

DYNAMIC MATCHED FILTERS FOR GRAVITATIONAL WAVES DETECTION

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Abstract

The algorithms for the detection of gravitational waves are usually very complex due to the low signal to noise ratio. In particular the search for signals coming from coalescing binary systems can be very demanding in terms of computing power, like in the case of the classical Standard Matched Filter Technique. To overcome this problem, we tested a Dynamic Matched Filter Technique, still based on Matched Filters, whose main advantage is the requirement of a lower computing power. In this work this technique is described, together with its possible application as a pre-data analysis algorithm. Also the results on simulated data are reported.

INTRODUCTION

The detection of Gravitational Waves (GW) from coalescing binary systems requires high performances data analysis techniques for signal extraction from the detector noise. In fact, the signal-to-noise ratio of GW signals is in general very low, due to their intrinsic weakness with respect to the detectors instrumental noise[1], also for earth based interferometric antennas, like VIRGO[2], LIGO[3], GEO[4], TAMA[5].

Actually, when the shape of the expected signals is known, the optimal technique is the Standard Matched Filter Technique (*SMFT*), that integrates the classical optimum Wiener-Komolgorov filter[6] with a static template coverage of the parameters space whose grid density is defined by the requirement on the signal detection level of confidence. As an example, the detection of a GW signal from coalescing binary systems for masses ranging from $1.4 M_{\odot}$ to $50 M_{\odot}$ with a signal-to-noise recovery of 90%, requires a static grid of $\sim 9,8 \cdot 10^5$ templates and a computing power of ~ 300 GFlop for the VIRGO detector[7][8].

On the other hand, the *SMTF* has two weak points, that are its sensitivity to the template shape and the assumption of Gaussian White Noise. In particular, slight differences between the theoretical templates and the real signals largely reduces the quality of the signals and the validity of the *SMFT* solution.

To overcome this problem, a straight solution is to improve the template generation model, but it also

increases the number of its physical and geometrical parameters and, as consequence, the number of templates of the static grid necessary for an adequate data analysis. The practical consequence is that a full on-line data analysis would require computing powers not available now[9]. Therefore, there is a clear need for developing also different data analysis strategies, which may be a good compromise between the available computing power and the most efficient coverage of the space of parameters. A possible solution is a hierarchical strategy, consisting in a on-line rough data analysis to select all the frames which may contain a signal, followed by a refined off-line search, like the *SMTF* [10].

We decided to follow this strategy introducing a Dynamic Matched Filter Technique (*DMFT*), in which the static grid of templates of the *SMFT* is replaced with a dynamic one, on-line generated using a global optimization algorithm, a solution that fully overcomes the computing power problems, although intrinsically no more optimal. In the following sections we will describe the *DMFT* and discuss its possible application as a pre-data analysis algorithm and the results of preliminary tests on simulated data.

DYNAMIC MATCHED FILTER TECHNIQUE

The goal of the Standard Matched Filter Technique (*SMFT*) is that of computing the *SNRs* for all the static set of templates, characterized by the full coverage of the space of parameters using the level of confidence of the detection as requirement. In presence of a GW signal, the largest *SNR* corresponds to its template, in absence of a GW signal, the *SNRs* should lie below a threshold, evaluated from the knowledge of the detector output noise. Therefore, the GW search is a classical global optimization procedure in which the objective function, that is the function to maximize, is the *SNR*, while the physical and geometrical parameters of the system, used to generate the templates, are the solution.

The Dynamic Matched Filter Technique (*DMFT*) simply changes the static template grid with a dynamic one, defined by the evolution of the global optimization algorithm used for its management. Although the *DMFT* is still based on Matched Filters, it is clear that it cannot guarantee the level of confidence of detection as the

SMFT. In fact, it is not possible to demonstrate that an optimization algorithm, although global, always reaches the global maximum of the objective function, while it is possible to demonstrate that it always converges toward a local maximum. Critical in the *DMFT* is the choice of the optimization algorithm. In fact, it must be robust, constrained and global, that are very important characteristics in a difficult search like this one, especially when the number of physical and geometrical parameters is very high and the objective function is not smooth. On the other hand the real advantage of the *DMFT* is that it requires a number of *SNR* evaluations that is orders of magnitude less than the *SMFT*. This characteristic allows us using it as an on-line pre-data analysis technique, since only small computer farms are required and the detection of candidates to be analyzed with more refined off-line techniques can be very useful.

For sake of completeness, it is important to underline that each *SNR* evaluation requires the on-line template generation, which must be included in the global computing power required by the *DMFT*, requiring template generation algorithms optimized for an on-line application.

THE PRICE ALGORITHM

For our tests we decided to use the Price Algorithm as global optimization algorithm[11]. This algorithm, also known as Controlled Random Search (*CRS*), is a very powerful extension of the simplex algorithm, that we have applied to very difficult optimization procedures in astrophysics with up to 30 degrees of freedom, developing and applying also improved versions[12].

In the following we will provide a short description of the way of working of the original version.

Let be $f(x_1, x_2, \dots, x_m)$ a function of m variables whose maximum (or minimum) has to be found within a fixed hypervolume, V . An array of N test points is randomly generated in V , computing the function f at each point.

A subset of $m+1$ points, P_1, P_2, \dots, P_{m+1} , is extracted from the search array and a new test point is determined according to the relation

$$P = 2G - P_{m+1}$$

where G is the centroid of the first m points. The algorithms check that the new test point P is within the hypervolume V , otherwise it generates a new one. If the value of $f(P)$ is lower than the value of P_{\max} , the maximum value of the objective function stored in the search array and the point P will replace P_{\max} in the search array, otherwise a new test point is generated. The algorithm iterates with this scheme, and the array of N points tends to cluster around the maximum of f , but it can randomly reach also zones far from the detected maximum.

This is very useful to get out of the local maxima. The original Price algorithm does not define precise stop

criteria, which can be defined in an effective way by the user[13].

The application of the Price optimization algorithm within the matched filters scenario is easy and effective. In fact, clear advantages of this procedure are the lower number of templates needed to build the search array and its independence from the number of parameters of a template. On the other hand, problems arise due to the lack of statistical information about the level of confidence of the detection (its statistical convergence to a maximum was instead demonstrated[11] and the need of a dynamic evaluation of the templates, instead of their static storage as in the *SMFT*. Finally, it is important and critical the choice of a suitable number of templates in the starting search array, to avoid a poor definition of the maximum (too small search arrays) or too long computational time (too large search arrays).

ALGORITHM IMPLEMENTATION

The basic implementation of algorithm has been written in C and is supplied of two criteria of arrest, one on the number of iterances and the second on the maximum difference of the values assumed from the function in the points of the matrix.

We have also write a parallel implementation of algorithm realized by using the Message Passing Interface (MPI) in order to increase the speed of the *DMFT*.

The parallel algorithm have a stellar architecture, where the role of the slave nodes is to compute the *SNR* of the current data set with respect to a select template, while the master node executes the update on the basic matrix.

This behavior does not corresponds to an exact Price algorithm, because it may happen that a point in the matrix is removed be the master while it is still used in the *SNR* computation on another node. In any case, if the number of points in the matrix is enough large respect to the number of computing slaves, no significant effect is reflected on the procedure effectiveness as confirmed by our result.

TEST OF THE ALGORITHM

We tested the Dynamic Matched Filter Technique using of a signal simulating the output of an GW interferometric detector with $SNR = 20$.

The GW signal was a chirp computed using the PN2 approximation, simulating the coalescence of a binary system with masses equal to $5 M_{\odot}$ and $7 M_{\odot}$, within Gaussian White Noise.

The length of the data set was 16.384 s, sampled at 4 kHz. The lower cut-off frequency was fixed at 40 Hz. The Price algorithm used an array of 100 points with a mass range from $4 M_{\odot}$ to $25 M_{\odot}$. Although statistical stop criteria should be used for stopping the optimization procedure[11], in the example reported here we preferred to use, for clarity, the number of iterations, fixed in 3000,

as stop criterion. The evolution of the optimization procedure is shown in Figure 1.

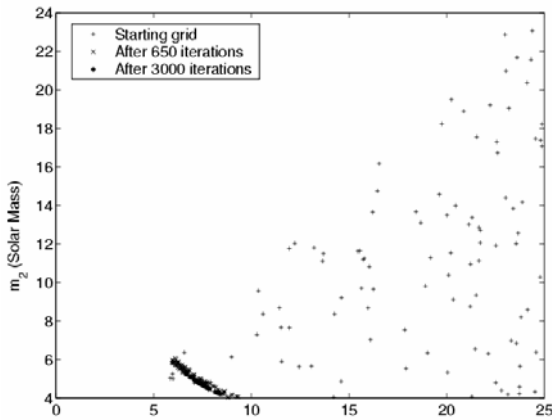


Fig 1
Price Evolution

The clustering around the true solution is evident, as is the presence of a signal according to the values of the SNR. This clearly shows that the *DMFT* can be effectively used as an on-line pre-analysis tool and as a trigger, by checking for the presence of a GW signal and giving the possibility of reducing the amplitude of the volume of the space of parameters, making the application of the *SMFT* easier.

CONCLUSION

In this paper we discussed on the possibility to use dynamic algorithm for GW detection from coalescing binaries. We tested the *DMFT* in connection with the the Price Algorithm in our basical implementation and by using a parallel algorithm, and we discussing its clear advantages in term of computing power, but outlining also its critical points.

Finally, we propose to use it as an on-line GW detection pre-analysis tool, in connection with the *SMFT*

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