Higgs Pairs Workshop 2022, Dubrovnik, Croatia

30 May 2022

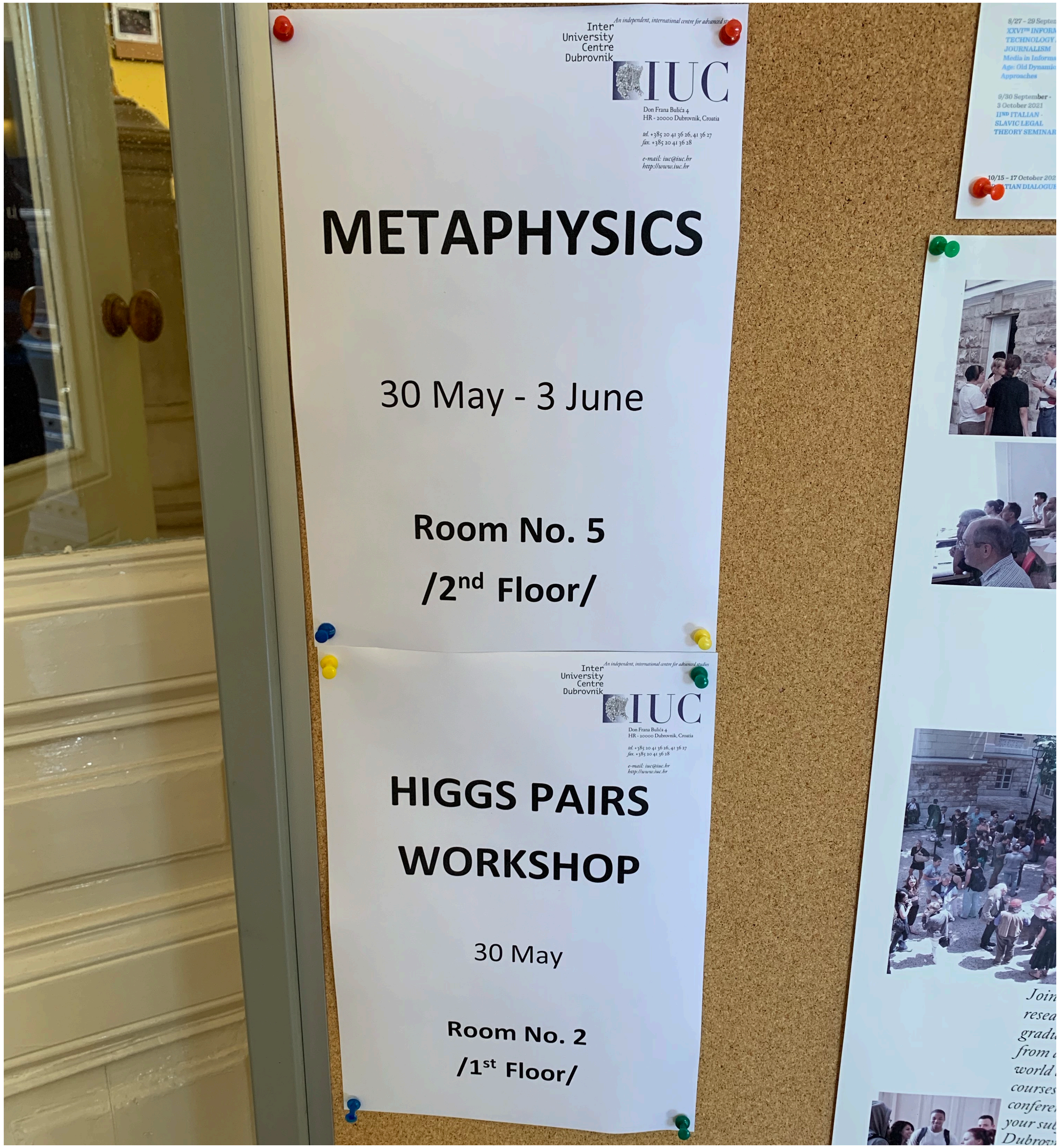
HH Production in the SM: Theoretical Status

Stephen Jones

IPPP, Durham / Royal Society URF

Durham
University

THE
ROYAL
SOCIETY



An independent, international centre for advanced st
Inter
University
Centre
Dubrovnik
IUC
Don Frana Bulića 4
HR - 20000 Dubrovnik, Croatia

tel. +385 20 41 36 26, 41 36 27
fax. +385 20 41 36 28

e-mail: iuc@iuc.hr
http://www.iuc.hr

METAPHYSICS

30 May - 3 June

Room No. 5
/2nd Floor/

An independent, international centre for advanced studies
Inter
University
Centre
Dubrovnik
IUC
Don Frana Bulića 4
HR - 20000 Dubrovnik, Croatia

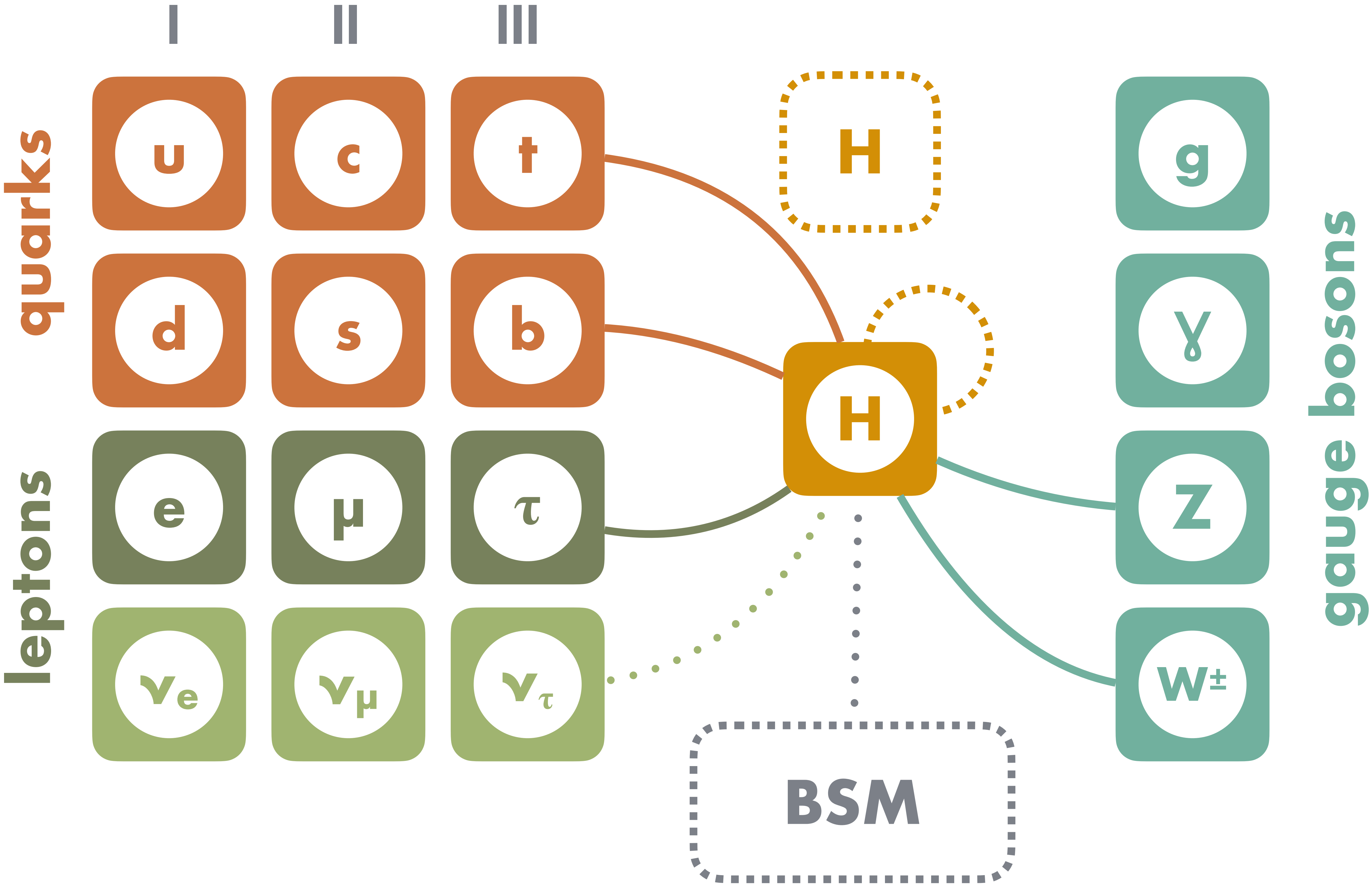
tel. +385 20 41 36 26, 41 36 27
fax. +385 20 41 36 28

e-mail: iuc@iuc.hr
http://www.iuc.hr

HIGGS PAIRS
WORKSHOP

30 May

Room No. 2
/1st Floor/


I
II
III
quarks
leptons
u
c
t
d
s
b
e
μ
τ
νe
νμ
ντ
H
H
g
γ
Z
W±
gauge bosons
BSM

Why measure Higgs pair production?

$$\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

EW symmetry breaking

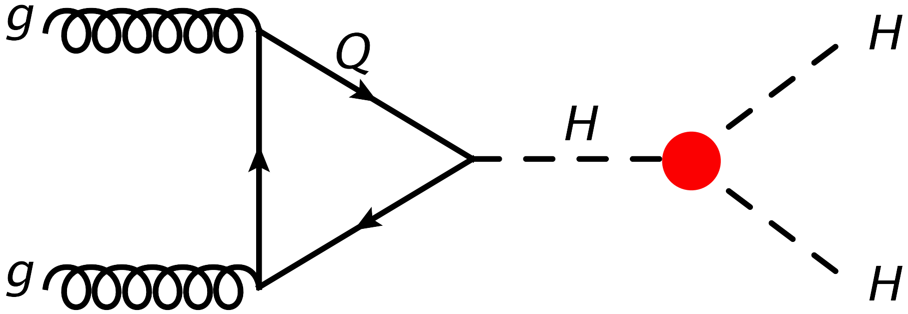
$$\mu^2 = \lambda v^2$$

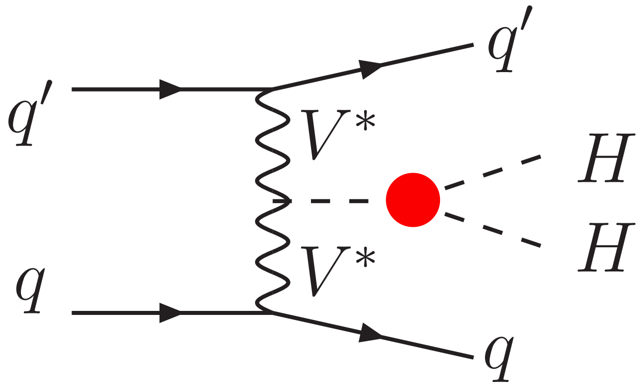
$$m_H^2 = 2\lambda v^2$$

$$V(H) = \frac{1}{2} m_H^2 H^2 + \boxed{\lambda v H^3} + \frac{\lambda}{4} H^4,$$

SM: self-couplings determined by m_H, v

EXP: need measurements to confirm/refute this





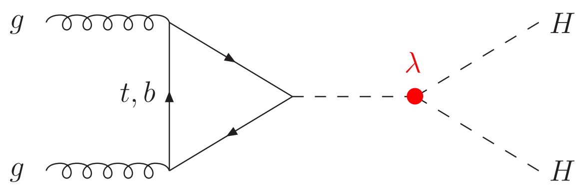
$+t\bar{t}HH$

$+VHH$

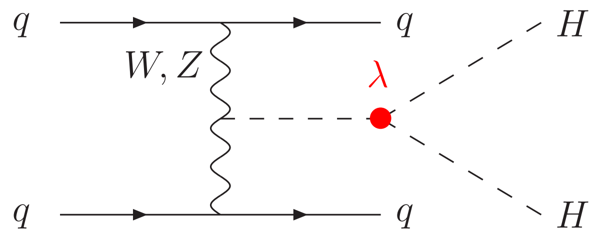
$+H$ (EW)

→ See: Stefano (Tue)

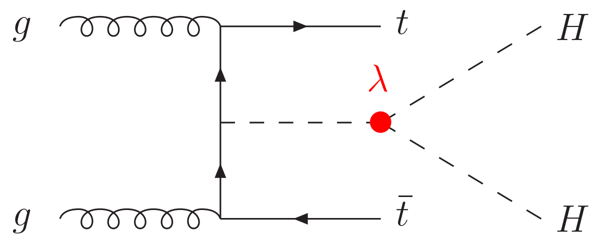
HH Production Channels at the LHC



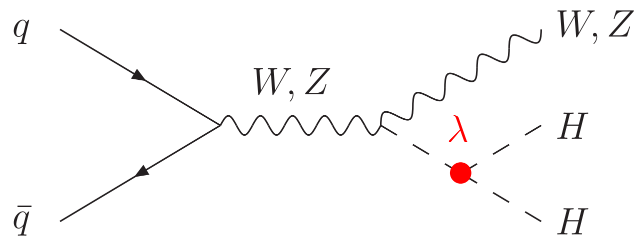
Gluon Fusion



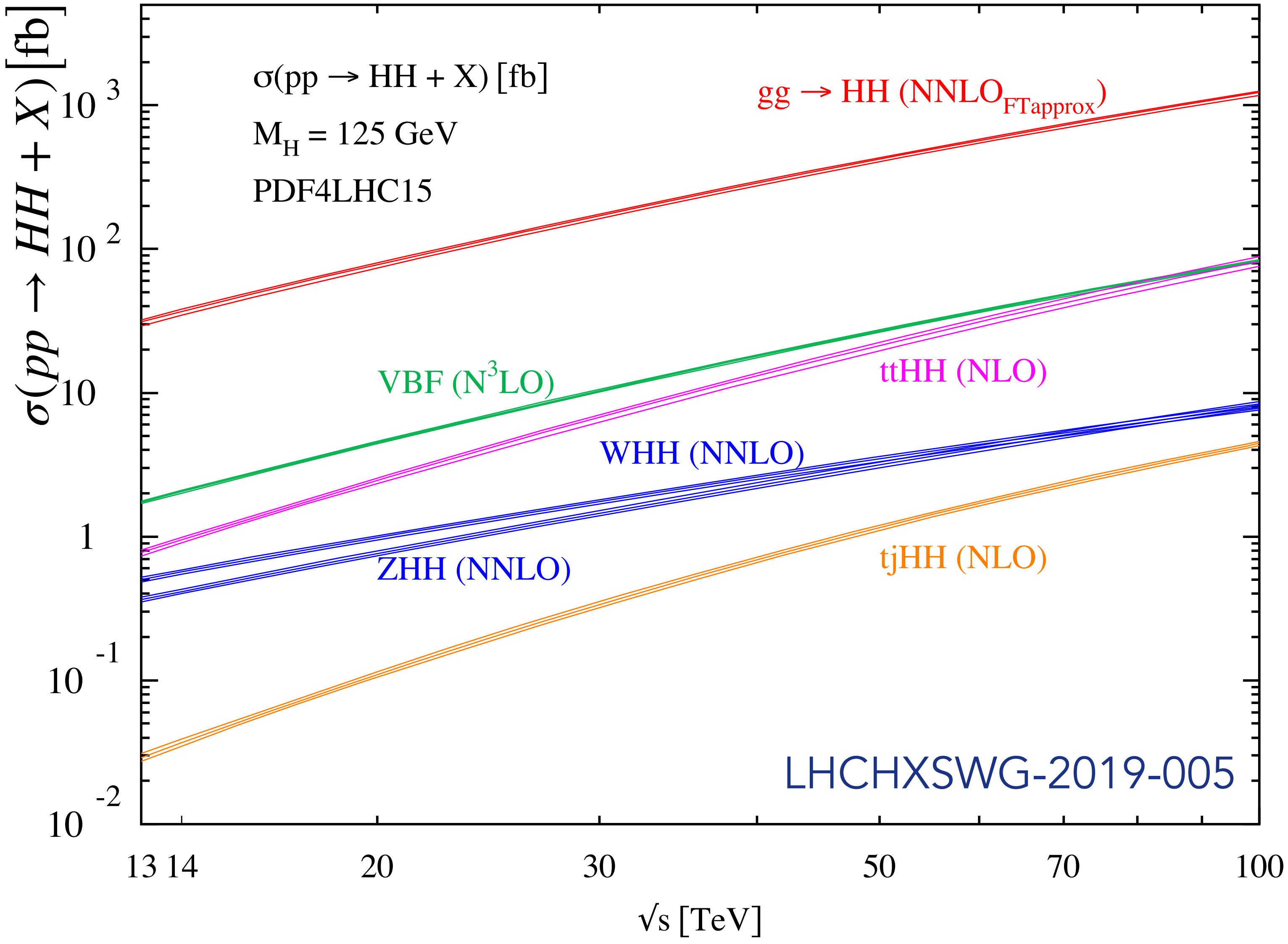
Vector Boson Fusion



Associated Production ($t\bar{t}$)



Associated Production (W,Z)



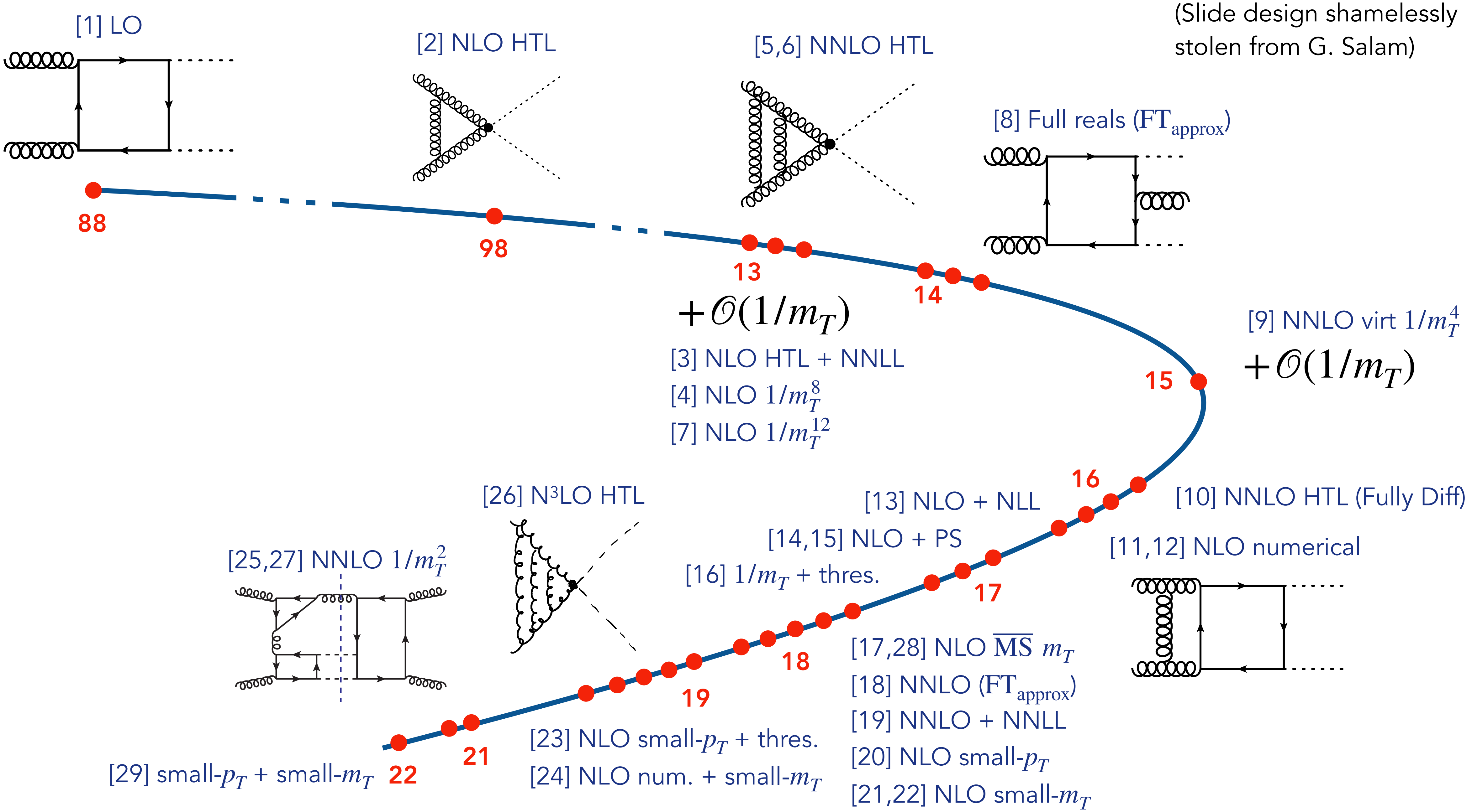
Production channels similar to H
A very important difference:

$$\sigma(pp \to HH) \sim \frac{\sigma(pp \to H)}{1000}$$

Gluon Fusion: State of the Art

An approximate history (30 years in 30 seconds)

(Slide design shamelessly stolen from G. Salam)



[1] LO

[2] NLO HTL

[5,6] NNLO HTL

[8] Full reals ($\mathrm{FT}_{\mathrm{approx}}$)

$+\mathcal{O}(1/m_T)$

[3] NLO HTL + NNLL

[4] NLO $1/m_T^8$

[7] NLO $1/m_T^{12}$

[9] NNLO virt $1/m_T^4$

$+\mathcal{O}(1/m_T)$

[10] NNLO HTL (Fully Diff)

[11,12] NLO numerical

[13] NLO + NLL

[14,15] NLO + PS

[16] $1/m_T$ + thres.

[26] N³LO HTL

[25,27] NNLO $1/m_T^2$

[17,28] NLO $\overline{\mathrm{MS}}$ m_T

[18] NNLO ($\mathrm{FT}_{\mathrm{approx}}$)

[19] NNLO + NNLL

[20] NLO small-p_T

[21,22] NLO small-m_T

[23] NLO small-p_T + thres.

[24] NLO num. + small-m_T

[29] small-p_T + small-m_T

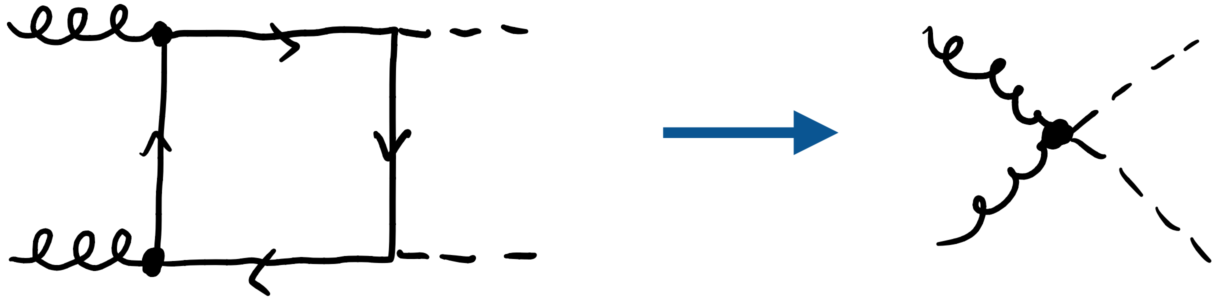
[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;

A useful approximation: Heavy Top Limit

Heavy Top Limit (HTL): integrate out top quarks ($m_T \to \infty$)

Introduces couplings c_h & c_{hh} between gluons and Higgs, matched to SM @ 4-loops

Spira 16; Gerlach, Herren, Steinhauser 18

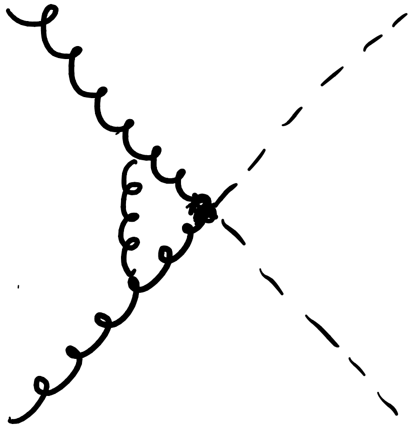


HTL valid for $\sqrt{\hat{s}} \ll 2m_T$

HH production $2m_H < \sqrt{\hat{s}}$

No internal masses, easier to compute higher-order corrections:

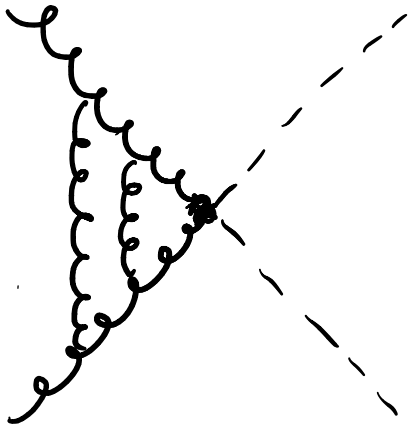
NLO



$K \approx 2$

Dawson, Dittmaier, Spira 98

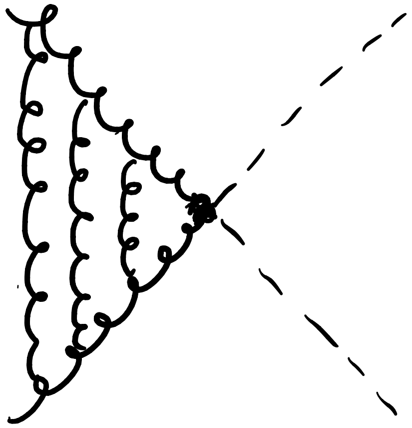
NNLO



$K \approx 1.2$

de Florian, Mazzitelli 13

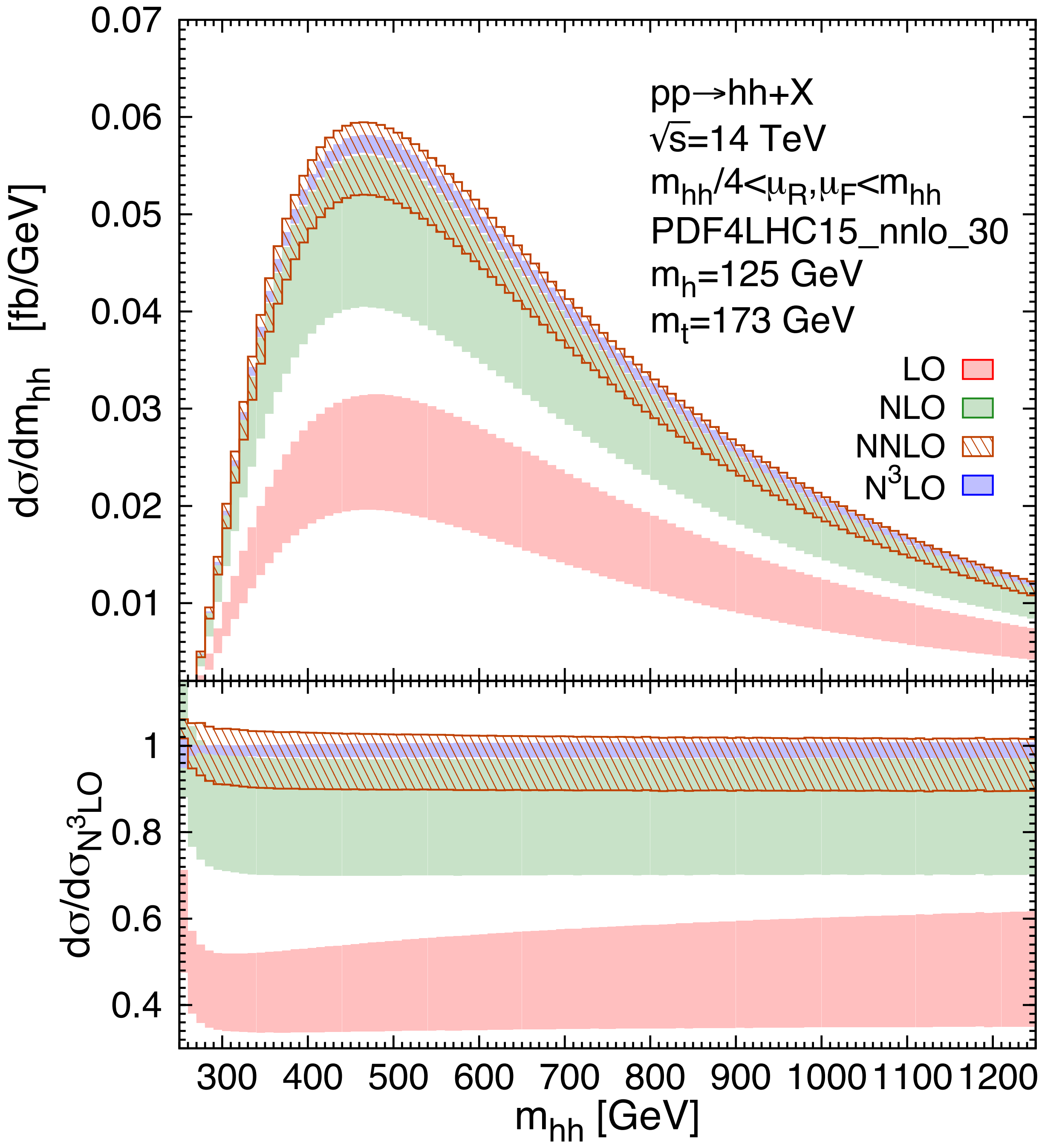
N³LO



$K \approx 1.03$

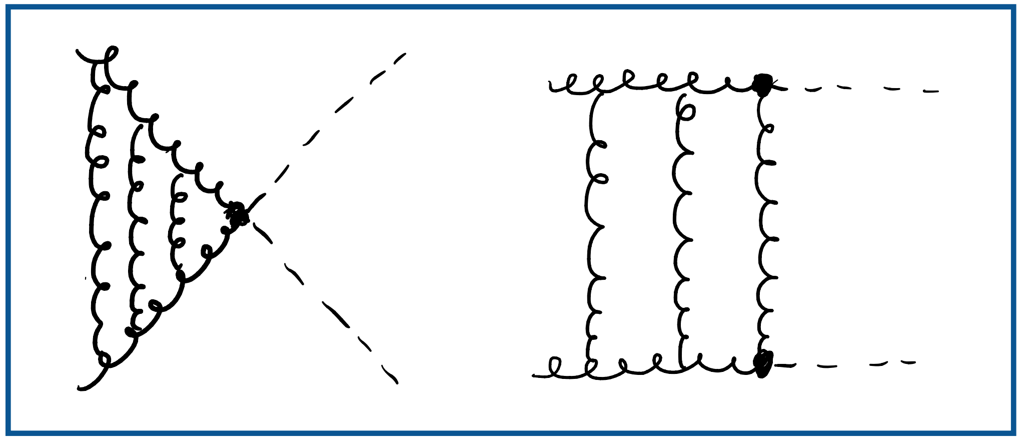
Chen, Li, Shao, Wang 19

N^3LO Heavy Top Limit



Chen, Li, Shao, Wang 19

→ See: Hua-Sheng (Tue)



Ingredients: N^3LO H calculation

Anastasiou, Duhr, Dulat, Herzog, Mistlberger 15; Dulat, Lazopoulos, Mistlberger 18

+ 2-loop 4-point functions

Banerjee, Borowka, Dhani, Gehrmann, Ravindran 18

order \ $\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
LO	$13.80^{+31\%}_{-22\%}$	$17.06^{+31\%}_{-22\%}$	$98.22^{+26\%}_{-19\%}$	$2015^{+19\%}_{-15\%}$
NLO	$25.81^{+18\%}_{-15\%}$	$31.89^{+18\%}_{-15\%}$	$183.0^{+16\%}_{-14\%}$	$3724^{+13\%}_{-11\%}$
NNLO	$30.41^{+5.3\%}_{-7.8\%}$	$37.55^{+5.2\%}_{-7.6\%}$	$214.2^{+4.8\%}_{-6.7\%}$	$4322^{+4.2\%}_{-5.3\%}$
N^3LO	$31.31^{+0.66\%}_{-2.8\%}$	$38.65^{+0.65\%}_{-2.7\%}$	$220.2^{+0.53\%}_{-2.4\%}$	$4438^{+0.51\%}_{-1.8\%}$

Very mild scale dependence

Beyond HTL @ NLO (Schematically)

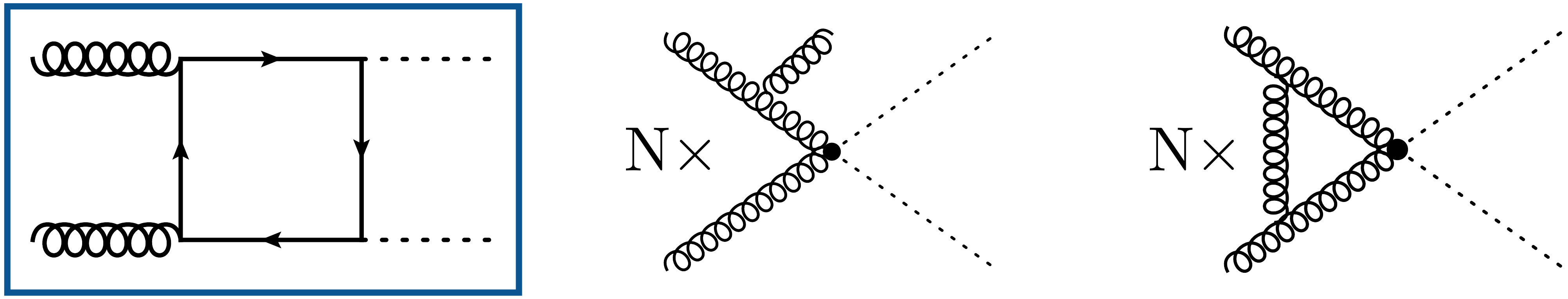
Born improved NLO HTL:

Spira et al. (HPAIR)

$$\mathrm{d}\sigma_{\mathrm{NLO}}(m_T) \approx \underbrace{\frac{\mathrm{d}\sigma_{\mathrm{LO}}(m_T)}{\mathrm{d}\sigma_{\mathrm{LO}}(m_T \to \infty)}}_{\mathrm{N}} \mathrm{d}\sigma_{\mathrm{NLO}}(m_T \to \infty)$$

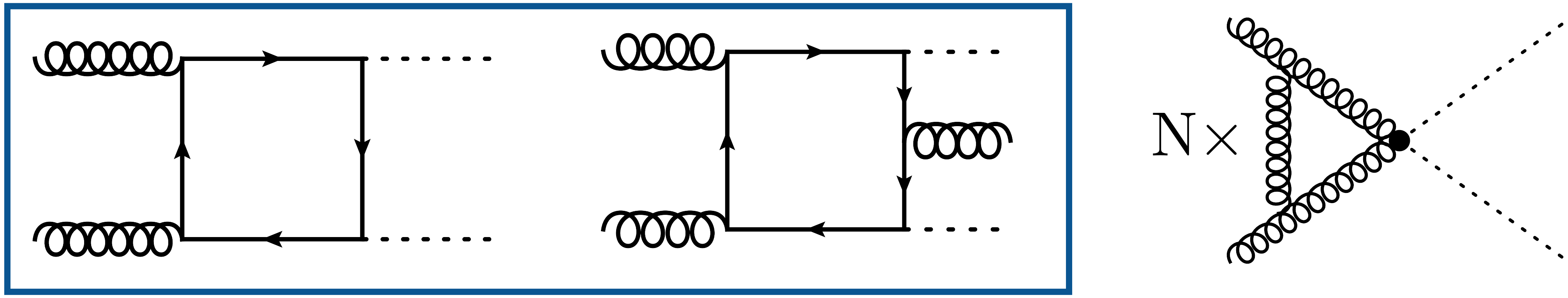
B-i HTL:

Dawson,
Dittmaier, Spira 98

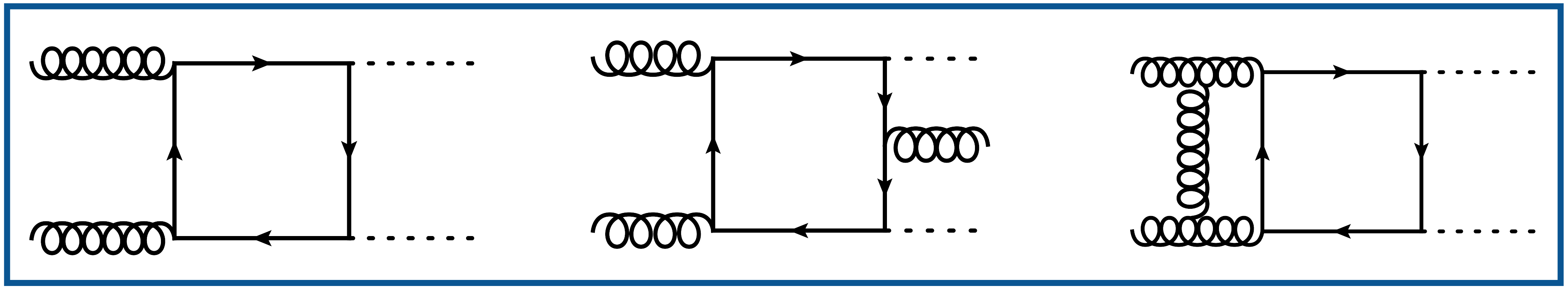


FTapprox:

Maltoni, Vryonidou,
Zaro 14



Full Theory:

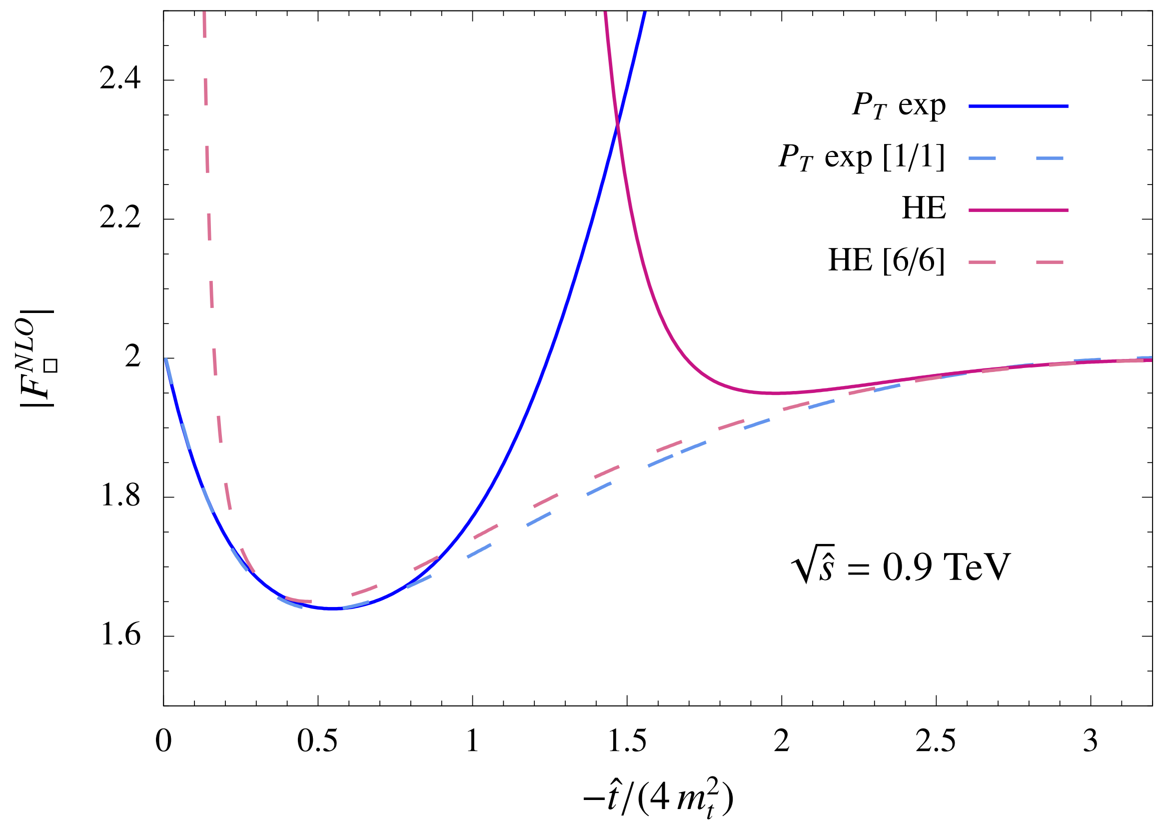


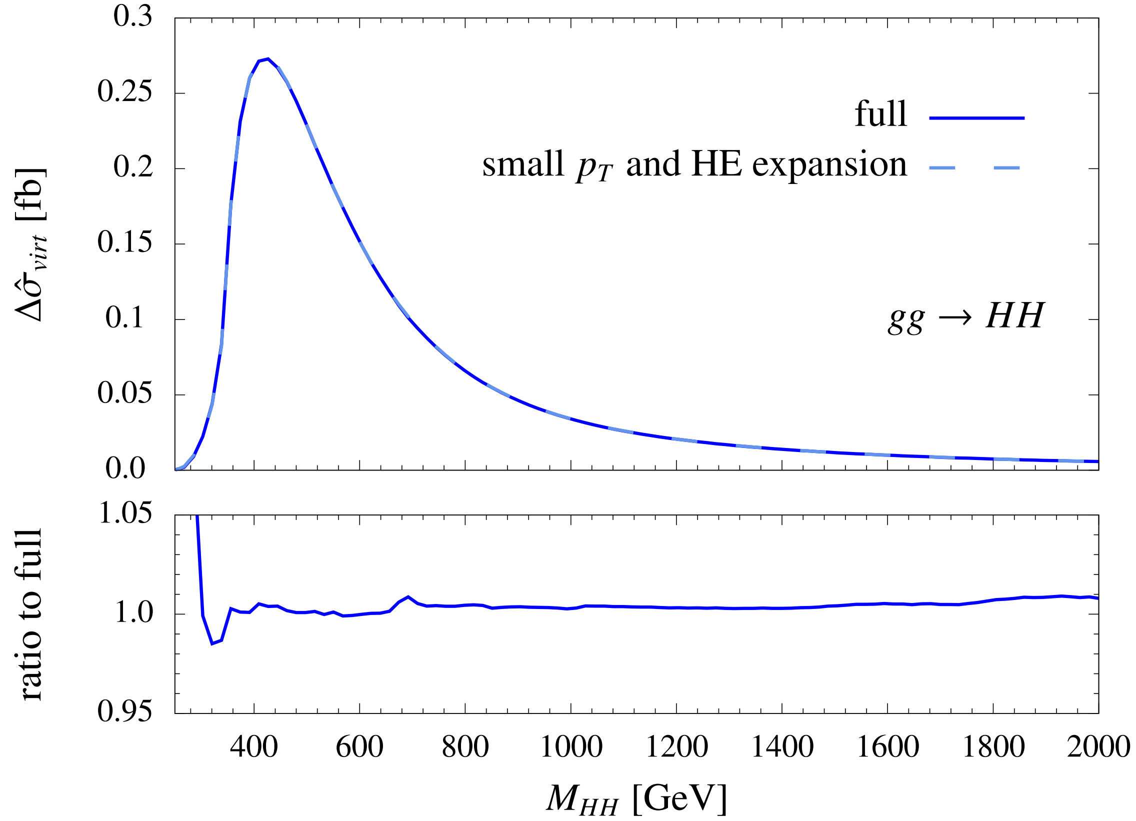
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, (Schubert), Zirke (16),16;
Baglio, Campanario, Glaus, Mühlleitner, (Ronca), Spira, (Streicher) 18, 20;

NLO: Combining small-p_T and small-m_T expansions

Expansion around $p_T^2 + m_H^2 \leq \hat{s}/4$ and also around $m_H \ll m_T \ll \hat{s}, |\,\hat{t}\,|$ are known

Bonciani, Degrassi, Giardino, Gröber 18; Davies, Mishima, Steinhauser, Wellmann 18, 18





Using Padé approximants: $\left[m/n\right](x) = \dfrac{p_0 + p_1 x + \cdots + p_m x^m}{1 + q_1 x + \cdots + q_n x^n}$

Can find some overlap where the two approximations agree for all relevant $\hat{s}, \hat{t}$

Bellafronte, Degrassi, Giardino, Gröber, Vitti 22

This approximation agrees well with the numerical NLO result

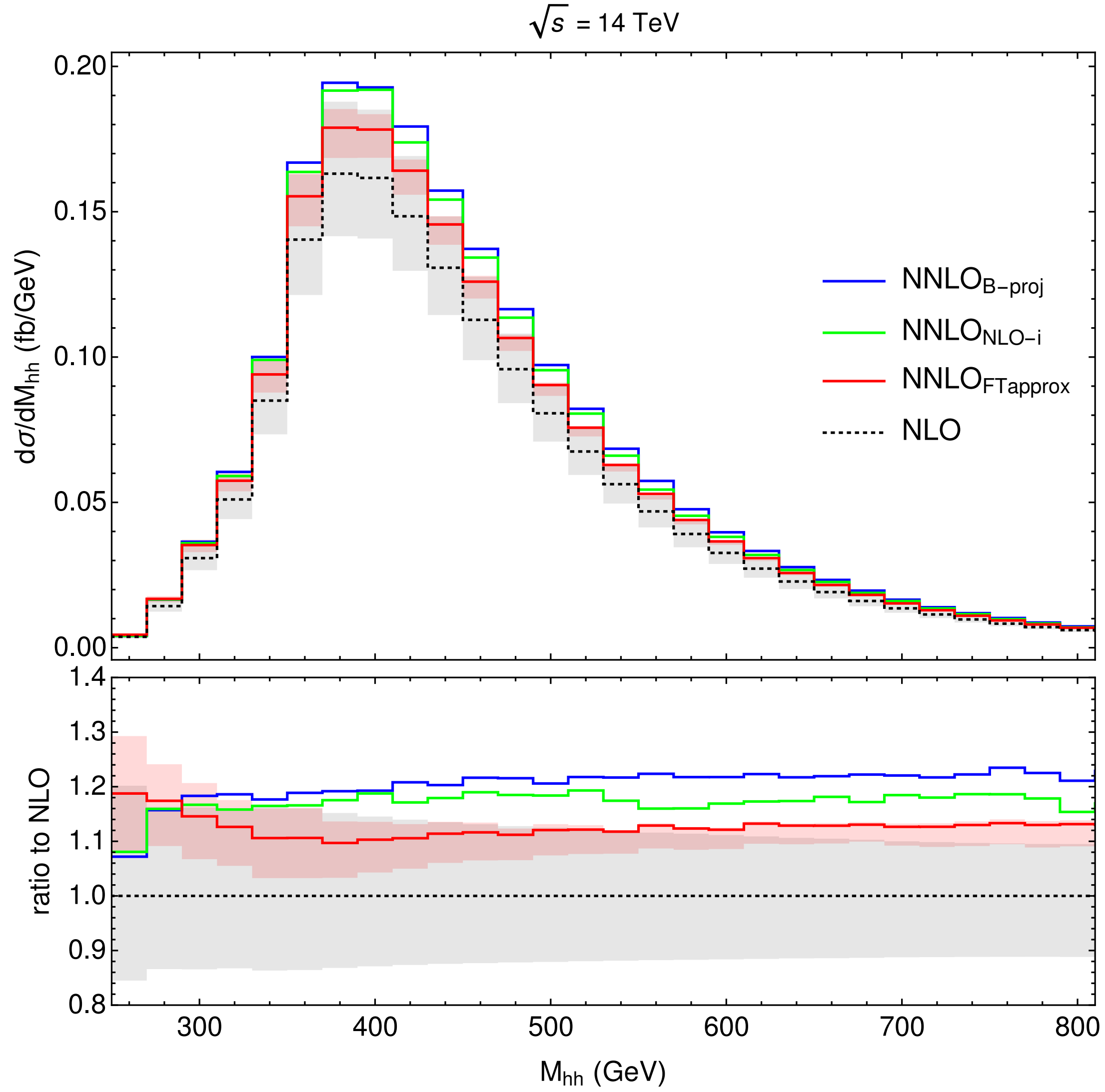
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, (Schubert), Zirke (16),16;

Baglio, Campanario, Glaus, Mühlleitner, (Ronca), Spira, (Streicher) 18, 20;

Beyond HTL @ NNLO

Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert, Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\mathrm{NLO_{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\mathrm{NNLO_{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\mathrm{NNLO_{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\mathrm{NNLO_{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\mathrm{NNLO_{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
$\mathrm{NNLO_{FTapprox}/NLO}$	1.118	1.116	1.096	1.067

1) NNLO$_{\text{NLO-i}}$

Rescale NLO by $K_{NNLO} = NNLO_{HTL}/NLO_{HTL}$

2) NNLO$_{\text{B-proj}}$

Project real radiation contributions to Born configurations, rescale by LO/LO$_{HTL}$

3) NNLO$_{\text{FTapprox}}$

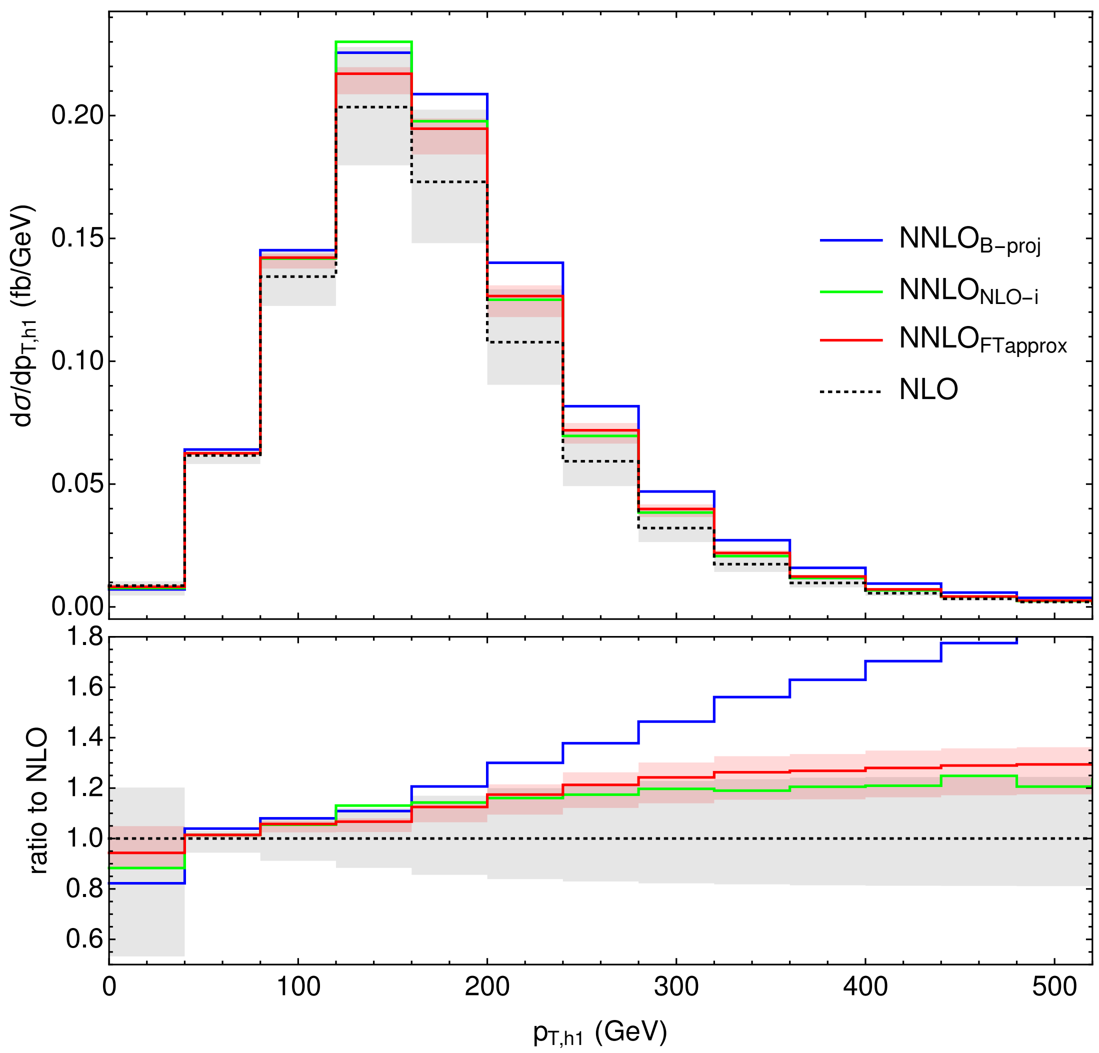
NNLO HTL squared amplitude rescaled for each multiplicity by:

$$\mathcal{R}(ij \to HH+X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH+X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH+X)}$$

Beyond HTL @ NNLO

Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert, Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\text{NLO}_{\text{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\text{NNLO}_{\text{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\text{NNLO}_{\text{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\text{NNLO}_{\text{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\text{NNLO}_{\text{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
$\text{NNLO}_{\text{FTapprox}}/\text{NLO}$	1.118	1.116	1.096	1.067

1) NNLO$_{\text{NLO-i}}$

Rescale NLO by $K_{\text{NNLO}} = \text{NNLO}_{\text{HTL}}/\text{NLO}_{\text{HTL}}$

2) NNLO$_{\text{B-proj}}$

Project real radiation contributions to Born configurations, rescale by $\text{LO}/\text{LO}_{\text{HTL}}$

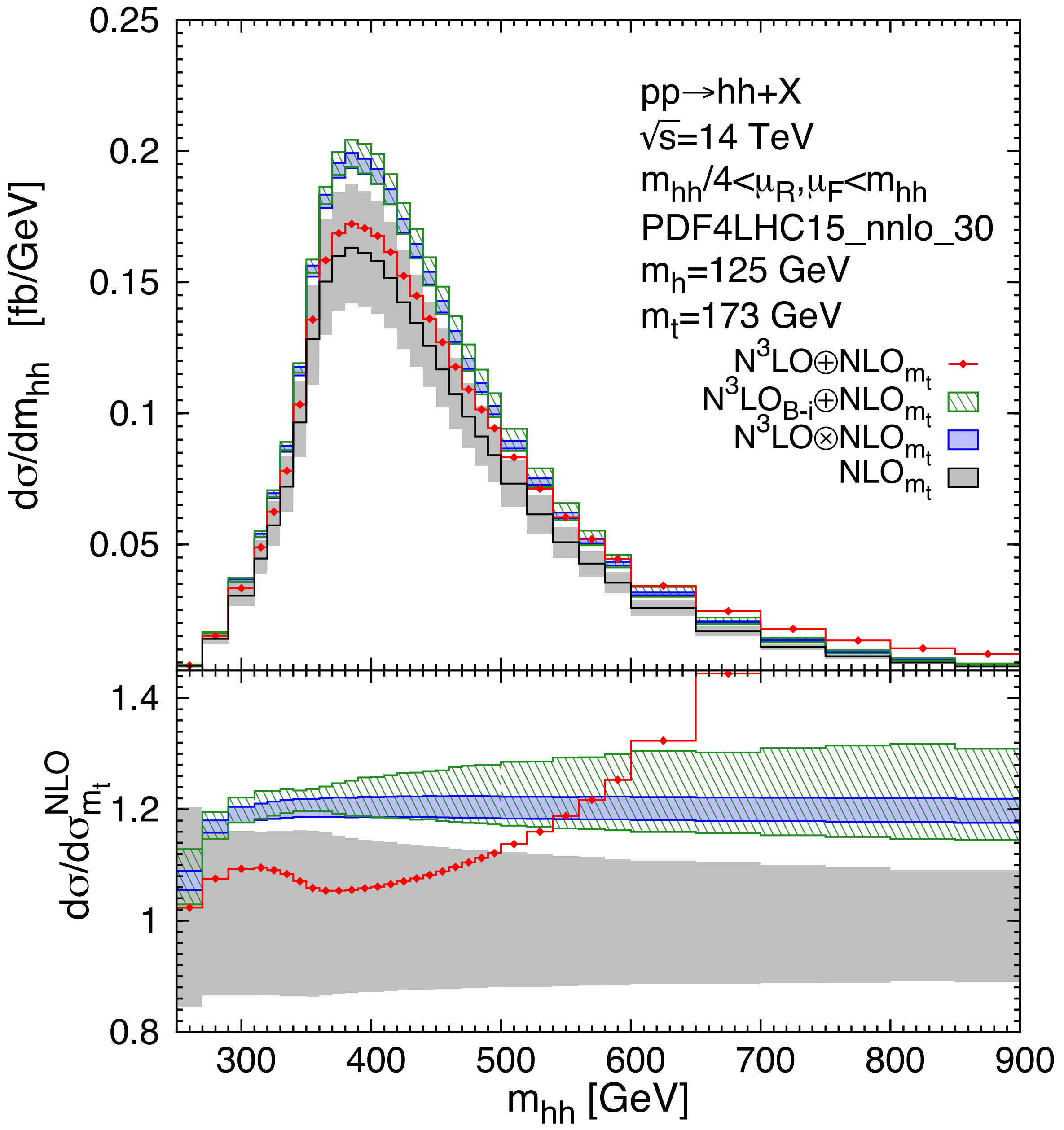
3) NNLO$_{\text{FTapprox}}$

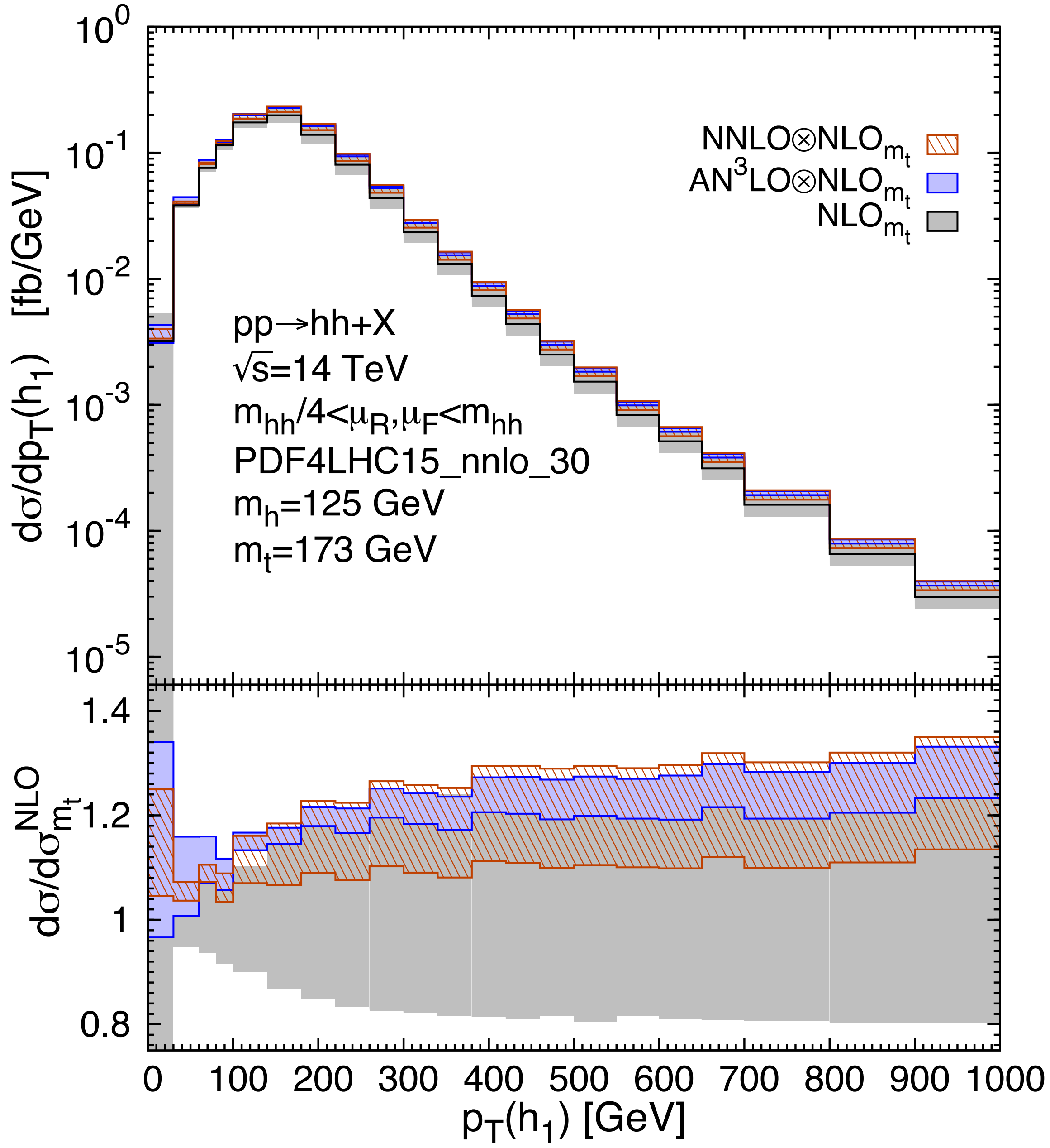
NNLO HTL squared amplitude rescaled for each multiplicity by:

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH + X)}$$

Beyond HTL @ N³LO

Top quark mass effects included in N³LO HTL (up to NLO)





Chen, Li, Shao, Wang 19

Results agree with NNLO result but with reduced scale uncertainty

$\mathrm{FT}_{\mathrm{approx}}$-like result not known, requires m_T in reals

→ See: Hua-Sheng (Tue)

Total Cross Section & Scale Uncertainty @ 14 TeV

Note: papers @ 13/14 TeV (not 13.6 TeV)

	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	σ_{N3LO} (fb)
Basic HTL	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$	$38.65^{+0.65\%}_{-2.7\%}$
B-i/proj HTL	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$40.44^{+1.9\%}_{-4.7\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$\boxed{36.69^{+2.1\%}_{-4.9\%}}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–	–
NLO-i. HTL	–	$32.88^{+13.5\%}_{-12.5\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$39.56^{+0.64\%}_{-2.7\%}$

Chen, Li, Shao, Wang 19, 19;
Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18;
de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16;
Maltoni, Vryonidou, Zaro 14 (recalculated);
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
Dawson, Dittmaier, Spira 98 (recalculated); Glover, van der Bij 88 (recalculated)

PDF4LHC15_nlo/nnlo

$m_H = 125$ GeV $\quad m_T = 173$ GeV

Uncertainty:

$$\mu_R = \mu_F = \frac{m_{HH}}{2}$$

$$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$$

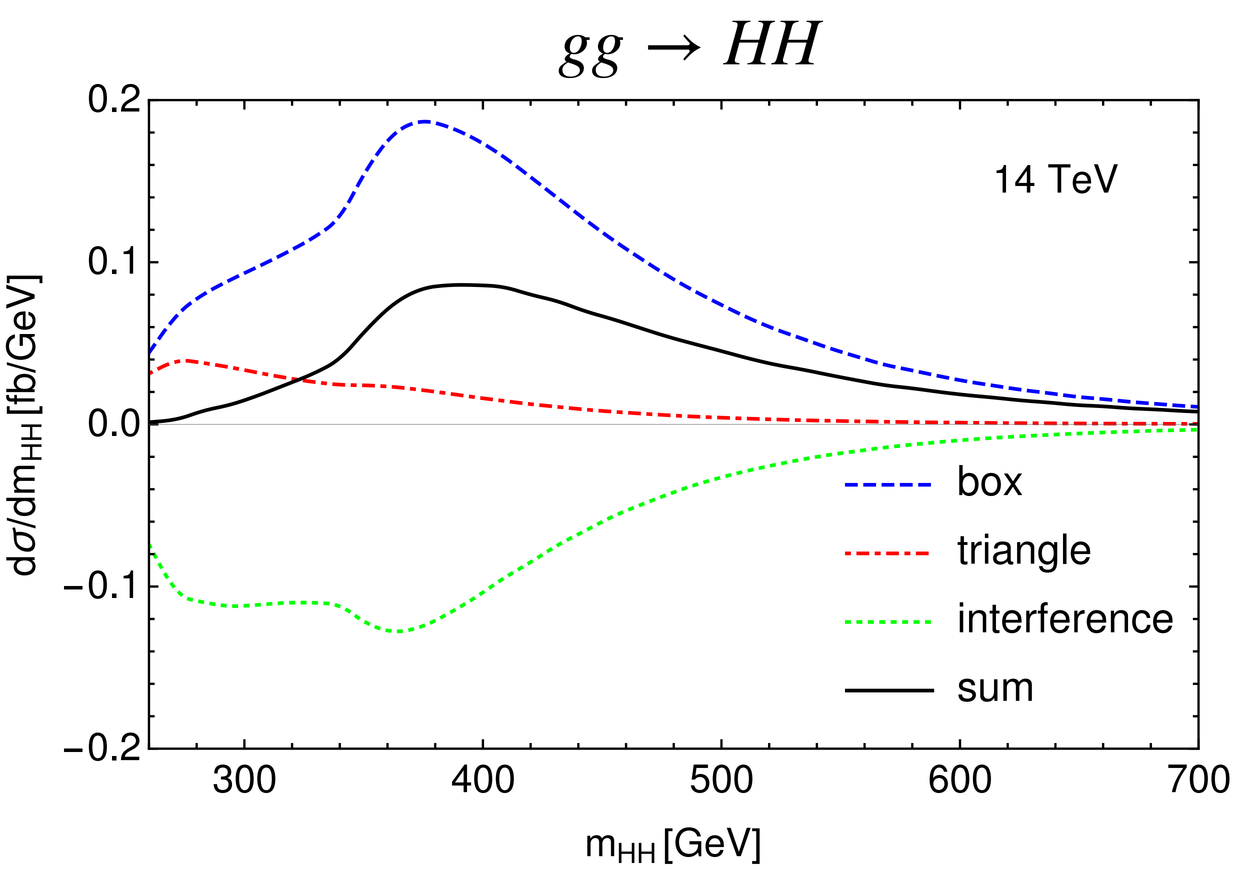
If we trust the NLO + N^{m}LO HTL combinations

Scale: $+2.2\,\%\,/-5.0\,\%$ PDF+α_s: $\pm 3.0\,\%$

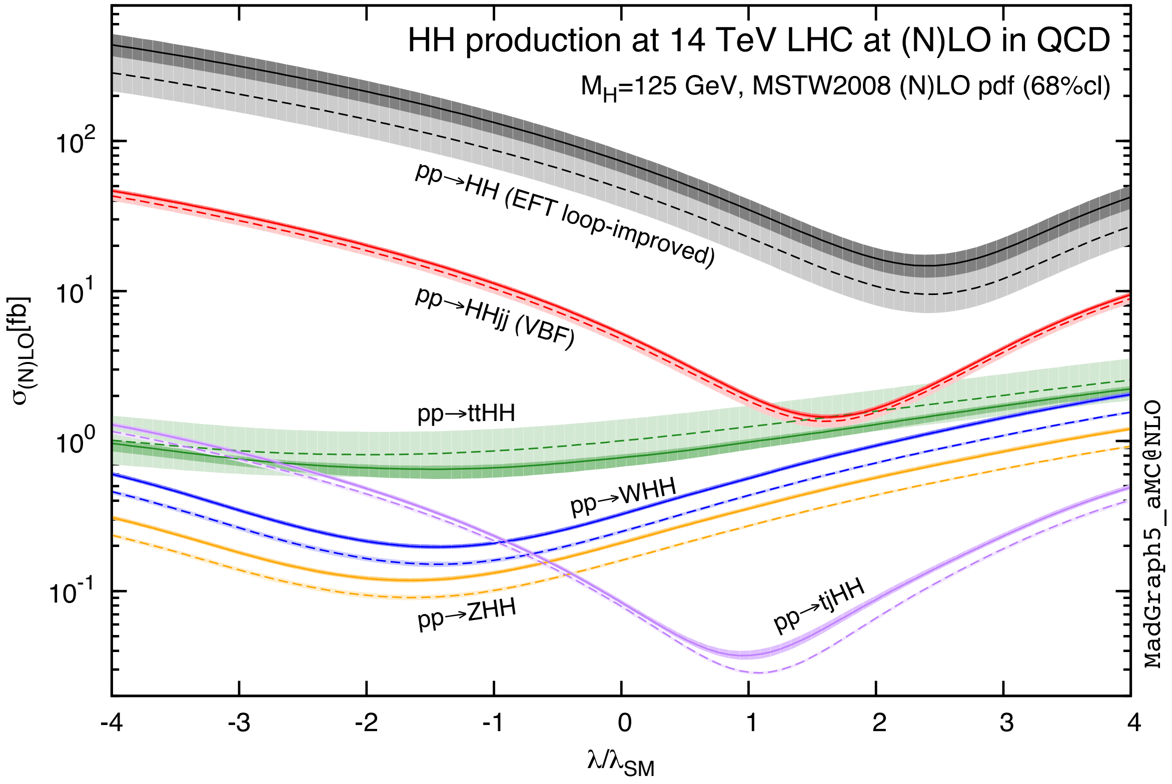
m_T approx: $\pm 2.7\,\%$ m_T scheme: $+4.0\,\%\,/-18.0\,\%$

See: HH Twiki

Status: HH Self Coupling



LHCHXSWG-2019-005



Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, M. Zaro 14

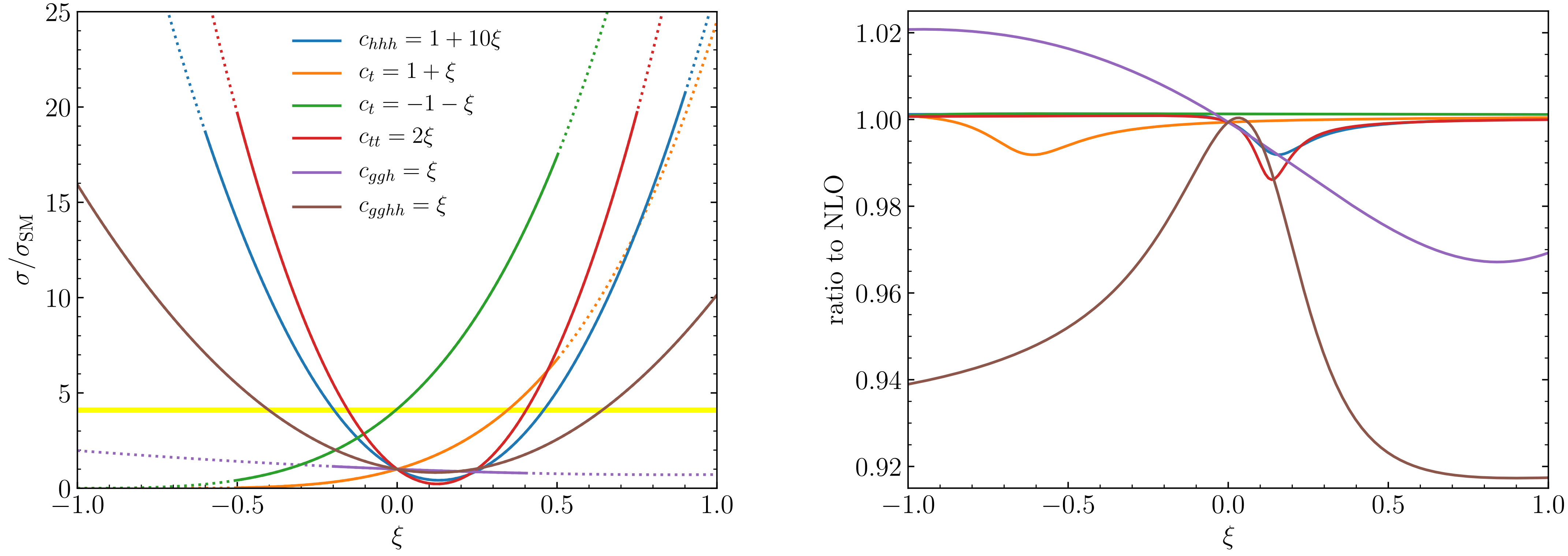
Theory uncertainties on cross section translate into uncertainties on the self-coupling extraction: for $gg \to HH$ close to SM λ_{hhh} we have $\dfrac{\Delta\sigma}{\sigma} \sim -\dfrac{\Delta\lambda}{\lambda}$

Self coupling dependence known at:

NLO+PS (full theory) Borowka, et al. 16; Baglio et al. 18,20; Heinrich, et al. 19, 20;

N^{3}LO (re-weighted HTL) Chen, Li, Shao, Wang 19

Status: HH EFT



de Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21

NNLO corrections have typically ~10% effect on $(\sigma/\sigma_{\mathrm{SM}})_{\mathrm{NNLO}'}/(\sigma/\sigma_{\mathrm{SM}})_{\mathrm{NLO}}$

Significantly reduce scale uncertainties

EFT Results in various approximations:

B.I. NLO HTL	Gröber, Mühlleitner, Spira, (Streicher) (15), 17;
NLO (HEFT)	Buchalla, Capozi, Celis, Heinrich, Scyboz 18;
+ PS	Heinrich, SJ, Kerner, Scyboz 20;
NLO (SMEFT)	Heinrich, Lang, Scyboz 22;
B.I. NNLO HTL	de Florian, Fabre, Mazzitelli 17;
NLO + NNLO’	de Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21

→ See: Ludovic/ Raquel/Nicolas (Tue)

Gluon Fusion: Issues & Dangers

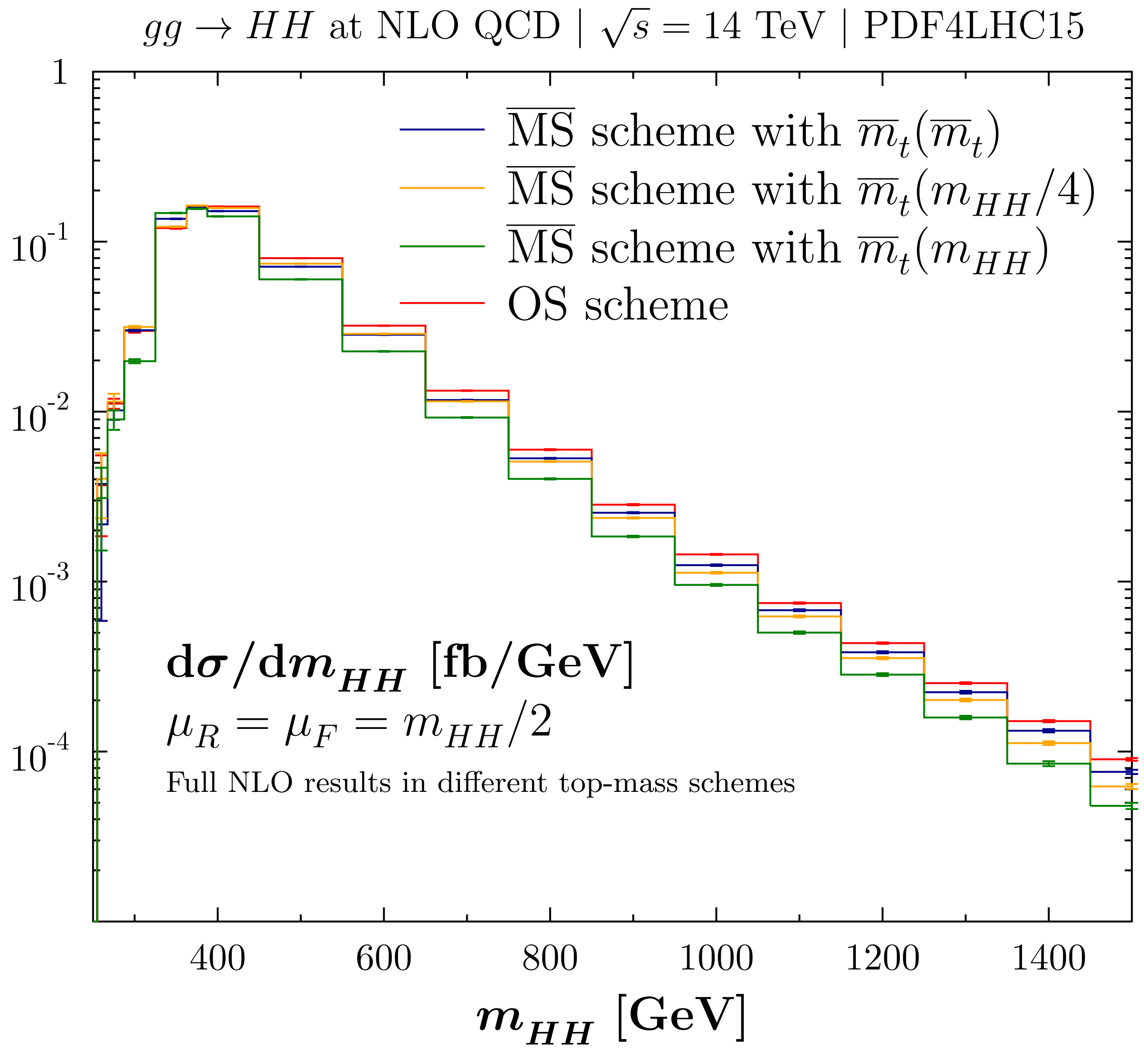
Mass Scheme Uncertainty

With such a tiny scale uncertainty, other sources of uncertainty become relevant

HH@NLO: m_T in the OS and $\overline{\mathrm{MS}}$ scheme

Baglio, Campanario, Glaus, Mühlleitner, (+Ronca), Spira, Streicher 18, (20)

OS to $\overline{\mathrm{MS}}$ mass conversion: $m_t \to \overline{m}_t(\mu_t)\left(1+\frac{\alpha_s(\mu_R)}{4\pi}C_F\left\{4+3\log\left[\frac{\mu_t^2}{\overline{m}_t(\mu_t)^2}\right]\right\}\right)$



Top quark mass scheme unc:

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=400\text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=600\text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV},$$

↑

Large uncertainty obtained comparing OS scheme with $\overline{\mathrm{MS}}$ scheme at scale m_{HH}

Mass Scheme Uncertainty (II)

Combination of scale (μ_R, μ_F) and top mass scheme (OS / $\overline{\mathrm{MS}}$) studied
Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 20

If we wish to take the **envelope** of the predictions as the uncertainty, then the two uncertainties should be added **linearly** (validated at NLO)

Scale (μ_R, μ_F)		**NLO Mass Scheme Unc.**		**Proposed Combination**
$\kappa_\lambda = -10: \quad \sigma_{tot} = 1680^{+3.0\%}_{-7.7\%}$ fb,		$\kappa_\lambda = -10: \quad \sigma_{tot} = 1438(1)^{+10\%}_{-6\%}$ fb,		$\kappa_\lambda = -10: \quad \sigma_{tot} = 1680^{+13\%}_{-14\%}$ fb,
$\kappa_\lambda = -5: \quad \sigma_{tot} = 598.9^{+2.7\%}_{-7.5\%}$ fb,		$\kappa_\lambda = -5: \quad \sigma_{tot} = 512.8(3)^{+10\%}_{-7\%}$ fb,		$\kappa_\lambda = -5: \quad \sigma_{tot} = 598.9^{+13\%}_{-15\%}$ fb,
$\kappa_\lambda = -1: \quad \sigma_{tot} = 131.9^{+2.5\%}_{-6.7\%}$ fb,		$\kappa_\lambda = -1: \quad \sigma_{tot} = 113.66(7)^{+8\%}_{-9\%}$ fb,		$\kappa_\lambda = -1: \quad \sigma_{tot} = 131.9^{+11\%}_{-16\%}$ fb,
$\kappa_\lambda = 0: \quad \sigma_{tot} = 70.38^{+2.4\%}_{-6.1\%}$ fb,		$\kappa_\lambda = 0: \quad \sigma_{tot} = 61.22(6)^{+6\%}_{-12\%}$ fb,		$\kappa_\lambda = 0: \quad \sigma_{tot} = 70.38^{+8\%}_{-18\%}$ fb,
$\kappa_\lambda = 1: \quad \sigma_{tot} = 31.05^{+2.2\%}_{-5.0\%}$ fb,	+	$\kappa_\lambda = 1: \quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%}$ fb,	=	$\kappa_\lambda = 1: \quad \sigma_{tot} = 31.05^{+6\%}_{-23\%}$ fb,
$\kappa_\lambda = 2: \quad \sigma_{tot} = 13.81^{+2.1\%}_{-4.9\%}$ fb,		$\kappa_\lambda = 2: \quad \sigma_{tot} = 13.2(1)^{+1\%}_{-23\%}$ fb,		$\kappa_\lambda = 2: \quad \sigma_{tot} = 13.81^{+3\%}_{-28\%}$ fb,
$\kappa_\lambda = 2.4: \quad \sigma_{tot} = 13.10^{+2.3\%}_{-5.1\%}$ fb,		$\kappa_\lambda = 2.4: \quad \sigma_{tot} = 12.7(1)^{+4\%}_{-22\%}$ fb,		$\kappa_\lambda = 2.4: \quad \sigma_{tot} = 13.10^{+6\%}_{-27\%}$ fb,
$\kappa_\lambda = 3: \quad \sigma_{tot} = 18.67^{+2.7\%}_{-7.3\%}$ fb,		$\kappa_\lambda = 3: \quad \sigma_{tot} = 17.6(1)^{+9\%}_{-15\%}$ fb,		$\kappa_\lambda = 3: \quad \sigma_{tot} = 18.67^{+12\%}_{-22\%}$ fb,
$\kappa_\lambda = 5: \quad \sigma_{tot} = 94.82^{+4.9\%}_{-8.8\%}$ fb,		$\kappa_\lambda = 5: \quad \sigma_{tot} = 83.2(3)^{+13\%}_{-4\%}$ fb,		$\kappa_\lambda = 5: \quad \sigma_{tot} = 94.82^{+18\%}_{-13\%}$ fb,
$\kappa_\lambda = 10: \quad \sigma_{tot} = 672.2^{+4.2\%}_{-8.5\%}$ fb		$\kappa_\lambda = 10: \quad \sigma_{tot} = 579(1)^{+12\%}_{-4\%}$ fb		$\kappa_\lambda = 10: \quad \sigma_{tot} = 672.2^{+16\%}_{-13\%}$ fb

@13 TeV

→ **See: Michael (Tue)**

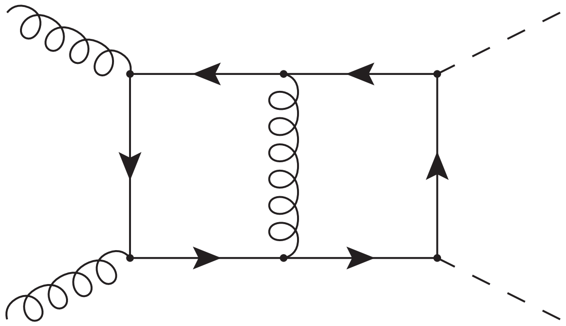
Mass Scheme Uncertainty (III)

Such mass scheme uncertainties show up in other processes (e.g. H*, HJ, ZH)
SPJ, Spira (Les Houches 19)

$$A_i^{\text{fin}} = a_s A_i^{(0),\text{fin}} + a_s^2 A_i^{(1),\text{fin}} + \mathcal{O}(a_s^3) \quad \text{with} \;\; a_s = \alpha_s/4\pi$$

$gg \to HH$

Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20



$$A_i^{(0)} \sim m_t^2 f_i(s,t)$$

$$A_i^{(1)} \sim 6 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right]$$

LO: m_t^2 from y_t^2

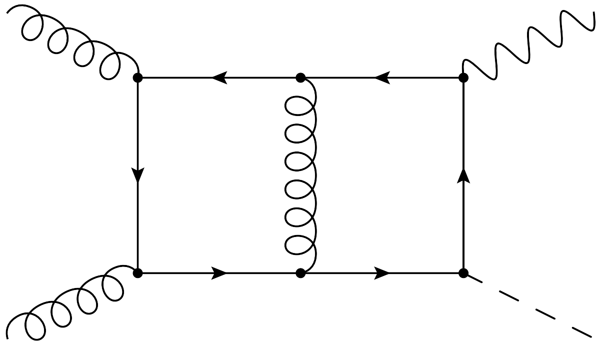
NLO: leading $\log(m_t^2)$ from mass c.t.

converting to $\overline{\text{MS}}$ gives $\log\left[\mu_t^2/s\right]$

motivating scale choice of $\mu_t^2 \sim s$

$gg \to ZH$

Davies, Mishima, Steinhauser 20; Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22



$$A_i^{(0)} \sim m_t^2 f_i(s,t) \;\; \log^2\left[\frac{m_t^2}{s}\right]$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{6} A_i^{(0)} \;\; \log^2\left[\frac{m_t^2}{s}\right]$$

LO: one m_t from y_t

NLO: leading $\log(m_t^2)$ not coming from mass c.t. (C_A)

EFT Choice

Several challenges/considerations with using Effective Field Theories (EFTs)

Can construct more than one EFT with different constraints/relations on operators

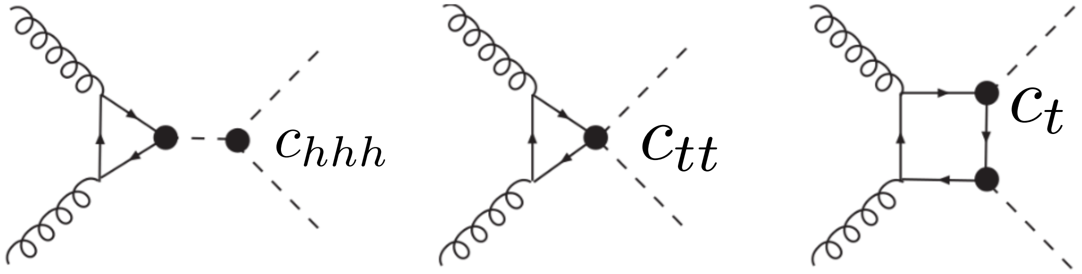
HEFT:

Higgs boson field $h(x)$ is $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ singlet

Expand in loop orders $\sim 1/(16\pi^2)$

$$\mathscr{L}_{\mathrm{HEFT}} = \mathscr{L}_2 + \sum_{L=1}^{\infty} \sum_i \left(\frac{1}{16\pi^2} \right)^L C_i^{(L)} O_i^{(L)}$$

A priori no relation between c_{ggh} & c_{gghh}



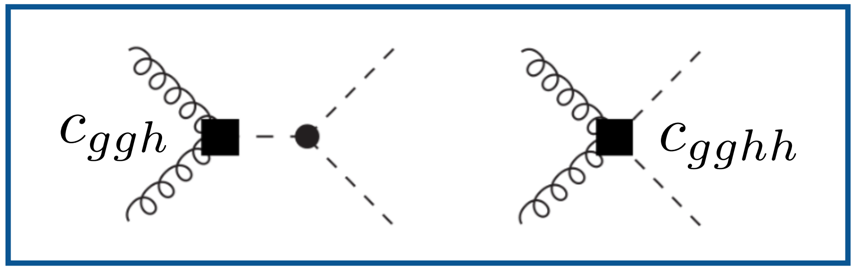
SMEFT ⊂ HEFT:

Higgs field complex doublet

Expand in canonical dimension $\sim 1/\Lambda^2$

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \mathscr{O}\left(\frac{1}{\Lambda^3} \right)$$

Relation $c_{ggh} \sim c_{gghh}$



Related in SMEFT

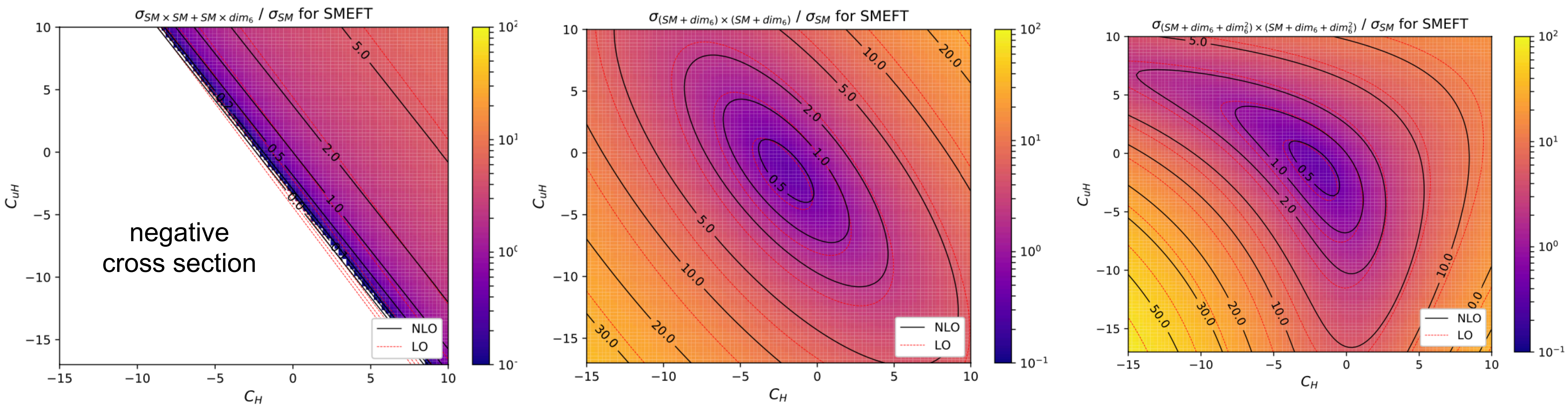
→ **See: Raquel/Ludovic/Nicolas (Tue), Alexandre (Fri)**

EFT Truncation

How the EFT expansion is truncated is not always innocent

$$\sigma \simeq \begin{cases} \sigma_{\mathrm{SM}} + \sigma_{\mathrm{SM}\times\mathrm{dim6}} & \text{(a)} \quad \text{i.e. Include } 1/\Lambda^2 \text{ at cross section level} \\ \sigma_{(\mathrm{SM}+\mathrm{dim6})\times(\mathrm{SM}+\mathrm{dim6})} & \text{(b)} \quad \text{i.e. Include } 1/\Lambda^2 \text{ at amplitude level} \\ \sigma_{(\mathrm{SM}+\mathrm{dim6})\times(\mathrm{SM}+\mathrm{dim6})} + \sigma_{(\mathrm{SM}\times\mathrm{dim6}^2)} & \text{(c)} \quad \text{i.e. } + 1/\Lambda^2 \cdot 1/\Lambda^2 \text{ at cross section level} \\ \sigma_{(\mathrm{SM}+\mathrm{dim6}+\mathrm{dim6}^2)\times(\mathrm{SM}+\mathrm{dim6}+\mathrm{dim6}^2)} & \text{(d)} \quad \text{i.e. } + 1/\Lambda^2 \cdot 1/\Lambda^2 \text{ at amplitude level} \end{cases}$$

Can get wildly different EFT limits both at LO & NLO Heinrich, Lang, Scyboz 22



linear
option (a)

quadratic
option (b)

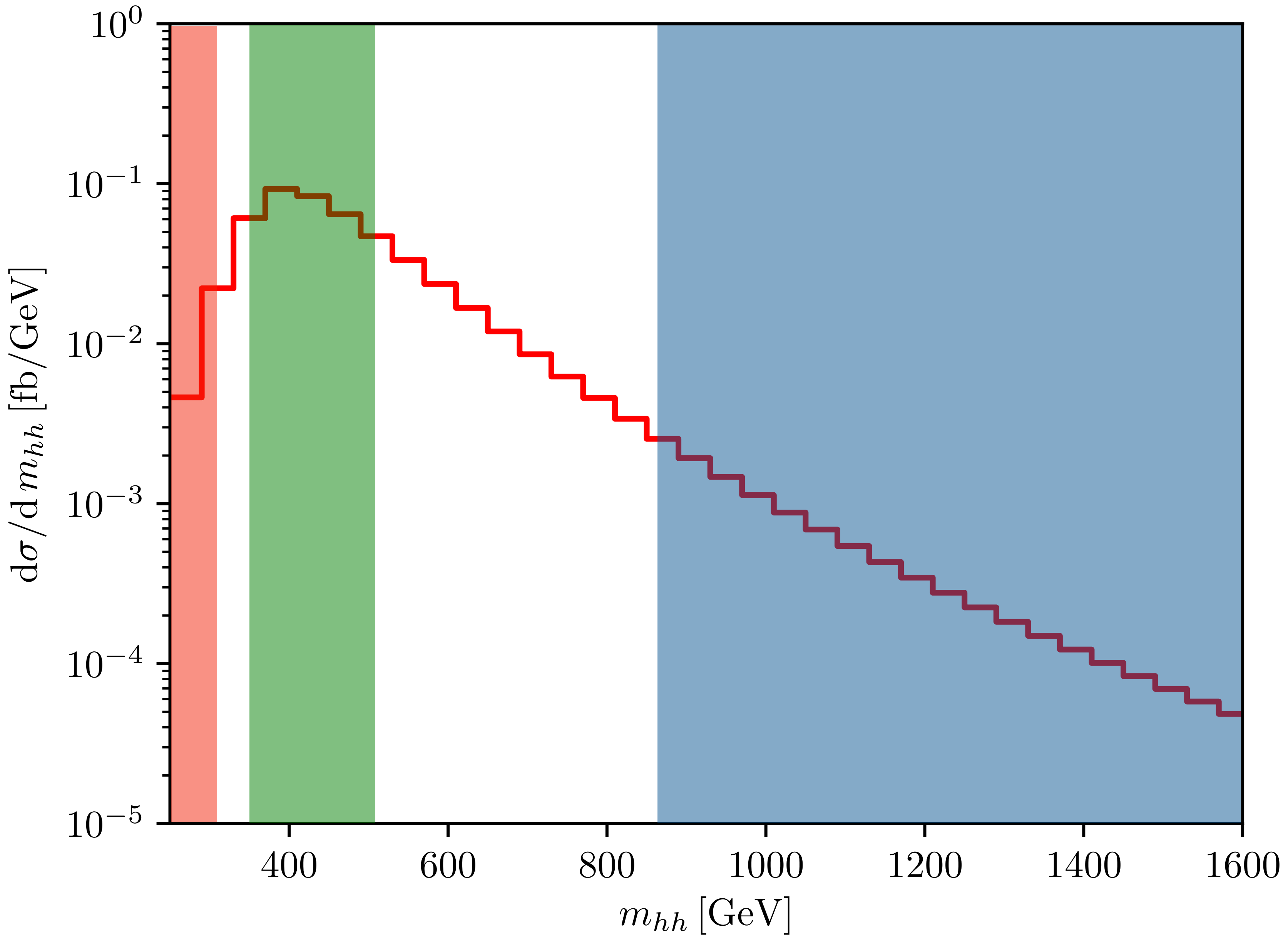
all squared
option (d)

Figure: Gudrun Heinrich LL2022

→ **See: Jannis (Fri)**

Gluon Fusion: Horizon

Tackling Mass Scheme Uncertainties



Low invariant mass:

expand in $1/m_t^2$ known to NNLO

Grigo, Hoff, Steinhauser 15;

Around Peak:

threshold expansion

Gröber, Maier, Rauh 17

High energy:

small-m_t expansion known at NLO

Davies, Mishima, Steinhauser, Wellmann 18, 19

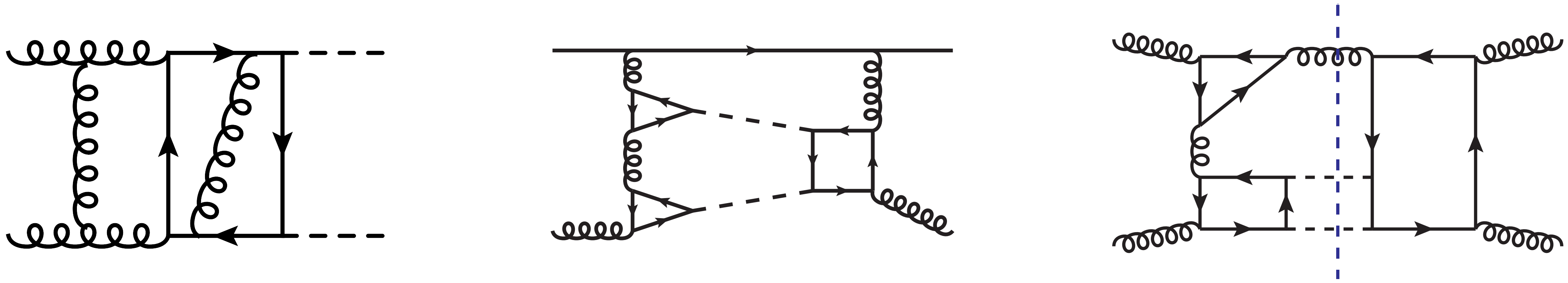
See also: Bonciani, Degrassi, Giardino, Gröber 18; Davies, Steinhauser 19; Davies, Herren, Mishima, Steinhauser 19; Davies, Gröber, Maier, Rauh, Steinhauser 19; Bellafronte, Degrassi, Giardino, Gröber, Vitti 22;

Options: 1) Try to understand structure of mass logarithms 2) Keep calculating 3) Other ideas(?)

Liu, Penin 17, 18;
Liu, Modi, Penin 22

NNLO Beyond HTL

NNLO Virtual & Real Corrections in a $1/m_t^2$ expansion:



3-loop virtual piece in large-m_t expansion (up to $1/m_t^8$)

Davies, Steinhauser 19

5-loop forward scattering amplitude (n_h^3) piece

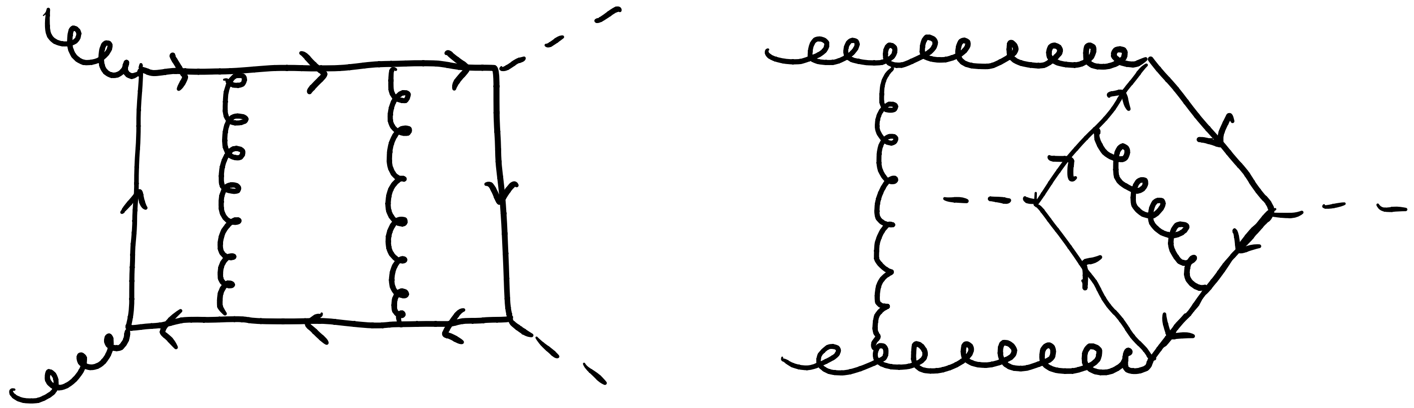
Davies, Herren, Mishima, Steinhauser 19

5-loop forward scattering amplitude (all pieces)

Davies, Herren, Mishima, Steinhauser 21

Would be extremely useful to have similar results in a small-m_t expansion

Feasible during Run 3 (?)



EW Corrections

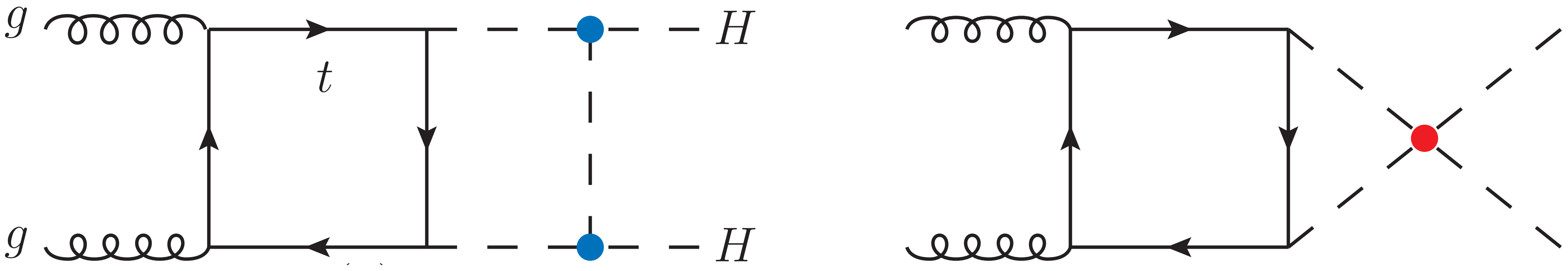
With N^{3}LO QCD (HTL) results known, could now be interesting to explore also the impact of EW corrections (in single Higgs for off-shell Higgs have $\pm 5\,\%$ impact)

Actis, Passarino, Sturm, Uccirati 08

Richer structure in the SM and much richer structure in the context of EFT

Example: Partial 2-loop EW corrections (involving λ_3 and λ_4)

Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 18



HL-LHC has only limited sensitivity to λ_4 , more relevant for FCC

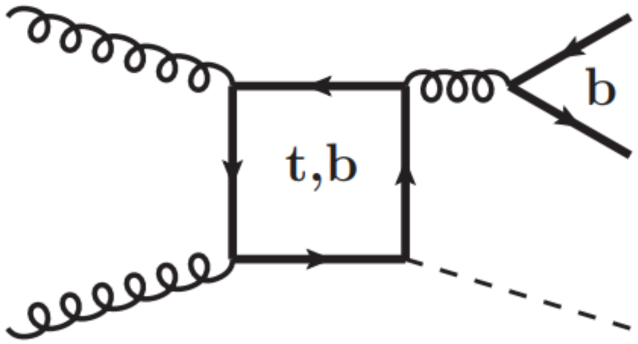
The complete EW corrections could potentially modify distributions and bounds in both SM and EFT frameworks

→See: Hantian (Fri)

Backgrounds

I am aware of two background studies taking place in the context of the HH WG

$b\bar{b}H$ (background to HH with $H \to bb$)

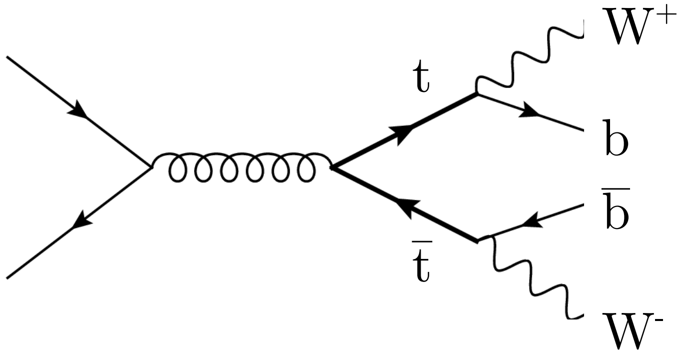


Included at LO in past searches (via NNLOPS ggF) w/ 100% uncertainty assigned ATLAS-CONF-2012-016

NLO corrections known in the HTL, large K-factors ($\sim 2 - 3$) depending on fiducial cuts
Deutschmann, Maltoni, Wiesemann, Zaro 18

Amplitudes for NNLO corrections are now also known
Badger, Hartanto, Krys, Zoia 21

$t\bar{t}$ (background to $b\bar{b}WW$, $bb\tau\tau$)



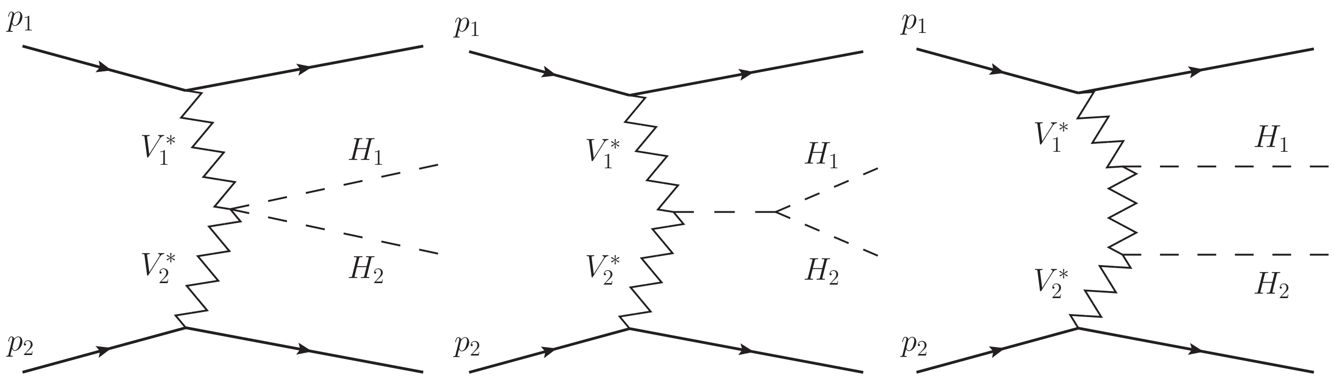
Simulated using NLO MC w/ large theoretical uncertainty

Results on $t\bar{t}$ with NNLOPS could be used to reduce the uncertainty
Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi 20, 21

VBF: State of the Art

VBF HH

VBF HH is sensitive not just to c_{hhh} but also c_{2v} and c_v



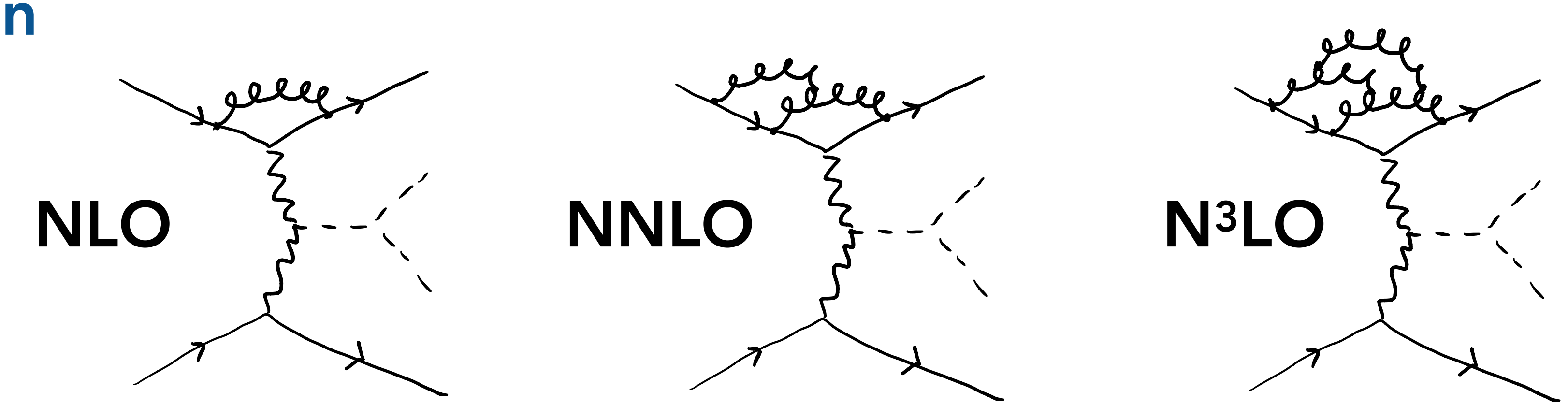
$T_1 \sim c_{2v}$ $T_2 \sim c_{hhh}\,c_v$ $B \sim c_v^2$

$$\mathscr{M}(V_L V_L \to HH) \simeq \frac{s}{v^2}(c_{2v} - c_v^2)$$

Higgs unitarises high-energy VBS behaviour in SM

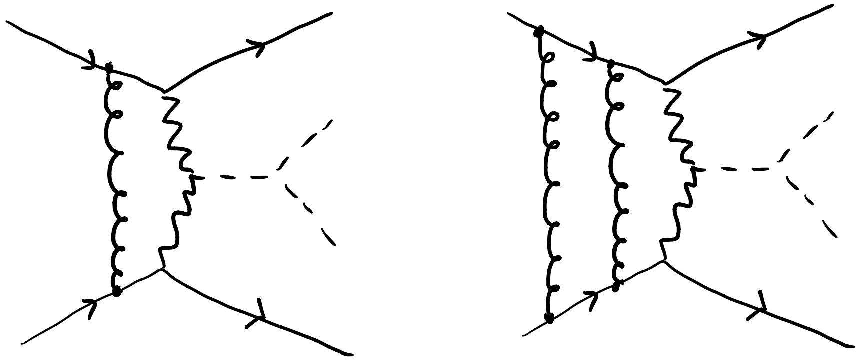
Approximations & Precision

Known to N³LO in the **structure function / DIS approximation**



NLO **NNLO** **N³LO**

NLO: approximation is exact due to colour conservation
NNLO: get colour suppressed non-factorisable contributions

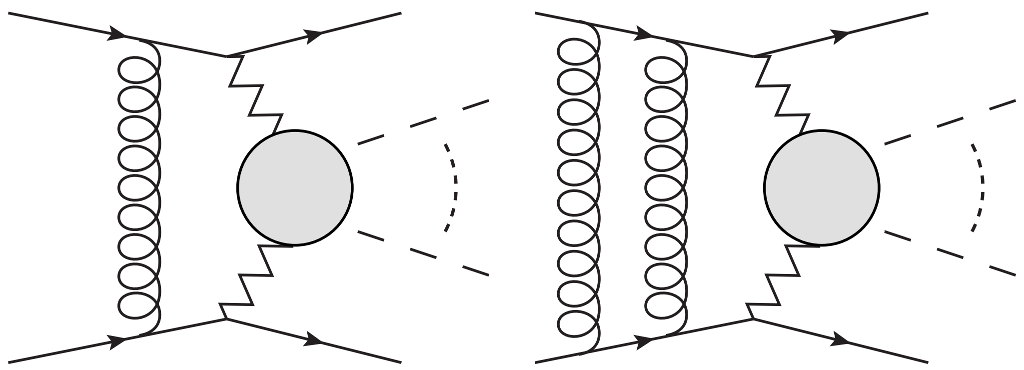


VBF HH: Non-factorisable contribution

Non-factorisable contributions recently studied using the eikonal approximation

Liu, Melnikov, Penin 19

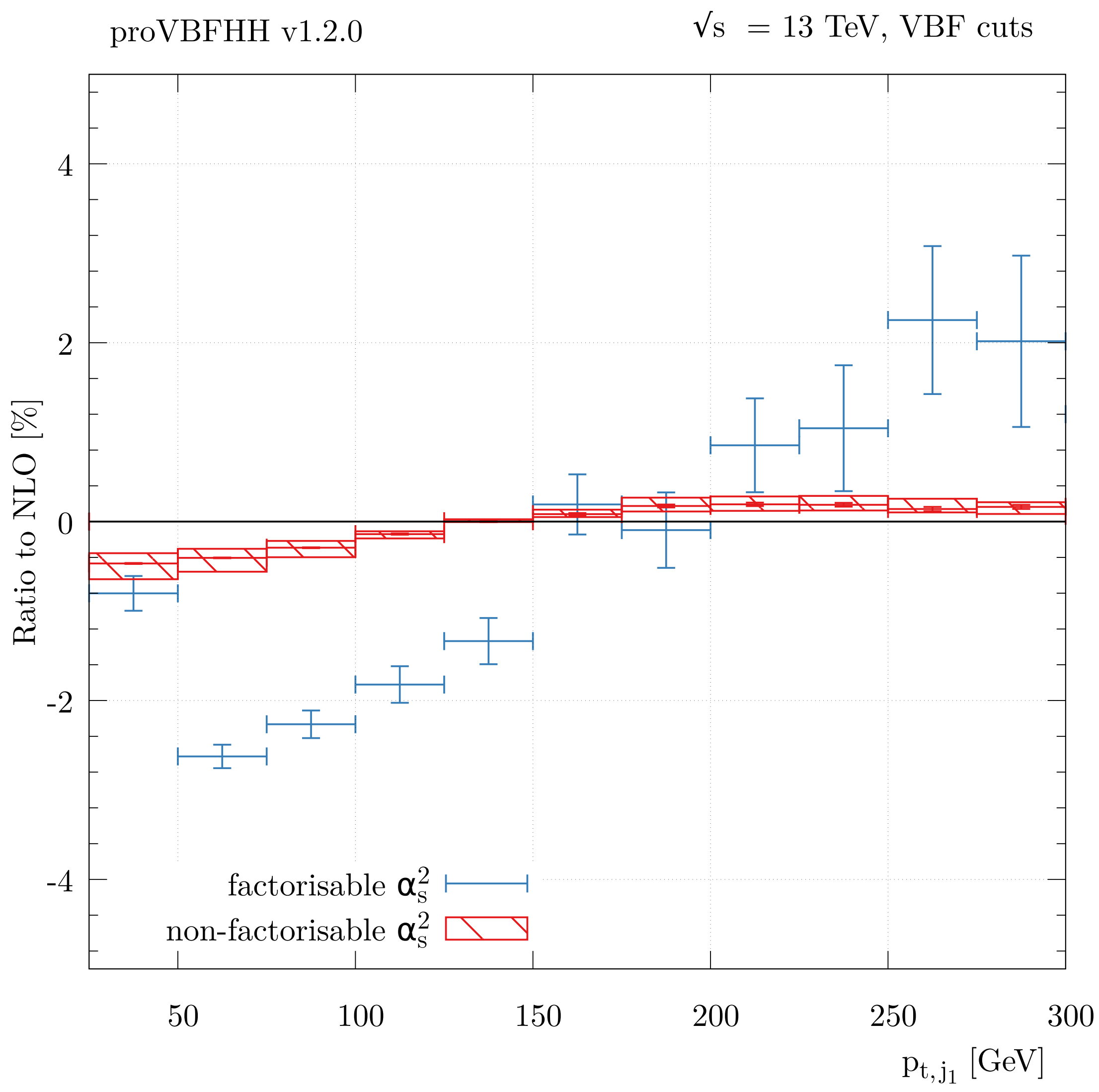
Dreyer, Karlberg, Tancredi 20, 22



~10% differences between non-fac contributions to T and B diagrams

$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{HH,nf} \sim -\tilde{\alpha}_s^2\Big[2.3\cdot \mathrm{d}\sigma^{\mathrm{LO}}_{TT} + 2.2\cdot \mathrm{d}\sigma^{\mathrm{LO}}_{TB} + 2.1\cdot \mathrm{d}\sigma^{\mathrm{LO}}_{BB}\Big]$$

Delicate cancellations between T and B diagrams conspires to preserve unitarity



Note: (As pointed out by authors) Eikonal approximation not trustworthy for too high $p_{t,j}$

HH VBF: NNLO QCD + NLO EW

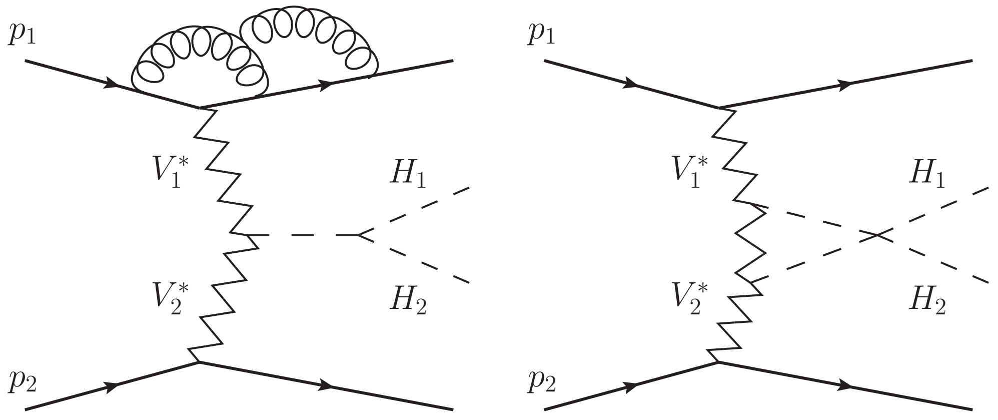
Inclusive known to N^{3}LO QCD

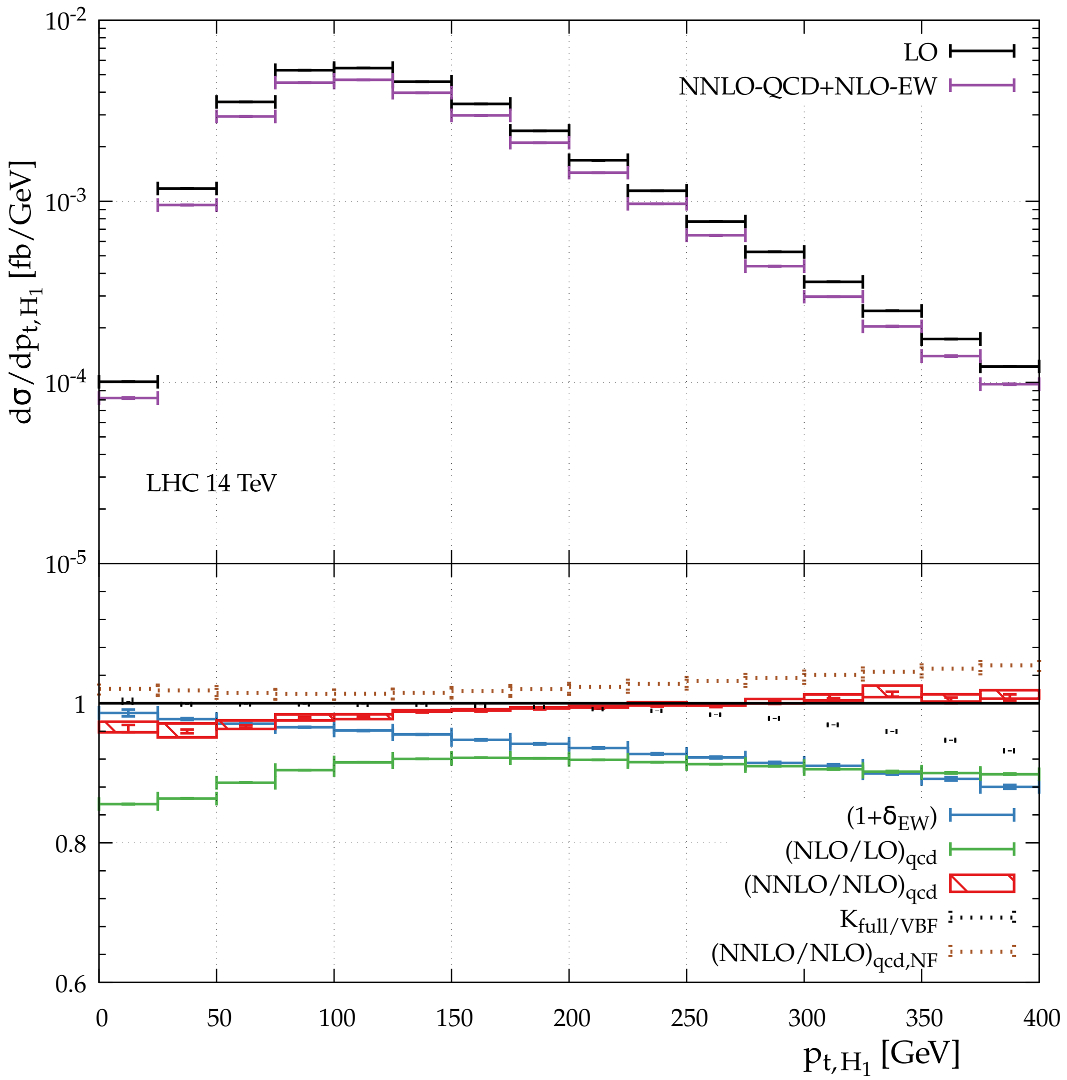
Dreyer, Karlberg 18

State of the art for differential predictions

NNLO QCD Dreyer, Karlberg 18

+ NLO EW Dreyer, Karlberg, Lang, Pellen 20





$\sigma^{\text{full}}_{\text{LO}}$	$\delta^{\text{full}}_{\text{NLO QCD}}$	$\delta^{\text{VBF}}_{\text{NNLO QCD}}$	$\delta^{\text{full}}_{\text{NLO EW}}$	$\sigma_{\text{NNLO QCD}\times\text{NLO EW}}$	$\delta^{\text{NF}}_{\text{NNLO QCD}}$ [fb]
$0.78444(9)^{+0.0825}_{-0.0694}$	$-0.07110(13)$	$-0.0115(5)$	$-0.0476(2)$	$0.6684(5)^{+0.002}_{-0.0004}$	$0.01237(2)$
$^{+10.5\%}_{-8.8\%}$	-9.1%	-1.5%	-6.1%	$-14.8\%^{+0.3\%}_{-0.06\%}$	$+1.7\%$

↑

EW corrections similar in size to NLO QCD corrections and to those in single Higgs case

All corrections available in public code **proVBFHH v1.2.0**

For non-SM: can rescale to include factorisable QCD corrections but currently have to take non-fac & EW corrections as uncertainty

Summary

Good progress in HH theory over the last few years

- Gluon fusion - Full SM result: NLO
- Gluon fusion - HTL result: N^{3}LO (also differential)
- Vector Boson Fusion - N^{3}LO inclusive, NNLO differentially + NLO EW
- Progress matched by amazing work from the experiments

Uncertainties beyond scale variations are now relevant

- Mass scheme uncertainties at the level of >10% @ NLO
- Motivates studies of m_T dependence beyond 2-loop

Many other fascinating developments

(e.g. efforts to tackle backgrounds, EW effects, EFT fits, …)

I hope/expect that the anticipated experimental progress during Run 3 is matched by exciting theory progress

Thank you for listening

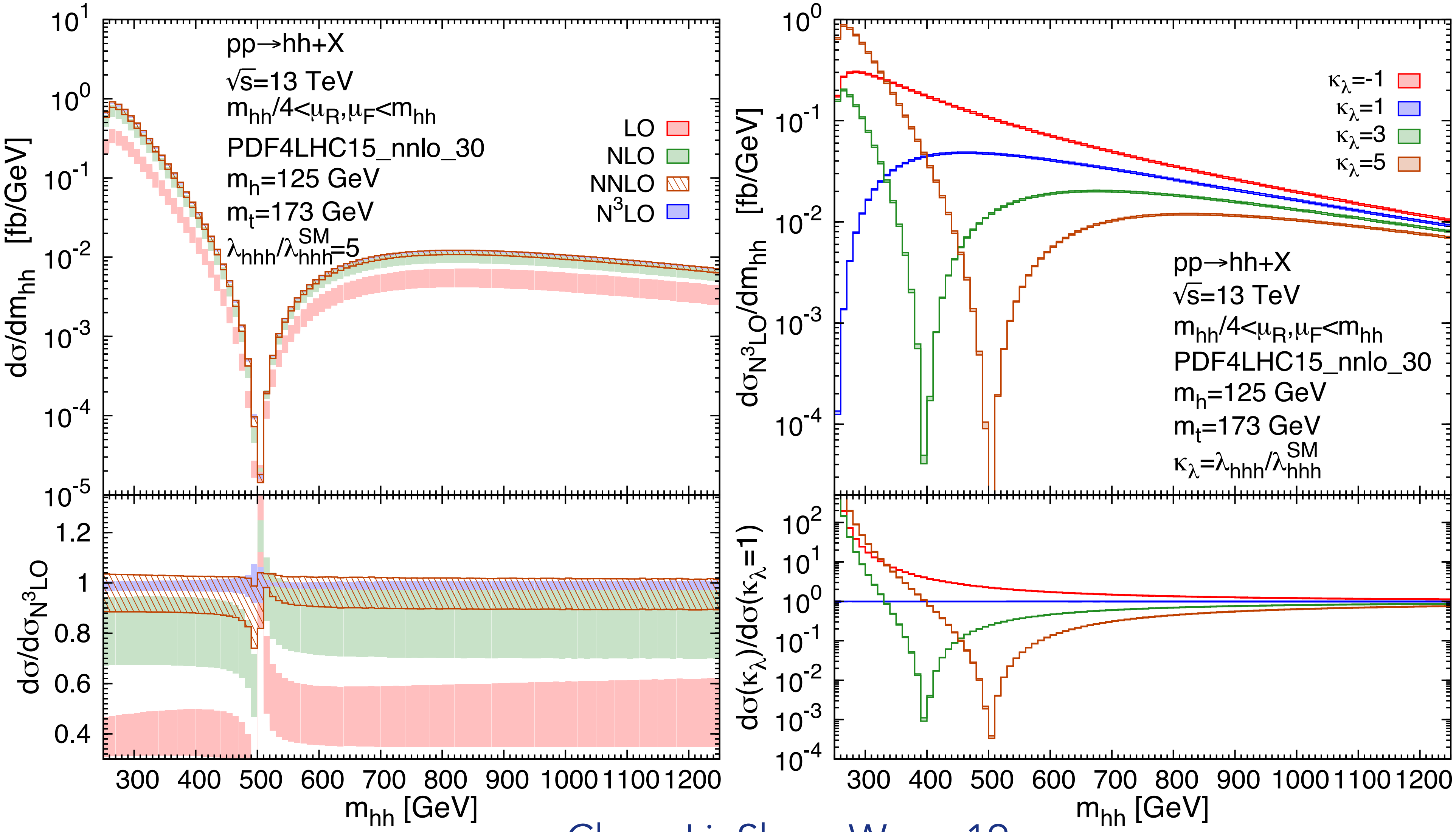
Backup

Self Coupling Considerations

For $gg \to HH$: m_{HH} is significantly modified by $\kappa_\lambda \neq 1$ due to large interference between ``boxes'' & ``triangles''

Results known to NLO (full), NNLO ($\mathbf{FT}_{\text{approx}}$), N^{3}LO (HTL)

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
de Florian, Fabre, Heinrich, Mazzitelli, Scyboz 21; Chen, Li, Shao, Wang 19;

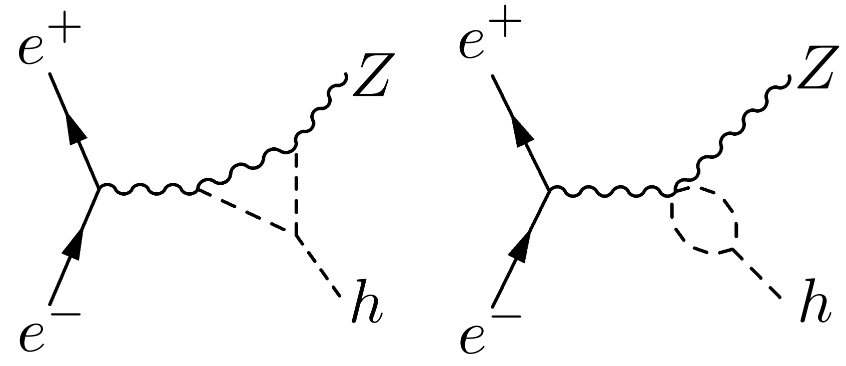


Chen, Li, Shao, Wang 19

Note: dip structure changes dramatically once m_T dependence is included (artefact of Born vanishing exactly at this point in HTL)

Higgs Self-Coupling from Single Higgs Production

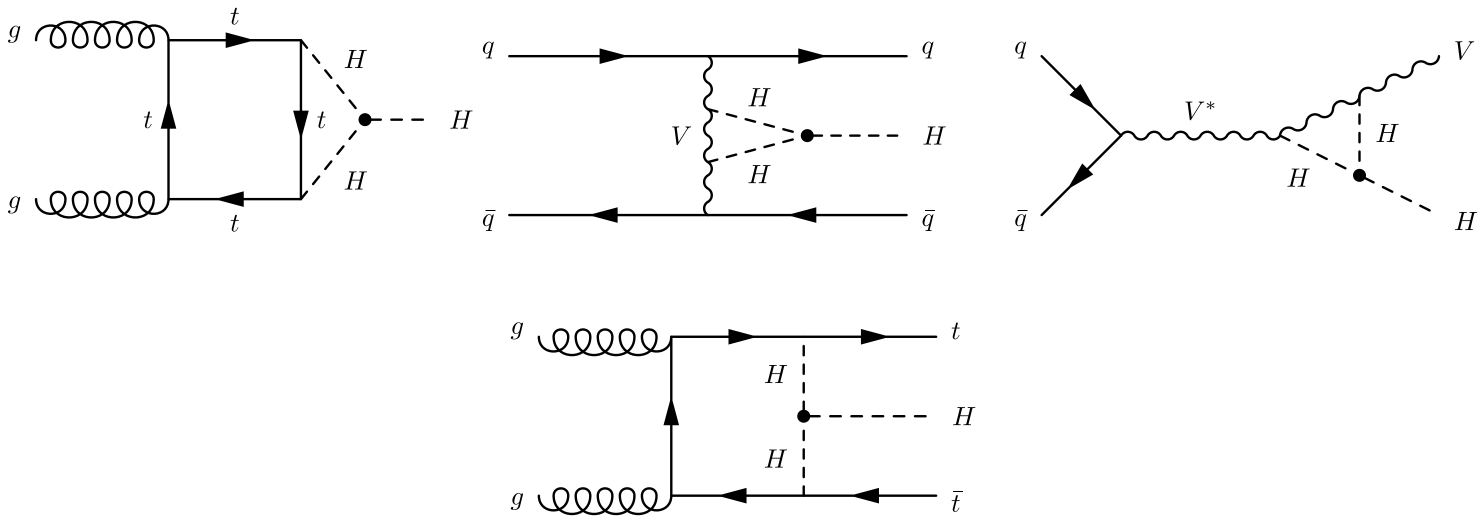
So far focused on HH production where λ_3 appears at LO
Can also constrain this coupling from high-order effects in single Higgs production

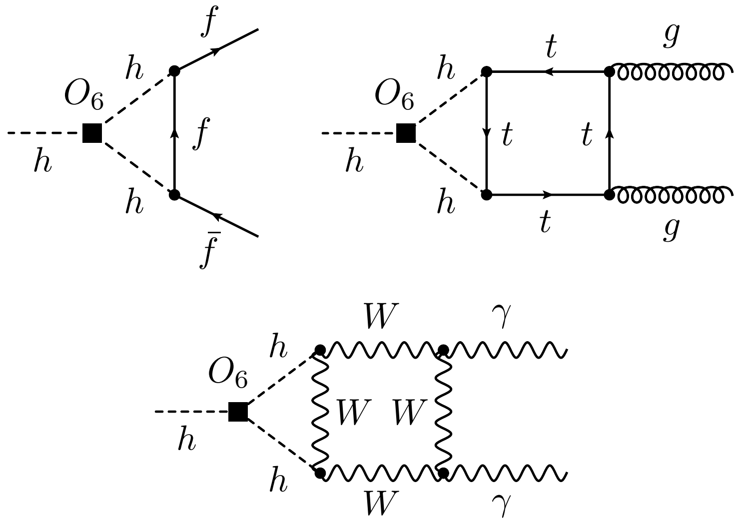


E.g. can constrain λ_3 below HH threshold from EW corrections to $e^+e^- \to ZH$

McCullough 13

At LHC, λ_3 appears in main Higgs production and decay channels





Gorbahn, Haisch 16, 19; Bizon, Gorbahn, Haisch, Zanderighi 16; Degrassi, Giardino, Maltoni, Pagani 16; Maltoni, Pagani, Shivaji, Zhao 17; Di Vita, Grojean, Panico, Riembau, Vantalon 17