

# ADDING KAONS TO THE BERTINI CASCADE MODEL

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## Abstract

A version of the Bertini cascade model for hadronic interactions is part of the Geant4 toolkit, and may be used to simulate pion-, proton-, and neutron-induced reactions in nuclei. It is typically valid for incident energies of 10 GeV and below, making it especially useful for the simulation of hadronic calorimeters. In order to generate the intra-nuclear cascade, the code depends on tabulations of exclusive channel cross section data, parameterized angular distributions and phase-space generation of multi-particle final states. To provide a more detailed treatment of hadronic calorimetry, and kaon interactions in general, this model is being extended to include incident kaons up to an energy of 15 GeV. Exclusive channel cross sections, up to and including seven-body final states, will be included for  $K^+$ ,  $K^-$ ,  $K^0$ ,  $\bar{K}^0$ ,  $\Lambda$ ,  $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ,  $\Xi^0$  and  $\Xi^-$ .  $K^+$ -nucleon and  $K^-$ -nucleon cross sections are taken from various cross section catalogs, while most of the cross sections for incident  $K^0$ ,  $\bar{K}^0$  and hyperons are estimated from isospin and strangeness considerations. Because there is little data for incident hyperon cross sections, use of the extended model will be restricted to incident  $K^+$ ,  $K^-$ ,  $K_S^0$  and  $K_L^0$ . Hyperon cross sections are included only to handle the secondary interactions of hyperons created in the intra-nuclear cascade.

## LOW ENERGY KAONS IN GEANT4

Simulating the propagation of low energy kaons (0 - 5 GeV) through materials is becoming an important part of the design of new high energy detectors and the validation of results from existing detectors. The Geant4 toolkit [1] currently provides three hadronic models which apply to pions, protons and neutrons in this energy range. These are the Low Energy Parameterized (LEP), Binary Cascade, and Bertini Cascade. Only one of these, the LEP model, may also be applied to incident kaons. However, the LEP model is not especially suited for these energies and is known to perform poorly for kaons. The Binary Cascade model may in the future deal with strange particles but this must await further theoretical developments. The remaining candidate, the Bertini Cascade model, is the most likely to be extended to incident kaons; it is relatively easy to extend to strange particles and is known to work reasonably well in this energy range.

## EXTENDING THE BERTINI CASCADE

The Bertini Cascade model [2] propagates incident particles through the nucleus in much the same way as Geant4 propagates particles through a material: an interaction cross section is found, the interaction length is calculated, an interaction occurs and a final state is generated. The final state particles are in turn propagated until they either interact or leave the nucleus.

### Cross Sections

The Bertini cascade assumes that particle-particle interaction cross sections and branching ratios within the nucleus are given by their free-space counterparts. So, in order to extend the model to include kaons,  $K^+p$ ,  $K^-p$ ,  $K^+n$ ,  $K^-n$ , cross section measurements are required. An extensive list of these cross sections and branching ratios is provided in the CERN catalog [3]. Above an incident momentum of about 15 GeV/c the data begin to thin out significantly, setting an upper bound on the applicability of the extended model. Incident  $K_L^0$  and  $K_S^0$  must also be included in the model, thus requiring the  $K^0$  and  $\bar{K}^0$  cross sections for intra-nuclear propagation. After the initial interaction with a nucleon, hyperons may be produced and subsequently interact with other nucleons before leaving the nucleus. Hence,  $\Lambda$ -,  $\Sigma$ - and  $\Xi$ -nucleon cross sections are also needed. Many of these were taken from Ref. [4].  $\Omega^-$  cross sections were not added due to their small values at these energies.

Of course many of the required cross sections have never been measured, and estimates or guesses are required to fill in the missing information. Where there are gaps in the energies at which measurements were made, a simple linear interpolation of the cross sections is employed in the extended model. If there are unmeasured channels from a given reaction, the channel cross sections are filled in by using the total cross section measurements as a constraint. As much as possible, missing cross sections were estimated by using isospin and strangeness conservation. For example, it was assumed that

$$\sigma_{K^0p} = \sigma_{K^+n} \quad (1)$$

and

$$\sigma_{\bar{K}^0n} = \sigma_{K^-p}. \quad (2)$$

## Final State Generation

For each interaction type, the model keeps a list of final state channels and particle types. For incident pions, protons and neutrons, the existing model keeps track of one-through six-body final states up to 10 GeV. In the extended model the number of particle types was increased to include kaons and the lowest mass hyperons. No resonances are included. The possible final state multiplicity was also increased (for incident strange particles) from six to seven. This reflects the fact that the sum of seven-body final states represents a significant fraction of the total cross section.

The final state momenta are sampled from parameterized angular and momentum distributions. The extended model uses the same distributions despite the fact that they were derived from pion, proton and neutron data.

## Intra-nuclear Propagation

In the extended model, strange particles are propagated through the nucleus using a very simple hard-sphere potential which is 7 MeV deep. This is identical to that used for propagating pions. This will be improved in a future version of the model since it is known that the real part of the optical potential for kaons and hyperons must be deeper.

Pauli blocking is implemented in the existing model for protons and neutrons, but is not required for strange particles.

In addition, there are no nucleon-nucleon correlations in the extended model. In the existing model pions may be absorbed by quasideuterons; this will be done for kaons in a similar manner in future versions.

## PRELIMINARY RESULTS

Early tests of the extended model have been made by comparing to inelastic  $K^+$  scattering data [5]. The data feature a study of the quasielastic peak in D, C, Ca and Pb targets, and should provide a useful test of the intra-nuclear kinematics in the model. Preliminary results for C and Pb targets are shown in Figs. 1 and 2, respectively.

As the  $43^\circ$  data indicate, the model gets the energy of the quasielastic peak more or less correct, while overestimating its width. This may be related to the too-shallow nuclear potential for kaons. The overall normalization is low by about 30%. The data have a systematic error of 11%, so most of the difference is likely due to the model or the value of the total inelastic cross section.

At  $24^\circ$  the model also seems to show the quasielastic peak in the right place. This is harder to estimate because the data include elastic scattering and scattering from low-lying collective levels in the nucleus, while the model deals only with inelastic, incoherent scattering. It also appears that the width is overestimated as at  $43^\circ$ .

Similar results were observed for the D and Ca targets.

## Low Energy Parameterized Model

The same targets were tested using the LEP model. In all cases, there were no  $K^+$  produced in the quasielastic peak at either angle. In fact, no  $K^+$  appeared in the spectrum at any value of the nuclear excitation energy. Instead the model converted the  $K^+$  into  $K_L^0$ ,  $K_S^0$  and pions. The incident  $K^+$  momentum had to be raised from 0.705 GeV/c to to 2 GeV/c before any  $K^+$  appeared at all in the final spectrum.

## CONCLUSIONS

The extension of the Bertini cascade model to include kaons is still in its early stages. Preliminary results are encouraging and provide an obvious improvement over the LEP model. Validation of the model must be extended to higher energies when data become available, and comparisons to  $K^-$ ,  $K_L^0$  and  $K_S^0$  data need to be made. As mentioned above, several improvements are needed:

- find better nuclear potentials for  $K^+$ ,  $K^-$  and  $\Lambda$
- add nucleon-nucleon correlations
- parameterize and use measured angular distributions for strange particle final states
- install pion-, proton- and neutron-induced strange particle production .

## REFERENCES

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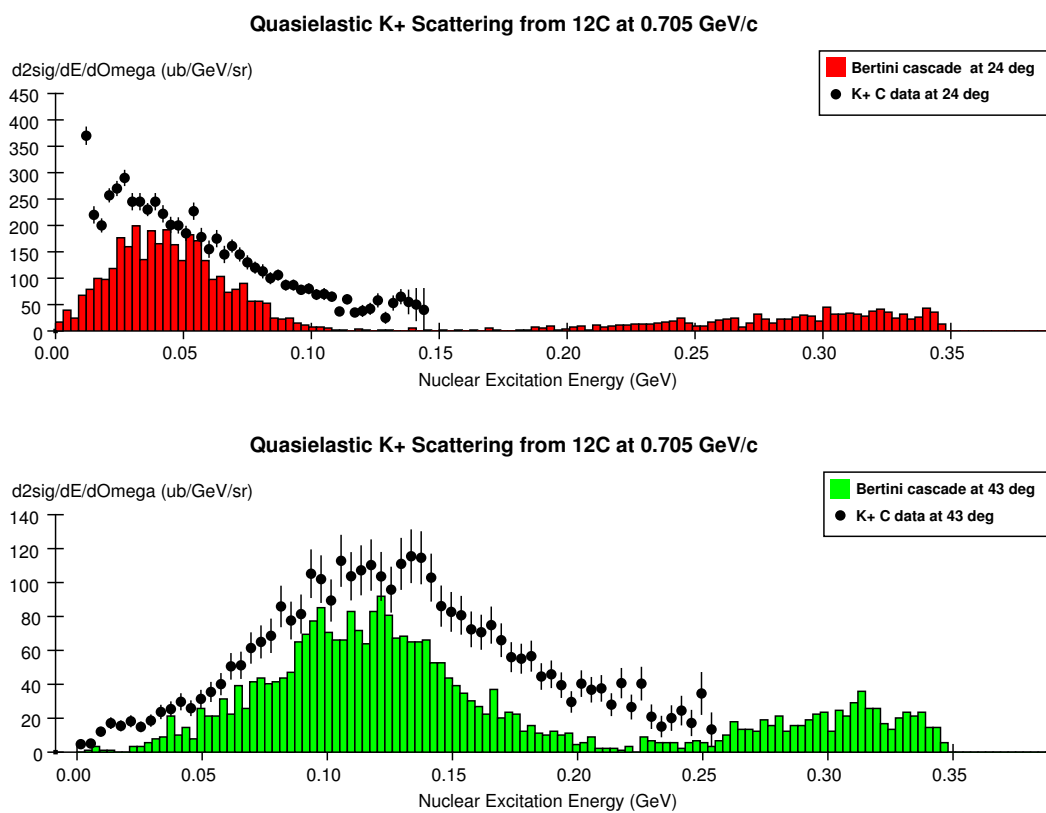


Figure 1: Inelastic K<sup>+</sup> scattering from C at 24 and 43 degrees. The incident K<sup>+</sup> momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section,  $d^2\sigma/dE/d\Omega$ , in  $\mu\text{b}/\text{GeV}/\text{sr}$ .

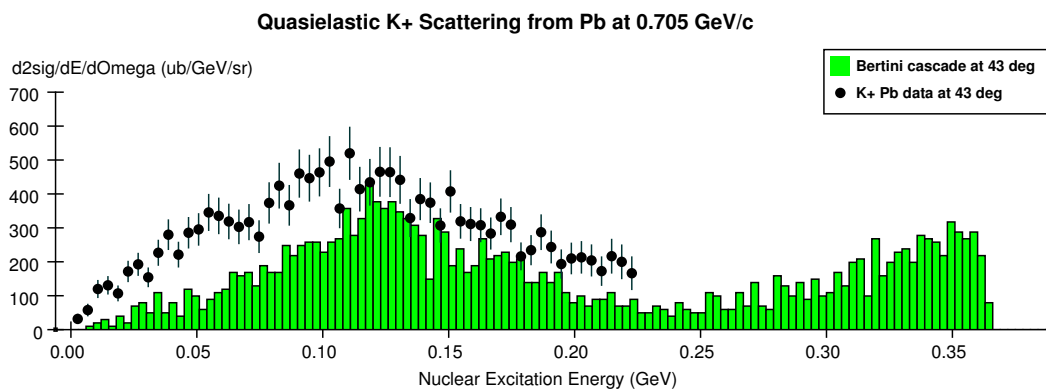
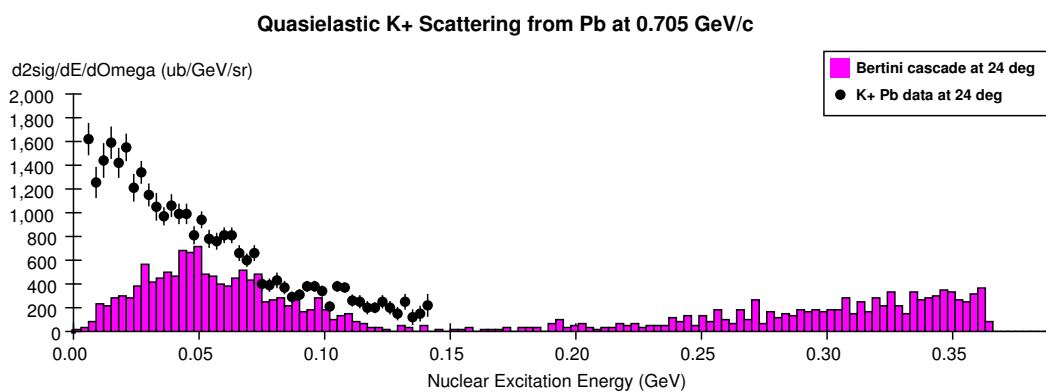


Figure 2: Inelastic  $K^+$  scattering from Pb at 24 and 43 degrees. The incident  $K^+$  momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section,  $d^2\sigma/dE/d\Omega$ , in  $\mu\text{b}/\text{GeV}/\text{sr}$ .