

# Holography & Dynamical Symmetry Breaking

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AdS/CFT Correspondence duality:

4d strongly coupled  $\mathcal{N}=4$  SYM = IIB strings on  $AdS_5 \times S^5$

3+1d  $\mathcal{N}=4$  gauge theory

$A^\mu$      $6\phi$      $4\psi$

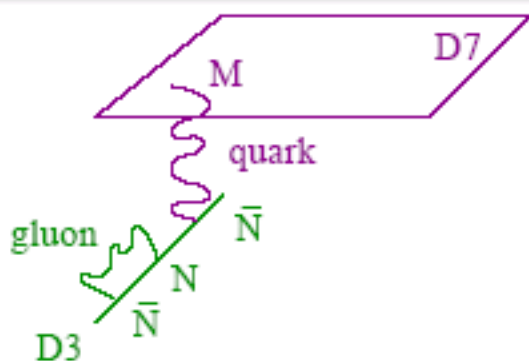
Large N

$Z(\text{classical SUGRA}) = Z(\text{quantum field theory})$

The radial direction in AdS is equivalent to  
RG scale

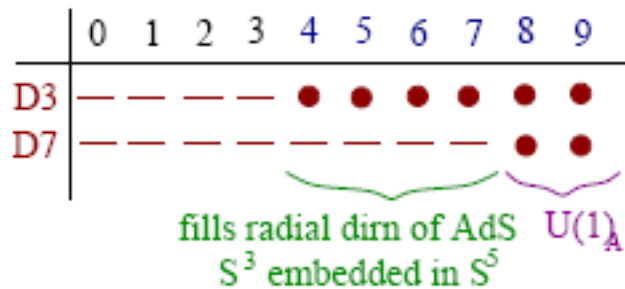
# Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...



Quarks can be introduced via D7 branes in AdS

The brane set up is



We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \quad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

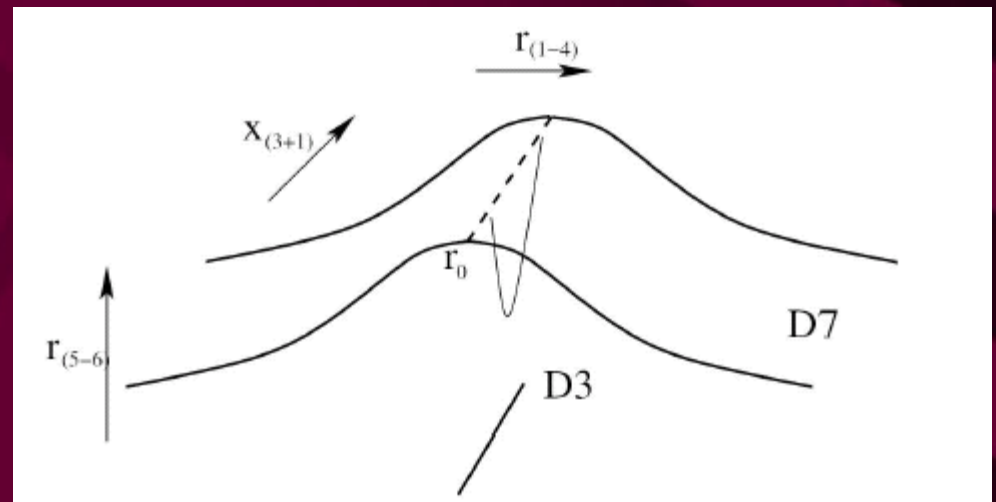
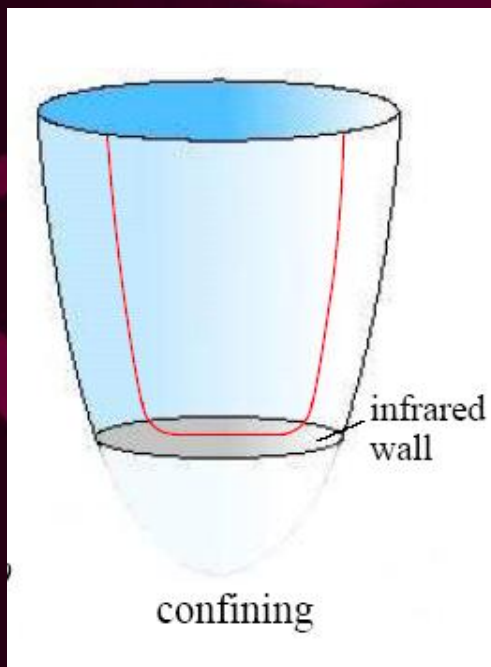
# Add Confinement and Chiral Symmetry Breaking

$$ds^2 = \frac{r^2}{R^2} A^2(r) dx_{3+1}^2 + \frac{R^2}{r^2} dr^2,$$

$$A(r) = \left(1 - \left(\frac{r_w}{r}\right)^8\right)^{1/4}, \quad e^\phi = \left(\frac{1 + (r_w/r)^4}{1 - (r_w/r)^4}\right)^{\sqrt{3/2}}$$

Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall....



BEEGK, Ghoroku..

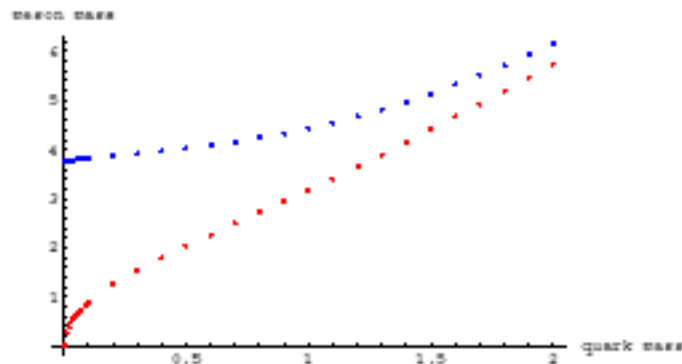
# Pion Physics

Seek pion solutions of the form

$$\pi(x, r) = f(\rho)e^{ikx}, \quad k^2 = -M^2$$

$f(\rho)$  must be smooth - normalizable - at all  $\rho$

The pion and sigma masses can thus be computed as a function of quark mass



There is a Goldstone in the massless limit.

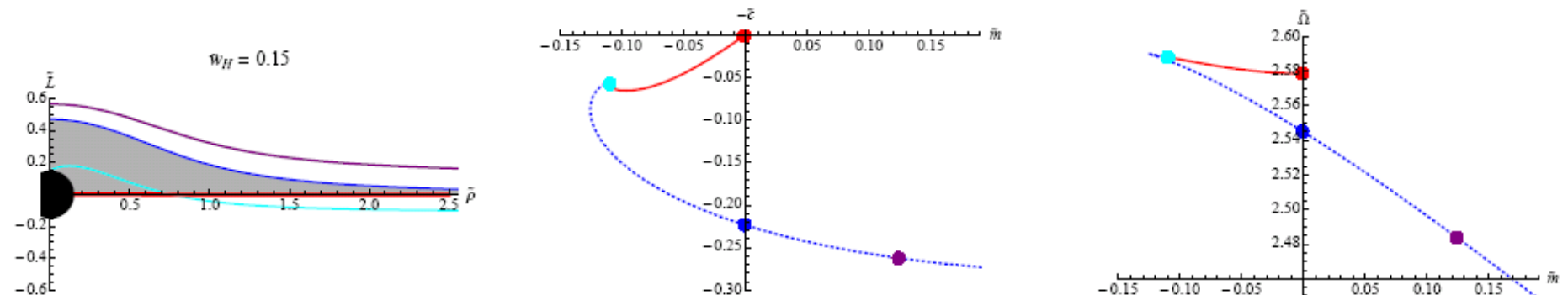
Expected  $\sqrt{m}$  behaviour

# Magnetic Field Induced Chiral Symmetry Breaking

Johnson, Filev, Kundu....

$$\mathcal{L} = \rho^3 \sqrt{1 + (\partial_\rho w_6)^2} \sqrt{1 + \frac{Bz R^4}{r^4}}$$

B “just” breaks  
conformal  
symmetry...



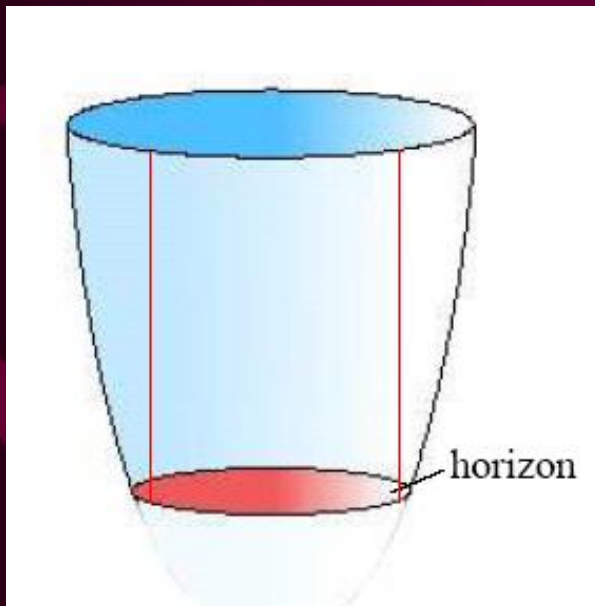
(a) Low temperature -  $\tilde{w}_H = 0.15$ . Here we see chiral symmetry breaking with the blue embedding thermodynamically preferred over the red at  $\tilde{m} = 0$ .

# Finite T - AdS-Schwarzschild

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2$$

where  $R^4 = 4\pi g_s N \alpha'^2$  and

$$f := 1 - \frac{r_H^4}{r^4}, \quad r_H := \pi R^2 T .$$



Quarks are  
screened by  
plasma

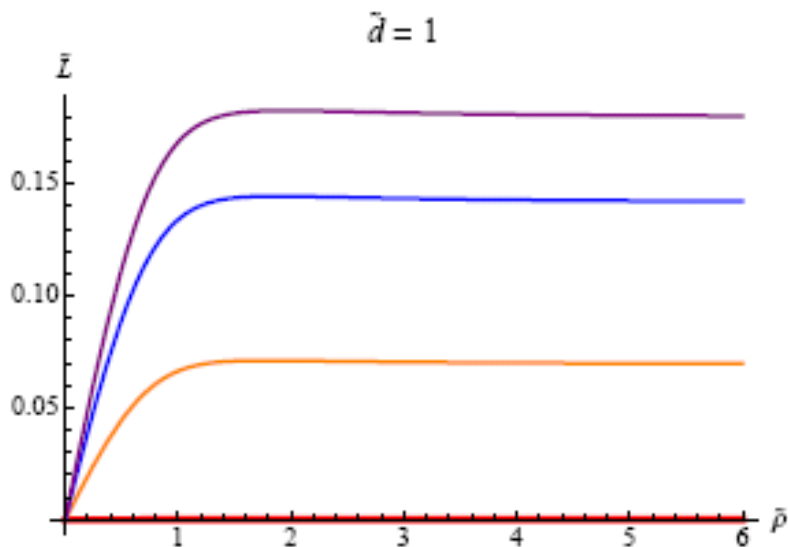
Asymptotically  
AdS, SO(6)  
invariant at all  
scales... horizon  
swallows  
information at  $r_H$   
.... Witten  
interpreted as finite  
temperature...  
black hole... has  
right  
thermodynamic  
properties...

# Chemical Potential

$$\bar{\psi}i(-iA^t\gamma_0)\psi \rightarrow \bar{\psi}\mu\gamma_0\psi$$

$$\mathcal{L} = \rho^3 \sqrt{1 + (\partial_\rho w_6)^2 - (\partial_\rho A_0)^2}$$

Myers,  
Mateos

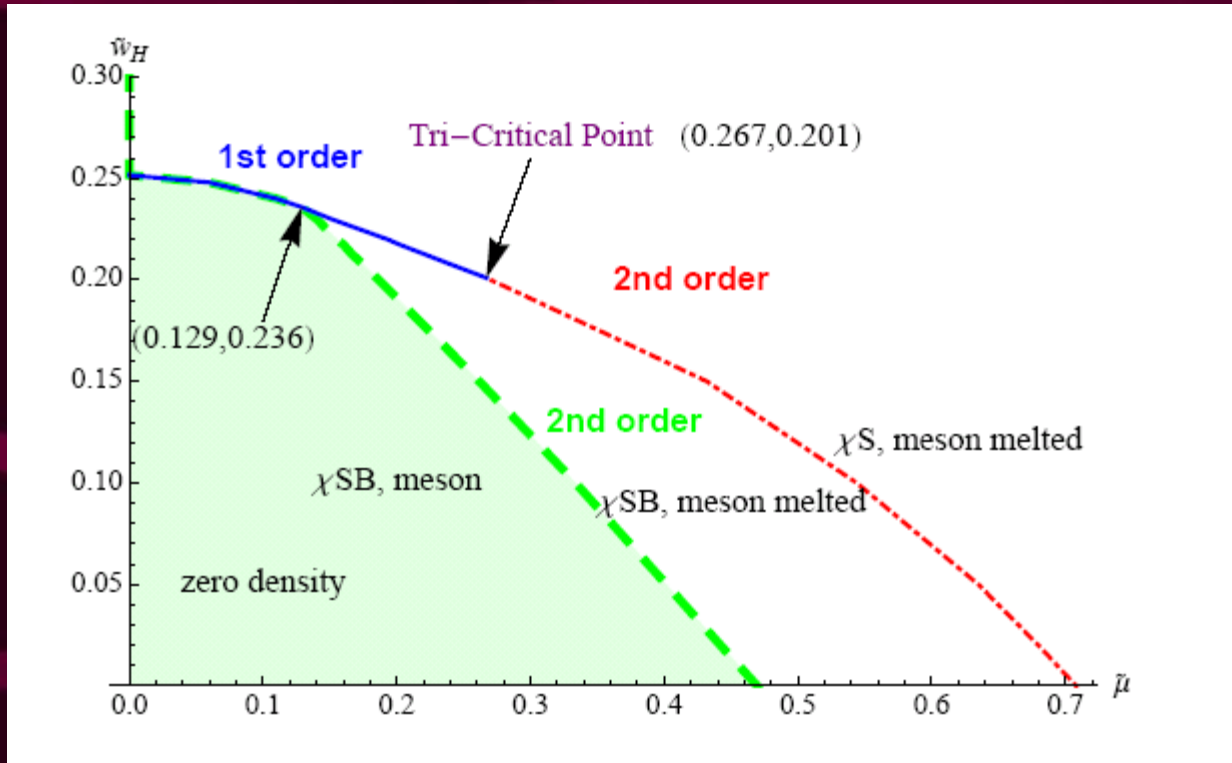


$\mu$  induces quarks to fill the vacuum....

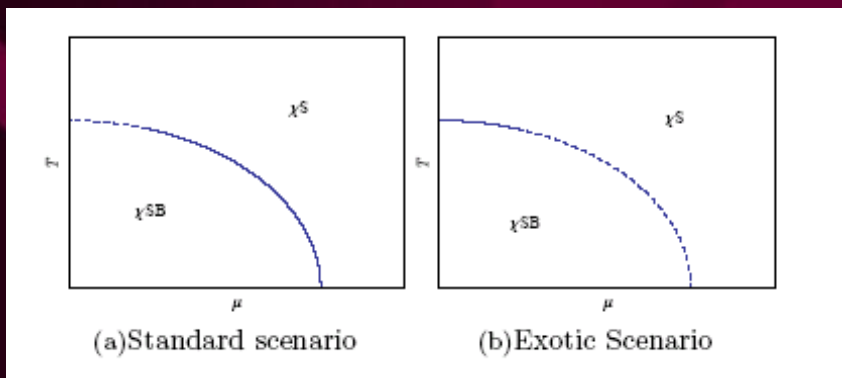
ie a spike of strings grows between the D7 and the D3...



# Phase Diagram for B Field Theory

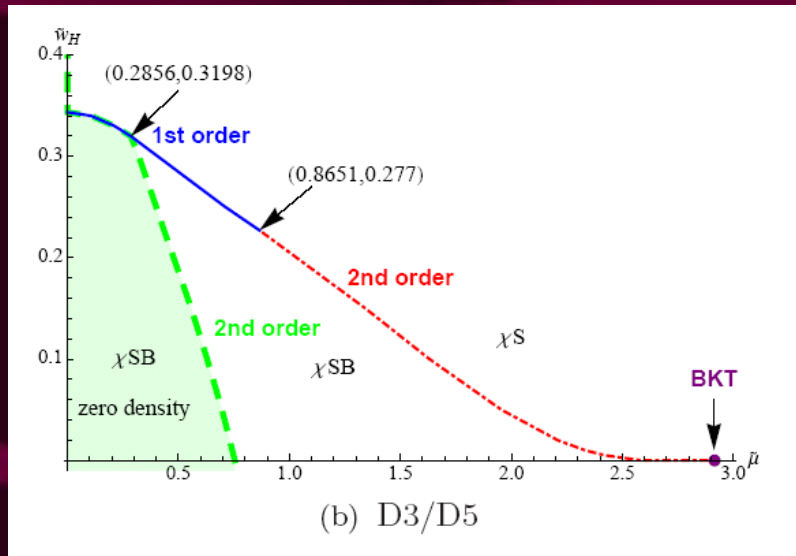


**JHEP**  
**1003:132,2010.**  
 e-Print:  
**arXiv:1002.1885**  
 [hep-th]



QCD scenarios for  
 comparison... Wilczek vs  
 Philipsen (Lattice)

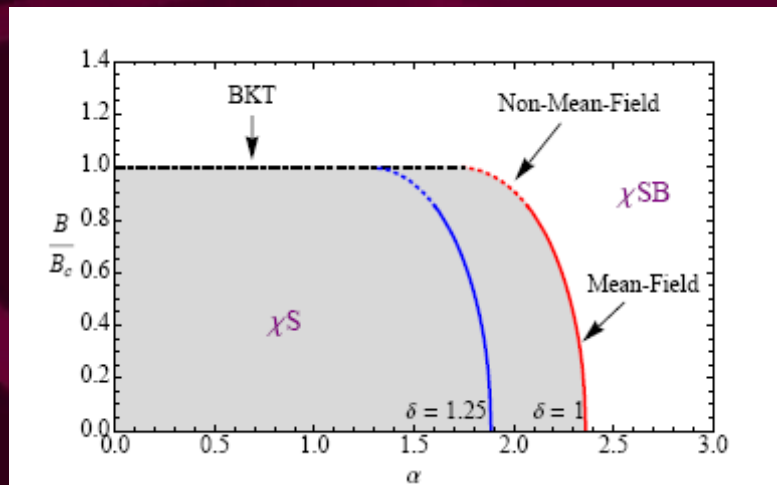
# Phase Diagram for B Field Theory – 2+1d



A BKT transition

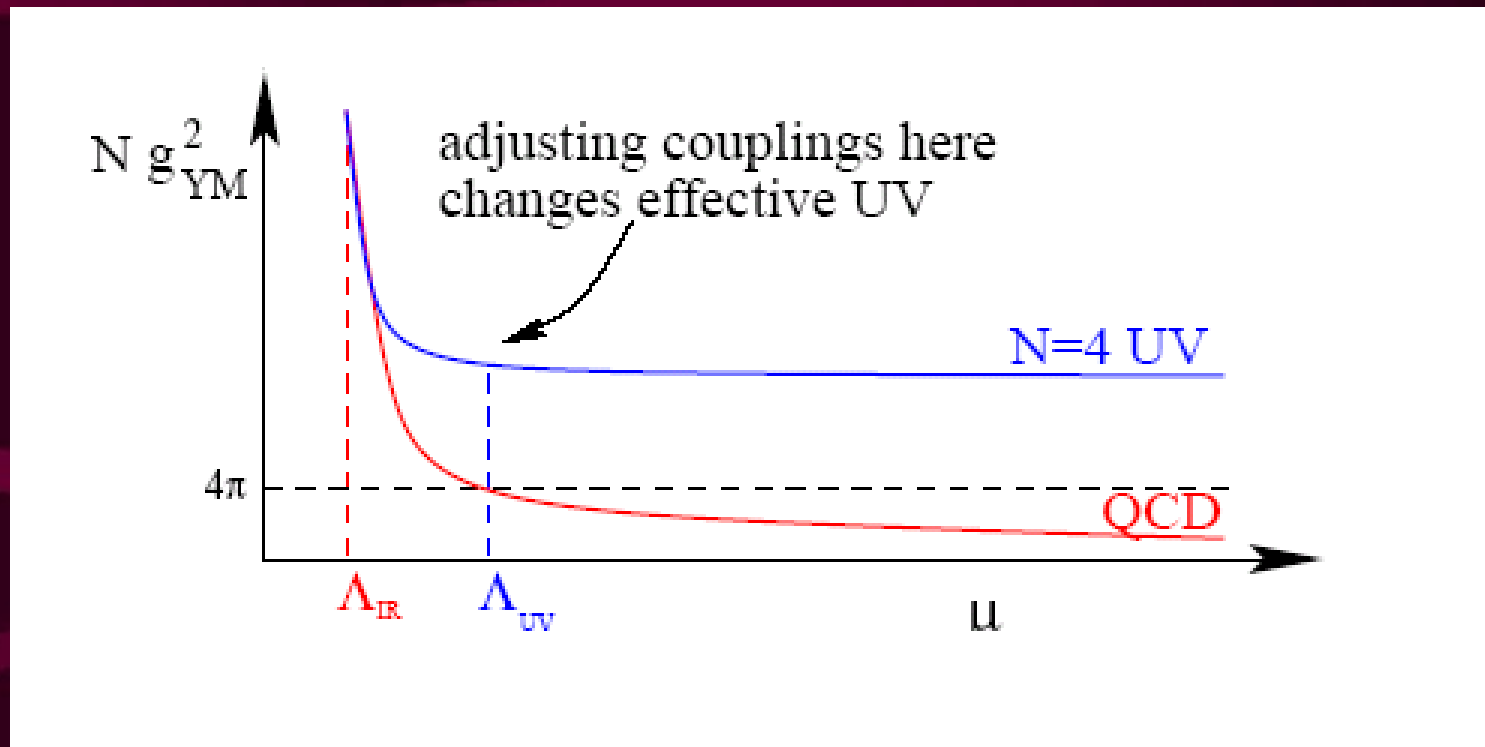
Exponential scaling of order parameters...

Karch, Son, Jensen



Second order non-mean field transitions

# QCD & Beyond



Not QCD... UV & IR cut offs separated... 10% success phenomenology – but no errors.

**These theories look like walking.... But they aren't...**

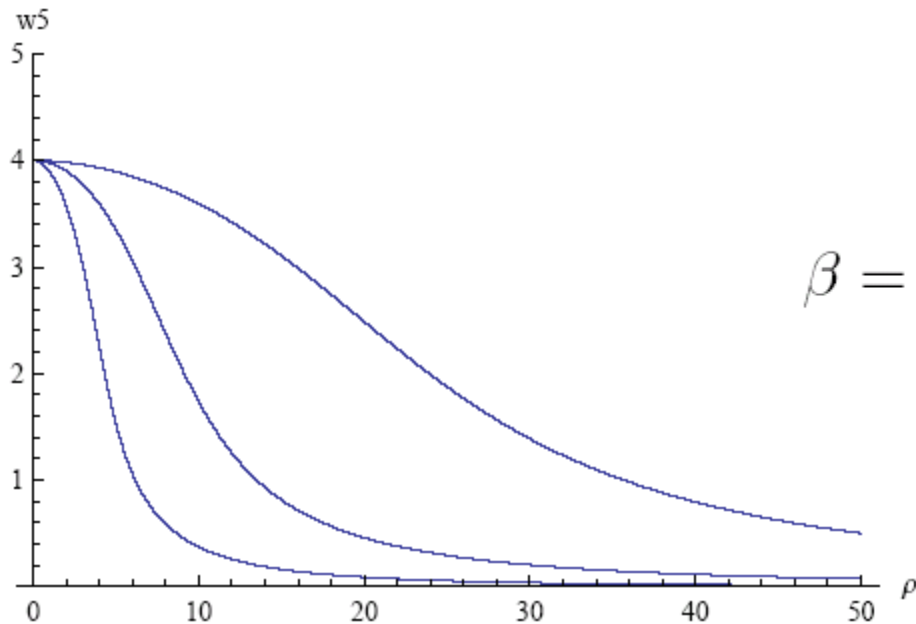
Can we test the idea of a critical coupling for XSBing? ... No ...

We would like to study models with very different running patterns for the coupling

This is very hard.. New gravity solutions... we're quenched...

So lets make it up.. Model... AdS+

$$e^\phi = g_{YM}^2(r^2) = g_{uv}^2 \beta(\rho^2 + w_5^2 + w_6^2)$$



$$\beta = a + 1 - a \tanh [\Gamma(r - \lambda)]$$

$\Gamma$  = walking?

Think of embedding as effective quark mass plot...

$f_\pi^2$ 

$$\frac{f_\pi^2}{\Lambda^2} = \frac{-4N}{\pi^2 \lambda^2} \frac{\left[ \int d\rho \rho^3 \beta \sqrt{1 + (\partial_\rho \Sigma_0)^2} \frac{\Sigma_0^2}{(\rho^2 + \Sigma_0^2)^2} \right]^2}{\left[ \int d\rho \frac{\Sigma_0^2}{(\rho^2 + \Sigma_0^2)^2} \partial_\rho \left( \frac{\beta \rho^3 \Sigma_0 (\partial_\rho \Sigma_0)}{\sqrt{1 + (\partial_\rho \Sigma_0)^2}} \right) \right]}$$

Cf Pagel Stokar

$$f_\pi^2 = \frac{N}{8\pi^2} \int q^3 dq \frac{\Sigma^2 - \frac{1}{2} q^2 \Sigma \Sigma'}{(q^2 + \Sigma^2)^2}$$

We have beta dependence... same large q fall off...

$$\langle \bar{q}q \rangle$$

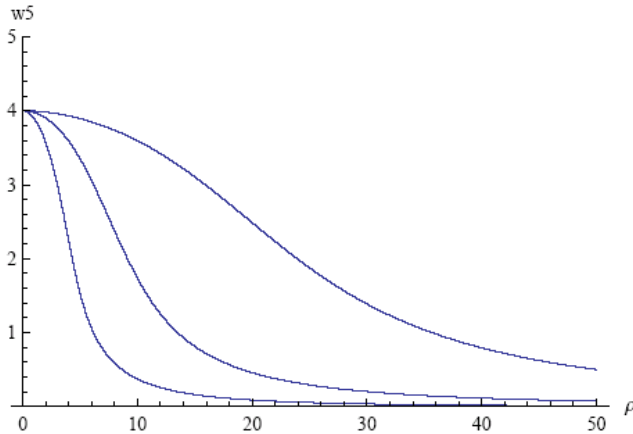
$$\frac{\langle \bar{q}_L q_R \rangle}{\Lambda^3} = \frac{-N}{4\pi\lambda^3 g_{uv}^2 N} \int d\rho \rho^3 \Sigma_0 \sqrt{1 + (\partial_\rho \Sigma_0)^2} \partial_{r^2} \beta \Big|_{r^2 = \rho^2 + \Sigma_0^2}$$

Cf constituent  
quark expression

$$\langle \bar{q}q \rangle = \frac{N}{2} \int q^3 dq \frac{\Sigma}{q^2 + \Sigma^2}$$

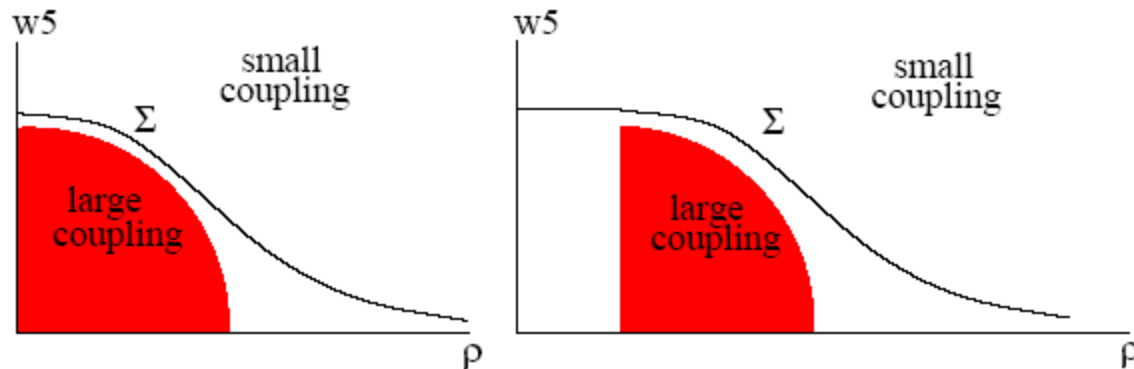
Depends on large  $q$ ... expect usual walking  
behaviour... enhance tail of Sigma to increase  
condensate at constant  $F_{pi}$ ?

$$\frac{\langle \bar{q}_L q_R \rangle}{\Lambda^3} = \frac{-N}{4\pi\lambda^3 g_{uv}^2 N} \int d\rho \rho^3 \Sigma_0 \sqrt{1 + (\partial_\rho \Sigma_0)^2} \partial_{r^2} \beta \Big|_{r^2 = \rho^2 + \Sigma_0^2}$$



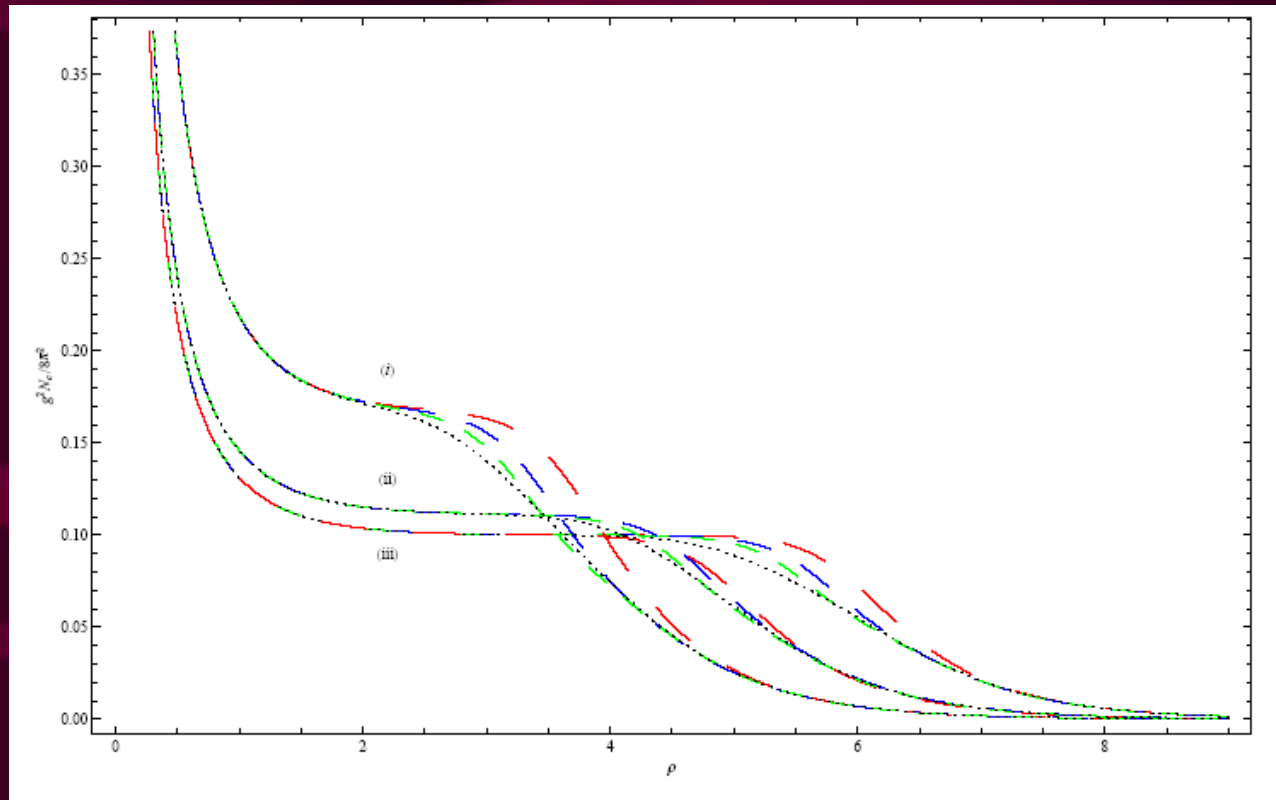
$$\beta = a + 1 - a \tanh [\Gamma(r - \lambda)]$$

Raising tail by spreading with Gamma does not work...



Sigma << Lambda  
does work...

# Swansea Group (Piai, Nunez)- backreaction



Walking profile (is rho mu?).... They don't know the field theory (!)..

With quarks chiral symmetry breaking would happen at the high scale...



# Time Dependent Problems

Inflaton – what running coupling induces a slow roll?

Disordered Chiral Condensates – can see this behaviour?

With Ed Threlfall (Durham)

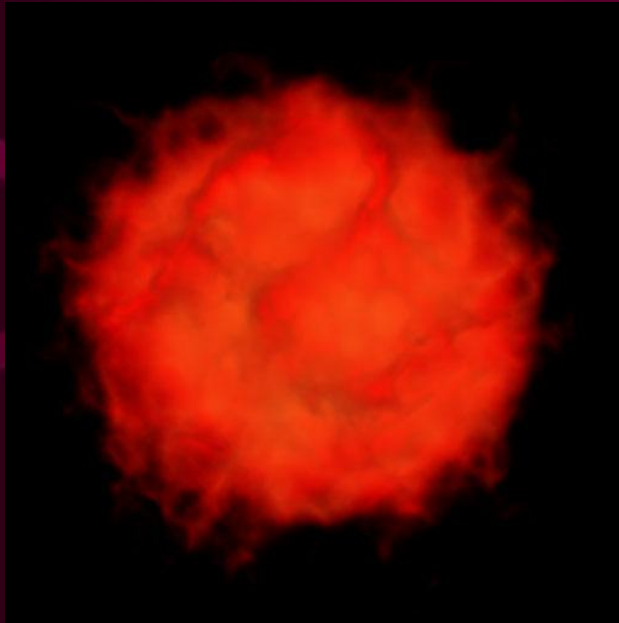
James French (Southampton)

Kristan Jensen (Seattle)

Andrew Tedder (Southampton)

# Hadronization

The hadrons emerging in an electron positron collider are not understood from first principles



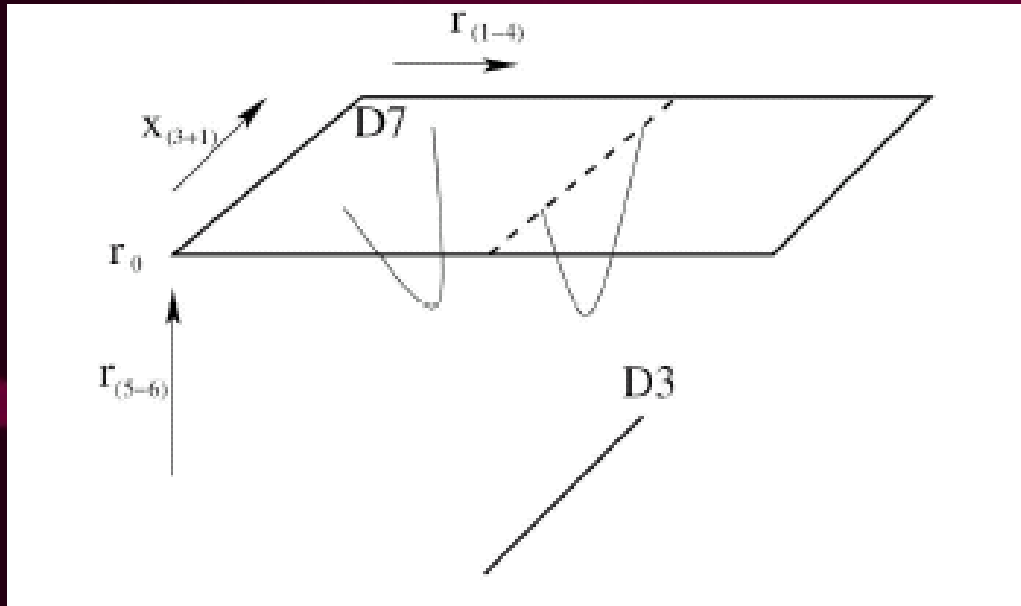
Monte Carlo event generators are tuned to data.

An average LEP jet event

Hadron	Measured
$\pi^+$	8.53
$\pi^0$	9.18
$K^+$	1.18
$K^0$	1.015
$\eta$	0.934
$\rho^0$	1.21
$K^{*+}$	0.357
$K^{*0}$	0.372
$\eta'$	0.13
$p$	0.488
$\Lambda$	0.185

Total > 38 particles!

# Separating Quarks in AdS/N=4 – Karch, Jensen,..



The D7 lies flat

Quark anti-quark pair represented by a string with two ends on the D7

Restrict to strings at closest approach 2+1d system...

Point-like initial conditions with separating ends

$$t(\sigma, 0) = 0, \quad x(\sigma, 0) = 0, \quad z(\sigma, 0) = z_0.$$

$$\dot{z}(\sigma, 0) = Cz_0 \sin \sigma, \quad \dot{x}(\sigma, 0) = Dz_0 \cos \sigma,$$

The equations of motion are

$$\begin{aligned} \partial_a [e^{\phi/2} \sqrt{-\eta} \eta^{ab} G_{MN} \partial_b X^N] \\ - \frac{1}{2} \partial_a X^P \partial_b X^N \frac{\partial}{\partial X^M} (e^{\phi/2} \sqrt{-\eta} \eta^{ab} G_{PN}) = 0. \end{aligned} \quad (6)$$

Note that the derivative in the final term acts on  $\eta^{ab}$  depending on the choice of  $\Sigma$ .

The open string boundary condition applied at the string end points  $\sigma = \sigma_*$  is

$$e^{\phi/2} \sqrt{-\eta} \eta^{\sigma b} G_{MN} \partial_b X^N = 0. \quad (7)$$

However, if the string is attached to a D-brane localized at  $x^M = c^M$ , then this condition is replaced in the directions transverse to the brane by the Dirichlet condition  $X^M = c^M$ .

There are also world sheet constraints from variation with respect to  $\eta_{ab}$  are

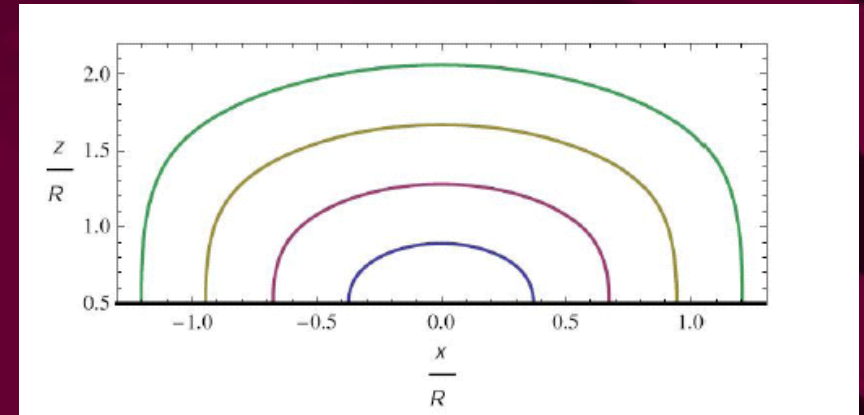
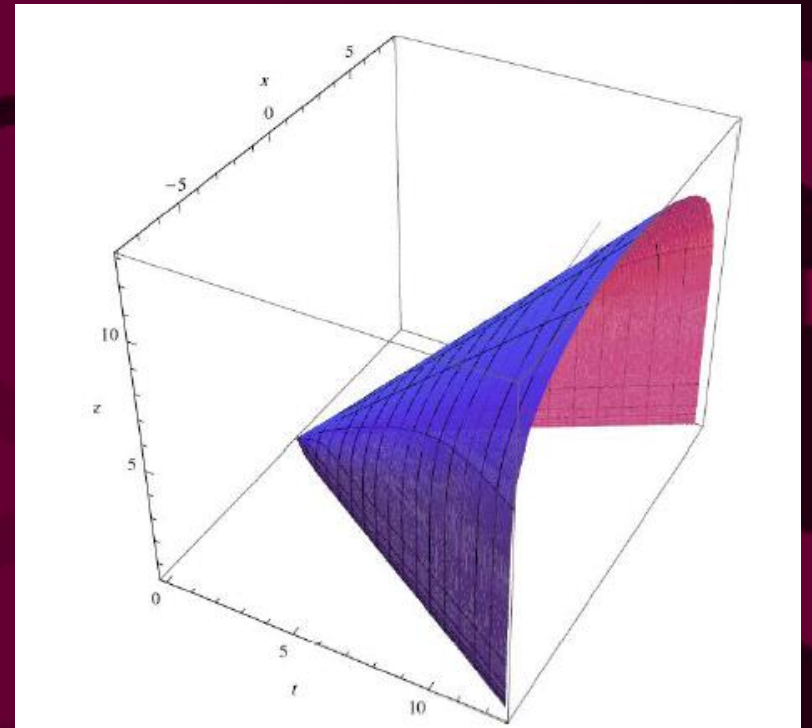
$$\partial_a X^M \partial_b X^N G_{MN} = \frac{1}{2} \eta_{ab} \eta^{cd} G_{MN} \partial_c X^M \partial_d X^N, \quad (8)$$

which, with the form of  $\eta_{ab}$  above become

$$\dot{X} \cdot X' = 0, \quad (9)$$

$$\dot{X}^2 + \Sigma^2 X'^2 = 0, \quad (10)$$

where  $\dot{X}$  indicates  $\partial_\tau X$  and  $X'$  denotes  $\partial_\sigma X$ .



Expected growth of  $1/r$  potential

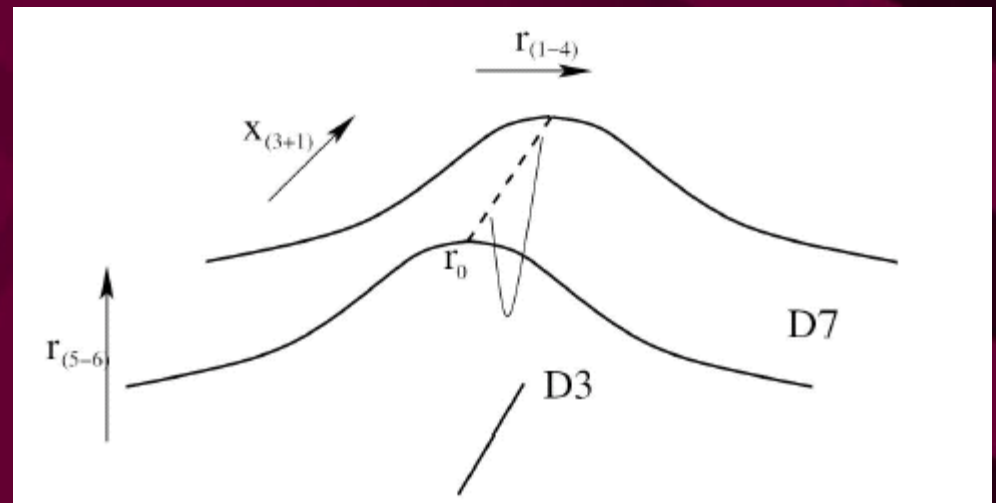
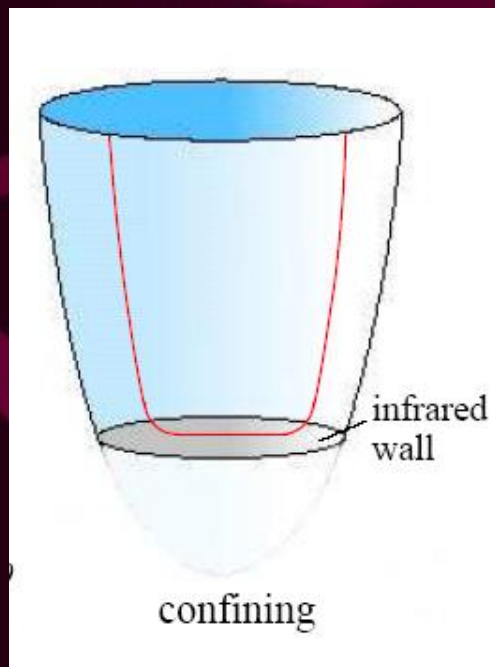
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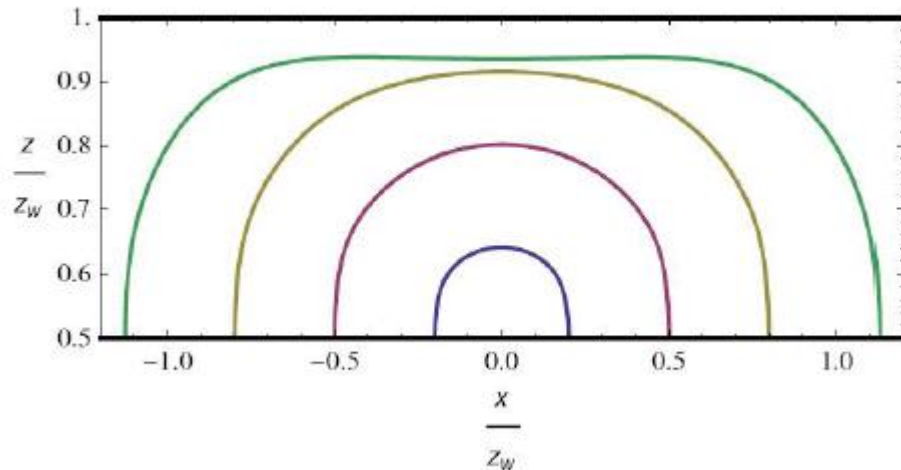
Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall....



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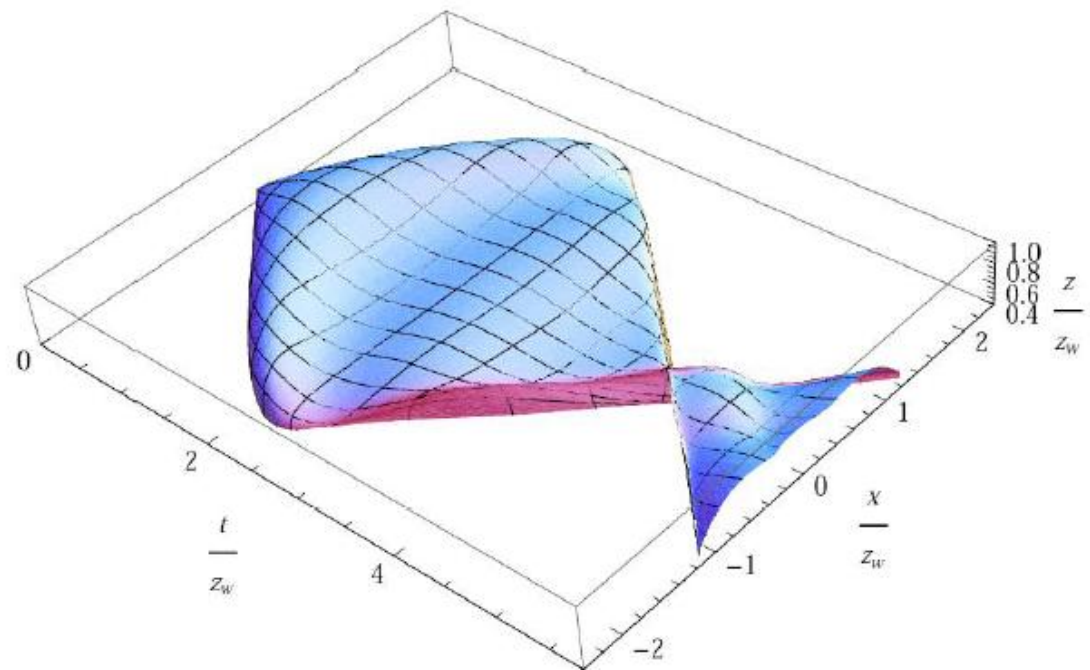
# Separating String Ends and the Wall



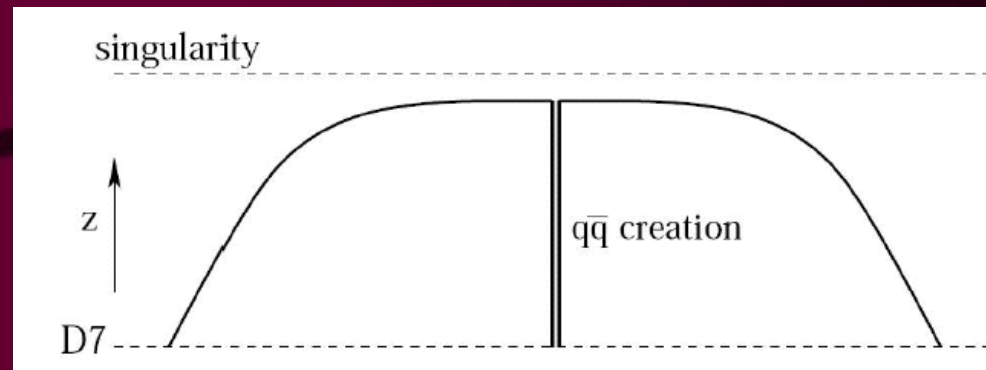
Same initial conditions but now a QCD-like string grows along the wall...

Evolve for longer and the string contracts back to a point before oscillating out again

ie No string breaking



# String Breaking



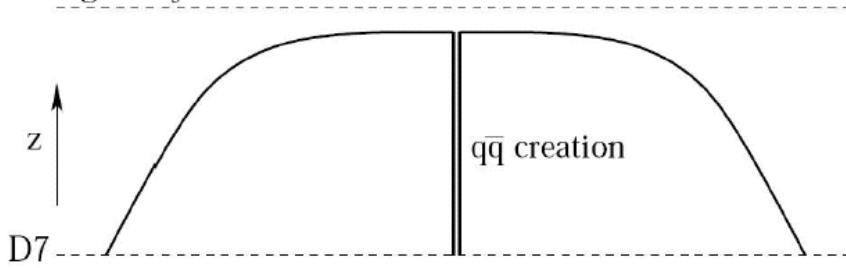
If the string lies within the D7 then the amplitude for the string to break can be computed see eg Dai & Polchinski or Bigazzi & Cotrone...

The amplitude is  $1/N$  suppressed....

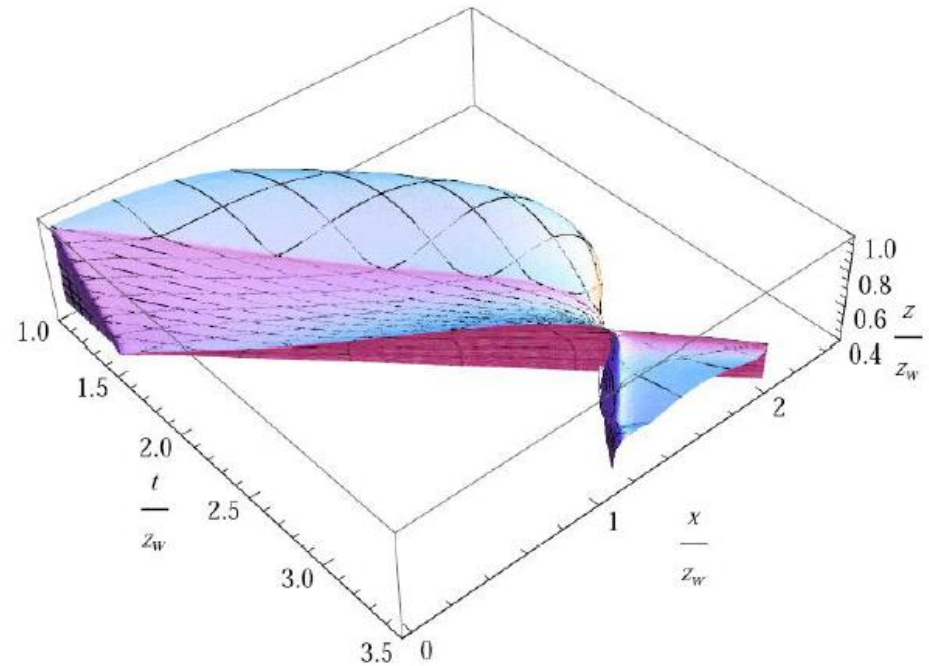
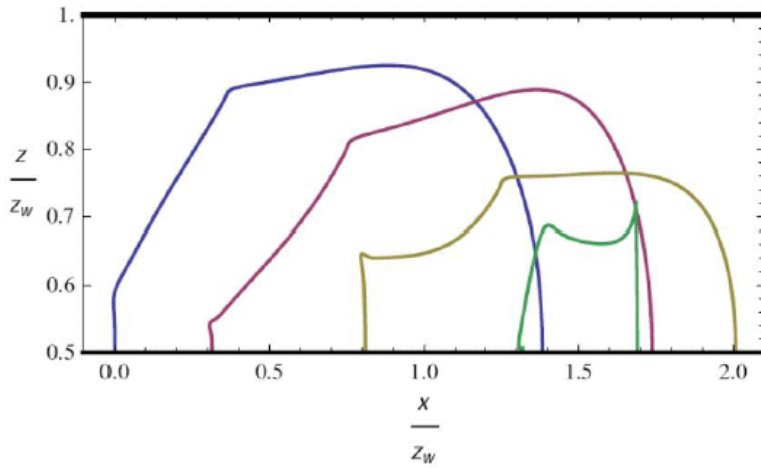
If the string lies off the D7 a quantum fluctuation must take it back to the D7 to break... a computation along these lines was done by Sonnenschien, Peeters and Zamaklar...

In QCD though the probability is basically one when sufficient energy is in the string(?)

singularity



We paste in a solution of the EoM for a straight static string... and evolve again...

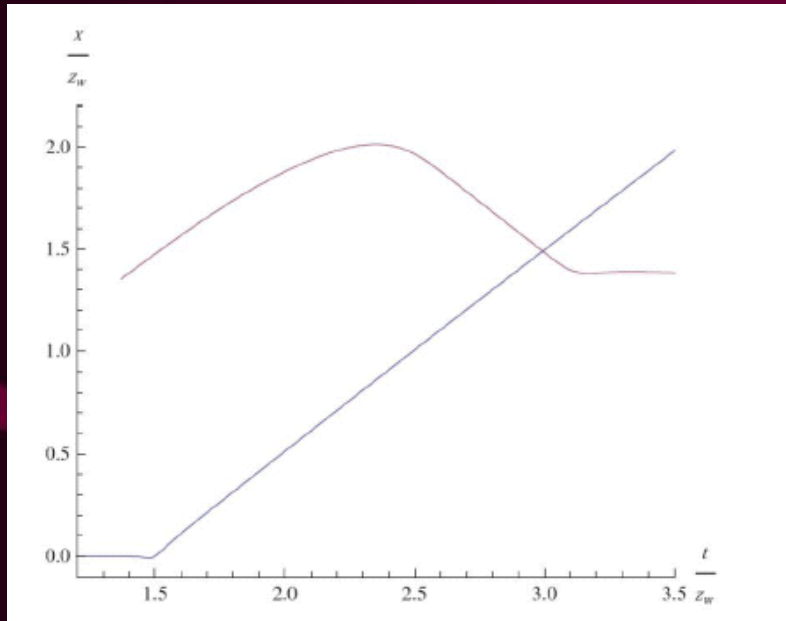


A kink propagates from the join and yanks the static end into motion

The two half string segments separate...



So there's extra evolution in the holographic coordinate relative to the Lund model but more importantly... an extra emission process



When  $j=0$  the solutions describe the rho mesons

$$A_{\mu,n} = \epsilon_{\mu} f_n(\rho) e^{ik_n \cdot x}$$

$$M_n^2 = 4(n+1)(n+2) \frac{r_0^2}{R^4}$$

## The EM Problem on the D7

There's a Chan-Paton flavour charge on the end of the string..

Jerking the string induces photon emission...

$$\frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} F^{ba}) = j^b,$$

$$f_n(\rho) = I_n \frac{{}_2F_1(-n, -n-1, 2; -\rho^2/r_0^2)}{(\rho^2 + r_0^2)^{n+1}}$$

(AdS solutions – Kruczenski, Myers, Mateos, Winters)

Expand in fn...

$$\int dr f_m(r) \square \sum_n A_n^\mu(x) f_n(r) = \int f_m J^\mu dr$$

then we find the potential induced by a moving charge sourcing a 4d massive gauge field... the retarded potential is:

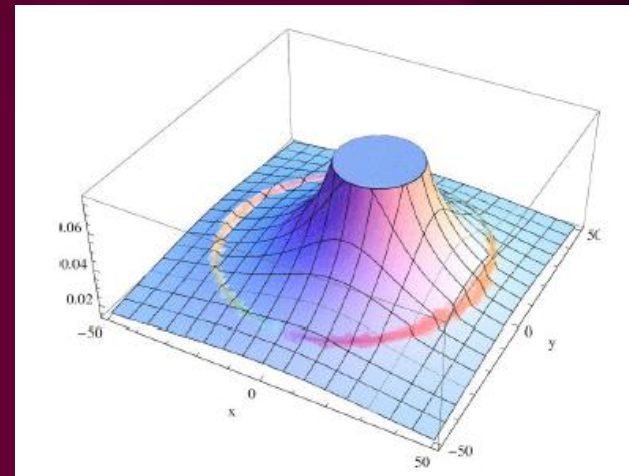
$$\bar{G}^{\mu\nu}_{\mu'} = \frac{1}{4\pi} \theta(t - t') (\delta(\sigma) + V(\sigma)\theta(-\sigma)) \delta^{\mu\nu}_{\mu'}$$

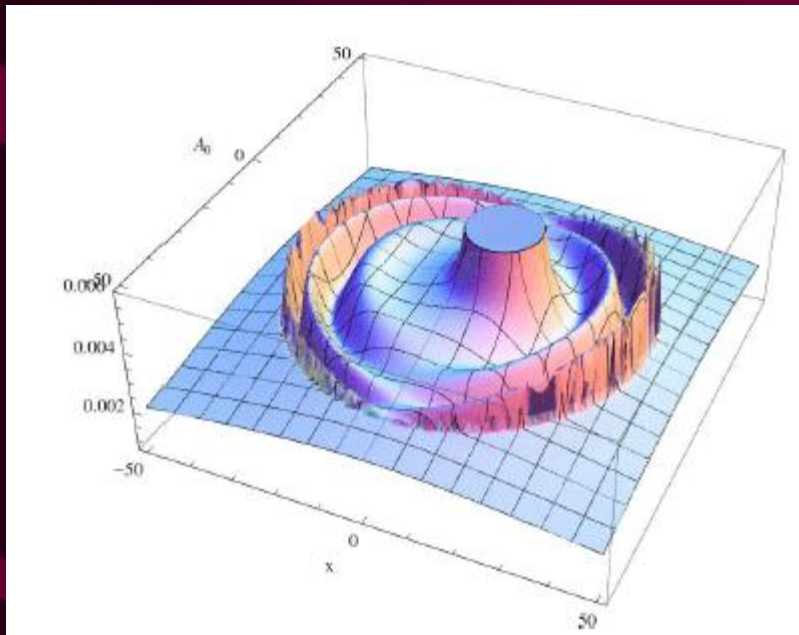
$$V(\sigma) = -\frac{M_n}{\sqrt{-2\sigma}} J_1(M_n \sqrt{-2\sigma})$$

$$\text{Synge world-function } \sigma = \frac{1}{2} \eta_{\mu\nu} (x - x')^\mu (x - x')^\nu$$

The string end point has a  $\exp(-M r)$  cloud of mesons about it..

jerking it induces the emissions of a wave of rho mesons – here some very light ones...





Emission of a very massive rho meson wave....

We see elements of the Lund model and the thermal model (???)

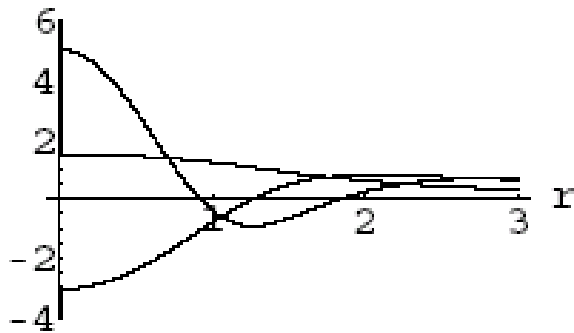
The strength of the source for each wave depends upon the spread of the source in the holographic coordinate  $r$  – basically the Fourier coefficient for expanding the source in the  $f(r)$ ...

In AdS the source is a delta function string end

In QCD it is presumably some wider smeared function

# Naïve Phenomenology

In **Phys.Rev.Lett.100:162003,2008** we assumed the radial distribution was Gaussian and estimated the production expected in QCD



Hadron	Experiment (MeV)	AdS/QCD (MeV)
$\rho$	776	776
$\rho^*$	1459	1742
$\rho^{**}$	1720	2533
$\rho^{***}$	1900	3305
$\rho^{****}$	2150	4059

$f(r)$  in a dilaton wall geometry

If we expand a Gaussian (centred on  $r=0$ , width 300MeV) initial condition in terms of these basis functions

$$c_n = \int_0^\infty \Psi(r) w(r) g_n(r) dr$$

We obtain the yields:  $\rho$  15     $\rho^*$  3     $\rho^{**}$  0....

# How to do the whole QCD spectra?

We simply rescale  $r$  coordinate by  $m_{\text{hadron}}/m_{\text{rho}}$  – the Gaussian does not stretch and we see a suppressed yield.

Spin factor of  $(2J+1)$  on production rates

We include all hadrons with physical mass below 1.7 GeV ie 125 hadrons!

Initial yields for 20 of the  
particle types.....

Parameters

Width = 150 MeV

$\gamma_s = 0.97$

R = 2.6

Average E = 4.96 GeV

Particle	Initial Yield
$\pi^0$	0.57
$\pi^+$	0.57
$K^+$	0.88
$K^0$	0.88
$\rho^0$	0.44
$\omega$	0.14
$p^+$	0.099
$n^0$	0.098
$\Lambda^0$	0.016
$a_1^+$	0.027
$\Delta^{++}$	0.036
$\Sigma^{*+}$	0.012
$K_2^{*+}$	0.0066
$\Xi^{*0}$	0.0040
$\omega_3$	0.0012

Particle	Initial Yield
$\rho(770)$	0.44
$\rho(1450)$	0.68
$\rho(1700)$	0.45
$\rho(1900)$	0.14
$\rho(2150)$	0.03

Most of these will decay in transit to the detector.... We use branching ratios from experiment (pdg via HERWIG) to get final yields

EG

The average results from a  
phi (1020) decay....

	phi
	1020
	0
pi0	0.0443
pi+	0.07
pi-	0.07
K+	0.48947
K-	0.48947
K0	0.34
Kbar0	0.34
eta	0.0126
rho0	0.043
rho+	0.043
rho-	0.043
omega	0.043
f0(600)	0
K*+	0
K*-	0
K*0	0
K*bar0	0
	0

# Giving final yields for LEP

(91 GeV):

## Parameters

Width = 150 MeV

$\gamma_s = 0.97$

R = 2.6

Average E = 4.96 GeV

Hadron	AdS-QCD	Expt
$\pi^+$	5.88	8.53
$\pi^0$	6.40	9.18
$K^+$	1.12	1.18
$K^0$	1.12	1.015
$\eta$	1.02	0.934
$\rho^0$	1.41	1.21
$K^{*+}$	0.334	0.357
$K^{*0}$	0.332	0.372
$\eta'$	0.040	0.13
$p$	0.446	0.488
$\phi$	0.02	0.1
$\Lambda$	0.167	0.185
$\frac{\Sigma^{++} + \Sigma^{*-}}{2}$	0.0129	0.0094
$\Xi^-$	0.0108	0.0122
$\Xi^{*0}$	0.0036	0.0033
$\Omega$	0.0008	0.0014

Decent match across orders of magnitude in production rates..

RMS of fit 37%

# Conclusion

Hadronization seems a field that holography might productively add to...

We can describe a quark anti-quark production event in a gravity dual...

Confinement leads to a QCD-like string forming... and we can add in string breaking by hand...

The string evolves in 5d and the main hadron emission is through the end points - rho emission is computable...

A toy piece of phenomenology seems to hit the right ball park numbers...

**THE HARD TASK** - Can we construct a useful phenomenological model for QCD?