Adaptive Multi-vertex Fitting

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Introduction

The Multi-Vertex Fitter is an algorithm that fits n vertices simultaneously. In each vertex fit, each track is weighted according to its distance to the current vertex. The Multi-Vertex Fitter (MVF,[1]) is very similar to the Adaptive Vertex Fitter (AVF,[2]) in that it has a soft assignment with a weight function that is a generalization of the AVF weight function. For the special case of a single vertex the two methods are equivalent.

The fit

The fitting procedure is iterative and can be described as follows:

The user supplies reconstructed tracks and (optionally) initial assignment probabilities between tracks and vertices.	
	Fit vertices, using the assignment probabilities.
	Recompute assignment probabilities.
Repeat until convergence.	

The weight function

Assume that there are n tracks to be fitted to m vertices. The weight of track i with respect to vertex j is given by

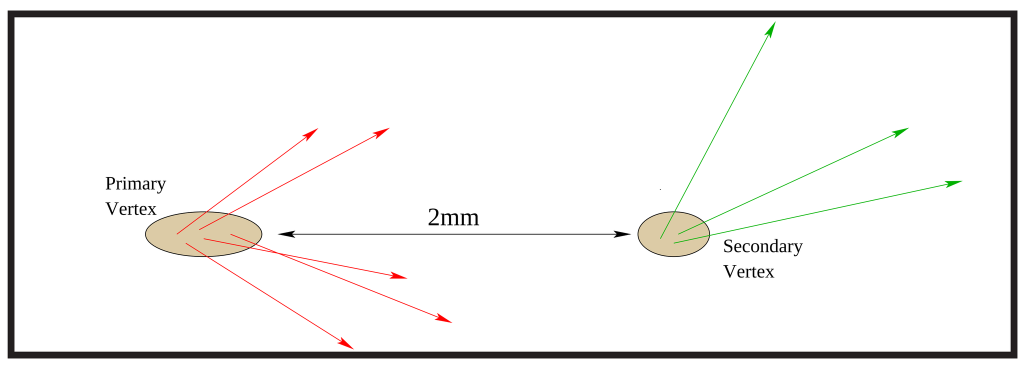
$$w_{ij} = \frac{\exp(-\chi^2_{ij}/2T)}{\exp(-\chi^2_{\rm cut}/2T) + \sum_{k=1}^{n} \exp(-\chi^2_{ik}/2T)}$$

χ^2_{ij} is the χ^2-compatibility between track i and vertex candidate j. $\chi^2_{\rm cut}$ is a cutoff which suppresses tracks which do not fit to any of the vertices. T is a temperature parameter which is lowered in the course of the iterations (annealing).

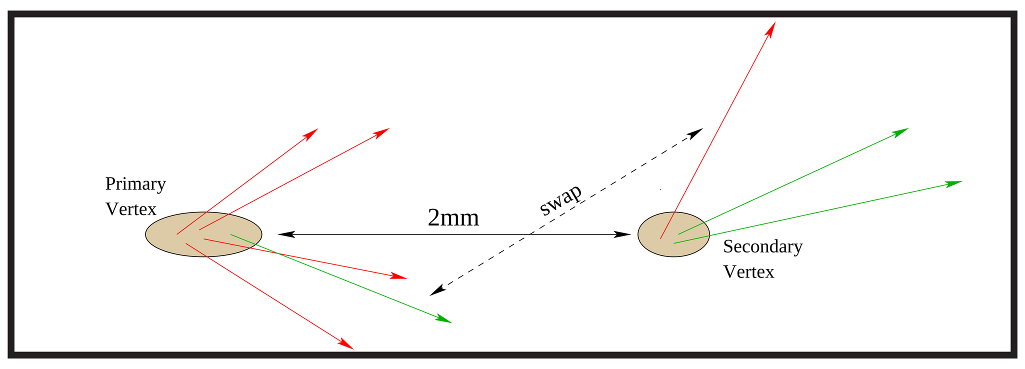
Verification strategy

The algorithm was implemented in the CMS reconstruction program ORCA [3]. The verification was done using the Vertex Gun developed for ORCA. The Vertex GUn is a fast simulation tool that generates reconstructed tracks, bypassing event generation, detector simulation and track reconstruction.

The verification sample consists of events with two vertices. Five tracks are attached to the primary vertex at $(0, 0, 0)$. The tracks form a "jet" with total jet momentum $\vec{p}_{\rm pr} = (0, 25, 25)$, and opening angle=0.5. A secondary vertex is positioned two millimeters from the primary vertex. It has three tracks, jet momentum $\vec{p}_{\rm sec} = (-15, 0, 20)$, and opening angle=0.5. All tracks are "perfect", i.e. the errors are Gaussian and correctly described by the covariance matrices attached to the tracks.



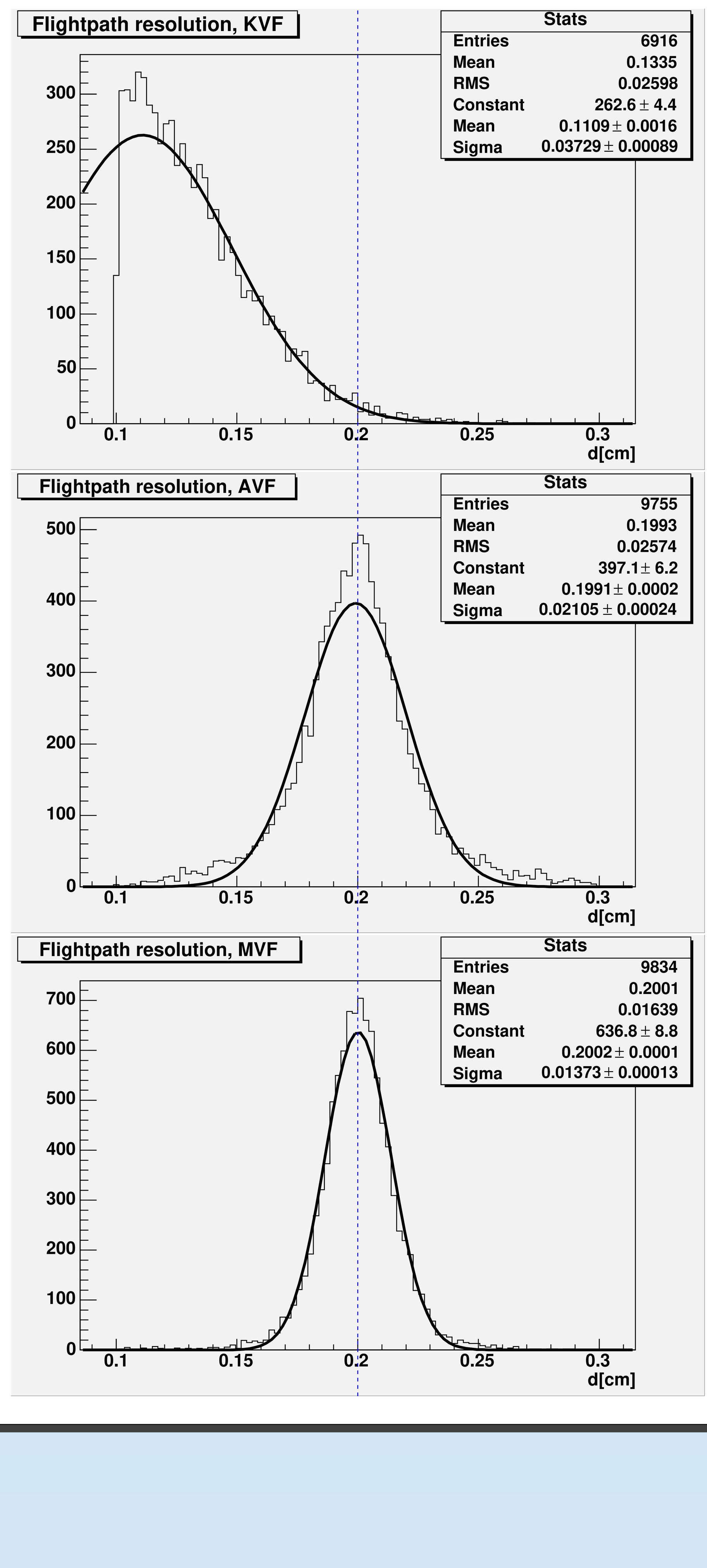
The the track of the primary vertex that is most compatible to the secondary vertex is selected, and vice versa. Those two tracks are moved to the "wrong" vertices — the primary track is assigned to the secondary vertex and vice versa. These two track bundles with one mis-associated track in each bundle are the input to the MVF.



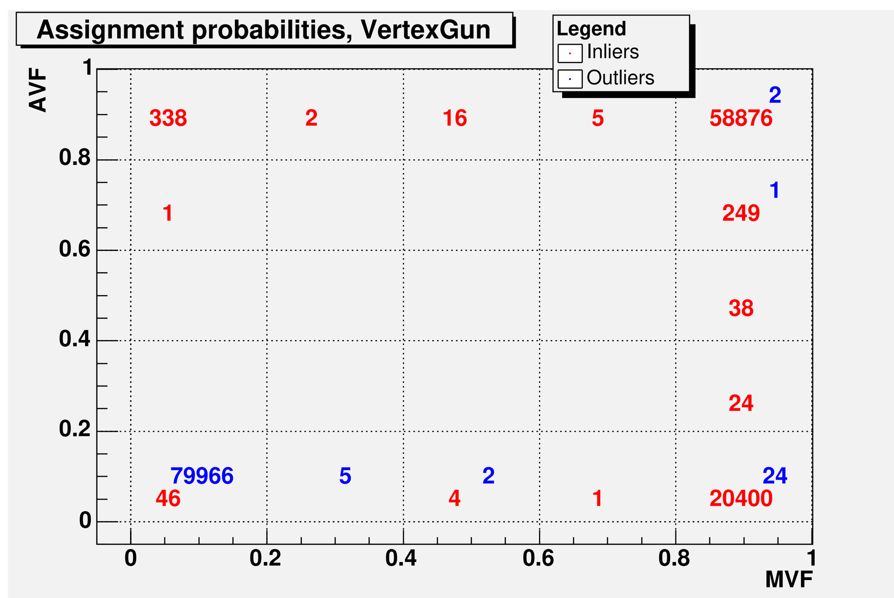
Results

We have compared the MVF to the AVF and to the Kalman Vertex Fitter (KVF). The most important indicator of the quality of vertex reconstruction is the resolution of the distance between the two reconstructed vertices, i.e. the decay length of a short-lived particle connecting the two vertices.

The KVF and the AVF cannot recover the mis-associated tracks. The KVF estimate shows a huge bias, because all tracks enter the fit with the same weight. The AVF downweights the outliers, but does not use the tracks that are associated to the wrong track bundle. Its estimate is therefore unbiased, but the resolution is worse than the one of the MVF.



It is instructive to directly compare the assignment probabilities of teh AVF and the MVF. The table shows 10000 events with 8 tracks and 2 vertices per track, resulting in 80000 correct track-to-vertex associations ("inliers") and 80000 wrong associations ("outliers"). It can be seen that both algorithm almost always identify the outliers. AVF fails to identify only three outliers; MVF assign about 30 tracks to the wrong vertex. In the case of inliers an ideal AVF can only do right in 60000 cases; 20000 tracks are misassociated from the very beginning and cannot be recovered. It can be seen that the AVF comes very close to the theoretical limit. Only about 450 inlying tracks (less than one percent) are downweighted when they should not be. The MVF performs similarly with respect to its own theoretical limit, only its theoretical limit is much higher, namely 80000. About 400 tracks have a weight close to zero where the right weight would be one or close to one.



An evaluation of the MVF in terms of physics, especially b-tagging, is under way. The preliminary results are very encouraging.

Future developments

A short term development of the MVF will be to introduce additional "hard assigned" tracks, i.e. tracks that are associated to one specific vertex, with the association being in a "frozen" state. This additional feature can be used to exploit knowledge that comes from external sources. As a longer term development a generalization of these "hard assigned" tracks seems to be desirable. We believe it would be useful to implement a framework for any kind of constraints, be it on the tracks, its weights, or a quantity related to the vertex. A way to achieve this could be to interact with the kinematic package [4]. This package implements kinematic constraints via the Lagrange multiplier formalism.

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