

Simulation of muons energy loss in lithium hydride for the neutrino factory design study

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Abstract

The project consisted of a study of the energy loss by muons passing through LiH with the Geant4 simulation package. Simulations have been done using lithium in different composition of ${}^6\text{Li}$ and ${}^7\text{Li}$ and then were compared with data from the Particle Data Group. The aim of the study was to try to understand the difference in the models used for the simulation of compounds and estimate the difference in energy loss when the properties of the compound are changed. The output from Geant4 simulations has been used as an input for the ROOT data analysis package.

I. Neutrino Physics

The Standard Model of physics describes the universe with the help of twelve particles, six quarks (up, down, top, bottom, charm and strange) and six leptons (electron, electron neutrino ν_e , muon, muon neutrino ν_μ , tau and tau neutrino ν_τ) and three forces, the strong force, the weak force and the electromagnetic force. The carriers of these forces are called bosons. The neutrino is neutral and is undergoing interactions through the electroweak interactions such as radioactive decays or nuclear reactions taking place in the sun. For each particle there is also an antiparticle carrying the opposite charge of the particle.

According to the Standard Model, neutrinos have no mass. But many years of solar, atmospheric, accelerators and reactors based neutrino experiments have demonstrated that neutrinos can change flavour and have a mass, thus challenging the very basic hypothesis of the Standard Model. This change of flavour is called neutrino oscillations e.g. an electron neutrino converting into a muon neutrino and vice versa. With the results of these experiments, we have been able to describe the neutrino flavour eigenstates (ν_e, ν_μ, ν_τ) as a function of the mass eigenstates (ν_1, ν_2, ν_3) and a mixing matrix called PMNS from the physicists who proposed the idea of this description (B. Pontecorvo, Z. Maki, M. Nakagawa, S. Sakata) :

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Where δ = the CP phase violation term, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.
 θ_{ij} is called the mixing angle between states i and j

The probability of measuring a particular flavour for a neutrino varies periodically as it propagates. The probability that a neutrino originally of a given flavour α will later be observed as having flavour β is given by:

$$P_{\alpha \rightarrow \beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right)$$

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

Where Δm_{ij}^2 = square mass difference between two neutrino states i and j

$\delta_{\alpha\beta}$ = Kronecker delta, $\delta_{\alpha\beta} = 1$ if $\alpha = \beta$
 0 if $\alpha \neq \beta$

E = energy of the neutrino

L = length over which the neutrino propagated

As we can see from the probability equation, if the neutrinos have a mass, they can oscillate. Therefore in order to describe completely this phenomenon we need to know precisely the mixing angles θ_{12} , θ_{13} and θ_{23} , two of the squared mass differences Δm_{12}^2 , Δm_{23}^2 and Δm_{32}^2 and the CP violation phase term δ . Experiments have been able to measure Δm_{21}^2 and $|\Delta m_{31}^2|$ and the mixing angles θ_{12} and θ_{23} . We do not know yet the sign of Δm_{31}^2 , θ_{13} and the phase violation term δ . To be able to make such precise measurements an intense neutrino beam is required. One of the proposed schemes is the neutrino factory [1].

II. The Neutrino Factory

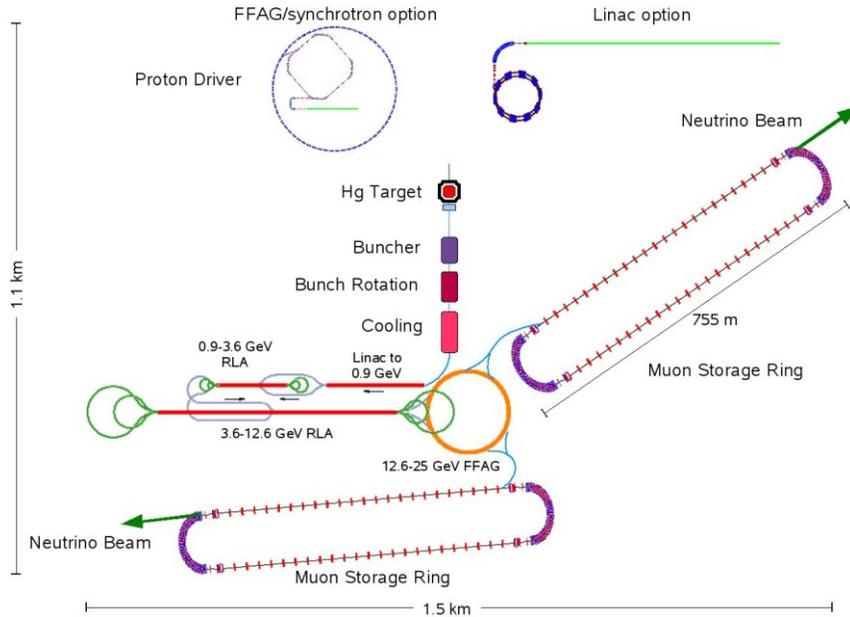


Figure 1: A schematic diagram of the proposed neutrino factory

It is a facility which will generate an intense beam of neutrinos coming from muon decay and be able to send 10^{21} neutrinos per year (1 accelerator year accounting for 10^7 s) to distant detectors located several kilometres away to observe if the composition of the beam has changed and hence study neutrino oscillations.

A neutrino factory (see figure 1) is composed of a proton driver which provides an intense 4MW beam of 5-15 GeV protons. The protons impinge on a Hg jet target to produce pions. Pions are first captured by a magnetic field before they enter drift region of the muon front end, pions decay to give muons in the drift region. Muons are then bunched, rotated and cooled by the process of ionization cooling in the front end. After cooling the muons are accelerated to higher energies using Recirculating Linear Accelerator (RLA's) and Fixed Field Alternating Gradient (FFAG), until they reach a final energy of 25 GeV. Two decay rings are also present to provide an intense beam on neutrinos by the decay of high energy muons, which are then sent to the far away detectors.

a) Ionization cooling

The muon transverse *emittance* is too large for the subsequent accelerating structures, and it needs to be reduced by a process known as ionization cooling. The principle is as follows: particles are passed through liquid hydrogen (H_2) or lithium hydride (LiH) absorbers and lose momentum in all directions as they ionize the material. Now to maintain the longitudinal momentum, we use RF cavities along with the absorber material, these RF cavities increase the longitudinal momentum for the same amount that has been reduced during ionization, compensating for the energy losses in the LiH absorbers, thus providing a beam with the reduced transverse momentum and the emittance. To demonstrate ionization cooling of muons for the first time, an experiment is

under construction at Rutherford Appleton Laboratory (RAL) and is known as the Muon Ionization Cooling Experiment (MICE) [2].

III. Energy loss simulations

a) Motivation behind the simulations

As the process of muon ionization cooling necessitates a delicate interplay between optimizing absorber length and RF cavity voltage for the muon momentum transformation, different absorber materials are under study. Simulations were done using LiH of different compositions.

b) Lithium hydride properties:

Lithium hydride is a salt-like crystalline compound with a face-centred cubic structure. It comes in nature as a mix of ${}^6\text{Li}$ (7.5%) and ${}^7\text{Li}$ (92.5%). Its fabrication and cost depends on the isotope used and purity composition [3]. We have simulated in Geant4 [4], dE/dx using LiH made either of 100% ${}^6\text{Li}$ or 100% ${}^7\text{Li}$ in order to compare the mean dE/dx and see if there is any difference.

element	atomic mass (gmol^{-1})	percentage (%)	LiH density (g/cm^3)
${}^6\text{Li}$	6.015121	100	0.685
${}^7\text{Li}$	7.016003	100	0.782
Li	6.941	7.5% ${}^6\text{Li}$ + 92.5% ${}^7\text{Li}$	0.82

c) Simulation results

Figure 2 shows a scatter plot of 1000 muons, each having momentum of 1000 MeV/c passing through a LiH box of dimensions 8 m.

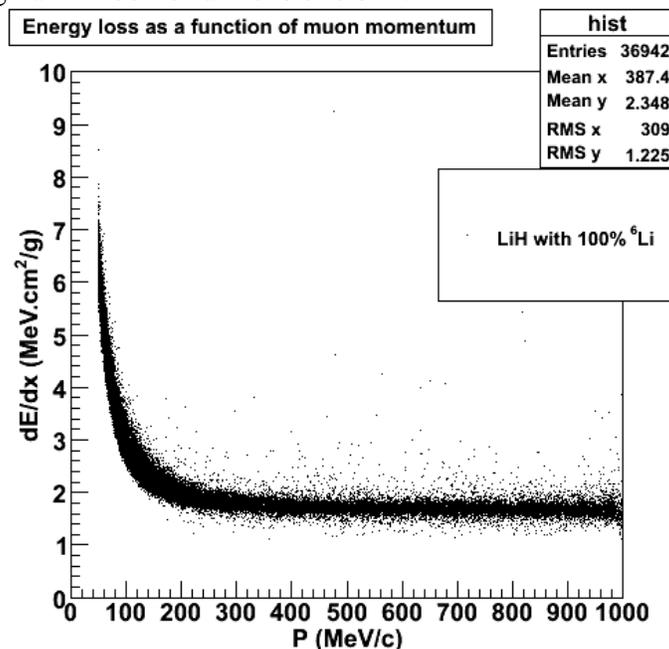


Figure 2: Scatter plot of 1000 muons in LiH made of 100% ${}^6\text{Li}$

Then in order to derive the mean $\langle dE/dx \rangle$ for further comparison with the Particle Data Group (PDG) [5] data, a code in ROOT [6] was written to form a loop which sums all the values of $\langle dE/dx \rangle$ over a range of momentum and then calculate the average.

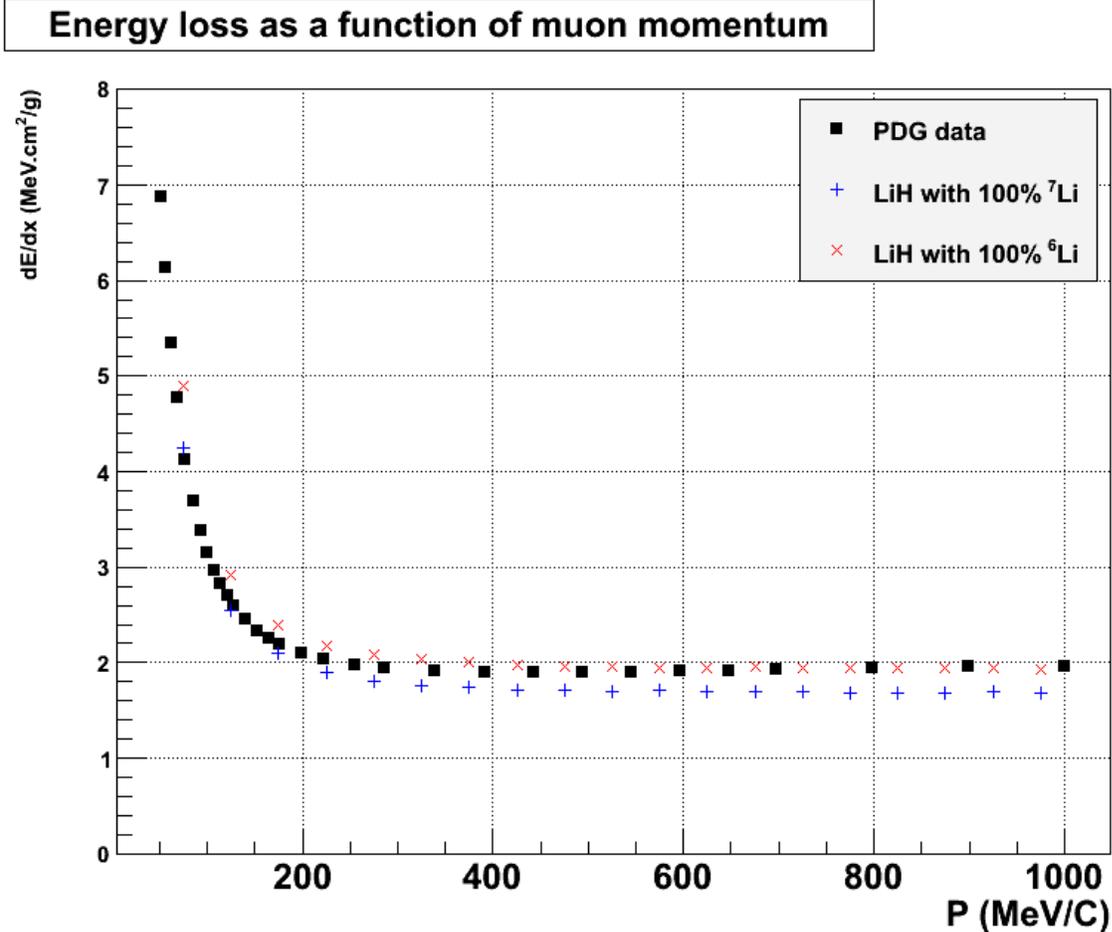


Figure 3: comparison of GEANT4 simulation results with the PDG data

IV. Result and Discussion

From Figure 4, it is quite evident that the energy loss by muons in LiH is different for different compositions of LiH. For all the momentum ranges, the difference in $\langle dE/dx \rangle$ between LiH with 100% ⁶Li and LiH with 100% ⁷Li was calculated and the maximum difference found was ~16%. A calculation of the errors on the $\langle dE/dx \rangle$ is underway in order to see if the difference seen between ⁷Li and ⁶Li is within the errors bars or not. To our surprise, it is a bit contradictory that the $\langle dE/dx \rangle$ values of LiH with ⁶Li are closer to the PDG data values, because the density of LiH used in PDG data is near the density of LiH with ⁷Li. So, it was expected that LiH with 100% of ⁷Li would give $\langle dE/dx \rangle$ values closer to the PDG data values.

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