Spontaneous breakdown of sterility

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We believe there exists a model of strong-coupling dynamical mass generation with one free parameter (mass scale). The model is thus either right or plainly wrong. Its neutrino sector is uniquely fixed. Weak point of strongly coupled field theories, however, is that there is no systematic way of their solution. We believe in signatures without knowing the spectrum.

The model

- Consider the $SU(2)_L x U(1)_Y$ SM chiral fermion fields $l^T_{fL}=(\nu_{fL}, e_{fL})$, e_{fR}, $q^T_{fL}=(u_{fL}, d_{fL})$, u_{fR}, d_{fR} and gauge the flavor index f=1,2,3.
- $SU(3)_f$ quantum flavor dynamics (QFD)
- coupling constant h, eight flavor gluons C_a^μ
- non-vector-like assignment: fermion mass bridges should be different

$$\bar{e}_R(3)\Sigma_e e_L(\bar{3}) \neq \bar{u}_R(3)\Sigma_u u_L(3) \neq \bar{d}_R(\bar{3})\Sigma_d d_L(3)$$

Anomaly freedom requires three flavor triplets of right-handed neutrino fields ν_{NfR} and brings into the model new global $U(3)_S=SU(3)_S x U(1)_S$ sterility symmetry

- In PT the model is asymptotically free and not vector-like. Nobody knows how to put in on the lattice.

$$\beta(h) = -[11 - \frac{1}{3}n_{chiral} - \frac{1}{3}n_{\nu_R}]\frac{h^3}{16\pi^2}$$

$$\frac{\bar{h}^2(q)}{4\pi} = \frac{12\pi}{(33 - n_{chiral} - n_{\nu_R})\ln(\mathrm{q}^2/\Lambda^2_{\mathrm{QFD}})}$$

Lagrangian of the world is $SU(3)_f x SU(3)_c x SU(2)_L x U(1)_Y$ gauge invariant and obeys global Abelian symmetries (besides $SU(3)_S$ to be discussed later):

$$J_B^\mu = \frac{1}{3}[\bar{q}_L \gamma^\mu q_L + \bar{u}_R \gamma^\mu u_R + \bar{d}_R \gamma^\mu d_R]$$

$$J_{B5}^\mu = \frac{1}{3}[-\bar{q}_L \gamma^\mu q_L + \bar{u}_R \gamma^\mu u_R + \bar{d}_R \gamma^\mu d_R]$$

$$J_L^\mu = \bar{l}_L \gamma^\mu l_L + \bar{e}_R \gamma^\mu e_R$$

$$J_{L5}^\mu = -\bar{l}_L \gamma^\mu l_L + \bar{e}_R \gamma^\mu e_R$$

$$J_S^\mu = \frac{1}{3} \sum \bar{\nu}_{NR} \gamma^\mu \nu_{NR}$$

Straightforward computation of anomalous triangles results in nonzero divergences of these currents

- current j^{μ}_{B-L-S} is conserved
- symmetries generated by currents j^{μ}_{B+L+S} and j^{μ}_{L5-3S} are broken merely by electroweak effects and are in fact rather good symmetries
- current j^{μ}_{B5-4S} is not conserved due to the strong QCD anomaly
- current j^{μ}_{S} is not conserved due to strong QFD anomaly. Spontaneous breakdown of this 'would-be' symmetry results in one heavy Majoron

Dynamical mass generation

- In PT masslessness of fermion fields is protected by chiral symmetries (electroweak and QFD)
- In PT masslessness of gauge fields is protected by gauge symmetries (electroweak and QFD)
- Massless fields can excite massive particles nonperturbatively.
- The idea: We heuristically argue that flavor gluons acquire self-consistently masses M_a of order O(1000 TeV). Their exchanges between left- and right-handed fermion fields generate fermion masses. Smallness of m_f is attributed to the proximity of nonperturbative IR fixed point. Fermion masses break spontaneously also $SU(2)_L x U(1)_Y$. Consequently, m_W and m_Z are expressed in terms of the fermion masses by sum rules.

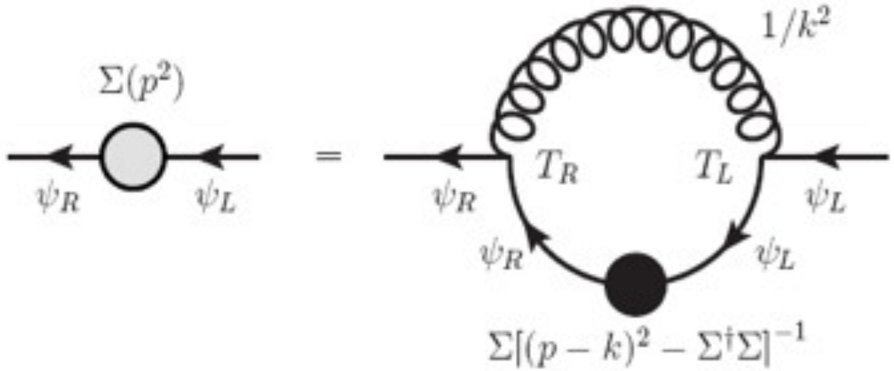
Fermion mass

- is spontaneously generated if the ground state (vacuum) is not invariant with respect to independent rotations of left-and right-handed fermion fields:

$$\langle 0|T\,\psi(x)\,\bar{\psi}(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, iS(p)e^{-ip\cdot(x-y)}$$

$$S(p) = (\not{p}+\Sigma^{+})(p^2-\Sigma\Sigma^{+})^{-1}P_L+(\not{p}+\Sigma)(p^2-\Sigma^{+}\Sigma)^{-1}P_R$$

Gap (Schwinger-Dyson) equation for fermion proper energy



$$\Sigma_f(p) = -4i \int \frac{d^4k}{(4\pi)^4} \frac{\bar{h}^2_{ab}(k^2)}{k^2} T_a(f_R)\Sigma_f(p-k)[(p-k)^2 - \Sigma_f^+(p-k)\Sigma_f(p-k)]^{-1} T_b(f_L)$$

Gauge boson mass

- is spontaneously generated if the ground state (vacuum) is not invariant with respect to global symmetry underlying the gauge one. The longitudinal polarization state is the composite 'would-be' Nambu-Goldstone (NG) boson:

$$\langle 0|TC_a^\mu(x)C_b^\nu(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} i\Delta_{ab}^{\mu\nu} e^{-ip.(x-y)}$$

- Mass manifests as a residue at the NG massless pole of polarization tensor $\Pi_{ab}(p^2)=M^2_{ab}/p^2$:

$$\mathrm{i}\Delta_{ab}^{\mu\nu}(q) = \frac{-\mathrm{i}}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\left(\mathbb{1} - \Pi(q^2)\right)_{ab}^{-1} + \alpha\frac{-\mathrm{i}}{q^2}\frac{q^\mu q^\nu}{q^2}\mathbb{1}_{ab}$$

Composite 'would-be' NG boson

manifests as a massless pole in the proper vertex corresponding to the Green's function of Noether symmetry current and two dynamically massive fields:

- fermion:

$$j^{\mu}_{a\psi} = \bar{\psi}\gamma^{\mu}[T_{aL}P_L + T_{aR}P_R]\psi$$

$$G^{\mu}_{a\psi}(x,y,z) = \langle 0|T j^{\mu}_{a\psi}(x)\psi(y)\bar{\psi}(z)|0\rangle$$

$$\boxed{\Gamma^{\mu}_{\psi;pole}(q) \sim \frac{q^{\mu}}{q^2}[\Sigma(q)T - T\Sigma(q)]}$$

- gauge boson:

$$j^{C\mu}_{a} = -f_{abc}F^{\mu\nu}_{b}C_{\nu c}$$

$$G^{\alpha\mu\nu}_{amn}(x,y,z) \equiv \langle 0|T j^{C\alpha}_{a}(x)C^{\mu}_{m}(y)C^{\nu}_{n}(z)|0\rangle$$

$$\boxed{\Gamma^{\mu}_{C;pole}(q) \sim \frac{q^{\mu}}{q^2}[\Pi(q)f - f\Pi(q)]}$$

Neutrino sector of QFD

$$\mathcal{L}_\nu = \bar{\nu}_L i\partial\!\!\!/\, \nu_L + \bar{\nu}_{NR} i\partial\!\!\!/\, \nu_{NR} + \mathcal{L}_{int} + \mathcal{L}_C$$

$$\mathcal{L}_{int} = h\,[\bar{\nu}_L\gamma_\mu(-\tfrac{1}{2}\lambda_a^*)\,\nu_L + \bar{\nu}_{NR}\gamma_\mu\tfrac{1}{2}\lambda_a\,\nu_{NR})\,]\,C_a^\mu$$

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^C \end{pmatrix}$$

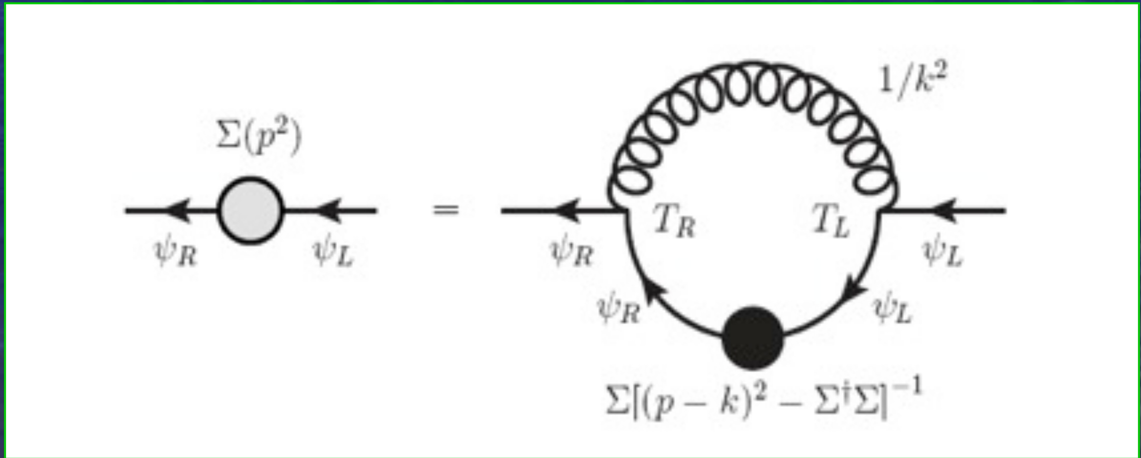
$$\bar{\nu}_{NR}\Sigma_{ND}\,\nu_L$$

$$\Sigma_\nu = \begin{pmatrix} \Sigma_L & \Sigma_D \\ \Sigma_D^T & \Sigma_R \end{pmatrix}$$

$$\mathcal{T}_a = \begin{pmatrix} T_{aL} & 0 \\ 0 & -T_{aR}^T \end{pmatrix}$$

$$\overline{(\nu_L)^C}\Sigma_L\,\nu_L$$

$$\bar{\nu}_{NR}\Sigma_{NMR}(\nu_{MR})^C$$



We optimistically assume that phenomenologically acceptable solution of the neutrino SD equation exists

- Upon diagonalization there are twelve massive Majorana neutrinos ν_M. Computing the numerical values of masses is a crystalline challenge.
- In general there is plenty of mixing angles and CP violating phases.
- Three SM neutrinos are the linear combinations of 12 massive ν_M. Hence, leptonic mixing matrix is not unitary.
- Global $U(1)_S$ symmetry is anomalous. In analogy with QCD we argue that the 'would-be' Abelian Majoron H should acquire large mass M_H due to QFD instanton effects of the order λ_{QFD}.
- Global $SU(3)_S$ symmetry is in general completely spontaneously broken. As a consequence, there should exist eight massless Majorons coupled directly with calculable effective couplings only to neutrinos.

The assumption need not be justified: Global $SU(3)_S$ can remain intact

- If for a good (experimental or theoretical) reason the sterility wants to remain conserved only the Majorana self-energy Σ_L of ν_L is dynamically generated by flavor gluon exchanges
- Nine massless neutrinos are practically decoupled
- "Standard" Majoron

Conclusion

- Rough framework: technical steps may undergo major changes
- Non-minimal neutrino scenario
- If sterility must be conserved only left-handed Majorana masses are allowed
- No generic Fermi scale
- Shining or dark, matter should be of one origin
- Phenomenology not developed
- "Elegance is an attitude"

To solve the SD equation for Σ we have to know low-momentum (t $\bar{h}^2_{ab}(q^2)$ momentum behavior we know)

- Ansatz

$$\frac{h^2}{k^2} = \frac{h^2}{k^2}[(1-\Pi(k^2))^{-1} - \Pi(k^2)(1-\Pi(k^2))^{-1}]$$

- Resulting

$$\bar{h}^2_{ab}(q^2) = h^2_*[-\Pi(q^2)(1-\Pi(q^2))^{-1}]_{ab}$$

- Expected nonperturbative IR fixed point

$$\beta(\bar{h}) \equiv d\bar{h}/d\ln(\mathrm{q}/\mathrm{M}) = \frac{1}{\mathrm{h}^2_*}\bar{\mathrm{h}}(\bar{\mathrm{h}}^2 - \mathrm{h}^2_*)$$

(i) Masses of charged leptons, u-type and d-type differ due to different vertices. (ii) For given charge fermion masses differ due to the mass matrix M_{ab} of flavor gluons. (iii) In finding numerical solution we have so far failed. (iv) Close to the fixed point fermion masses should be small.

- Drastic approximation: Neglect the matrix structure of both Π and Σ, neglect momentum dependence and replace Π/q^2 by M^2/q^2 and Σ by m.
- SD equation turns into

$$m = \frac{h_*^2}{16\pi^2} \int_0^\infty dk^2 \frac{M^2}{k^2+M^2} \frac{m}{k^2+m^2}$$

- Solution is: $m=M \exp[-8\pi^2/h_*^2]$
- For $M=10^3$ TeV

$m_\nu=10^{-9}$ GeV corresponds to $h_\nu^2/4\pi=2\pi/\,15 \ln 10$

$m_t=10^2$ GeV corresponds to $h_t^2/4\pi=2\pi/\,4 \ln 10$

Intermediate boson mass generation

$$\Gamma_W^\alpha(p+q,p) = \frac{g}{2\sqrt{2}}\{\gamma^\alpha(1-\gamma_5)-$$
$$-\frac{q^\alpha}{q^2}[(1-\gamma_5)\Sigma_U(p+q)-(1+\gamma_5)\Sigma_D(p)]\},$$

$$\Gamma_Z^\alpha(p+q,p) = \frac{g}{2\cos\theta_W}\{t_3\gamma^\alpha(1-\gamma_5)-$$
$$-2Q\gamma^\alpha\sin^2\theta_W - \frac{q^\alpha}{q^2}t_3[\Sigma(p+q)+\Sigma(p)]\gamma_5\},$$

$$m_W^2 = \frac{1}{4}g^2\sum(m_U^2 I_{U;D}(0) + m_D^2 I_{D;U}(0))$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2)\sum(m_U^2 I_{U;U}(0) + m_D^2 I_{D;D}(0))$$