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# HH Production in the SM: Theoretical Status

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## Why measure Higgs pair production?

$$
\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^{\dagger}\Phi) + \lambda(\Phi^{\dagger}\Phi)^2
$$
  
\nEW symmetry breaking  
\n
$$
\mu^2 = \lambda v^2
$$
\n
$$
V(H) = \frac{1}{2}m_H^2H^2 + \lambda vH^3 + \frac{\lambda}{4}H^4,
$$
\nSM: self-couplings  
\ndetermined by  $m_H, v$   
\n
$$
\mu = \frac{q'}{2} + \
$$

4

## HH Production Channels at the LHC q H



## Gluon Fusion: State of the Art

## An approximate history (30 years in 30 seconds)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus,<br>Annual Carlo Carlo Carlo Carlo Carlo Carlo Carlo Ca Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; computation of the radiation of  $\mathbb{R}^n$  radiation corrections including the asymptotic expansion and the asymptotic expansion and the asymptotic expansion and the set of  $\mathbb{R}^n$ 

## A useful approximation: Heavy Top Limit

**Heavy Top Limit (HTL):** integrate out top quarks ( $m_T \rightarrow \infty$ ) Introduces couplings  $c_h$  &  $c_{hh}$  between gluons and Higgs, matched to SM @ 4-loops

Spira 16; Gerlach, Herren, Steinhauser 18



No internal masses, easier to compute higher-order corrections:



#### N3LO Heavy Top Limit Graphs and  $\overline{X}$  in the performance of performance  $\overline{X}$  $T_{\rm obs}$  limit s scale-dependent part of the two-loop and its two-



culations. The two Wilson coecients are also expanded

→ See: Hua-Sheng (Tue) in our calculations. All the terms except for *<sup>O</sup>*(↵<sup>5</sup> → See: Hua-Sheng (Tue) **Very mild scale dependence** 



has been compared against the analytical result we cal-

Ingredients: N<sup>3</sup>LO H calculation LHAPDF6 [97], and the associated strong coupling ↵*s*. Anastasiou, Duhr, Dulat, Herzog, Mistlberger 15; Dulat, Lazopoulos, Mistlberger 18 + 2-loop 4-point functions variation of the factorization scale *µ<sup>F</sup>* and the renormalization scale *µ<sup>R</sup>* in the form of *µR,F* = ⇠*R,F µ*<sup>0</sup> with ⇠*R,* ⇠*<sup>F</sup>* 2 *{*0*.*5*,* 1*,* 2*}*. Ravindran 18 Banerjee, Borowka, Dhani, Gehrmann,



#### Table inclusive total cross sections  $\mathbb T$  inclusive total cross sections (in unit of fb) Vary mild scale denendence

Chen, Li, Shao, Wang 19 From Li Shan Wang 19  $r_{\rm{max}} = r_{\rm{max}} + r_{\rm{max}}$ 

## Beyond HTL @ NLO (Schematically)



Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, (Schubert), Zirke (16),16; Baglio, Campanario, Glaus, Mühlleitner, (Ronca), Spira, (Streicher) 18, 20;

# NLO: Combining small- $p_T$  and small- $m_T$  expansions

Expansion around  $p_T^2 + m_H^2 \leq \hat{s}/4$  and also around  $m_H \ll m_T \ll \hat{s}, |\hat{t}|$  are known Bonciani, Degrassi, Giardino, Gröber 18; Davies, Mishima, Steinhauser, Wellmann 18, 18



 $H1$ dill S.  $[m/n](x) = \frac{1}{1 + a_1x + \dots + a_nx^n}$  $\mathbf{1} + \mathbf{q}$ <sub>1</sub> $\mathbf{x}$  expansion and small  $\mathbf{q}_n \mathbf{x}$ Using Padé approximants: [*m*/*n*](*x*) =  $p_0 + p_1 x + \dots + p_m x^m$  $1 + q_1 x + \dots + q_n x^n$ 

Can find some overlap where the two approximations agree for all relevant  $\hat{s},\hat{t}$ Bellafronte, Degrassi, Giardino, Gröber, Vitti 22

aglio, Campanario, Glaus, Mühlleitner, (Ronca), Spira, (Str<br>. 2.5 Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, (Schubert), Zirke (16),16; This approximation agrees well with the numerical NLO result a,  $\sum_{i=1}^{n}$   $\binom{n}{i}$   $\binom{n}{i}$   $\binom{n}{i}$   $\binom{n}{i}$   $\binom{n}{i}$   $\binom{n}{i}$ Baglio, Campanario, Glaus, Mühlleitner, (Ronca), Spira, (Streicher) 18, 20;

# Beyond HTL @ NNLO

### Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert, Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;<br>  $\mathcal{A}^{\mathcal{C}}_{\textrm{H}\textrm{EFT}}(\imath\jmath\rightarrow HH+1)$  $t_{\rm 10}$ 



### $\vert$  1) NNLO<sub>NLO-i</sub>  $\vert$  1) NNLO<sub>NLO-i</sub>

Rescale NLO by  $K_{NNLO} = NNLO_{HTL}/NLO_{HTL}$  $\frac{1}{n}$  and  $\frac{1}{n}$  and  $\frac{1}{n}$  and  $\frac{1}{n}$  $\parallel$  - Rescale inlumby n $_{\rm NNLO}$  = ininlu $_{\rm HTL}$ /inlu $_{\rm HTL}$ NNLOFTapprox predictions is also presented. The uncertainties due to the *q<sup>T</sup>* -subtraction and

### $\vert$  2) NNLO <sub>B-proj</sub>

 $\parallel$  Project real radiation contributions to Born configurations, rescale by LO/LO $_{\sf HTL}$ stronger as we increase the collider energy, being close to a factor  $\alpha$  factor of  $\alpha$  $A$ s is well known, scale uncertainties can only provide a lower limit only provide a lower limit on the true perturbative  $\mathcal{A}$ 

#### 3) NNLO FTapprox **3**  $\sum_{i=1}^{\infty}$

MNLO HTL squared amplitude rescaled for each  $\frac{1}{2}$  multiplicity by: with the moderation of  $\alpha$  improvement is suggested in the suggests and in the significant in the significa

rt, 
$$
\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \to HH + X)}{\mathcal{A}_{\text{HEFF}}^{(0)}(ij \to HH + X)}
$$
8;

# Beyond HTL @ NNLO

### Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;  $\mathcal{A}_{\mathrm{HEFT}}^{\mathcal{F}}(i,j\rightarrow HH+X)$ Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert,



### 1: 1) NNLO<sub>NLO-i</sub> and Higgs boson pair productions for Higgs boson pair production for diagnosis sections for diagnosis sections for  $\mathbf{I}$

Rescale NLO by  $K_{NNLO} = NNLO_{HTL}/NLO_{HTL}$ energies at NLO and NNLO with the three considered approximations. Scale uncertain $t_1$  - Rescale inlumby  $K_{NNLO}$  = ininlu $H_{\rm TL}$ /inlu $H_{\rm TL}$ NNLOFTapprox predictions is also presented. The uncertainties due to the *q<sup>T</sup>* -subtraction and

### $\vert$  2) NNLO <sub>B-proj</sub>

 $\parallel$  Project real radiation contributions to Born configurations, rescale by LO/LO $_{\sf HTL}$ stronger as we increase the collider energy, being close to a factor  $\alpha$  factor of  $\alpha$  $A$ s is well known, scale uncertainties can only provide a lower limit only provide a lower limit on the true perturbative  $\mathcal{A}$ 

#### 3) NNLO FTapprox **Execute 1 we see that the distribution of the NNLO** and Table 1 we see that the NNLO and NNLO an  $\sum_{i=1}^{\infty}$  always larger than the NNLO scale uncertainties (although within  $\sum_{i=1}^{\infty}$

MNLO HTL squared amplitude rescaled for each  $\pm$  multiplicity by:  $\frac{1}{2}$  with the moderation of  $\frac{1}{2}$  suggests a significant in the significant in

rt, 
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\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \to HH + X)}{\mathcal{A}_{\text{HEFF}}^{(0)}(ij \to HH + X)}
$$
8;

#### Beyond HTL @ N3LO  $\overline{\phantom{a}}$

Top quark mass effects included in N3LO HTL (up to NLO)  $\mathcal{L}$  $\sim$  10  $\sim$  10  $\sim$  10  $\sim$  2  $\sim$  2



KHOWH  $\text{FT}_{\text{approx}}$ -like result not known, requires  $m_T$  in reals  $\longrightarrow$  See: Hua-Sheng (Tue)<br><sup>14</sup>  $\sim$ Results agree with NNLO result but with reduced scale uncertainty

nena l

## Total Cross Section & Scale Uncertainty @ 14 TeV

#### Note: papers @ 13/14 TeV (not 13.6 TeV)



Chen, Li, Shao, Wang 19, 19; Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; Maltoni, Vryonidou, Zaro 14 (recalculated); Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; Dawson, Dittmaier, Spira 98 (recalculated); Glover, van der Bij 88 (recalculated)

$$
\begin{cases}\n\text{PDF4LHC15_nlo/nnlo} \\
m_H = 125 \text{ GeV} & m_T = 173 \text{ GeV} \\
\text{Uncertainty:} \\
\mu_R = \mu_F = \frac{m_{HH}}{2} \\
\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right]^{2} (7 - \text{point})\n\end{cases}
$$

#### If we trust the  $NLO + NmLO$  HTL combinations

Scale:  $+2.2\%$  /  $-5.0\%$  *PDF+* $\alpha_s$ *:*  $\pm 3.0\%$  $m_{T}$  approx:  $\pm 2.7\,\% \quad m_{T}$  scheme:  $+4.0\,\%$  /  $-~18.0\,\%$ 

See: HH Twiki

## Status: HH Self Coupling



Grant Agreement numbers PITN-GA-2010-264564 (LHCPhenslate into uncertainties Ford complete by the IIISN contractor  $\Lambda_{\sigma}$  $\sum_{i=1}^n$  $SO(5)$ NI  $A_{III}$  we have  $\longrightarrow$ coupling extraction: for  $gg \to HH$  close to SM  $\lambda_{hhh}$  we have  $\frac{1}{\sqrt{g}}$  $\ln$  the solf Theory uncertainties on cross section translate into uncertainties on the self- $\mathbf{A}$  $\Lambda$   $\lambda$  $\Delta \lambda$ [14] T. Binoth, S. Karg, N. Kauer, and R. Ruckl, Phys.Rev. **D74**,  $\overline{1}$  $\overline{1}$  $\mathcal{L}$ Self coupling dependence known at: Δ*σ σ*  $\sim -\frac{\Delta\lambda}{\lambda}$ *λ*

[1] F. Englert and R. Brout, Phys.Rev.Lett. **13**, 321 (1964). pagno <del>e</del>t al. 10,20, 11  $\overline{10}$ N<sup>3</sup>LO (re-weighted HTL) Chen, Li, Shao, Wang 19 CONF-2013-012. ATLAS- CONF-2013-013. (2013). https://launchpad.net/madgraph5,http://amcatnlo.cern.ch. [18] U. Baur, T. Plehn, and D. L. Rainwater, Phys.Rev.Lett. **89**, Borowka, et al. 16; Baglio et al. 18,20; Heinrich, et al. 19, 20; NLO+PS (full theory)

## Status: HH EFT



NNLO corrections have typically ~10% effect on  $(\sigma/\sigma_{\rm SM})_{\rm NNLO}/(\sigma/\sigma_{\rm SM})_{\rm NLO}$ Significantly reduce scale uncertainties<br>

#### **EFT Results in various approximations:**

- B.I. NLO HTL Gröber, Mühlleitner, Spira, (Streicher) (15), 17;
- NLO (HEFT) Buchalla, Capozi, Celis, Heinrich, Scyboz 18; NLO (HEFT)
- that, with the coupling of the anomalous couplings coupling coupling coupling coupling coupling coupling coupling<br>the coupling coupl  $+ PS$ Heinrich, SJ, Kerner, Scyboz 20;
- $\begin{array}{ccc} \text{NIL} & \text{$ larger than the current experimental limit. While Eq. (3.1) describes the allowed region  $\bigcap$  (CNAEFT), in Eq. (3.1), generate variations in the di-Higgs cross section which are discussed w LO (JIVILI I) The current experimental limit. While  $\epsilon$ NLO (SMEFT) Heinrich, Lang, Scyboz 22;
- **in the EFT** parameter space of the results in Fig. 3 (left) clearly in Fig. 5 (left) clearly in Fig. 5 (left) c B.I. NNLO HTL de Florian, Fabre, Mazzitelli 17; → See: Luc
- NLO + NNLO' de Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21 **Kaquel/NIC** NLO + NNLO'

**→** See: Ludovic/ Raquel/Nicolas (Tue)

# Gluon Fusion: Issues & Dangers

## Mass Scheme Uncertainty

With such a tiny scale uncertainty, other sources of uncertainty become relevant

(TRONCa), Spira, Sueicher To, (20)  $\textsf{HH@NLO}\text{:}\ m_{T}$  in the  $\textsf{OS}$  and  $\overline{\textsf{MS}}$  scheme  $\begin{array}{c} \textsf{Baglio}, \textsf{Campanario}, \textsf{Glaus}, \textsf{Mühlleitner}, \ \textsf{H}\textsf{H@NLO}\text{:}\ m_{T} \textsf{in the OS} \textsf{and } \overline{\textsf{MS}} \textsf{Scheme} \end{array}$ (+Ronca), Spira, Streicher 18, (20)

OS to 
$$
\overline{\text{MS}}
$$
 mass conversion:  $m_t \to \overline{m_t}(\mu_t) \left( 1 + \frac{\alpha_s(\mu_R)}{4\pi} C_F \left\{ 4 + 3 \log \left[ \frac{\mu_t^2}{\overline{m_t}(\mu_t)^2} \right] \right\} \right)$ 



mass in the range between *Q/*4 and *Q* we obtain the follow-Top quark mass scheme unc:

$$
\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300 \text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%} \text{fb/GeV},
$$
\n
$$
\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=400 \text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%} \text{fb/GeV},
$$
\n
$$
\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=600 \text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%} \text{fb/GeV},
$$
\n
$$
\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200 \text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%} \text{fb/GeV},
$$
\nLarge uncertainty obtained  
\ncomparing OS scheme with  $\overline{\text{MS}}$   
\nscheme at scale  $m_{HH}$ 

#### Mass Scheme Uncertainty (II) 4 Uncertainties for di↵erent Higgs self-interactions <sup>=</sup> 1 : *tot* = 131*.*9+2*.*5% 6*.*7% fb*,* =0: *tot* = 70*.*38+2*.*4% 6*.*1% fb*,*

5*.*0% fb*,*

=1: *tot* = 31*.*05+2*.*2%

Combination of scale  $(\mu_R, \mu_F)$  and top mass scheme (OS / MS) studied Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 20 Raqlio Campanario Glaus Mühlleitner Ronca Spira Bagilo, Campanario, Giaus, iviunileither, Ronca, Spira 20 =2: *tot* = 13*.*81+2*.*1% 4*.*9% fb*,* = 2*.*4 : *tot* = 13*.*10+2*.*3% 5*.*1% fb*,*

If we wish to take the envelope of the predictions as the uncertainty, then the two uncertainties should be added linearly (validated at NLO) 6% at NLO for large and small values of [17] such that the change with respect to the uncertainties should be added **ilnearly** (val **linearly** (validated at NLO) *nimearly* (validated at NLO)



#### ichame Linc Proposed Combination Eq. (9) *linearly* we arrive at the central values with combined uncertainties, Proposed Combination



 $\bigcirc$  final numbers showld serve as the recommended values for the total cross sections  $\bigcirc$ and uncertainties at the LHC with <sup>p</sup>*<sup>s</sup>* = 13 TeV as a function of . @13 TeV

**→** See: Michael (Tue)

## Mass Scheme Uncertainty (III)

Such mass scheme uncertainties show up in other processes (e.g. H\*, HJ, ZH) SPJ, Spira (Les Houches 19)

 $A_i^{\text{fin}} = a_s A_i^{(0),\text{fin}} + a_s^2 A_i^{(1),\text{fin}} + \mathcal{O}(a_s^3)$  with  $a_s = \alpha_s / 4\pi$ 

Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20



$$
A_i^{(0)} \sim m_t^2 f_i(s, t)
$$
  

$$
A_i^{(1)} \sim 6C_F A_i^{(0)} \log \left[\frac{m_t^2}{s}\right]
$$

LO:  $m_t^2$  from  $y_t^2$ NLO: leading  $log(m_t^2)$  from mass c.t. converting to  $\overline{\mathrm{MS}}$  gives  $\log\left[\mu_t^2/s\right]$ motivating scale choice of  $\mu_t^2 \thicksim s$ 

Davies, Mishima, Steinhauser 20; Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22



$$
A_i^{(0)} \sim m_t^2 f_i(s, t) \log^2 \left[ \frac{m_t^2}{s} \right]
$$

$$
A_i^{(1)} \sim \frac{(C_A - C_F)}{6} A_i^{(0)} \log^2 \left[ \frac{m_t^2}{s} \right]
$$

LO: one  $m_t$  from  $y_t$ NLO: leading  $log(m_t^2)$  not coming from mass c.t.  $(C_{\!A})$ 

## EFT Choice

### Several challenges/considerations with using Effective Field Theories (EFTs)

Can construct more than one EFT with different constraints/relations on operators

#### HEFT:

Higgs boson field  $h(x)$  is  $SU(2)_L \times U(1)_Y$  singlet Expand in loop orders  $\sim 1/(16\pi^2)$ 

$$
\mathcal{L}_{\text{HEFT}} = \mathcal{L}_2 + \sum_{L=1}^{\infty} \sum_{i} \left( \frac{1}{16\pi^2} \right)^L C_i^{(L)} O_i^{(L)}
$$

A priori no relation between & *cggh cgghh*

#### $SMEFT \subset HEFT:$

Higgs field complex doublet Expand in canonical dimension  $\sim 1/\Lambda^2$ 

$$
\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i} \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)
$$

Relation *cggh* ∼ *cgghh*

#### **→** See: Raquel/Ludovic/Nicolas (Tue), Alexandre (Fri)



## **EFT Truncation**

How the EFT expansion is truncated is not always innocent

$$
\sigma \simeq \begin{cases}\n\sigma_{\text{SM}} + \sigma_{\text{SM} \times \text{dim}6} & \text{(a)} \qquad \text{i.e. Include } 1/\Lambda^2 \text{ at cross section level} \\
\sigma_{\text{(SM+dim6)} \times (\text{SM+dim6})} & \text{(b)} \qquad \text{i.e. include } 1/\Lambda^2 \text{ at amplitude level} \\
\sigma_{\text{(SM+dim6)} \times (\text{SM+dim6})} + \sigma_{\text{(SM} \times \text{dim6}^2)} & \text{(c)} \qquad \text{i.e. } + 1/\Lambda^2 \cdot 1/\Lambda^2 \text{ at cross section level} \\
\sigma_{\text{(SM+dim6+dim6^2)} \times (\text{SM+dim6+dim6^2})} & \text{(d)} \qquad \text{i.e. } + 1/\Lambda^2 \cdot 1/\Lambda^2 \text{ at amplitude level}\n\end{cases}
$$

 $\sim$  act wildly different EET limite both ot LO 8, NILO Heinrich Lang Sevi Can get wildly different EFT limits both at LO & NLO Heinrich, Lang, Scyboz 22



## Gluon Fusion: Horizon

## Tackling Mass Scheme Uncertainties

![](_page_24_Figure_1.jpeg)

Low invariant mass:

expand in  $1/m_t^2$ known to NNLO Grigo, Hoff, Steinhauser 15;

Around Peak: threshold expansion Gröber, Maier, Rauh 17

#### High energy:

small- $m_{\tilde{t}}$  expansion known at NLO Davies, Mishima, Steinhauser, Wellmann 18, 19

See also: Bonciani, Degrassi, Giardino, Gröber 18; Davies, Steinhauser 19; Davies, Herren, Mishima, Steinhauser 19; Davies, Gröber, Maier, Rauh, Steinhauser 19; Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;

**Options:** 1) Try to understand structure of mass logarithms 2) Keep calculating 3) Other ideas(?)

Liu, Penin 17, 18; Liu, Modi, Penin 22

#### $NNLO$  Beyond HTL such as those shown in (c) lead to *n*<sup>3</sup> **have a contributions which have a computed in the computed in the computed in the computations of the computed in** Ref. [25]. The *n*<sup>3</sup>

and have not been computed in Ref.  $\alpha$  in Ref. [25]; they are considered in Ref. [25]; they are considered here.

#### NNLO Virtual & Real Corrections in a  $1/m_t^2$  expansion: NNLO Virtual & Real Corrections in a  $1/m_t^2$  expansion:

![](_page_25_Figure_2.jpeg)

![](_page_25_Figure_3.jpeg)

3-loop virtual piece in large- $m_e$  expansion (up to  $1/m<sup>8</sup>$ ) 3-loop virtual piece in large- $m_t$  expans 3-loop virtual piece in large- $m_t$  expansion (up to  $1/m_t^8$ ) in  $\text{large-}m_t$ , expansion (up to  $1/m_t^8$ ) Davies, Steinhauser 19

5-loop forward scattering amplitude  $(n_h^3)$  piece Figure 2: Sample Feynman diagrams in the forward-scattering kinematics. Three- and

J-loop forward scattering amplitude (all pieces) 5-loop forward scattering amplitude (all pieces) but the source between wishima,

![](_page_25_Figure_7.jpeg)

Davies, Steinhauser 19 Davies, Herren, Mishima, Steinhauser 19 four-particle cuts are shown by  $\eta$ ,  $\beta$  is the *n*<sup>3</sup> second data lines, respectively. The *n*<sup>3</sup> second data lines, respectively.

Davies, Herren, Mishima, Steinhauser 21

current current quarks, Higgs bosons and gluons,  $\mathcal{L}$ Lo nave similar results in a  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  for  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  for  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  $\left\{ \begin{array}{rcl} \text{small-} & m_t \text{ expansion} & \text{if} & \left\{ \begin{array}{rcl} \text{small-} & m_t \text{ expansion} & \text{if} & \left\{ \begin{array}{rcl} \text{small-} & m_t \text{ is a positive} \end{array} \right\} \end{array} \right\} \end{array}$ **Example 2** Feasible during Run 3 (?) Would be extremely useful and  $\mathcal{C}$  $F_{\text{res}}$   $\begin{array}{ccc} \text{Snian } m_t \text{ expansion} & \text{if} & \text$ to have similar results in a

![](_page_25_Figure_11.jpeg)

![](_page_25_Picture_12.jpeg)

## EW Corrections

With N<sup>3</sup>LO QCD (HTL) results known, could now be interesting to explore also the impact of EW corrections (in single Higgs for off-shell Higgs have  $\pm 5 \, \%$  impact) Actis, Passarino, Sturm, Uccirati 08

Richer structure in the SM and much richer structure in the context of EFT

**Example:** Partial 2-loop EW corrections (involving  $\lambda_3$  and  $\lambda_4$  )

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 18

![](_page_26_Figure_5.jpeg)

HL-LHC has only limited sensitivity to  $\lambda_4$  , more relevant for FCC

The complete EW corrections could potentially modify distributions and bounds in both SM and EFT frameworks

→ See: Hantian (Fri)

## Backgrounds

I am aware of two background studies taking place in the context of the HH WG  $\mathcal{L}$ 

 $b\bar{b}H$  (background to  $HH$  with  $H \to bb$ )  $\quad$   $t\bar{t}$  (background to  $b\bar{b}WW$ ,  $bb\tau$ )

![](_page_27_Picture_3.jpeg)

Included at LO in past searches (via NNLOPS ggF) w/ 100% uncertainty assigned ATLAS-CONF-2012-016 NUCLO COLO CONTROLS IN THE LARGE-MANUSING BEEN CONTROLLED IN THE LARGE-MELTING WAS ARRESTED FOR LARGE-MANUSING  $\frac{1}{2}$ 

NLO corrections known in the HTL, large K-factors (  $\sim 2 - 3$ ) depending on fiducial cuts Amplitudes for NNLO corrections are now also known Deutschmann, Maltoni, Wiesemann, Zaro 18 [ATLAS non-resonant bbWW search, 1908.06765] with the recent recent<br>New York and recent rece Badger, Hartanto, Krys, Zoia 21

 $\mathcal{L}$  $t\bar{t}$  (background to  $b\bar{b}WW,\,bb\tau$ )

![](_page_27_Figure_7.jpeg)

theoretical uncertainty Simulated using NLO MC w/ large

ased to reduce the uncertainty Zanderighi 20, 21 region of HTL in the PS region of HTL in the PS region of  $\sim$ Results on  $t\bar{t}$  with NNLOPS could be used to reduce the uncertainty Mazzitelli, Monni, Nason, Re, Wiesemann,

## VBF: State of the Art

## VBF HH

VBF HH is sensitive not just to  $c_{hhh}^{}$  but also  $c_{\rm 2v}^{}$  and  $c_{v}^{}$ 

![](_page_29_Figure_2.jpeg)

#### araz *G*<sup>2</sup> *<sup>F</sup> m*<sup>4</sup> *V A* Precision **districts** where  $\alpha$ Approximations & Precision

Known to N<sup>3</sup>LO in the  $\overline{\phantom{a}}$ (*q*<sup>1</sup> + *k*1)<sup>2</sup> *m*<sup>2</sup> approximation structure function / DIS

> *k k*<sup>1</sup> annrovimation is a satisfy *k*<sup>1</sup> + *k*<sup>2</sup> = *q*<sup>1</sup> + *q*2, is the trilinear Higgs selfcoupling and  $\alpha$  is the value of the value of

to colour conservation

*<sup>i</sup>* ) = ⇣ ⌘

*F <sup>V</sup>* <sup>1</sup> (*xi, Q*<sup>2</sup> *i* )

 $U \cap \mathfrak{c}$  as  $\mathfrak{c}$  and  $\mathfrak{c}$ 

*x* footoriaablo

 $\cap$ 

![](_page_29_Figure_5.jpeg)

#### VBF HH: Non-factorisable contribution **Factorisas Counterparts. For this relation is relatively** corrections, as depicted in Fig. 6, where we provide two providences in Fig. 6, where we provide two providence ble contribution on

Liu, Melnikov, Penin 19 eikonal approximation eilen with the eigenvalue of the eigenvalue of the eigenvalue of Dreyer, Karlberg, Tancredi 20, 22  $\epsilon$  atoriaable contributions resemtly. Non-factorisable contributions recently d using the eikonal annroximation was denig the enternal approximation.  $r_{\text{c}}$  reflection can be expanded in the value of  $r_{\text{c}}$ studied using the eikonal approximation

#### twee ~10% differences between non-fac contributions to T and B diagrams conclude that the non-factorisable corrections received a  $\sim$

$$
d\sigma_{HH,nf}^{NNLO} \sim -\tilde{\alpha}_s^2 \left[ 2.3 \cdot d\sigma_{TT}^{LO} + 2.2 \cdot d\sigma_{TB}^{LO} + 2.1 \cdot d\sigma_{BB}^{LO} \right]
$$

ute cancellations hetween T and R Delicate cancellations between T and B ams conspire In order to see how the NNLO non-factorisable cordiagrams conspires to preserve unitarity

(As pointed out by authors) Fikonal  $f(x)$  to point to arbitrary and in principal in principal  $f(x)$ ximation not trustworthy for too high  $p<sub>r</sub>$ approximation not trustworthy for too high  $p_{t,j}$ **Note:** (As pointed out by authors) Eikonal

![](_page_30_Figure_6.jpeg)

## HH VBF: NNLO QCD + NLO EW

Inclusive known to N3LO QCD

Dreyer, Karlberg 18

State of the art for differential predictions

NNLO QCD Dreyer, Karlberg 18

+ NLO EW Dreyer, Karlberg, Lang, Pellen 20

![](_page_31_Figure_6.jpeg)

![](_page_31_Figure_7.jpeg)

![](_page_31_Picture_605.jpeg)

Table 1: The fiducial cross section for the process pp  $\mathbf{A}$  and in performance in computed according to Eq. (4) at 14 TeV and under the selection cuts given in Sec. 2. The

the factorisable ones. We compute the factorisable NNLO QCD corrections using the  $FW\ corrections\ similar\ in\ size\ to\ NLO OCD$  $\downarrow$  corrections Forections and to those in single ringgs case EW corrections similar in size to NLO QCD corrections. and to these in single Higgs sase corrections and to those in single Higgs case

#### ↵5 . At the same order, the virtual corrections are obtained by inserting EW particles anywhere possible in the tree-level topologies, and the tree-level topologies, and the contributions are the t Note that at the order *<sup>O</sup>* ↵5 Ī. , photon-induced contributions also arise. The second-induced contributions also arise. The second-induced contributions are  $\mathbf{r}$ neglected in the provision work as the provision of shown to be rather small for similar small for similar sma processes [42, 43, 38]. Note that EW corrections to single-Higgs production have been All corrections available in public code  $\mathbf{S}$ proVBFHH v1.2.0 indicate the statistical error while the additional information on full

Earnon CMI: can roccale to include **HOLLO BEEN CALLESCAN** As mentioned previously, all LO and NLO predictions are based on the full computation, *i.e.*  $\mathbb{R}$  bounted the van face  $\mathbb{R}$ FIAI corrections. The Monte Carlo from the Monte Carlo fr  $\Box$  alrave to take for each  $\alpha$  LVV corrections as  $\frac{400}{400}$  and  $\frac{1}{200}$  and  $\frac{1}{200}$  for  $\frac{1}{200}$ unich iamuy  $E_{\alpha r, n, \alpha n}$  CM $\theta$  can rescale to include For non-SM: can rescale to include  $f_{\text{actorical}}$  of  $\cap$  corrections but querently factorisable QCD corrections but currently  $\alpha$  corrected taken as a factor  $\Omega$   $\Gamma$  h  $\alpha$  corrections as have to take non-fac & EW corrections as We note that the non-factorisable NNLO  $\alpha$  corrections are the only positive corrections are the only positive corrections, and uncertainty cancels the factorisable NNLO QCD cancels the factorisable NNLO QCD corrections. This is is in the factorisable NNLO QCD corrections. This is in this is in the factorisable NNLO QCD corrections. This is in this

## Summary

### Good progress in HH theory over the last few years

- Gluon fusion Full SM result: NLO
- Gluon fusion HTL result: N<sup>3</sup>LO (also differential)
- Vector Boson Fusion N<sup>3</sup>LO inclusive, NNLO differentially + NLO EW
- Progress matched by amazing work from the experiments

#### Uncertainties beyond scale variations are now relevant

- Mass scheme uncertainties at the level of >10% @ NLO
- Motivates studies of  $m_T^{}$  dependence beyond 2-loop

#### Many other fascinating developments

(e.g. efforts to tackle backgrounds, EW effects, EFT fits, …)

I hope/expect that the anticipated experimental progress during Run 3 is matched by exciting theory progress

### Thank you for listening

![](_page_33_Picture_0.jpeg)

#### Self Coupling Considerations 1SINE λhhh/λhhh SM=3 10-5  $\overline{\phantom{a}}$

For  $gg \to HH$ :  $m_{HH}$  is significantly modified by  $\kappa_{\lambda} \neq 1$  due to large interference between ``boxes'' & ``triangles''  $\overline{a}$  $\overline{c}$  v v  $\mathsf{m}$ 

Results known to NLO (full), NNLO (FT<sub>approx</sub>), N<sup>3</sup>LO (HTL)  $300$   $(0.000)$   $(0.000)$   $(0.000)$  $\sqrt{2}$  $\rightarrow$  approx $\prime$ ,  $\rightarrow$   $\rightarrow$   $\rightarrow$   $\rightarrow$   $\rightarrow$ 

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; de Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21; Chen, Li, Shao, Wang 19;

![](_page_34_Figure_4.jpeg)

Note: dip structure changes dramatically once  $m_T^2$ dependence is included (artefact of Born vanishing exactly at this point in HTL)

# Higgs Self-Coupling from Single Higgs Production

So far focused on HH production where  $\lambda_3$  appears at LO Can also constrain this coupling from high-order effects in single Higgs production 3

![](_page_35_Picture_2.jpeg)

 $\overline{5}$  t E.g. can constrain  $\lambda_3$  below HH threshold from EW  $\overline{\phantom{a}}$ corrections to *e*+*e*<sup>−</sup> → *ZH*

McCullough 13 **McCullough 13** 

 $\Delta t$  l HC  $\lambda$  annears in main Higu At LHC, *λ*<sub>3</sub> appears in main Higgs production and decay channels *H*

![](_page_35_Figure_6.jpeg)

diagrams with internal Goldstone lines. Gorbahn, Haisch 16, 19; Bizon, Gorbah Pagani 16: Maltoni Pagani Shivaji Zhar Pagani 16; Maltoni, Pagani, Shivaji, Zhao 17; Di Vita, Grojean, Panico, Riembau, Vant package [18, 19]. With these definitions the full form of Gorbahn, Haisch 16, 19; Bizon, Gorbahn, Haisch, Zanderighi 16; Degrassi, Giardino, Maltoni, Pagani 16; Maltoni, Pagani, Shivaji, Zhao 17; Di Vita, Grojean, Panico, Riembau, Vantalon 17 <sup>12</sup>↵(*q*<sup>2</sup> Haisch Zandorighi 16: Dograssi Giardino Maltoni  $b$  rabon $\mu$  zahading into  $\mu$  bograss. $\mu$  Grafan  $\mathcal{F}$  . Examples of one loop HH -dependent diagrams for the Higgs boson self-energy (a) and the single-Higgs boson self-energy (a) and the single-Higgs boson self-energy (a) and the single-Higgs boson self-energy (a) a  $\widetilde{\mathsf{a}}$ orbahn, Haisch 16, 19; Bizon, Gorbahn, Haisch, Zanderighi 1 circle.