createHessePdf(RooArgSet(frac,mean,sigma));
```

  - Returned pdf represents HESSE parabolic approximation of fit



- Extract correlation, covariance matrix

```
TMatrixDSym cov = fr->covarianceMatrix() ;
TMatrixDSym cov = fr->covarianceMatrix(a,b) ;
```

  - Can also retrieve partial matrix (Schur compl.)

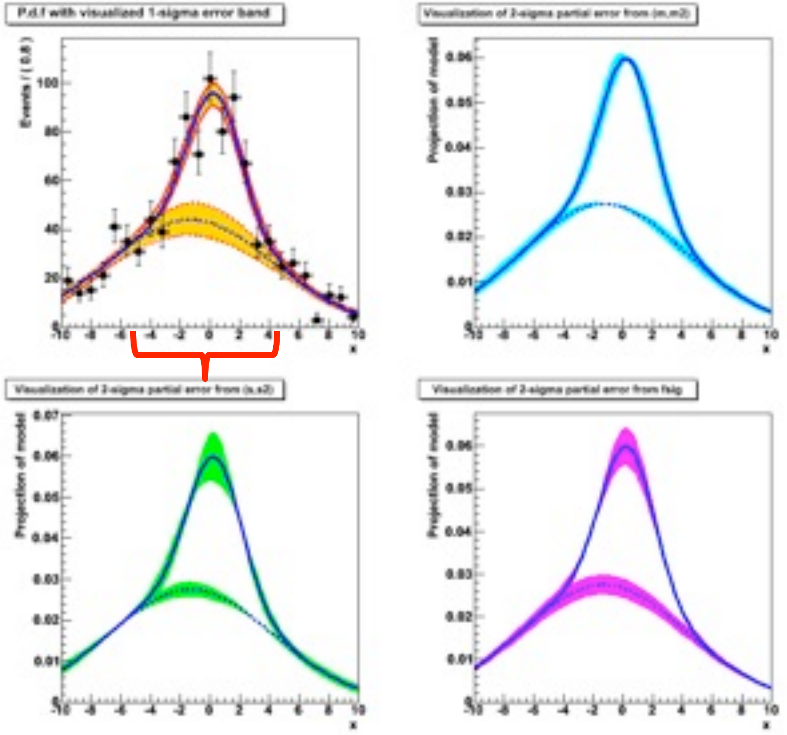
Using the fit result output – Error propagation

- Can (as visual aid) propagate errors in covariance matrix of a fit result to a pdf projection

```
w::model.plotOn(frame,VisualizeError(*fitresult)) ;
w::model.plotOn(frame,VisualizeError(*fitresult,fsig)) ;
```

  - Linear propagation on pdf projection $\Delta = \vec{E} V^{-1} \vec{E}$

- Propagated error can be calculated on arbitrary function
  - E.g fraction of events in signal range



```
RooAbsReal* fracSigRange =
     w::model.createIntegral(x,x,"sig") ;

Double_t err =
    fracSigRange.getPropagatedError(*fr);
```

Adding parameter pdfs to the likelihood

- Systematic/external uncertainties can be modeled with regular RooFit pdf objects.
- To incorporate in likelihood, simply multiply with orig pdf

```
w.factory(“Gaussian::g(x[-10,10],mean[-10,10],sigma[3])”) ;

w.factory(“PROD::gprime(f,Gaussian(mean,1.15,0.30))”) ;
```

$$-\log L(\mu,\sigma) = -\sum_{data} -\log(f(x_i;\mu,\sigma) - \log(Gauss(\mu,1.15,0.30))$$

– Any pdf can be supplied, e.g. a **RooMultiVarGaussian** from a **RooFitResult** (or one you construct yourself)

```
w.import(*fr->createHessePdf(w::mean,w::sigma),”parampdf”) ;
w.factory(“PROD::gprime(f,parampdf)”) ;
```

Working with profile likelihood

- A profile likelihood ratio $$\lambda(p) = \frac{L(p,\hat{\hat{q}})}{L(\hat{p},\hat{q})}$$

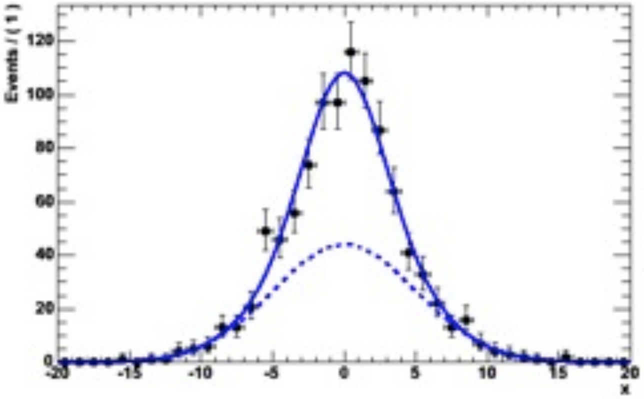
  ← **Best L for given p**

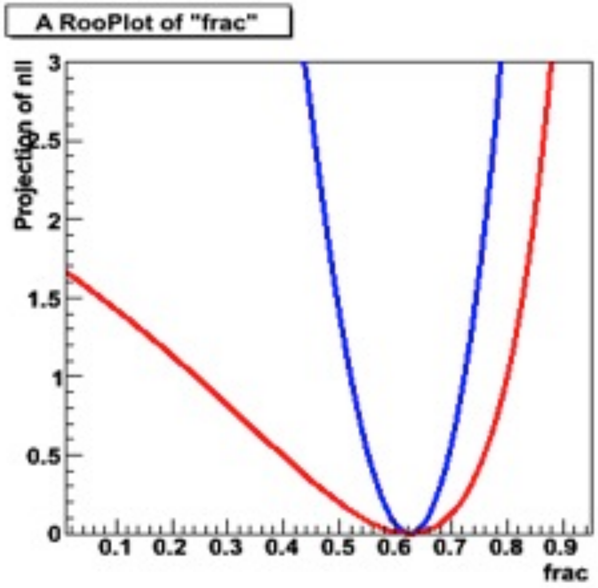
  ← **Best L**

  can be represent by a regular RooFit function (albeit an expensive one to evaluate)

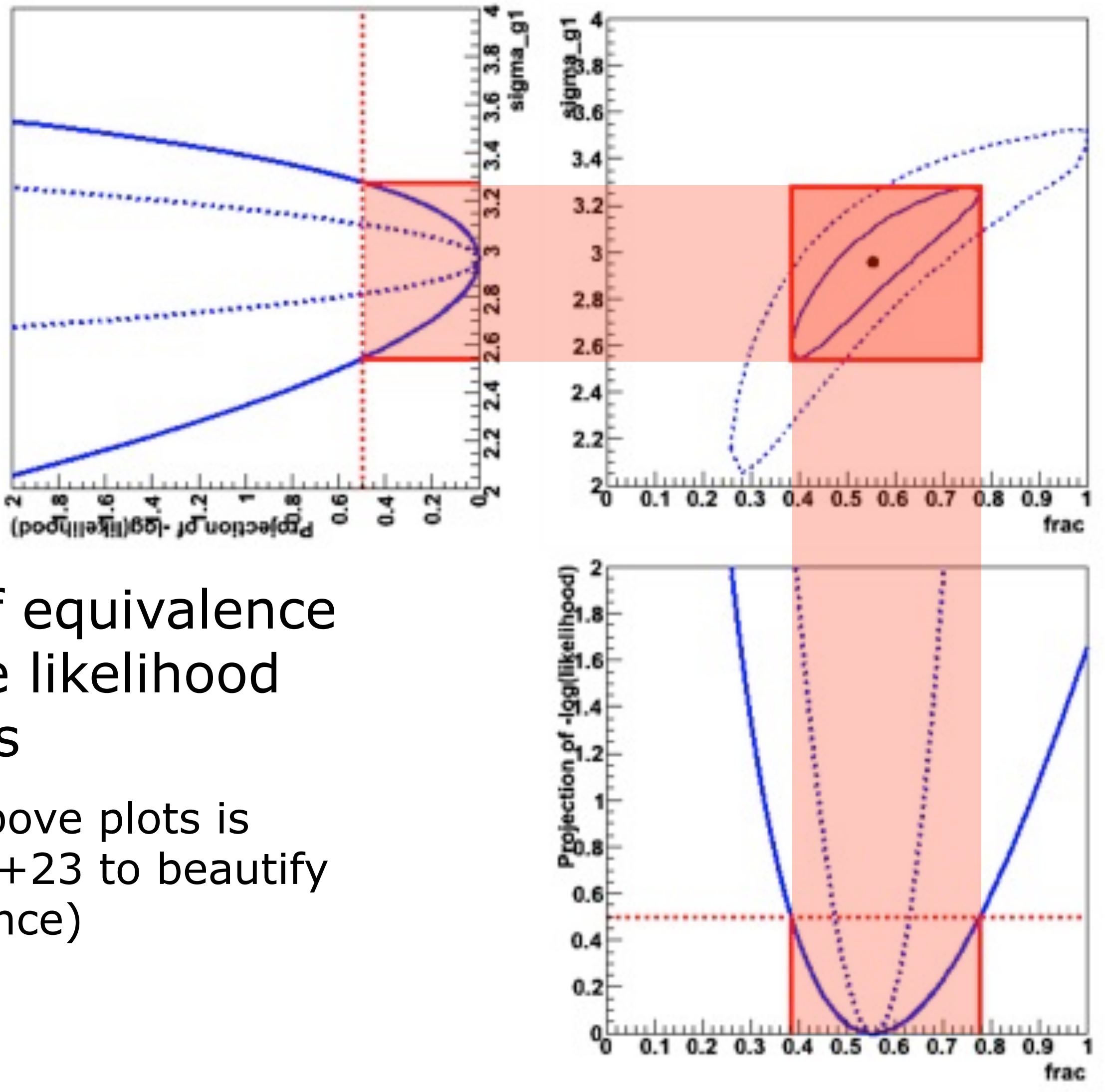
```
RooAbsReal* ll = model.createNLL(data,NumCPU(8)) ;
RooAbsReal* pll = ll->createProfile(params) ;
```

```
RooPlot* frame = w::frac.frame() ;
nll->plotOn(frame,ShiftToZero()) ;
pll->plotOn(frame,LineColor(kRed)) ;
```





On the equivalence of profile likelihood and MINOS



- Demonstration of equivalence of (RooFit) profile likelihood and MINOS errors
  - Macro to make above plots is 34 lines of code (+23 to beautify graphics appearance)

5 Simultaneous fits and combinations

Constructing joint pdfs

- Operator class SIMUL to construct joint models at the pdf level

```
// Pdfs for channels ‘A’ and ‘B’
w.factory(“Gaussian::pdfA(x[-10,10],mean[-10,10],sigma[3])”) ;
w.factory(“Uniform::pdfB(x)”) ;

// Create discrete observable to label channels
w.factory(“index[A,B]”) ;

// Create joint pdf
w.factory(“SIMUL::joint(index,A=pdfA,B=pdfB)”) ;
```

- Can also construct joint datasets

```
RooDataSet *dataA, *dataB ;
RooDataSet dataAB(“dataAB”,”dataAB”,Index(w::index),
                  Import(“A”,*dataA),Import(“B”,*dataB)) ;
```

Constructing joint likelihood

- Can then construct the joint likelihood as usual

```
RooAbsReal* nllJoint = w::joint.createNLL(dataAB) ;
```

- Also possible to make likelihood first and then join

```
RooAbsReal* nllA = w::A.createNLL(*dataA) ; w.import(nllA) ;
RooAbsReal* nllB = w::B.createNLL(*dataB) ; w.import(nllB) ;
w.factory(sum::nllJoint(nllA,nllB)) ;
```

  - But then there is no definition of joint pdf
    and cannot execute frequentist techniques on joint models...

Using joint models

- When constructing joint models and likelihoods:

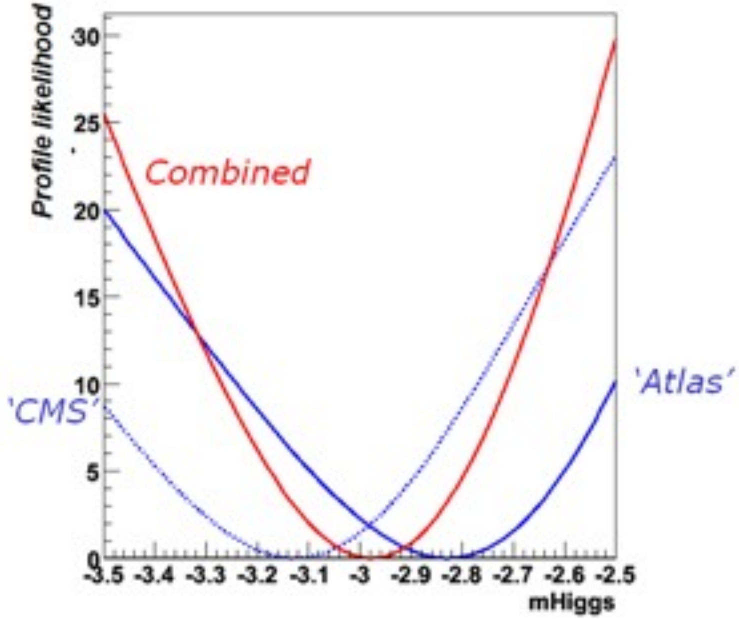
  parameters with the same name = same parameter

- If intentional, you are done at this point.

```
RooAbsReal* pllJoint = nllJoint->createProfile(paramOfInterest) ;
```

- Takes all parameter correlations fully into account
- To add additional correlations, simply multiply joint pdf with appropriate **`RooMultiVarGaussian`** pdf in parameters of choice

```
w.factory("MultiVarGaussian::corr
                ({a,b},{0,0},COV)");
w.factory("PROD::jointc(joint,corr)") ;
```



Tools to aid logistics of building a joint model

- Multiple experiments / analysis groups are unlikely to be organized to an extent where parameter naming schemes match exactly
  - The workspace has tools to manage this
  - These tools are the basis for (future) high level combination tools that will be part of the RooStats project
- Import model from another workspace
  - Example:: rename all variables of import model to unique names by appending a suffix **`_aHZZ`**, and rename mHiggs to **`MH`**

```
w.import(atlasHiggsZZ,
         RenameAllVariablesExcept(“mHiggs”,”aHZZ”),
         RenameVariable(“mHiggs”,”MH”) ;
```

  - Can also import straight from file using **`fileName:wspaceName:objName`** syntax

```
w.importFromFile(“ahzz.root:w:atlasHiggsZZ”,…) ;
```

Summary

- Brief overview of RooFit functionality, tailored to serve as introductory to RooStats
  - Many features were not mentioned here
  - No discussion of how this work internally (optimization, analytical deduction abilities)
  - About 90% of the details were omitted
- Documentation
  - Starting point: http://root.cern.ch/drupal/content /roofit
  - Users manual (134 pages ~ 1 year old)
  - Quick Start Guide (20 pages, recent)
  - Link to 84 tutorial macros (also in $ROOTSYS/tutorials/roofit)
- Support
  - Post your question on 'Stat & Math Forum' of ROOT (root.cern.ch → Forum → Stat & Math tools)
  - I aim for <24h response (but I don't manage every day!)

6 Hands-on exercises

Getting started – ROOT setup

- Start a ROOT 5.25/02 session
  - On lxplus (SLC4) or lx64slc5 (SLC5) choose appropriate line below

```
lxplus> source ~verkerke/public/setup_slc4.csh
lxplus> source ~verkerke/public/setup_slc4.sh
lxplus> source ~verkerke/public/setup_slc5.csh
lxplus> source ~verkerke/public/setup_slc5.sh
```

- Now move to your personal working area
- Load the roofit & roostats libraries

```
root> gSystem->Load(“libRooStats”) ;
```

- If you see a message that RooFit v3.10 is loaded you are (almost) ready to go.
- Import the namespace RooFit in CINT

```
root> using namespace RooFit ;
```

- Recommendation: put the last two lines in your ROOT login script to automate the loading
  - At least for the duration of the tutorial

Getting started – Online reference material

- RooFit class documentation (from code)
  - http://root.cern.ch/root/html/ROOFIT_ROOFITCORE_Index.html
  - http://root.cern.ch/root/html/ROOFIT_ROOFIT_Index.html
- RooFit home page at ROOT web site
  - http://root.cern.ch/drupal/content/roofit
  - Has links to manual and tutorial macros

Exercise 1 – A simple fit

- Copy ~verkerke/public/ex1.C and run it.
  - This macro uses the ‘w::’ shortcut syntax only available in CINT
  - Look at ex1var.C to see the solution written in pure C++
- This macro does the following for you:
  - Creates a workspace “w”, and uses the factory to fill it with a Gaussian g(x,mean,sigma)
  - Generates an unbinned dataset in x with 10K events from the pdf
  - Performs an unbinned ML fit of the pdf to the data
  - Makes a plot of the data with the pdf overlaid
  - Calls the Print() function on the parameter to see that the parameter estimate and its error have been propagated to the variable
- Modify the macro to generate a binned dataset instead of an unbinned dataset and run again
  - Use generateBinned() instead of generate()

Exercise 2 – Making a composite model

- Rename ex1.C to ex2.C

- Add a 2rd order Chebychev pdf to the workspace with coefficients a1=0 and a2=0.1 (each with range [-1,1])
  - See page 21 of presentation for help

- Using the SUM operator create a new pdf model that adds the Gaussian and the Chebychev.
  - Give each component a coefficient (e.g. Nsig and Nbkg) with a range [0,10000]
  - See page 27 of presentation for the syntax of SUM.
  - You can create Nsig and Nbkg in the same command as the SUM constructions following the logic explained on page 12 of the presentation

- Call the Print("t") method on the workspace to see the new contents

- Generate a dataset with 1000 events from model, fit it, and plot the data, model, as well as the background component of model
  - Use the Components() method to specify the background component.
  - If you like you can add LineStyle(kDashed) option
  - *If you get ROOT error messages that 'Components()' is not defined, you have forgotten your 'using namespace RooFit'*

Exercise 2 – Making a composite model (cont’d)

- This part is optional – do it only when you feel you are progressing quickly, otherwise do it when you have completed the other exercises

- Redo the fit, adding a Save() argument to fitTo() and save the returned RooFitResult* pointer
  - See page 17 of presentation for help

- Visualize the correlation matrix from the fit result
  - gStyle->SetPalette(1) ;
  - myFitResult->correlationHist()->Draw(“colz”) ;

- Plot the fitted pdf with the error band defined by the fit result
  - Add a VisualizeError(*myFR) option in RooAbsPdf::plotOn().
  - Do the same for the background component plot
  - NB: You can change the color of the band using e.g. FillColor(kYellow), and have the band placed at the bottom of the draw stack with the additional MoveToBack() command

Demo 1 – FFT convolution of arbitrary pdfs

- Copy ~verkerke/public/fftdemo.C and run it
- This macro demonstrates how the FCONV fourier convolution operator is used to convolute a Landau pdf with a Gaussian resolution model
- A binned likelihood fit of the numerically convoluted pdf with three floating parameters takes ~1 second

Exercise 3 – Persisting your model

- Copy ex2.C to ex3a.C
- At the end of the macro, import the toy data you generated into the workspace as follows
  - w.import(data,Rename("data")) ;
- Write your workspace to file
  - using the method w.writeToFile("model.root").
- Now quit your ROOT session
- Copy ~verkerke/public/ex3b.C.
  - This macro will read in your model.root file and plot the pdf and dataset contained in it
- Look at the macro and run it

Demo 2 – simultaneous fitting

- Copy ~verkerke/public/simfitdemo.C and run it
- This macro demonstrates techniques to make simultaneous fits to a ‘signal’ and ‘control’ samples in multiple ways

1. Plain fit to signal sample with sigPdf+BkgPdf
2. Plain fit to control sample with sigPdf+BkgPdfCtrl
3. Simultaneous fit to signal and control samples
4. Construct a pdf on sigPdf *parameters* from fit 2), multiply with pdf for signal sample.
   - Equivalent to 3) in the approximation of a parabolic likelihood for the control sample

Exercise 4 – Working with the likelihood

- Copy ex3b.C to ex4.C
- Remove the plotting code and add a line to create a function object that represents the –log(likelihood)
  - Use method RooAbsPdf::createNLL(RooAbsData&), the returned object is of type RooAbsReal*
  - See page 41 in the presentation for help
- Minimize the likelihood function 'by hand' by passing it to a RooMinuit object and calling its methods migrad() and hesse()
  - See page page 42 in the presentation for help (also for below)
  - Now call the minos() function only for parameter Nsig.
  - Call w::Nsig.Print() afterwards to see that the asymmetric error has been propagated
  - Fix the width of the Gaussian and run minos again and observe the effect. (use w::sigma.setConstant(kTRUE))
- Make a plot of –log(L) vs Nsig
  - First create a plot frame in the parameter using RooPlot* frame = w::Nsig.frame() ;
  - Now plot the likelihood function on the frame, using plotOn() as usual
  - If you like you can add a ShiftToZero() argument to the plotOn() call and see what that does
  - You can adjust the virtual range of the plot frame with SetMinimum() and

Demo 3 – n-Dim models and likelihood ratio plot

- Copy ~verkerke/public/llrplot.C and run it
- This macro builds a 3-dimensional model
  - Flat background in (x,y,z)
  - Gaussian signal in (x,y,z) with correlations
- It plots three 2D projections (x,y), (x,z) and (y,z)
- Then it makes three varieties of 1D plots of model and data
  - Plain projection on x (shows lots of background)
  - Projection on x in a 'signal box' in (y,z)
  - Projection on x with a cut on the LR(y,z)>68%, where LR(y,z) is defined as

$$LR(y,z) = \frac{\int f_{sig} \times S(x,y,z)dx}{\int S(x,y,z) + B(x,y,z)dx}$$

(i.e. the signal probability according to the model using the (y,z) observables only)

Exercise 5 – Profile likelihood

- Copy ~verkerke/public/ex4.C (standard solution to ex4) to ex5.C
- Adjust the *horizontal* plot range of the likelihood plot so that it just covers the interval ΔLL=+25 units
  - Make a new plot frame that zooms in on that range and plot the likelihood again (you can use myparam.frame(pmin,pmax) to control the plot range)
- Create the *profile* likelihood function in Nsig
  - Call createProfile(w::Nsig) on the likelihood and save the returned pointer to the profile likelihood function (of type RooAbsReal*)
  - Plot the profile likelihood ratio on the Nsig frame too (make it red by adding a LineColor(kRed))
- Find the profile likelihood ratio interval of Nsig : find the points at which the PLR rises by +0.5 units
  - Compare the interval to that of the MINOS error of exercise Ex 4.

Exercise 6 – Parallelizing the likelihood calculation

- Check the number of CPU cores available on the current host (‘cat /proc/cpuinfo’)
- Modify the createNLL() call of ex5 to take an extra NumCPU(N) argument
  - The likelihood calculation will now be parallelized over N cores
- Rerun ex5 and observe the difference in wall-time execution speed.
  - The speedup is best demonstrated on an empty worker node (your best is lx64slc5)