New results from NA61/SHINE

intermittency

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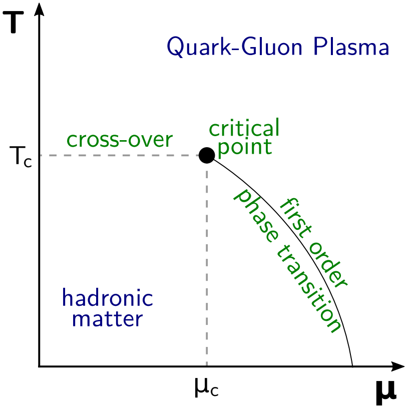
Outline

1. **Critical point search strategies**
2. **Experimental measures**
3. **Experimental results**
4. **Reference power-law toy model**
5. **Comparison with data**

Critical point of QGP

Critical point search strategies



Critical point (CP) – a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition.

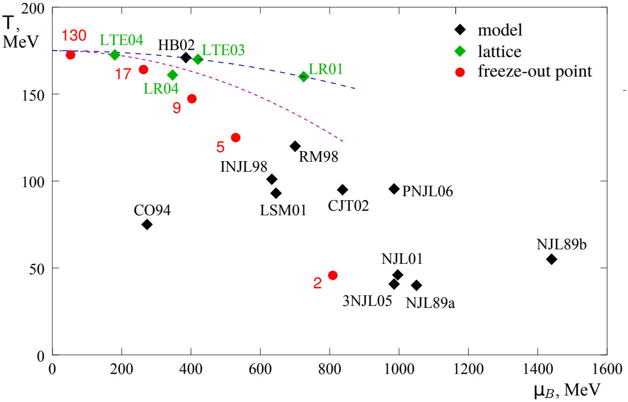
2nd order phase transition ⟶ scale invariance ⟶ power-law form of correlation function.

These expectations are for fluctuations and correlations in the configuration space which are expected to be projected to the momentum space via quantum statistics and/or collective flow.

Asakawa, Yazaki NPA 504 (1989) 668
Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

Critical point of QGP

Critical point search strategies



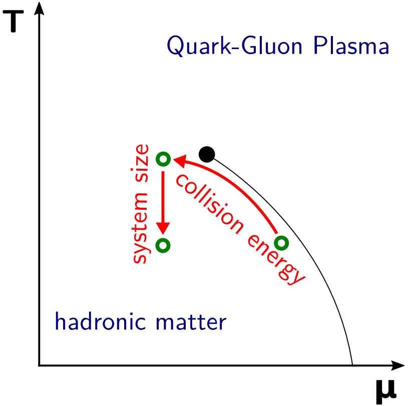
The main signal of the CP is anomaly in fluctuations in a narrow domain of the phase diagram.

However predictions on the CP existence, its location and what and how should fluctuate are model-dependent.

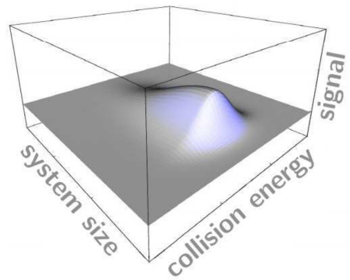
Stephanov, PoS LAT2006 (2006) 024

Exploring the phase diagram with heavy-ion collisions

Critical point search strategies



Search for the critical end point in heavy-ion collisions is performed by a scan in the parameters controlled in laboratory (collision energy and nuclear mass number, centrality). By changing them, we change freeze-out conditions (T, μ_B).



The experimental search for the critical point requires a two-dimensional scan in collision energy and size of the colliding nuclei (centrality).

1 **Critical point search strategies**

2 Experimental measures

- Scaled factorial moments of order r
- Second scaled factorial moment
- In practice...
- Cumulative variables

3 **Experimental results**

4 **Reference power-law toy model**

5 **Comparison with data**

Scaled factorial moments of order r

Experimental measures

In the Grand Canonical Ensemble the correlation length diverges at the second order phase transition and the system becomes scale invariant. This phenomenon leads to enhanced multiplicity fluctuations with special properties, that can be revealed by scaled factorial moments $F_r(\delta)$ of order r:

$$F_r(\delta) = \frac{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i(n_i-1)...(n_i-r+1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i \right\rangle^r}$$

- δ - is the size of each of the M subdivision intervals of the momentum phase-space region Δ
- n_i - number of particles in i-th bin
- $\langle ... \rangle$ - averaging over events

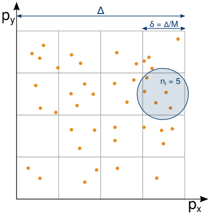
When the system is a simple fractal and $F_r(\delta)$ follows a power law dependence:

$$F_r(\delta) = F_r(\Delta) \cdot (\Delta/\delta)^{\phi_r}.$$

Additionally, the exponent (intermittency index) ϕ_r obeys the relation:

$$\phi_r = (r-1) \cdot d_r,$$

where the anomalous fractal dimension d_r is independent of r.



Wosiek, APPB 19 (1988) 863
Satz, NPB 326 (1989) 613
Bialas, Peschanski, NPB 273 (1986) 703

Second scaled factorial moment

Experimental measures

Second factorial moment ($r = 2$):

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i(n_i - 1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^{M} n_i \right\rangle^2}$$

At the second order phase transition the system is a simple fractal and the factorial moment exhibits a power law dependence on M:

$$F_2(M) \sim (M)^{\phi_2}$$

Prediction for critical point: $\phi_2 = 5/6$.

Wosiek, APPB 19 (1988) 863
Bialas, Hwa, PLB 253 (1991) 436
Bialas, Peschanski, NPB 273 (1986) 703
Antoniou, Diakonos, Kapoyannis, Kousouris, PRL 97 (2006) 032002

Second scaled factorial moment

Experimental measures

However, to cancel the $F_2(M)$ dependence on the single particle inclusive momentum distribution, one needs a uniform distribution of particles in bins.
One can either subtract $F_2(M)$ for mixed events:

$$\Delta F_2(M) = F_2^{data}(M) - F_2^{mixed}(M)$$

or use cumulative quantities, and then:

$$\Delta F_2(M) = F_2(M) - F_2(1)$$

Bialas, Gazdzicki, PLB 252 (1990) 483

In practice...

Experimental measures

Modified, equivalent formula

$$F_r(M) = \frac{r!(M^2)^{r-1}}{\langle N\rangle^r}\left\langle \sum_{i=1}^{M^2} \binom{n_i}{r} \right\rangle$$

for r=2:

$$F_2(M) = \frac{2M^2}{\langle N\rangle^2}\left\langle N_{pp}(M) \right\rangle$$

- M - number of bins in p_x and p_y
- n_i - number of particles in i-th bin
- N - event multiplicity
- $\langle ... \rangle$ - averaging over events
- $N_{pp}(M)$ - (a single event property) total number of particle pairs in M^2 bins in an event

$$\frac{\sigma_{F_2}}{|F_2|} = \sqrt{\frac{(\sigma_{N_{pp}})^2}{\langle N_{pp}\rangle^2} + 4\frac{(\sigma_N)^2}{\langle N\rangle^2} - 4\frac{(\sigma_{N_{pp}N})^2}{\langle N\rangle\langle N_{pp}\rangle}}$$

Cumulative variables

Experimental measures

Volume 252, number 3 PHYSICS LETTERS B 20 December 1990

A new variable to study intermittency

A. Bialas [1] and M. Gazdzicki [2]
CERN, CH-1211 Geneva 23, Switzerland

Received 17 September 1990

It is proposed to study intermittency properties of particle spectra using the variables for which the single-particle distribution is constant. The construction of such a set of variables is described. It is shown that this method drastically reduces distortions of intermittency due to a non-uniform single-particle density. A method of systematic analysis of intermittency in three dimensions based on the new variable is suggested.

To define the new variables, let us first discuss the one-dimensional case. Assume that the single-particle distribution in a variable x is measured and given by a (non-negative) function $\rho(x)$. Our new variable $X=X(x)$ is defined as

$$X=\int_a^x \rho(x)\mathrm{d}x \Big/ \int_a^b \rho(x)\mathrm{d}x\,, \qquad (4)$$

where a and b are the lower and upper phase space limits of the variable x.

This new "integral" variable has the following, very useful properties:

(1) its value for a given particle does not depend on the choice of the original variable x, but is uniquely determined by the shape of the single-particle spectrum,

(2) the single particle distribution in the variable X is uniform in the interval from 0 to 1.

- gives a new way to compare meaningfully the data obtained in different experiments
- removes the dependence of the intermittency parameters on the shape of the single-particle distribution
- intermittency index of an ideal critical system described in two dimensions in momentum space was proven to remain approximately invariant after transformation to cumulative variables

Nikos Antoniou, Fotis Diakonis, https://indico.cern.ch/event/818624/

Cumulative variables

Experimental measures

1) normalization:

$$P(x,y) = \rho(x,y) \Big/ \int_{x,y} \rho(x,y)\,dxdy$$

2) projection on x:

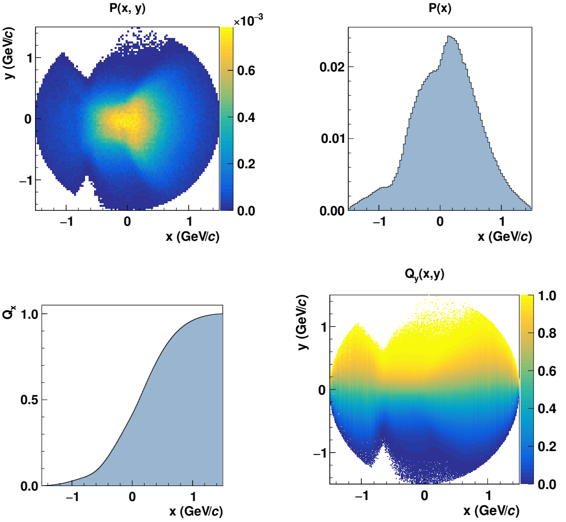
$$P(x) = \int_{y} P(x,y)\,dy$$

3) cumulative x:

$$Q_x = \int_{x_{min}}^{x} P(x)\,dx$$

4) cumulative y:

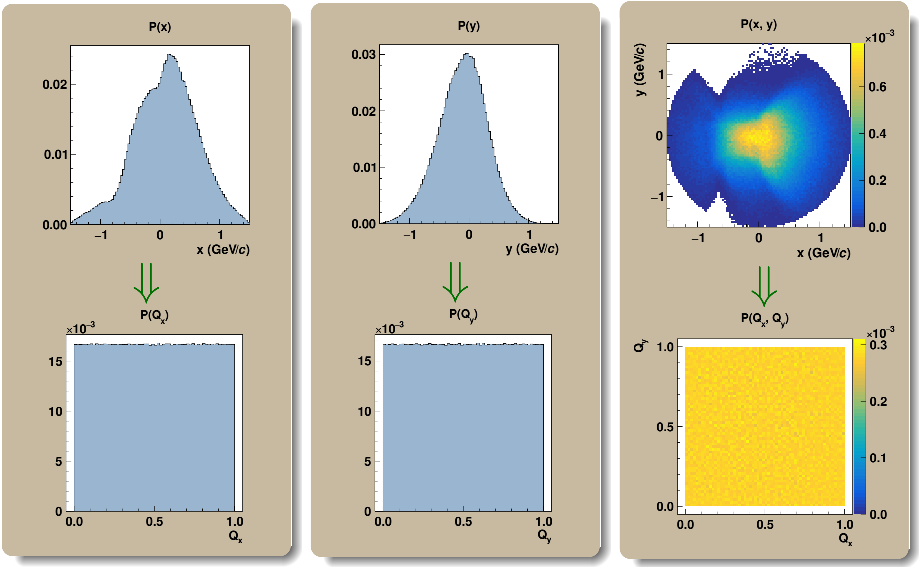
$$Q_y(x) = \int_{y_{min}}^{y} P(x,y)\,dy / P(x)$$



(examples for 0-5% Ar+Sc at 150*A* GeV/*c*, $x=p_x$, $y=p_y$)

Cumulative variables

Experimental measures

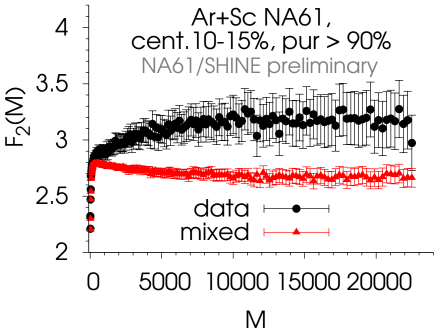


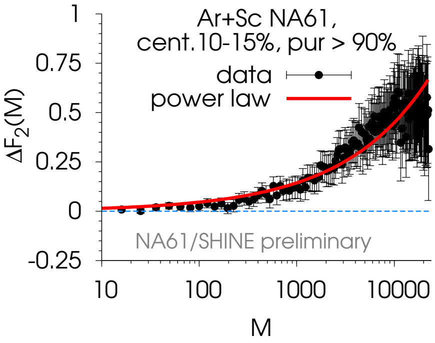
(examples for 0-5% Ar+Sc at 150*A* GeV/*c*, $x=p_x$, $y=p_y$)

Already published results

Experimental results

Mid-rapidity protons at 17 GeV

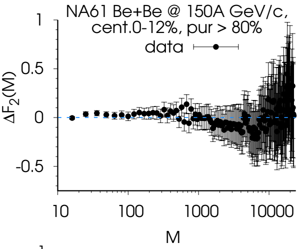


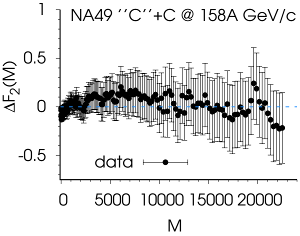


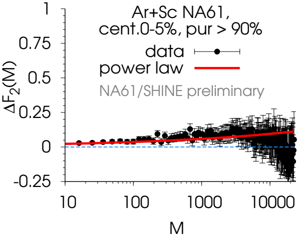
Note that points are strongly correlated.

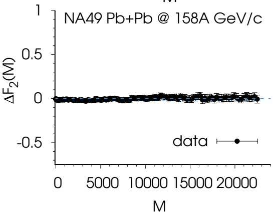
NA61/SHINE: PoS(CPOD2017) 054

ΔF_2 for mid-rapidity protons at 17 GeV







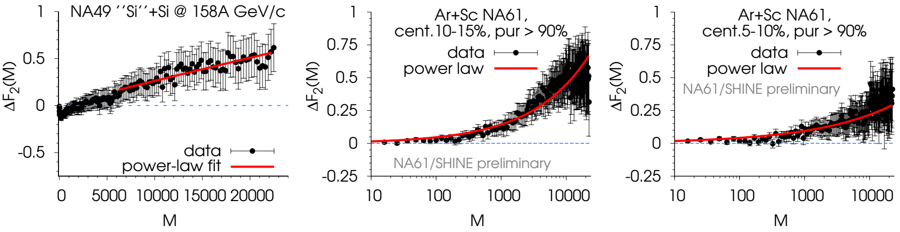


No signal visible in central Be+Be, C+C, Ar+Sc and Pb+Pb

NA49: EPJC 75 (2015) 587
NA61/SHINE: PoS(CPOD2017) 054

Already published results
Experimental results

ΔF_2 for mid-rapidity protons at 17 GeV

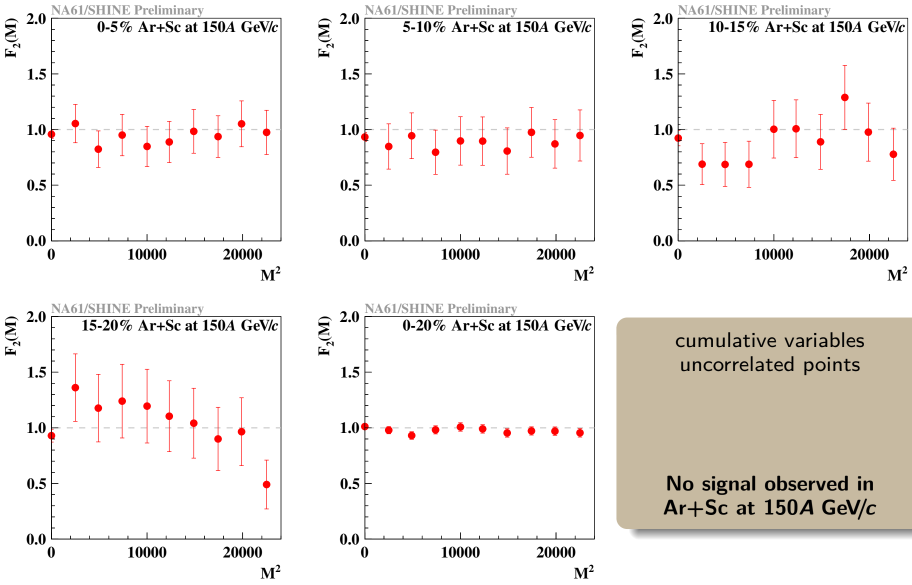


A deviation of ΔF_2 from zero seems apparent in central Si+Si and mid-central Ar+Sc

NA49: EPJC 75 (2015) 587
NA61/SHINE: PoS(CPOD2017) 054

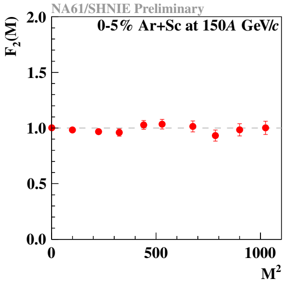
New results

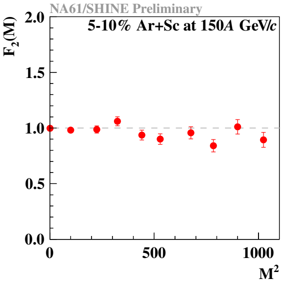
Experimental results

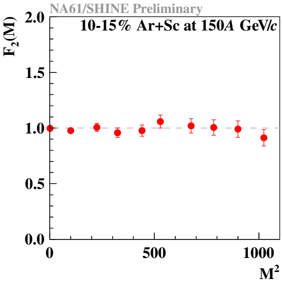


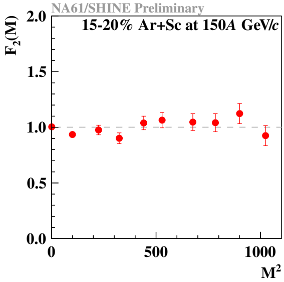
cumulative variables
uncorrelated points

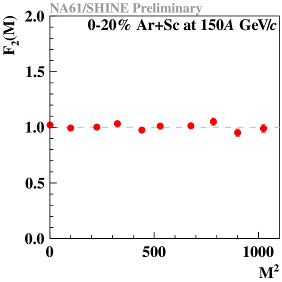
No signal observed in Ar+Sc at 150*A* GeV/*c*











cumulative variables
uncorrelated points
limited M range

No signal observed in Ar+Sc at 150*A* GeV/*c*

Generating particles

Reference power-law toy model

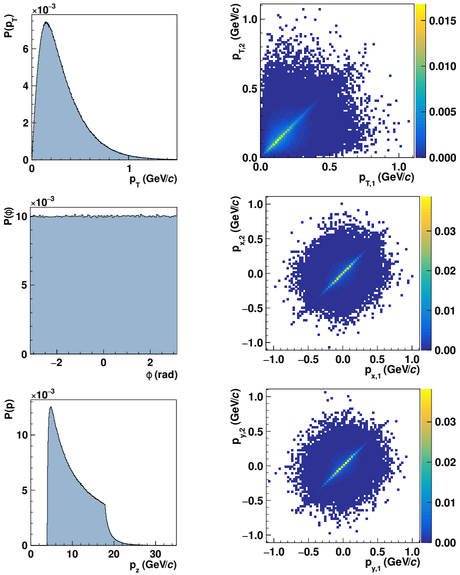
Uncorrelated particles (background)

- $\rho_B(p_T) = p_T \cdot e^{-6p_T}$
- $\phi = \text{Uniform}(-\pi, +\pi)$
- $y_{LAB}^{proton} = \text{Uniform}(-0.75, 0.75) + y_{CMS}^{beam}$
- $p_x = p_T \cos(\phi)$
  $p_y = p_T \sin(\phi)$
  $p_z = m_T \sinh(y_{LAB}^{proton})$

Correlated pairs (signal)

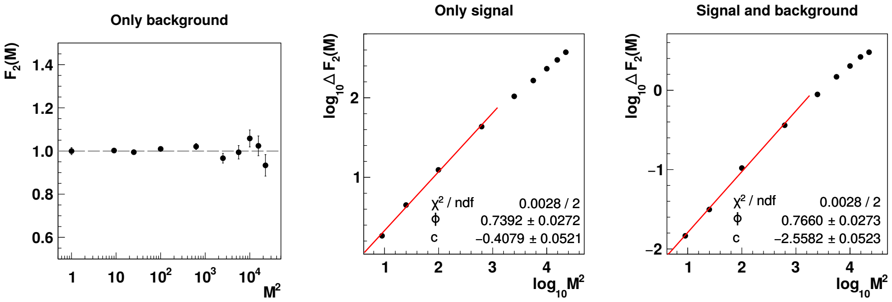
- $\rho_S(p_{T,1}, p_{T,2}) = \rho_B(p_{T,1}) \cdot \rho_B(p_{T,2}) \cdot \left[|\Delta p_x|^{\phi} + \epsilon \right]^{-1} \cdot \left[|\Delta p_y|^{\phi} + \epsilon \right]^{-1}$

Example for: $\phi = 0.80$
$\epsilon = 1e^{-5}$
$N_B = \text{Poisson}(30)$
$N_S = 2$



Example of $F_2(M)$ for the model data

Reference power-law toy model



Procedure

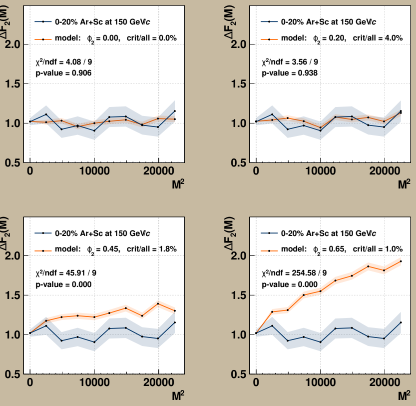
Comparison with data

1. generate lots of toy model data sets:
   - with 10 times more events as in the Ar+Sc data
   - with multiplicity distribution identical to the Ar+Sc data
   - critical-to-all ratio: vary from 0.0 to 4.0% (with 0.2 step)
   - φ_2: vary from 0.00 to 1.00 (with 0.05 step)
   - transform to cumulative p_x and p_y
2. calculate F_2
3. compare F_2 for each of the toy-model data set with F_2 for Ar+Sc

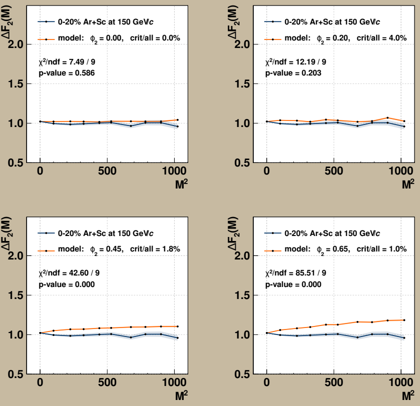
Example of $F_2(M)$ results

Comparison with data

full M range



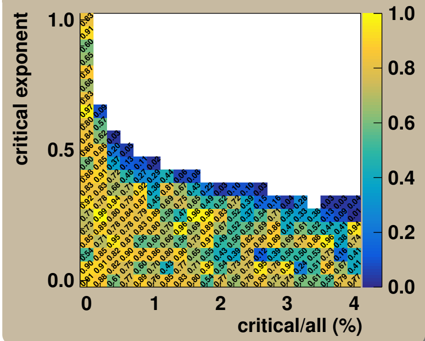
limited M range



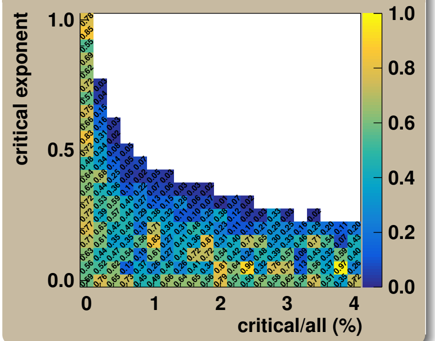
p-value results

Comparison with data

full M range



limited M range

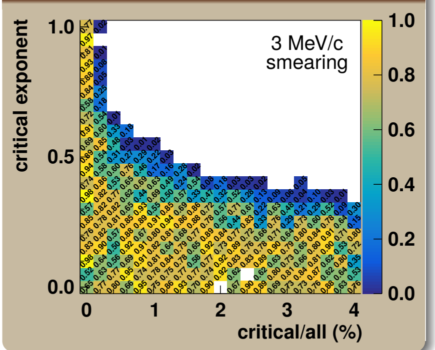


white area:
p-value < 0.01

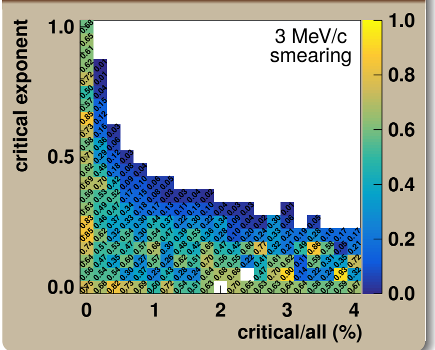
p-value results

Comparison with data

full M range



limited M range



white area:
p-value < 0.01

Thank You!