

D-Branes at Finite Temperature in TFD

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Abstract

We review the construction of the *D*-branes at finite temperature as boundary states in the Fock space of thermal perturbative closed string. This is a talk presented by I. V. V. at Common Trends in Cosmology and Particle Physics June 2003, Balatonfüred, Hungary.

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Since their discovery in 1995 [1], D -branes have played a major role in the development of the string theory and the models based on it. They have given a first basis to unify the five superstring theories, offered powerful tools to understand the nonperturbative limit of string theory, made possible the connection between field theory and gravity and have been used to address some fundamental problems as the black hole entropy and hierarchy problem, to mention just some of the applications of the D -branes. Also, new mathematical tools have had to be employed and new mathematical problems emerged in the investigation of the properties of the D -branes. By its richness in concepts and applications, D -brane theory represents one of the most fruitful research direction in the modern mathematical physics.

One of the most interesting problems concerning the D -branes is the development of a microscopic description of their thermodynamical properties. In the low energy limit of string theory, i. e. when the string tension is infinite and strings behave as particles, the D -branes are described by solutions of supergravity. Thus, the thermodynamics of the D -branes can be computed using the Masubara formalism. However, one can describe the D -branes as boundary states in the Fock space of the closed string theory by interpreting the one-loop diagram in open string channel as the tree-level diagram in the closed string channel or by using the T-duality. Then, one can naturally ask: What is the state corresponding to the D -brane if the string is thermal? Answering this question gives us the description of the D -brane at finite temperature as a quantum state in the Fock space of the thermal closed string [3, 4, 5, 6, 7]. We note, however, that in the perturbative limit of string theory the D -branes can be viewed as (physical) submanifolds of the space time where the open strings can end. In this case, one can compute the free energy of the open strings in the presence of the D -branes and obtain the self-energy of the D -brane through the finite temperature dualities [2]. However, this method does not give us the representation of the D -brane as a thermal state.

In order to answer the above question, we have to be able to manipulate the transformation from zero temperature to finite temperature at the Fock space level. A simpler way to do that is to use the Thermo Field Dynamics (TFD) formalism [10] in which the statistical average of an observable \mathcal{O} is expressed as a "thermal vacuum" expectation value

$$\langle \mathcal{O} \rangle = \langle \langle 0(\beta_T) | \mathcal{O} | 0(\beta_T) \rangle \rangle. \quad (1)$$

The "thermal vacuum" state $|0(\beta_T)\rangle\rangle$ belongs to the Fock space that is a direct product between the original Fock space and an independent identical copy of it, a thermodynamic reservoir. This second copy of the original string is denoted by a tilde and does not represent a physical system. The β_T states for the inverse of the temperature in units such that the Boltzmann constant is one. The temperature is

implemented in the Fock space of the doubled system by a Bogoliubov transformation $\Gamma(\beta_T)$ that mixes operators from the two copies

$$|0(\beta_T)\rangle\rangle = e^{-i\Gamma(\beta_T)}|0\rangle\rangle \quad (2)$$

$$\mathcal{O}(\beta_T) = e^{-i\Gamma(\beta)}\mathcal{O}e^{i\Gamma(\beta_T)}, \quad (3)$$

where $|0\rangle\rangle = |0\rangle|\widetilde{0}\rangle$ is the product vacuum state between the two vacua of the string and the tilde string. In general, the possible Bogoliubov operators form a group of transformations, which for bosons is $SU(1,1)$, while for fermions is $SO(2)$. For internal consistency of the theory, the thermal vacuum should be invariant under the tilde operation that transforms the two copies of the Fock space one into another. One can select an unique Bogoliubov operator that guarantees the tilde invariance and generates an unitary transformation [10]. This represents our choice.

Our purpose now is to apply the TFD method to string theory in order to find the state corresponding to the D -brane at finite temperature. The Fock space of the closed string is a direct product between the Fock spaces of the left- and right-moving oscillators as well as the Hilbert space of the center of mass degrees of freedom. If the string is supersymmetric on the world-sheet, one should tensor this Fock space with two more spaces, corresponding to the left- and right-moving two-dimensional Majorana spinors. Since all modes are independent of each other, one can apply the TFD construction separately to each Fock space. If we consider for simplicity just the bosonic string, the Bogoliubov transformation that takes the zero temperature Fock space

$$\hat{\mathcal{H}} = \mathcal{H} \otimes \tilde{\mathcal{H}}, \quad (4)$$

where tilde denotes the second copy of the bosonic string Fock space, to finite temperature, has the following form

$$\Gamma = \Gamma^\alpha + \Gamma^\beta = -i \sum_{n=1}^{\infty} \theta_n(\beta_T)(A_n \cdot \tilde{A}_n - A_n^\dagger \cdot \tilde{A}_n^\dagger + B_n \cdot \tilde{B}_n - B_n^\dagger \cdot \tilde{B}_n^\dagger). \quad (5)$$

Here, the superscripts α and β denote the left- and right-moving modes. The operators A_n^μ and $A_n^{\mu\dagger}$ annihilate and create string left-moving excitations of frequency n and in the space-time direction $\mu = 1, 2, \dots, 24$. All the directions are space-like since we work in the light-cone gauge in order to eliminate the unphysical degrees of freedom. The same considerations apply to the right-moving modes B . The Bogoliubov operator depends on the temperature through the parameters θ which, for bosons, are given by the following equation

$$\cosh \theta_n(\beta_T) = (1 - e^{-\beta_T n})^{-\frac{1}{2}}. \quad (6)$$

The Bogoliubov operator (5) acts on $X^\mu(\tau, \sigma)$ which represents the most general solution of the closed string equations of motion. ($X^\mu(\tau, \sigma)$ is a linear combination between a linear term in the world-sheet time-like parameter τ and independent left- and right-moving oscillations [8]). The resulting operator, denoted by $X^\mu(\tau, \sigma)(\beta_T)$ is still a solution of the string equations of motion with the creation and annihilation operators now at finite temperature given by (3). The coordinates and momenta of the center of mass are inert under the Bogoliubov transformation. Since we work with two copies of the original string, we have to apply (5) to the tilde string as well and we obtain the second copy of the string $\tilde{X}^\mu(\tau, \sigma)(\beta_T)$ at finite temperature. The creation and annihilation operators $A_n^\mu(\beta_T)$, $\tilde{A}_n^\mu(\beta_T)$, $A_n^{\mu\dagger}(\beta_T)$, $\tilde{A}_n^{\mu\dagger}(\beta_T)$, $B_n^\mu(\beta_T)$, $\tilde{B}_n^\mu(\beta_T)$, $B_n^{\mu\dagger}(\beta_T)$ and $\tilde{B}_n^{\mu\dagger}(\beta_T)$ act on the thermal vacuum given by (2). All the oscillator properties of each string mode are preserved at finite temperature.

Let us take a look at the conformal symmetry. By using the Bogoliubov transformations (2) and (3) we have constructed solutions of the string equations of motion at finite temperature. These solutions are obtained basically by mapping the creation and annihilation operators for each string mode at finite temperature. Then the generators of the conformal transformations, the Virasoro operators which can be expressed in terms of creation and annihilation operators are also mapped at finite temperature. Since the Bogoliubov transformation is unitary, the Virasoro operators at finite temperature $L_n(\beta_T)$ satisfy the Virasoro algebra. However, if one computes physical quantities, one has to employ observables at zero temperature according to the assumption (1). It is a simple exercise to show that the thermal vacuum does not vanish under the action of the Virasoro operators at zero temperature L_n and thus the thermal vacuum breaks the zero temperature Virasoro algebra. Similarly, the true vacuum (at zero temperature) breaks the Virasoro algebra at finite temperature. Breaking the symmetries at finite temperature by the zero temperature operators is a general property of the thermal systems and it is not specific to strings, as we have used the general arguments of TFD do derive it. This remarks were also made in [9] in the case of the supersymmetric oscillator.

Let us turn now to the discussion of the D -branes. If one imposes the Neumann boundary conditions along the directions $a = 1, 2, \dots, p$ and the Dirichlet boundary conditions along $i = p + 1, \dots, 24$ in the open string theory, one obtains a hypersurface that exchanges momenta with the string and defines the Dp -brane. By employing the T-duality, the Dp -brane is mapped into the closed string channel. This mapping changes the Dirichlet and the Neumann boundary conditions among themselves. It also suggests the following interpretation of the two types of strings in D -brane theory: the open string channel describes the degrees of freedom of the D -brane, while the closed strings describe the interactions of two or more D -branes with one another. If we denote by $X^a(\tau, \sigma)$ and $X^i(\tau, \sigma)$ the closed string coordi-

nate operators along the Dirichlet and the Neumann directions, respectively, the equations defining a Dp -brane are the following ones:

$$\begin{aligned}\partial_\tau X^a|_{\tau=0}|B\rangle &= 0 \\ X^i|_{\tau=0}|B\rangle &= x^i,\end{aligned}\tag{7}$$

where x^i denote the coordinates of the hyperplane in the transverse space. If one Fourier expands the string coordinate operators, the equations (7) become conditions on the Fock space of the closed string. The solution to this set of equations is given by the following relation:

$$|B\rangle = N_p \delta^{(d_\perp)}(q-x) e^{-\sum_{n=1}^{\infty} A_n^{\dagger\mu} S_{\mu\nu} B_n^{\dagger\nu}} |0\rangle,\tag{8}$$

where q^μ are the operators corresponding to the center of mass of the closed string, $S_{\mu\nu}$ is the diagonal matrix $S_{\mu\nu} = (\eta_{ab}, -\delta_{ij})$ and $|0\rangle$ is the translationally invariant closed string vacuum. The same boundary conditions should be imposed on the copy of the Fock space and they generate a Dp -brane solution in the tilde operators.

The counterpart of the Dp -brane at finite temperature should be obtained by applying the same idea that has led to the coherent state (8) at zero temperature. Indeed, if one varies continuously the temperature T from zero to a small but finite value, the hypersurface that defines the Dp -brane at zero temperature should not change. Therefore, one should be able to use the same geometrical condition, i. e. the same Dirichlet and Neumann boundary conditions, to write down the equations that define the thermal coherent state in the Fock space of the thermal string and to interpret the solution to these equations as a thermal Dp -brane. As we have already seen, the fields $X^\mu(\tau, \sigma)(\beta_T)$ and $\tilde{X}^\mu(\tau, \sigma)(\beta_T)$ satisfy the string equations of motion that can be obtained from the variational principle applied to the Lagrangian density $\hat{\mathcal{L}} = \mathcal{L} - \tilde{\mathcal{L}}$ of the two copies of the closed string. If we imposed the Dirichlet and the Neumann boundary conditions to the action \hat{S} , we obtain two copies of the relations (7) but with the string operators at finite temperature instead of those at zero temperature:

$$\begin{aligned}\partial_\tau X^a|_{\tau=0}(\beta_T) |B(\beta_T)\rangle\rangle &= \partial_\tau \tilde{X}^a|_{\tau=0}(\beta_T) |B(\beta_T)\rangle\rangle = 0 \\ X^i|_{\tau=0}(\beta_T) |B(\beta_T)\rangle\rangle &= \tilde{X}^i|_{\tau=0}(\beta_T) |B(\beta_T)\rangle\rangle = x^i,\end{aligned}\tag{9}$$

where the coordinates of the centers of mass of the string and the tilde string are the same. The Dp -brane state $|B(\beta_T)\rangle\rangle$ has now contributions from the two sectors of the Fock space \hat{H} . In order to find the solution of the equations (9) we note that the algebra of the string oscillators at finite temperature is preserved by the

Bogoliubov transformations, which means that the temperature dependent creation and annihilation operators and tilde operators are independent. The thermal Dp -brane solution has the following form

$$|B(\beta_T)\rangle\rangle = N_p^2 \delta^{2(d_\perp)}(q-x) e^{-\sum_{n=1}^{\infty} [A_n^{\dagger\mu}(\beta_T) + \tilde{A}_n^{\dagger\mu}(\beta_T)] S_{\mu\nu} [B_n^{\dagger\nu}(\beta_T) + \tilde{B}_n^{\dagger\nu}(\beta_T)]} |0(\beta_T)\rangle\rangle, \quad (10)$$

where $\delta^{2(d_\perp)}(q-x) = \delta^{(d_\perp)}(q-x) \delta^{(d_\perp)}(\tilde{q}-x)$. One can easily check that the same solution can be obtained by mapping the product of Dp -brane state and tilde state at zero temperature to $T \neq 0$. Also, we can obtain partial Dp -brane-like solutions in either thermal string or tilde string sectors by solving the constraints (9) in the respective variables.

As an immediate application of the TFD formalism in solving the thermal Dp -branes is the calculation of their "internal" entropy as follows. In the TFD approach, one can define the entropy operator of an infinite set of independent oscillators. If we consider the closed string as being the set of independent oscillation modes, its entropy in k_B units is given by the following relation

$$K = \sum_{\mu} \sum_n \left[(A_n^{\mu\dagger} A_n^{\mu} + B_n^{\mu\dagger} B_n^{\mu}) \log \sinh^2 \theta_n + (A_n^{\mu} A_n^{\mu\dagger} + B_n^{\mu} B_n^{\mu\dagger}) \log \cosh^2 \theta_n \right]. \quad (11)$$

This operator commutes with the Bogoliubov generator. A similar entropy can be defined for the tilde string, but it does not describes a physical quantity according to the TFD principles. At the boundary of the world-sheet, the string modes generate a coherent state which represents the Dp -brane. Thus, the entropy of the Dp -brane is given by the expectation value of the entropy operator (11) in the state (10)

$$K_{Dp} = \langle\langle B(\beta_T) | K | B(\beta_T) \rangle\rangle = 48 \sum_{m=1}^{\infty} \left[\log \sinh^2 \theta_m - \sinh^2 \theta_m \log \tanh^2 \theta_m \right] + 2 \prod_{m=1}^{\infty} \prod_{\mu=1}^{24} \prod_{\nu=1}^{24} \sum_{k=0}^{\infty} \cosh^2 \theta_m \frac{(-)^{2k+2} S_{\mu\nu}^{2k+2}}{k! (k+1)!}. \quad (12)$$

The contribution of the oscillators to K_{Dp} diverges in the $T \rightarrow 0$ limit and behaves as $\log(-1)$ as $T \rightarrow \infty$. This might be an indication that the notion of temperature breaks down for arbitrary large temperature due to the similar phenomenon which occurs in string theory at Hagedorn temperature.

In conclusion, we have shown that there are thermal states in the Fock space of the thermal closed bosonic string corresponding to the Dp -branes. These state represent the natural generalization of zero temperature coherent states. Although

divergent, an entropy can be calculated for closed strings in this state. In the same way, one can introduce world-sheet fermions into the formalism and construct thermal states corresponding to super D -branes. However, we expect that the supersymmetry be broken at some stage at least in the case of the space-time supersymmetry. We hope to report on this topic in some future work [11].

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