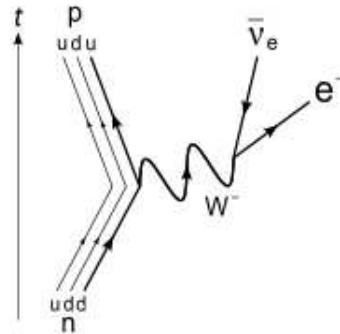


4. The electroweak sector of the Standard Model.

4.1. Gauge group and matter content.

Standard model = "unified" description of weak and electromagnetic interactions. From the Fermi theory of weak interactions



with $G_F/\sqrt{2} = g^2/8M_w^2$, we know that we need at least a charged gauge boson W_m^\pm and the photon A_m .

Experimentally there also exists **neutral currents** (neutral massive gauge boson) and **coloured strong interactions** → gauge group

Gauge bosons : G_m^A A_m^a B_m

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y .$$

- the **Higgs mechanism** generates the breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$.
- Only **left-handed** quarks/leptons interact with $SU(2)_L$ gauge fields. The SM lagrangian has the symbolic form

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{kin}} - V(\Phi) + \mathcal{L}_{\text{Yuk.}}$$

where

$$\begin{aligned}\mathcal{L}_{\text{kin}} = & -\frac{1}{4}F_{mn}^2 - \frac{1}{4}(F_{mn}^a)^2 + |D_m\Phi|^2 \\ & + \bar{\Psi}_L i\gamma^m D_m \Psi_L + \bar{\Psi}_R i\gamma^m D_m \Psi_R ,\end{aligned}$$

where

$$D_m \Psi_L = (\partial_m - ig \frac{\tau_a}{2} A_m - ig' \frac{Y_L}{2} B_m) \Psi_L$$

$$D_m \Psi_R = (\partial_m - ig' \frac{Y_R}{2} B_m) \Psi_R ,$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 ,$$

and $\mathcal{L}_{\text{Yuk.}}$ will be discussed later on. With our conventions

$$Q = T_3 + \frac{Y}{2} . \tag{52}$$

Matter content of the Standard Model:

$$\text{Leptons : } l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L : (\mathbf{1}, \mathbf{2})_{Y=-1} \quad , \quad e_{iR} : (\mathbf{1}, \mathbf{1})_{Y=-2}$$

$$\text{Quarks : } q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L : (\mathbf{3}, \mathbf{2})_{Y=1/3}$$
$$u_{iR} : (\mathbf{3}, \mathbf{1})_{Y=4/3} \quad d_{iR} : (\mathbf{3}, \mathbf{1})_{Y=-2/3}$$

$$\text{Higgs field : } \Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} : (\mathbf{1}, \mathbf{2})_{Y=1}$$

Standard Model of FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model".

FERMIONS

Leptons spin = 1/2		
Flavor	Mass GeV/c ²	Electric charge
ν_e electron neutrino	$<1 \times 10^{-8}$	0
e^- electron	0.000511	-1
ν_μ muon neutrino	<0.0002	0
μ^- muon	0.106	-1
ν_τ tau neutrino	<0.02	0
τ^- tau	1.7771	-1

Spin is the intrinsic angular momentum of particles. Spin is given in units of \hbar , which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58 \times 10^{-25}$ GeV s = 1.05×10^{-34} J s.

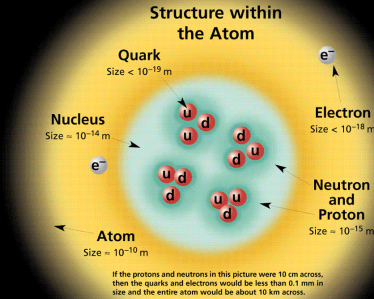
Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c² (remember $E = mc^2$), where $1 \text{ GeV} = 10^9 \text{ eV} = 1.60 \times 10^{-10}$ joule. The mass of the proton is $0.938 \text{ GeV}/c^2 = 1.67 \times 10^{-27} \text{ kg}$.

matter constituents

spin = 1/2, 3/2, 5/2, ...

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c ²	Electric charge
u up	0.003	2/3
d down	0.006	-1/3
c charm	1.3	2/3
s strange	0.1	-1/3
t top	175	2/3
b bottom	4.3	-1/3



BOSONS

Unified Electroweak bosons	
Name	Mass GeV/c ²
γ photon	0
W^-	80.4
W^+	80.4
Z^0	91.187

Color-charged particles interact by exchanging gluons. Colorless particles interact by exchanging photons and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons. They are confined in **hadrons**. This confinement (binding) is due to the exchange of color-charged constituents. As only colorless particles can exist, only colorless combinations of quarks and antiquarks (see figure) can exist. These are the particles we observe: **mesons** (quark-antiquark pairs) and **baryons** (three quarks).

Residual Strong Interaction
The strong binding of color-neutral particles is due to the strong interaction between the quarks. This interaction is viewed as the exchange of mesons.

PROPERTIES OF THE INTERACTIONS

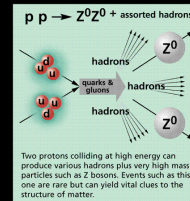
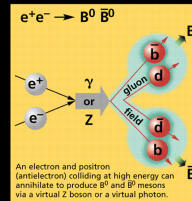
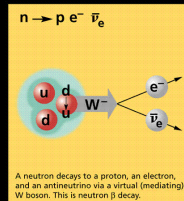
Property	Interaction		Weak (Electroweak)		Electromagnetic		Strong	
	Gravitational		Flavor		Electric Charge		Fundamental	
Acts on:	Mass – Energy		Flavor		Electric Charge		Color Charge	
Particles experiencing:	All		Quarks, Leptons		Electrically charged		Quarks, Gluons	
Particles mediating:	Graviton (not yet observed)		W^+ W^- Z^0		γ		Gluons	
Strength relative to electromagnetism for two u quarks at:	10^{-41}		0.8		1		25	
for two protons in nucleus	10^{-41}		10^{-4}		1		60	
	10^{-36}		10^{-7}		1		Not applicable to hadrons	

Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0 , γ , and η_c and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures

These diagrams are an artist's conception of physical processes. They are not exact and have no meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.



The Particle Adventure
Visit the award-winning website at
<http://ParticleAdventures.org>

This chart has been prepared by the U.S. Department of Energy, U.S. National Science Foundation, Lawrence Berkeley National Laboratory, Stanford Linear Accelerator Center, and the American Physical Society.

©2000 Content of this chart is the property of the Particle Adventure project. For more information, contact the Particle Adventure project at <http://CPEP.org>

4.2. Weak mixing angles and gauge boson masses.

With an $SO(4)$ rotation, the Higgs vev can be written as

$$\Phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \text{ where } v^2 = \frac{\mu^2}{\lambda} \simeq (246 \text{ GeV})^2 \quad (\text{exp.}) \quad (53)$$

Gauge boson **masses** arise from

$$\begin{aligned} |D_m \Phi|^2 &\rightarrow \frac{g^2 v^2}{8} |A_m^{(1)} - i A_m^{(2)}|^2 + \frac{v^2}{8} |g A_m^{(3)} - g' B_m|^2 \\ &= \frac{g^2 v^2}{4} W^{+,m} W_m^- + \frac{(g^2 + g'^2) v^2}{8} Z^m Z_m \end{aligned}$$

where the definitions and masses of gauge bosons are

$$\begin{aligned}
W_m^\pm &= \frac{1}{\sqrt{2}}(A_m^{(1)} \mp iA_m^{(2)}) \quad , \quad M_w = \frac{gv}{2} \\
Z_m &= \frac{gA_m^{(3)} - g'B_m}{\sqrt{g^2 + g'^2}} \quad , \quad M_z = \frac{v}{2}\sqrt{g^2 + g'^2} \\
A_m &= \frac{g'A_m^{(3)} + gB_m}{\sqrt{g^2 + g'^2}} \quad , \quad M_A = 0
\end{aligned} \tag{54}$$

We introduce the electroweak angle

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{M_w}{M_z} \quad , \quad \tan \theta_w = \frac{g'}{g} \tag{55}$$

that rotates from the weak basis to the mass basis

$$\begin{pmatrix} Z_m \\ A_m \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} A_m^{(3)} \\ B_m \end{pmatrix} \tag{56}$$

Notice that

$$\rho \equiv \frac{M_w^2}{M_z^2 \cos^2 \theta_w} = 1 \text{ in the SM} \quad (57)$$

The electric charge is $e = g \sin \theta_w$.

4.3. Neutral and charged currents

Ex: With the definitions above, show that

$$\begin{aligned} D_m = \partial_m - ig A_m^a \frac{\tau_a}{2} - ig' \frac{Y}{2} B_m = \partial_m - ie Q A_m \\ - \frac{ig}{2\sqrt{2}} (W_m^+ \tau_+ + W_m^- \tau_-) - \frac{ig}{\cos \theta_w} Z_m (T_3 - \sin^2 \theta_w Q) \end{aligned}$$

We define the currents by

$$\mathcal{L} = \bar{\Psi}_i i \gamma^m \partial_m \Psi_i + g (W_m^+ J_W^{m,+} + W_m^- J_W^{m,-} + Z_m J_Z^m) + e A_m J_{em}^m \quad (58)$$

Ex : Using the quantum numbers of the quarks/leptons,
show that

$$\begin{aligned}
J_W^{m,+} &= \frac{1}{\sqrt{2}}(\bar{\nu}_L^i \gamma^m e_L^i + \bar{u}_L^i \gamma^m d_L^i) , \\
J_W^{m,-} &= \frac{1}{\sqrt{2}}(\bar{e}_L^i \gamma^m \nu_L^i + \bar{d}_L^i \gamma^m u_L^i) , \\
J_{em}^m &= -\bar{e}^i \gamma^m e^i + \frac{2}{3}\bar{u}^i \gamma^m u^i - \frac{1}{3}\bar{d}^i \gamma^m d^i , \\
J_Z^m &= \frac{1}{\cos \theta_w} \left[\frac{1}{2}\bar{\nu}_L^i \gamma^m \nu_L^i + \left(-\frac{1}{2} + \sin^2 \theta_w\right)\bar{e}_L^i \gamma^m e_L^i + \sin^2 \theta_w \bar{e}_R^i \gamma^m e_R^i \right. \\
&\quad + \left(\frac{1}{2} - \frac{2}{3}\sin^2 \theta_w\right)\bar{u}_L^i \gamma^m u_L^i - \frac{2}{3}\sin^2 \theta_w \bar{u}_R^i \gamma^m u_R^i \\
&\quad \left. - \left(-\frac{1}{2} + \frac{1}{3}\sin^2 \theta_w\right)\bar{d}_L^i \gamma^m d_L^i + \frac{1}{3}\sin^2 \theta_w \bar{d}_R^i \gamma^m d_R^i \right] . \tag{59}
\end{aligned}$$

4.4. Fermion masses and the CKM matrix

Yukawas generate quarks/lepton masses after EWSB :

$$-\mathcal{L}_{\text{mass}} = m_{ij}^u \bar{u}_L^i u_R^j + m_{ij}^d \bar{d}_L^i d_R^j + m_{ij}^l \bar{e}_L^i e_R^j + \text{c.c.} , \quad (60)$$

where $m_{ij}^{u,d,l} = h_{ij}^{u,d,l} v / \sqrt{2}$. We use for compactness a matrix notation

$$-\mathcal{L}_{\text{mass}} = \bar{u}_L m^u u_R + \bar{d}_L m^d d_R + \bar{e}_L m^l e_R + \text{c.c.} . \quad (61)$$

Obs: No neutrino masses here, see lectures of B. Kayser.

We can define the mass eigenstate basis (as compared to the weak eigenstate basis) with the help of the 3×3 unitary transformations

$$u_{L,R} = V_{L,R}^u u'_{L,R} , \quad d_{L,R} = V_{L,R}^d d'_{L,R} , \quad e_{L,R} = V_{L,R}^e e'_{L,R} ,$$

such that

$$(V_L^u)^\dagger m^u V_R^u = \text{diag} (m_u, m_c, m_t) , \text{ etc}$$

In the mass basis, the charged and e.m. currents remain the same, whereas the hadronic charged current becomes

$$(J_W^{m,+})_{\text{quarks}} \rightarrow \frac{1}{\sqrt{2}} \bar{u}'_L \gamma^m V_{CKM} d'_L \equiv \frac{1}{\sqrt{2}} \bar{u}'_L \gamma^m \tilde{d}_L$$

where $V_{CKM} = (V_L^u)^\dagger V_L^d$ is the (unitary) [CKM matrix, 1973](#).

We also defined

$$\tilde{d}_L = V_{CKM} d'_L \leftrightarrow \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}$$

There are therefore **flavor changing transitions** in the SM : $s \rightarrow uW^-$, etc. Experimental measurements give a **hierarchical form** of V_{CKM} of the type (Wolfenstein parametrization)

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (62)$$

where $\lambda = \sin \theta_c \simeq .0.22$ is the Cabibbo angle.

Cabibbo wrote first in 1962 the 2×2 version of the CKM matrix

$$\begin{pmatrix} \sin \theta_c & \cos \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \quad (63)$$

- V_{CKM} contain three rotation angles and a CP violating phase.

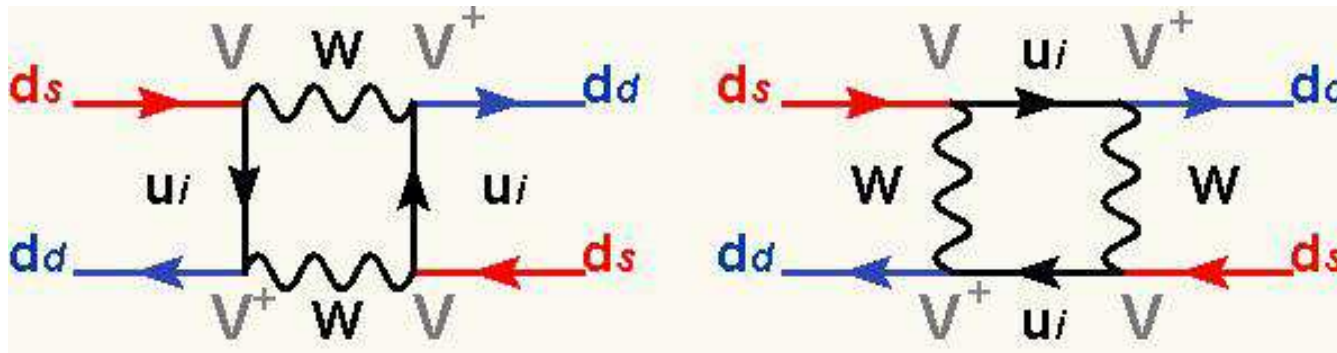
The unitarity of the CKM matrix

$$V_{ik}V_{jk}^* = \delta_{ij} \quad , \quad V_{ki}^*V_{kj} = \delta_{ij}$$

has various important consequences. One of them is the GIM mechanism (Glashow-Iliopoulos-Maiani, 1972).

The GIM mechanism

The FCNC (flavor changing neutral currents) effects were measure to be small. This was puzzling in the 1970's, but it is explained in the SM. Consider for ex. the $K^0 - \bar{K}^0$ mixing, which can arise at the loop-level :



In the limit of equal or vanishing quark masses, the amplitude vanishes due to the unitarity of V_{CKM} :

$$A_{K^0\bar{K}^0} \sim \frac{g^4}{M_W^4} (\sum_i V_{id}^* V_{is}) (\sum_i V_{ss} V_{jd}^*) = 0 \quad (64)$$

The main contribution turn out to be proportional to $(m_c^2 - m_u^2)^2/M_W^4$ and is in excellent agreement with the experimental result.

HR : In 1972, only the u, d and s quarks were known. The GIM mechanism is considered to be the first proof of the existence of the charm quark.

Ex: Write down explicitly the diagrams for the $K^0 - \bar{K}^0$ mixing in the two generation case, with u and c quarks in the loop.

4.5. The custodial symmetry.

(Sikivie, Susskind, Voloshin, Zakharov, 1980)

The tree-level relation $\rho = M_W^2 / (M_Z^2 \cos^2 \theta_w) = 1$ is the result of an (approximate) symmetry.

In any theory of electroweak interactions which conserves the electric charge and has an approximate global $SU(2)$ symmetry under which A_m^a transform as a triplet, $\rho = 1$ at tree-level.

Approximate : in the limit of $g' = 0$ and in the absence of the Yukawa couplings.

Proof: The gauge boson mass matrix is then of the form

$$\begin{pmatrix} M^2 & 0 & 0 & 0 \\ 0 & M^2 & 0 & 0 \\ 0 & 0 & M^2 & m_1^2 \\ 0 & 0 & m_1^2 & m_2^2 \end{pmatrix} \quad (65)$$

No photon charge $\rightarrow M^2 m_2^2 - m_1^4 = 0$. The $W_3 - A$ mass matrix is then of the form : **Exercice :**

$$\begin{pmatrix} M_W^2 & \pm M_W \sqrt{M_Z^2 - M_W^2} \\ \pm M_W \sqrt{M_Z^2 - M_W^2} & M_Z^2 - M_W^2 \end{pmatrix} \quad (66)$$

It is then easy to check that $M_W = \cos \theta_w M_Z$.

The Higgs potential $V(\Phi^\dagger \Phi)$ is invariant under an $SO(4)$ symmetry. Indeed,

$$\Phi = \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix}, \quad \Phi^\dagger \Phi = \sum_{i=1}^4 \Phi_i^2 \quad \rightarrow$$

$SO(4) = SU(2)_L \times SU(2)_R$ symmetry. The Higgs vev

$$\Phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ breaks } SO(4) \rightarrow SO(3) = SU(2)_D$$

Other Higgs representations ? Exercice :

Consider Higgs triplets. Show that the Higgs vev generate the breaking $SO(3) \rightarrow SO(2)$. In this case there is no custodial symmetry and $\rho \neq 1$.

A useful parametrization :

$$\mathcal{H} = \begin{pmatrix} i\tau_2 \Phi^* & \Phi \end{pmatrix} = \begin{pmatrix} \Phi_0^* & \Phi_+ \\ -\Phi_+^* & \Phi_0 \end{pmatrix}, \quad \Phi^\dagger \Phi = \text{Tr} \mathcal{H}^\dagger \mathcal{H}$$

$V(\Phi^\dagger \Phi)$ is invariant under $\mathcal{H} \rightarrow U_L \mathcal{H} U_R^\dagger$, with $U_{L,R}$ unitary matrices implementing $SU(2)_L \times SU(2)_R$ transformations. Symmetry breaking

$$\langle \mathcal{H} \rangle = \frac{v}{\sqrt{2}} I_{2 \times 2} \quad \text{breaks} \quad SU(2)_L \times SU(2)_R \rightarrow SU(2)_D$$

$U(1)_Y$ and Yukawas **break** the custodial symmetry. However

$$\mathcal{L}_{\text{Yuk}} = h \begin{pmatrix} \bar{t}_L & \bar{b}_L \end{pmatrix} \mathcal{H} \begin{pmatrix} t_R \\ b_R \end{pmatrix}$$

is invariant under $SU(2)_D$ (if $h_t = h_b$).

A one-loop computation in the SM gives

$$\delta\rho = \frac{3g^2(m_t^2 - m_b^2)}{64\pi^2 M_W^2} - \frac{3g^2}{32\pi^2} \ln \frac{m_H}{M_Z} + \dots$$

where \dots are subleading contributions from the SM (or eventual new physics contributions, see lectures B. Dobrescu) are smaller than 10^{-3} .

5. QUANTUM CORRECTIONS AND RENORMALIZATION.

5.1. UV divergences and regularization.

Perturbation theory in QFT is plagued with **UV divergences**. We have to keep an UV cutoff Λ in computing physical quantities. There are three cases that arise :

- **Super-renormalizable theories** : only a finite number of Feynman diagrams diverge.
- **Renormalizable theories** : a finite number of amplitudes diverge. Divergences at all orders in pert. theory.
- **Non-renormalizable theories** : All amplitudes are divergent at a certain order in perturbation theory.

- In (super)renormalizable theories, UV divergences can be absorbed into rescaling of fields and redefinitions of the various couplings and masses. Taking the couplings/masses from experience, the UV cutoff disappears from physical quantities → the theory is predictive at any energy scale.
- In non-renormalizable theories, we need an infinite number of couplings and masses in order to absorb UV divergences. We would need an infinite amount of experimental data to determine all these couplings → at high-energies $E > \Lambda$ the theory loses its predictive power. At low-energy the theory is perfectly predictive.

- 5.2. Relevant, marginal and irrelevant couplings

Consider a scalar theory of the form

$$S_\Lambda = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 + \frac{m^2\phi^2}{2} + \sum_n \lambda_n \phi^n \right), \quad (67)$$

where S_Λ is the euclidian action defined with a cutoff Λ . The couplings λ_n have (classical) mass dimensions $[\lambda_n] = 4 - n$. Let us consider the theory with two different maximal euclidian momenta/cutoffs:

i) $0 < p < \Lambda$

ii) $0 < p < \Lambda' = \epsilon \Lambda$, where $\epsilon < 1$.

The theory ii) has therefore a **lower cutoff**.

It is interpreted as a theory where the high-momenta of theory i) were **integrated out**. The theory i) has the action (67). In the theory ii) the cutoff can be redefined to be the same as in i) with the help of a **scale transformation**

$$x' = \epsilon x \quad , \quad p' = \epsilon^{-1} p \quad , \quad \phi' = \epsilon^{-1} \phi \quad (68)$$

In terms of the rescaled field and coordinates, the action of theory ii) become

$$S_{\Lambda'} = \int d^4 x' \left(\frac{1}{2} (\partial' \phi')^2 + \frac{m'^2 (\phi')^2}{2} + \sum_n \lambda'_n (\phi')^n \right) , \quad (69)$$

where

$$m'^2 = \frac{1}{\epsilon^2} m^2 \quad , \quad \lambda'_n = \epsilon^{n-4} \lambda_n \quad (70)$$

Notice that the new mass and couplings scale with their classical dimension. We see therefore that the mass and couplings with positive dimension **grow** in the IR, whereas couplings with negative dimension **decrease** in the IR. It is said that

$[\lambda_n] > 0 \quad \rightarrow \quad \text{relevant coupling}$

$[\lambda_n] = 0 \quad \rightarrow \quad \text{marginal coupling}$

$[\lambda_n] < 0 \quad \rightarrow \quad \text{irrelevant coupling}$

5.3. (Non)renormalizability and couplings dims.

There is a straight connection between renormalizability and the three type of couplings above:

- relevant couplings \rightarrow super-renormalizability.
- marginal couplings \rightarrow renormalizability.
- irrelevant couplings \rightarrow non-renormalizability.

It is easy to argue for this by **dimensional arguments**.

Take some simple examples.

a) - Relevant coupling

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2\phi^2}{2} - \lambda_3\phi^3. \quad (71)$$

The coupling has dimension $[\lambda_3] = +1$, so it is relevant.

At one-loop, the UV divergent terms lead to (Ex:)

$$\delta\mathcal{L}_1 \sim \lambda_3 \Lambda^2 \phi + \lambda_3^2 \phi^2 \ln \Lambda ,$$

which are both of super-renormalizable type. The first lead to mass renormalization, whereas the second leads to a scalar tadpole.

At two loops, the only UV divergences are a cosmological constant and a scalar tadpole. At three loops, there is only a log UV divergence in the cosmological constant. No UV divergences exist at higher loops.

Dim. argument : The highest UV divergent term in

the coupling is the three-loop vacuum energy

$$\lambda_3^4 \ln \Lambda \quad (72)$$

Higher loops have higher powers in λ_3 and cannot contribute to the UV divergent terms in the effective lagrangian

Obs: $1/m^2$ terms are IR, not UV contributions).

b) - Irrelevant coupling

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2\phi^2}{2} - \lambda_6\phi^6. \quad (73)$$

The coupling has dimension $[\lambda_6] = -2$, so it is irrelevant. At one-loop, the UV divergent terms in the

eight-point amplitude lead to (Ex:)

$$\Gamma_{1\text{-loop}}^{(8)}(p_i) \sim c \lambda_6^2 \ln \Lambda + \dots .$$

To cancel this divergence, one has to add a new coupling to the original action

$$\delta \mathcal{L}_1 \sim \lambda_8 \phi^8 ,$$

and to adjust the coupling λ_8 such that

$$\lambda_8 + c \lambda_6^2 \ln \Lambda = \text{finite}$$

At two-loops, we get new **new UV divergences**, like the one in the six-point amplitude, prop. to

$$\Gamma_{2\text{-loops}}^{(6)}(p_i) \sim c' (p_i p_j) \lambda_6^2 \ln \Lambda ,$$

which can be canceled by adding **another coupling**

$$\delta\mathcal{L}_2 \sim \lambda'_8 \phi^4 (\partial\phi)^2 ,$$

such that

$$\lambda'_8 + c' \lambda_6^2 \ln \Lambda = \text{finite}$$

The UV divergences **proliferate** at higher loop orders, generating an infinite tower of operators of higher and higher dimension.

Dimensional argument: Terms of the type $\lambda_6^n \phi^{4+2n} \ln \Lambda$, $\lambda_6^n (\partial\phi)^2 \phi^{2n} \ln \Lambda$ have the correct dimension to be generated for any n . Predictivity at high-energy is **lost**.

- However, let us define $\lambda_6 \sim 1/M^2$. Then :

In the IR $E < M$, the effect of non-renormalizable operators on physical quantities is prop. to some power of E/M and/or m/M , so their effects is negligible.

Effective theories with cutoff Λ (ex. General relativity, $\Lambda = M_P$) are predictive at energies $E \ll \Lambda$.

Another viewpoint: for $\mathcal{L}_{\text{int}} = \sum_n \lambda_n \phi^n$, leading cross-section for $2 \rightarrow 2$ particle scattering is

$$\sigma = \sum_n c_n \lambda_n^2 E^{2n-4} \sim \frac{1}{E^2} \sum_n c_n \left(\frac{E}{M}\right)^{2n}$$

for $\lambda_n \sim 1/M^{n-4} \rightarrow$ **predictive power lost** for $E \geq M$.

Ex : Coupling renormalization for ϕ^4 theory.

Consider the ϕ^4 theory

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4$$

and compute the four-point function at one-loop

$$\Gamma(k_1 k_2 k_3 k_4) = -i\lambda_0 + \frac{(-i\lambda_0)^2}{2} \times \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m_0^2} \frac{i}{(p - k_1 - k_2)^2 - m_0^2} + \text{two crossing terms}$$

After the Wick rotation to euclidian momenta

$$\Gamma(k_1 k_2 k_3 k_4) = -i\lambda_0 + \frac{i\lambda_0^2}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_0^2} \frac{1}{(p - k_1 - k_2)^2 + m_0^2} + \text{two crossing terms}$$

The integral is log divergent in the UV. There are various ways to "renormalize" the integral. Here is a simple way : Define

$$\begin{aligned} V(s) &\equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m_0^2} \frac{1}{(p - k_1 - k_2)^2 + m_0^2} \\ &= \int_{p^2 \geq \mu^2}^{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2} + \text{finite} , \end{aligned}$$

where the energy scale μ is arbitrary. We find

$$\Gamma(k_1 k_2 k_3 k_4) = -i\lambda_0 + \frac{3i\lambda_0^2}{16\pi^2} \ln \frac{\Lambda}{\mu} + \text{finite} = -i\lambda(\mu) + \text{finite}$$

What is the **physical interpretation** of this manipulation?

i) λ_0 is not a physical parameter. It can be chosen to depend on Λ such that

$$\lambda(\mu) = \lambda_0(\Lambda) - \frac{3\lambda_0^2}{16\pi^2} \ln \frac{\Lambda}{\mu}$$

is independent of Λ .

ii) Any value of μ leads to the same physical result. λ_0 is independent of μ ? Therefore

$$\frac{d\lambda}{d\ln \mu} = \frac{3\lambda^2}{16\pi^2} = \beta(\lambda) \quad (74)$$

describes the [renormalization group equation \(RGE\)](#) of λ at one-loop. (74) is then a differential eq., whose solution is

$$\lambda(\mu) = \lambda(\mu_0) + \frac{\lambda(\mu_0)}{1 - \frac{3\lambda(\mu_0)}{16\pi^2} \ln \frac{\mu}{\mu_0}}$$

There is an equivalent prescription : add a local "counterterm" to the lagrangian

$$\mathcal{L} + \delta\mathcal{L} = \mathcal{L}_0 ,$$

which cancels the UV divergence.

In renormalizable theories, a **finite number** of counterterms are needed in order to render the theory UV finite. In non-renormalizable theories, an **infinite number** of counterterms are needed.

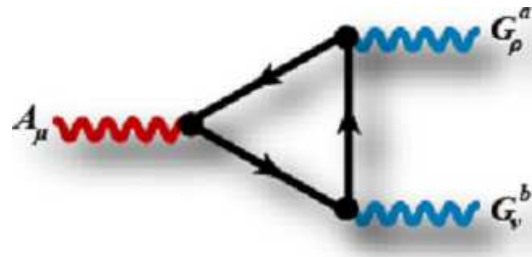
5.4. Quantum anomalies*

Symmetries of the classical action can have **anomalies** at the quantum level. They are generated by one-loop **triangle diagrams**.

- For global symmetries, this does not create problems.

Ex: $\pi^0 \rightarrow \gamma\gamma$ is related to the axial $U(1)_A$ anomaly.

- For gauge symmetries, if present, they generate inconsistencies.



The corresponding currents are of chiral type

$$J_m^A = \bar{\Psi} \gamma_m \gamma_5 T^a \Psi = \bar{\Psi}_R \gamma_m T^a \Psi_R - \bar{\Psi}_L \gamma_m T^a \Psi_L$$

The resulting gauge anomaly that has to vanish is

$$A^{ABC} = \text{tr} [\{T^A, T^B\} T^C]_L - \text{tr} [\{T^A, T^B\} T^C]_R = 0 ,$$

where the trace is taken over **all the fermions**. For the SM, the only possible anomalies are (**To check:**) $SU(2)_L^2 U(1)_Y$, $U(1)_Y^3$ and $SU(3)_c^2 U(1)_Y$. The results in the SM are

$$\begin{aligned}
tr \left[\left\{ \frac{\tau^a}{2}, \frac{\tau^b}{2} \right\} Y \right]_L &= \frac{1}{2} \delta^{ab} (tr Y)_L = 3 \times \left(N_c \times \frac{1}{3} - 1 \right) = 0 , \\
tr \left[\{Y, Y\} Y \right]_{L-R} &= \dots = 6(-2N_c + 6) = 0 \\
tr \left[\left\{ \frac{\lambda^A}{2}, \frac{\lambda^B}{2} \right\} Y \right]_{L-R} &= \frac{1}{3} \delta^{AB} (tr Y)_{L-R} = \dots = 0
\end{aligned}$$

- Anomaly cancelation happens **precisely for three colors** $N_c = 3$!
- Anomaly cancelation provides a deep connection between quarks and leptons in the SM, maybe a hint towards Grand Unified Theories ? (Bogdan lectures ?)

6. The Higgs / Symmetry breaking sector of the Standard Model.

6.1.1 Perturbativity bounds

The RGE for the [Higgs self-coupling](#) in the SM is

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 24\lambda^2 - (3g'^2 + 3g^2 - 12h_t^2) \lambda + \frac{3}{8}(g'^4 + 2g^2g'^2 + 3g^4) - 6h_t^4 + \dots,$$

where \dots denote smaller Yukawas. In the [large Higgs mass limit](#) $\lambda \gg g^2, h_t^2$, it reduces to

$$\frac{d\lambda}{\lambda^2} = \frac{3}{2\pi^2} d\ln\mu \rightarrow \frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \ln \frac{\Lambda}{\mu}.$$

This can be interpreted in two alternative ways :

i) If the **Higgs mass is known**, SM has a Landau pole (non-pert. regime) $\lambda(\Lambda) \gg 1$ for

$$\Lambda = v e^{\frac{2\pi^2}{3\lambda}} = v e^{\frac{4\pi^2 v^2}{3M_h^2}}$$

ii) Conversely, asking for **perturbativity** up to scale Λ (say M_{GUT}), we obtain an **upper bound** on the Higgs mass

$$M_h^2 \leq \frac{4\pi^2 v^2}{3 \ln \frac{\Lambda}{v}} .$$

6.1.2 Stability bounds

SM has another instability in the **small Higgs mass limit**, since λ can **become negative** at high-energy.

If $\lambda \ll h_t^2$, the leading RGE's are

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = -6h_t^4, \quad 16\pi^2 \frac{dh_t}{d\ln\mu} = \frac{9h_t^3}{2}$$

which integrates to

$$\lambda(\mu) = \lambda(\Lambda) + \frac{\frac{3h_t^4(\Lambda)}{8\pi^2} \ln \frac{\Lambda}{\mu}}{1 + \frac{9h_t^2(\Lambda)}{16\pi^2} \ln \frac{\Lambda}{\mu}},$$
$$h_t^2(\mu) = \frac{h_t^2(\Lambda)}{1 + \frac{9h_t^2(\Lambda)}{16\pi^2} \ln \frac{\Lambda}{\mu}}.$$

This can be interpreted in two ways :

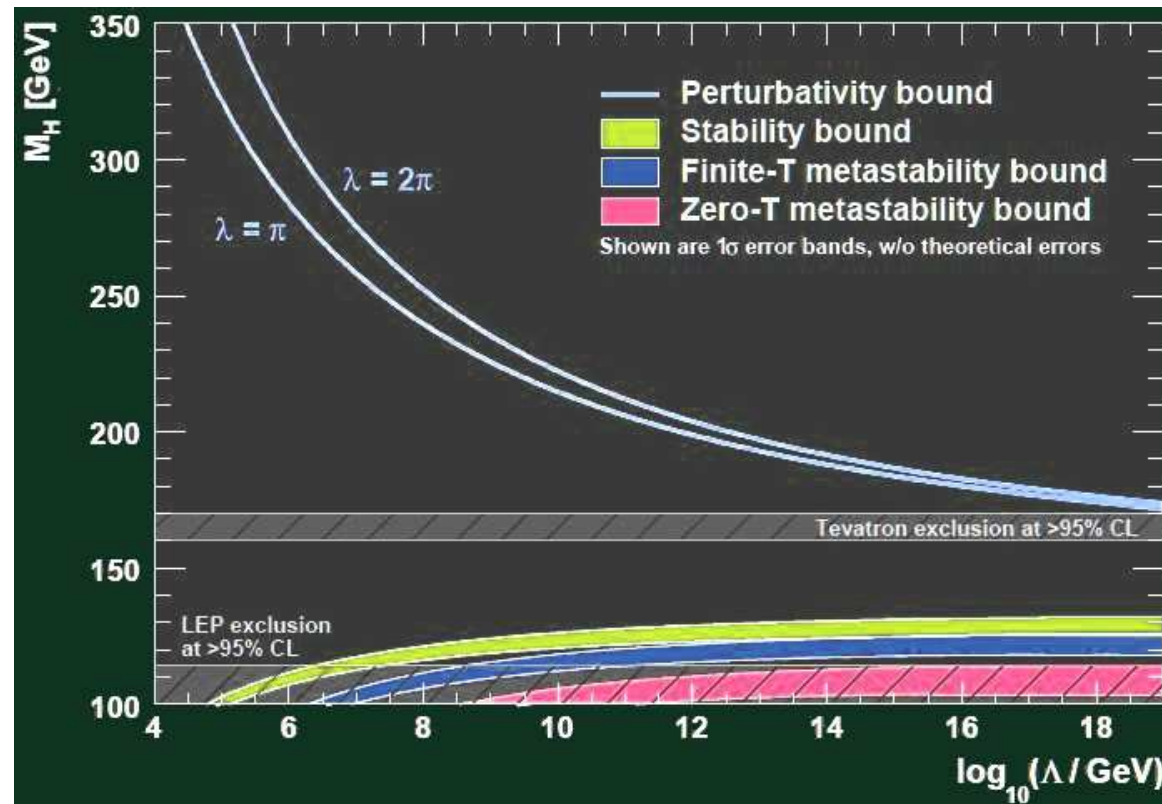
i) For a fixed, **known value of the Higgs mass** : take $\mu = v$. Then, new physics should show up before the scale Λ where $\lambda(\Lambda) = 0$

$$\Lambda \leq v e^{\frac{8\pi^2\lambda}{3h_t^4}} = v e^{\frac{4\pi^2 M_h^2}{3h_t^4 v^2}}$$

ii) For a **fixed Λ** , we get a **lower bound** on the Higgs mass

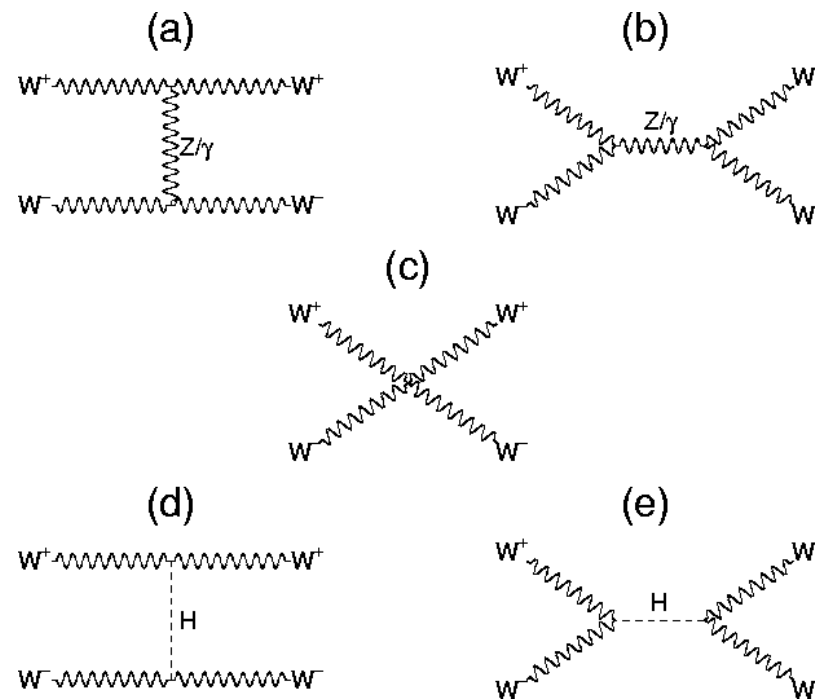
$$M_h^2 \geq \frac{3h_t^4 v^2}{4\pi^2} \ln \frac{\Lambda}{v} = \frac{3m_t^4}{\pi^2 v^2} \ln \frac{\Lambda}{v}$$

These theoretical **Higgs mass limits** are summarized in the following plot



- **6.2. $W W$ scattering and unitarity.**

Let us consider the longitudinal $W_L W_L \rightarrow W_L W_L$ scattering



For a massive gauge particle of momentum k and mass M_W , $A_m = \epsilon_m e^{ikx}$, the three polarizations satisfy $\epsilon_m \epsilon^m = -1$, $k_m \epsilon^m = 0$. For $k^m = (E, 0, 0, k)$, they are

$$\begin{aligned} \text{transverse} & : \epsilon_1^m = (0, 1, 0, 0) \quad , \quad \epsilon_2^m = (0, 0, 1, 0) \quad , \\ \text{longitudinal} & : \epsilon_L^m = \left(\frac{k}{M_W}, 0, 0, \frac{E}{M_W} \right) \sim \frac{k^m}{M} + \mathcal{O}\left(\frac{E}{M_W}\right) . \end{aligned}$$

Since the longitudinal polarization is proportional to the energy, we expect a tree-level amplitude behaving as

$$\mathcal{A} = \mathcal{A}^{(4)} \left(\frac{E}{M_W} \right)^4 + \mathcal{A}^{(2)} \left(\frac{E}{M_W} \right)^2 + \dots$$

Actually, the diagrams a), b) and c) give $\mathcal{A} = g^2 \left(\frac{E}{M_W} \right)^2$.

On the other hand, **unitarity constrains the amplitude** to stay small enough at any energy.

Start with the unitarity of the S-matrix $S^\dagger S = 1$. Then

$$S = 1 + i\mathcal{A} \quad \rightarrow \quad i(\mathcal{A} - \mathcal{A}^\dagger) + \mathcal{A}^\dagger \mathcal{A} = 0$$

Let us sandwich this eq. between a two-particle state $|i\rangle$:

$$i(\mathcal{A} - \mathcal{A}^\dagger)_{ii} + \sum_f |\mathcal{A}_{fi}|^2 = 0 \quad (75)$$

which is the **optical theorem** : the imaginary part of the forward amplitude of the process $i \rightarrow i$ is proportional to the total cross section of $i \rightarrow$ anything.

Let us decompose the scattering amplitude into partial waves

$$\mathcal{A} = \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) a_l ,$$

where a_l are partial wave amplitudes of elastic scattering of two particles. Projecting (75) into the partial wave l gives $Im\ a_l = |a_l|^2$. This is only possible if

$$|Re\ a_l| \leq 1/2 , \ 0 \leq Im\ a_l \leq 1 \quad \rightarrow \quad |a_l|^2 \leq 5/4 ,$$

which is the unitarity bound we were searching for.

- For the SM without the Higgs boson

$$a_0 = \frac{g^2 E^2}{M_W^2} \quad \rightarrow \quad \text{unitarity breaks down for } \sqrt{s} \sim 1.2\ TeV$$

With the Higgs boson, amplitudes d),e) cancel the raising energy term, such that

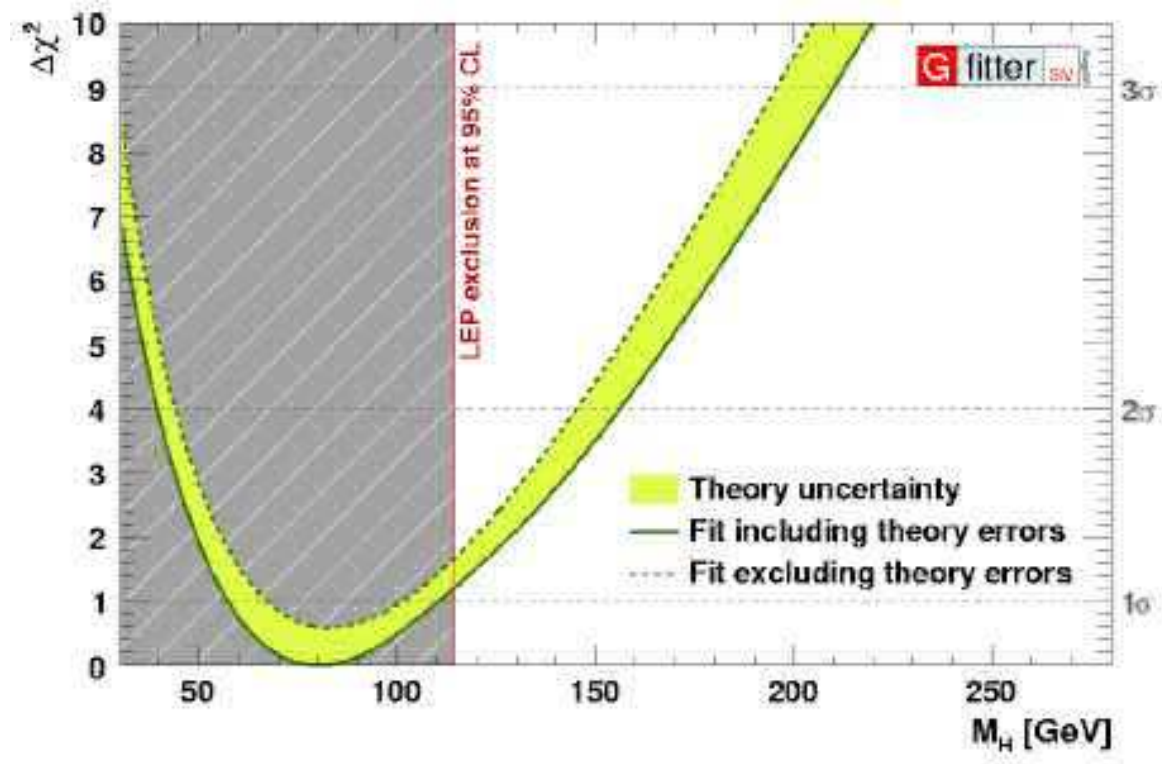
$$a_0 = \frac{g^2 M_H^2}{4M_W^2} \rightarrow \text{unitarity breaks down unless } M_H \leq 1.2 \text{ TeV}$$

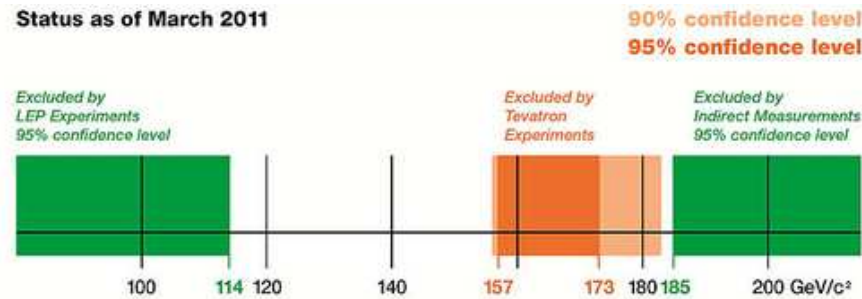
By considering other channels, one get the stronger bound $M_H \leq 800 \text{ GeV}$.

Intepretation :

- If LHC finds no Higgs with a mass $M_H \leq 800 \text{ GeV}$, unitarity of S-matrix will be violated ! New light degrees of freedom should exist in order to restore unitarity \rightarrow the no-loose "theorem" for LHC.

Most theories have a bias towards a **light Higgs**, since it provides a better fit for the SM precision tests.





Latest news ("Lepton-Photon", august 2011): Both ATLAS+CMS exclude the SM Higgs at 95 % CL for $145 \leq M_H \leq 446 \text{ GeV}$ except $288 - 296 \text{ GeV}$

M. Peskin (LP2011) "There is therefore strong evidence that either :

- Higgs is light, compatible with electroweak precision tests and theoretical prejudice, or
- the Higgs boson is very heavy and strongly self-coupled".