Lecture 3

With Big Bang nucleosynthesis theory and observations
we are confident of the theory of the early Universe
at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the LHC, we hope to be able to go
up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

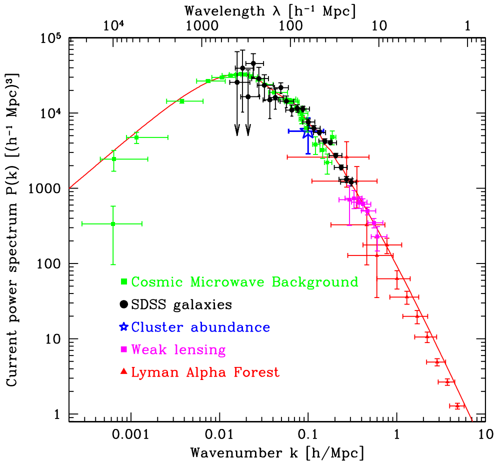
$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Linear analysis appropriate.

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 370 000 yrs, $T = 3000$ K
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

Overall consistency



NB: density perturbations = random field.
k = wavenumber
$P(k)$ = power spectrum transfered to present epoch using linear theory

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We know that this is not the whole story!

Properties of perturbations in conventional (“hot”) Universe.

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^{\,2}$$

$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV, $t \simeq 60$ thousand years)
$a(t) \propto t^{2/3}$ at matter domination stage (until recently).

Cosmological horizon at time t (assuming that nothing preceeded hot epoch): distance that light travels from Big Bang moment,

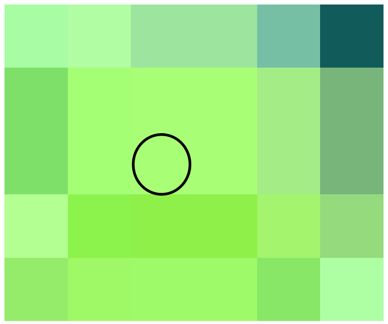
$$l_{H,t} \sim H^{-1}(t) \sim t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

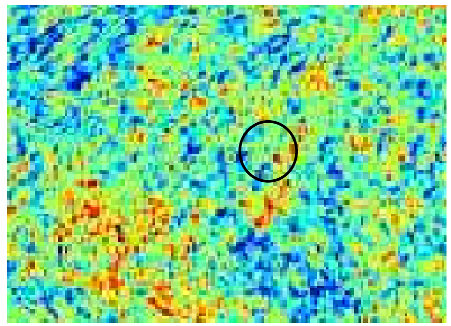
$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_{H,t} \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



superhorizon mode



subhorizon mode

In other words, physical wavenumber (momentum) gets redshifted,

$$q(t) = \frac{2\pi}{\lambda(t)} = \frac{k}{a(t)}\,, \qquad k = \text{const} = \text{coordinate momentum}$$

Today

$$q > H \equiv \frac{\dot{a}}{a}$$

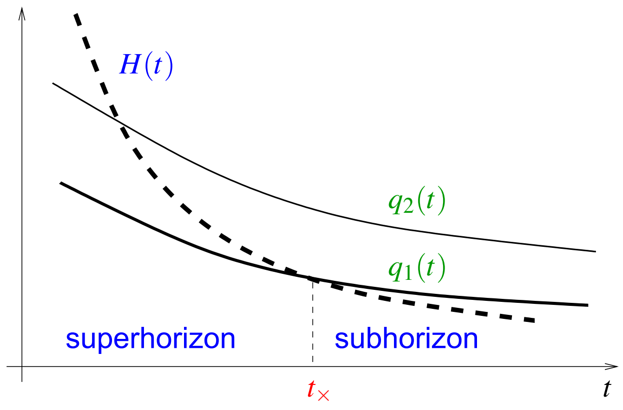
Early on

$$q(t) < H(t)$$

Very different regimes of evolution.

NB: Horizon entry occured after Big Bang Nucleosynthesis epoch for modes of all relevant wavelengths $\Longleftrightarrow$ no guesswork at this point.

Regimes at radiation (and matter) domination



$$q_2 > q_1$$

Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  The only possibility, if expansion started from hot Big Bang.

  No longer an option!

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase.

Prototype example: wave equation in expanding Universe (not exactly the same as equation for sound waves, but captures main properties).

Massless scalar field ϕ in FLRW spacetime: action

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$g_{\mu\nu} = (1, -a^2, -a^2, -a^2)$: spacetime metric;
$g^{\mu\nu} = (1, -a^{-2}, -a^{-2}, -a^{-2})$: its inverse;
$g = \det\,(g_{\mu\nu}) = a^6$: its determinant
($d^4x\sqrt{-g}$: invariant 4-volume element).

$$S = \frac{1}{2}\int d^3x dt\ a^3(t)\left(\dot\phi^2 - \frac{1}{a^2}\vec\partial\phi\cdot\vec\partial\phi\right)$$

Field equation

$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi - \frac{1}{a^2}\Delta\phi = 0$$

NB. $\dot a/a = H$: Hubble parameter.

Fourier decomposition in 3d space

$$\phi(\vec x, t) = \int d^3k\ \mathrm{e}^{i\vec k\vec x}\phi_{\vec k}(t)$$

NB. $\vec k$: coordinate momentum, constant in time.
Physical momentum $q = k/a(t)$ gets redshifted.

Wave equation in momentum space:

$$\ddot\phi + 3H(t)\dot\phi + \frac{k^2}{a^2(t)}\phi = 0$$

- Redshift effect: frequency $\omega(t) = k/a(t)$.
- Hubble friction: the second term.

As promised, evoltion is different for $k/a > H$ (subhorizon regime) and $k/a < H$ (superhorizon regime).

Subhorion regime (late times): damped oscillations

$$\phi_{\vec k}(t) = \frac{A_{\vec k}}{a(t)}\cos\left(\int_0^t \frac{k}{a(t)}\,dt + \psi\right)$$

NB. Subhorizon sound waves in baryon-photon plasma:
– Amplitude of $\delta\rho/\rho$ does not decrease
– Sound wave v_s different from 1 ($v_s \approx 1/\sqrt{3}$).
All the rest is the same

Solution to wave equation in superhorizon regime (early times) at radiation domination

$$\phi = \text{const} \qquad \text{and} \qquad \phi = \frac{\text{const}}{t^{3/2}}$$

Constant and decaying modes.
NB: decaying mode is sometimes called growing, it grows as $t \to 0$.

Same story for density perturbations.

$\delta\rho/\rho \propto t^{-3/2}$: very inhomogeneous Universe at early times $\Longrightarrow$ inconsistency

Under assumption that modes were superhorizon, the initial condition is unique (up to overall amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \Longrightarrow \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\Longrightarrow$ phase of oscillations uniquely defined; $\psi = 0$.

Perturbations come to the time of photon last scattering (= recombination) at different phases, depending on wave vector:

$$\delta(t_r) \equiv \frac{\delta\rho}{\rho}(t_r) \propto \cos\left(k\int_0^{t_r} dt\ \frac{v_s}{a(t)}\right) = \cos(kr_s)$$

r_s: sound horizon at recombination.

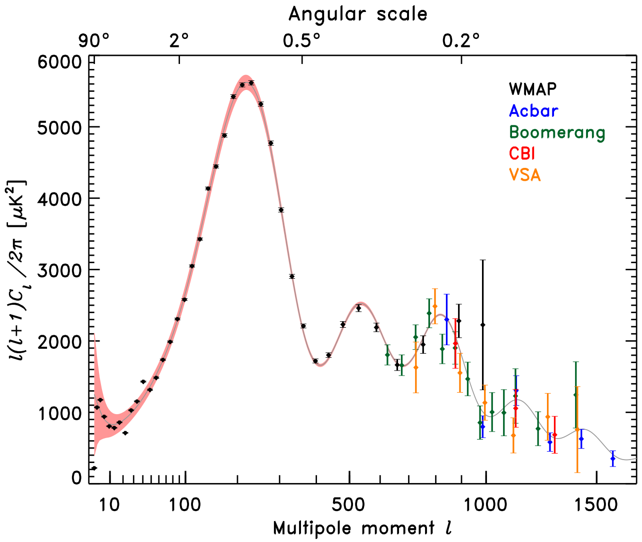
Waves with $k = \pi/r_s$ have large $|\delta\rho|$, while waves with $k = (\pi + 1/2)/r_s$ have $|\delta\rho| = 0$ in baryon-photon component.
This translates into oscillations in CMB angular spectrum

Fourier decomposition of temperatue fluctuations:

$$\delta T(\theta,\varphi) = \sum_{l,m} a_{lm}Y_{lm}(\theta,\varphi)$$

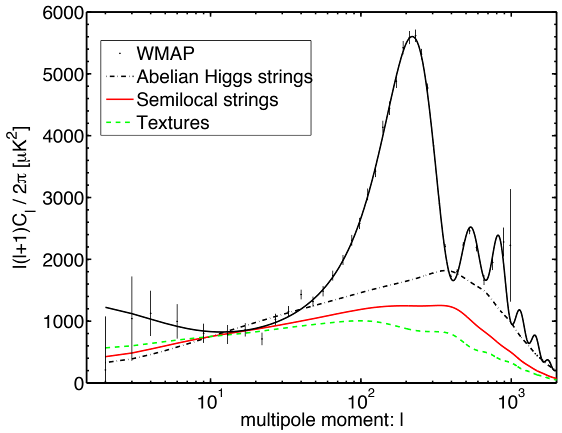
$\langle a^*_{lm}a_{lm}\rangle = C_l$, temperature angular spectrum;

larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths



Furthermore, there are perturbations which were superhorizon at the time of photon last scattering (low multipoles, $l \lesssim 50$)

These properties would not be present if perturbations were generated at hot epoch in causal manner: phase ψ would be random function of k, no oscillations in CMB angular spectrum.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long and unusual:
perturbations were subhorizon early at that epoch,
our visible part of the Universe was in a causally connected region.

- Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

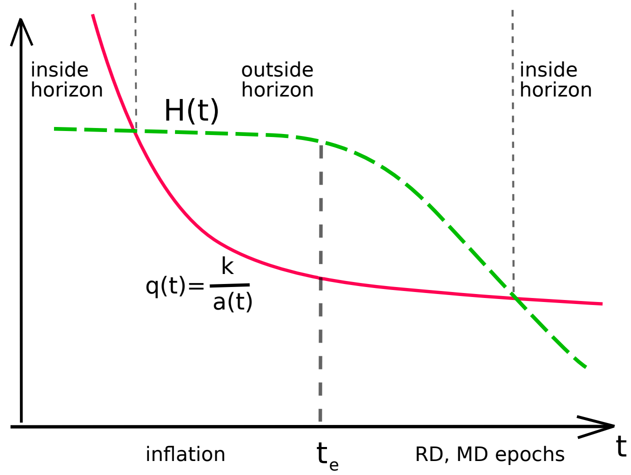
Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt}\ , \qquad H \approx \text{const}$$

Perturbations subhorizon early at inflation:

$$q(t) = \frac{k}{a(t)} \gg H$$

Physical wave number and Hubble parameter at inflation and later:



Alternatives to inflation:
Contraction — Bounce — Expansion, Start up from static state.
Difficult, but not impossible.

Other suggestive observational facts about density perturbations
(valid within certain error bars!)

- Perturbations in overall density, not in composition (jargon: "adiabatic")

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: "isocurvature" or "entropy") $\Longrightarrow$ wrong initial condition for acoustic oscillations $\Longrightarrow$ wrong prediction for CMB angular spectrum.

- Primordial perturbations are Gaussian.

Gaussian random field $\delta(\mathbf{k})$: correlators obey Wick's theorem,

$$\begin{aligned}
\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0 \\
\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle\cdot\langle\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle \\
&+ \text{permutations of momenta}
\end{aligned}$$

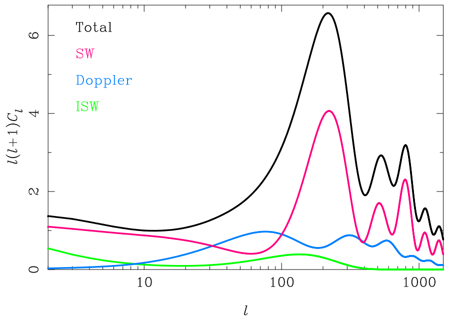
- $\langle\delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.
- Hint on the origin: enhanced vacuum fluctuations of free quantum field
Free quantum field

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{kx}}\left(f_{\mathbf{k}}^{(+)}(t)a_{\mathbf{k}}^\dagger + e^{i\mathbf{kx}}f_{\mathbf{k}}^{(-)}(t)a_{\mathbf{k}}\right)$$

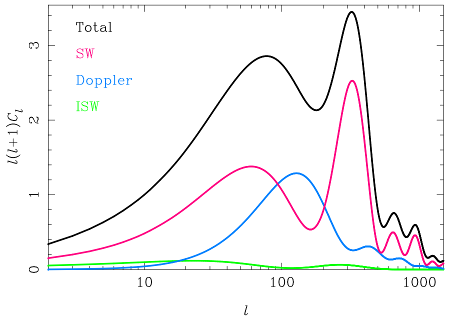
In vacuo $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$

Enhanced perturbations: large $f_{\mathbf{k}}^{(\pm)}$. But in any case, Wick's theorem valid

CMB angular spectra



homogeneous chemical composition



inhomogeneous chemical composition

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation $\Longrightarrow$ watch out Planck!

- Inflation does the job very well: fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

- Enhancement of vacuum fluctuations is less automatic in alternative scenarios

- Non-Gaussianity: big issue
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum (3-point function; vanishes for Gaussian field)

$$\langle\delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$$

    Shape of $G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$ different in different models $\Longrightarrow$ potential discriminator.
  - In some models bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.

Non-Gaussianity has not been detected yet

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

  Symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda\vec{x}\,, \quad t \to t - \frac{1}{2H}\log\lambda$$

  Inflation automatically generates nearly flat spectrum.

- Alternative: conformal symmetry

  Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.

  $\Longrightarrow$ No scale, good chance for flatness of spectrum

  Model-building has begun recently

- Primordial power spectrum is flat (or almost flat).

  Homogeneity and anisotropy of Gaussian random field:

$$\langle\delta(\vec{k})\delta(\vec{k}')\rangle = \frac{1}{4\pi k^3}\mathscr{P}(k)\delta(\vec{k}+\vec{k}')$$

  $\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle\left(\frac{\delta\rho}{\rho}(\vec{x})\right)^2\rangle = \int_0^\infty \frac{dk}{k}\,\mathscr{P}(k)$$

  Flat spectrum: $\mathscr{P}$ is independent of k

  Harrison' 70; Zeldovich' 72

  Parametrization

$$\mathscr{P}(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$

  $A =$ amplitude, $(n_s-1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow$ $n_s = 1$.

- Statistical anisotropy

$$\mathscr{P}(\mathbf{k}) = \mathscr{P}_0(k)\left(1 + \frac{\vec{u}\vec{k}}{k} + w_{ij}(k)\frac{k_ik_j}{k^2} + \dots\right)$$

  $\vec{u}$, w_{ij}: fundamental vector, tensor in our part of the Universe.
  - Anisotropy of the Universe at pre-hot stage
  - Possible in inflation with strong vector fields (rather contrived).
  - Natural in some other scenarios, including conformal models.
  - Would show up in correlators

$$\langle a_{lm}a_{l'm'}\rangle \quad \text{with} \quad l' \neq l \quad \text{and/or} \quad m' \neq m$$

Observational data: controversy at the moment

- Tensor modes = primordial gravitational waves

  Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models but not alternatives to inflation

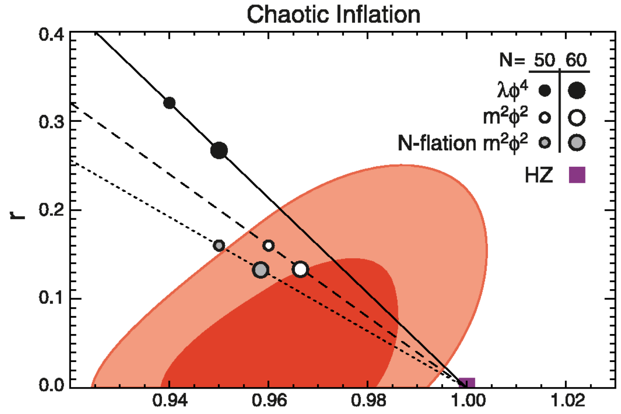
  May make detectable imprint on CMB temperature anisotropy

  and especially on CMB polarization

  Smoking gun for inflation

Until now: search via effect on CMB temperature anisotropy.

Scalar tilt vs tensor power



NB:

$$r = \left(\frac{\text{amplitude of gravity waves}}{\text{amplitude of density perturbations}} \right)^2$$

Opportunity for observing tensor modes

CMB POLARIZATION

- CMB is polarized,
  because photons of different polarizations scatter off electrons differently.
- Scalar and tensor modes lead to different types of polarization (so called E- and B-modes, respectively).

  Most promising way to search for tensor modes = gravity waves

- Planck, dedicated baloon experiments.

To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceeded by some other epoch, at which these perturbations were generated.
- Inflation is consistent with all data. But there are competitors: the data may rather be viewed as pointing towards early conformal epoch of the cosmological evolution.

  More options:
  Matter bounce,
  Negative exponential potential,
  Lifshitz scalar, . . .

- Only very basic things are known for the time being.

Good chance for future

- Detection of B-mode (partity odd) of CMB polarization $\Longrightarrow$ effect of primordial gravity waves $\Longrightarrow$ simple inflation
  - Together with scalar and tensor tilts $\Longrightarrow$ properties of inflaton
- Non-trivial correlation properties of density perturbations (non-Gaussianity) $\Longrightarrow$ contrived inflation, or something entirely different.
  - Shape of non-Gaussianity $\Longrightarrow$ choice between various alternatives
- Statistical anisotropy $\Longrightarrow$ anisotropic pre-hot epoch.
  - Shape of statistical anisotropy $\Longrightarrow$ specific anisotropic model
- Admixture of entropy (isocurvature) perturbations $\Longrightarrow$ generaion of dark matter and/or baryon asymmetry befor the hot epoch

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

Good chance to learn
what preceeded the hot Big Bang epoch

Barring the possibility that Nature is dull