Large-x PDFs at LHC, HERA, and other experiments

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and

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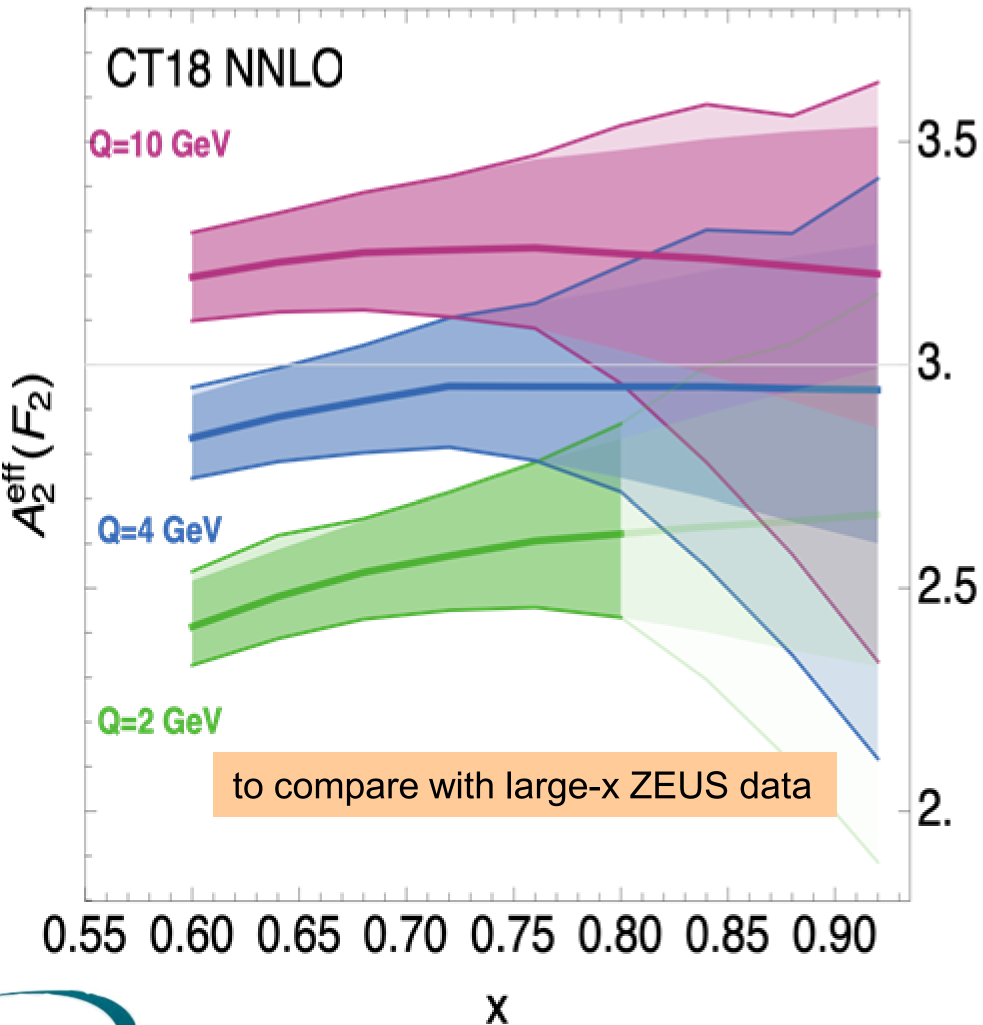
With T. Hobbs, X. Jing, B. T. Wang, A. Accardi, F. Olness

and

CTEQ-TEA (Tung Et. Al.) working group

[S. Dulat, T.-J. Hou, J. Gao, M. Guzzi, J. J. Huston, D. Stump, C. Schmidt, I. Sitiwaldi, K. Xie, C.-P. Yuan]

Phys. Rev. D 103, 054029



"FORDECYT-PRONACES"

Proton PDFs at $x > 0.5$…

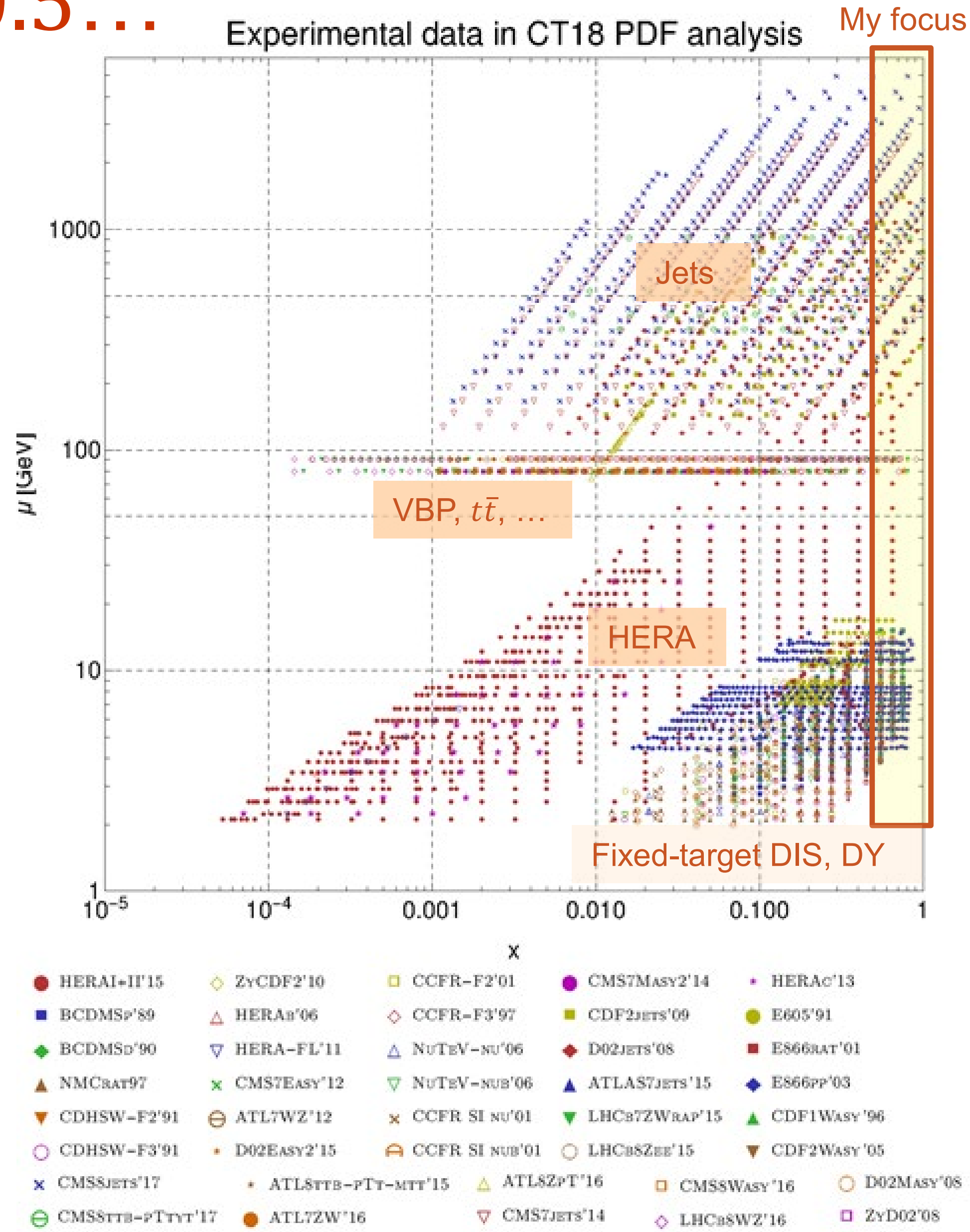
… are important for LHC new physics searches

… are of interest for theoretical models of nucleon structure and lattice QCD

…are **still** constrained predominantly by fixed-target DIS and Drell-Yan experiments
`[Hou et al., arXiv:1912.10053]`

…are likely to be sensitive to nuclear effects
`(⇒ T. Hobbs's talk)`

… can be accessed at large Q at colliders



Interconnected questions

1. What can the PDFs at $x > 0.5$ tell us about nonperturbative QCD dynamics?
2. What is the best strategy to confront large-x data with predictions of nonperturbative QCD?
3. Are experimental constraints on large-x PDFs mutually consistent?
4. How can we access the $x > 0.5$ region at colliders (HERA, LHC, EIC)?

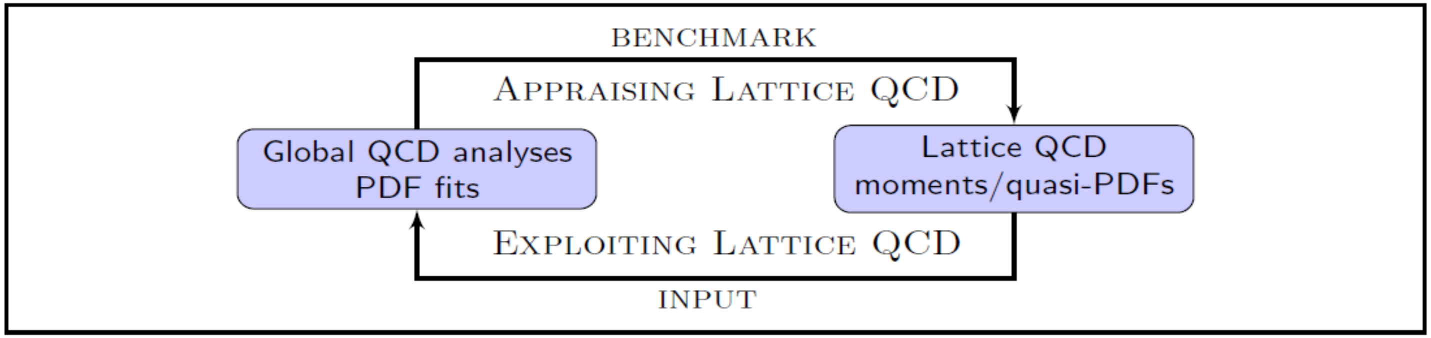
Strategy issues are relevant to studies of large-x pion PDFs at JLAB, AMBER@CERN, EIC
[Aguilar et al., 1907.08218; Roberts et al. 2102.01765]

We will use tests of quark counting rules (QCRs) as an example
[See also Ball, Nocera, Rojo, 1604.00024; Barry et al., 1804.01965; Novikov et al., 2002.02902]

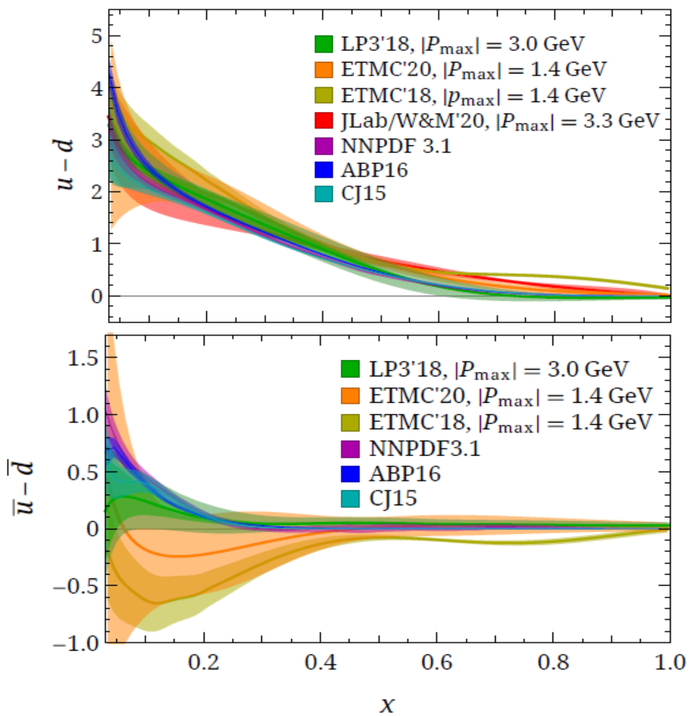
What can the PDFs at $x > 0.5$ tell us about nonperturbative QCD dynamics?

Confronting predictions from nonperturbative QCD approaches and lattice QCD with pheno PDFs

Lattice QCD: ab initio computations of PDFs



Talk by E.Nocera Snowmass EF06 Topical group



Lattice QCD computes nonperturbative functions for the hadron structure (Mellin moments, quasi-PDFs, pseudo-PDFs) by discretizing the QCD Lagrangian density

This is a rapidly progressing field: computations of PDFs in several lQCD approaches have been compared against phenomenological PDF models at two workshops:

- PDFLattice2017, Oxford, March 2017
- PDFLattice2019, Michigan State University, Sept. 2019

[Prog.Part.Nucl.Phys. 100 (2018) 107; arXiv:2006.08636]

Pheno PDFs provide empirical benchmarks for lattice QCD computations. Lattice QCD has the potential to predict PDF combinations not accessible in the experiment.

Proton and pion PDFs

PDFs in nonperturbative QCD

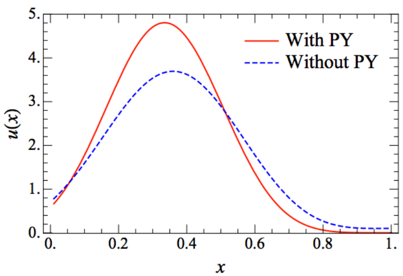
$@\ \mu_0^2 < 1\ GeV^2$

Phenomenological PDFs

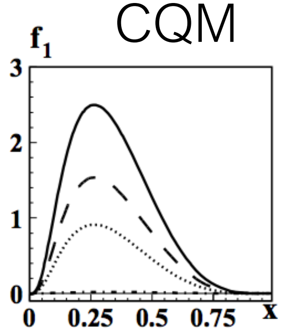
$@\ \mu^2 > 1\ GeV^2$

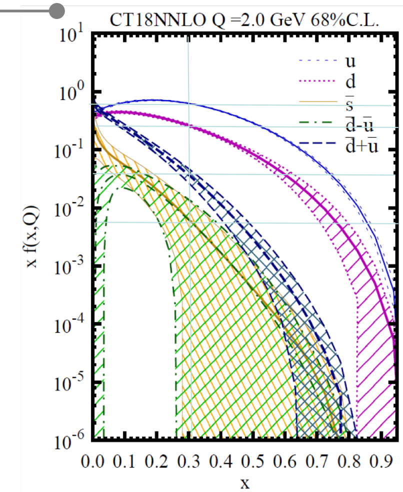
Proton

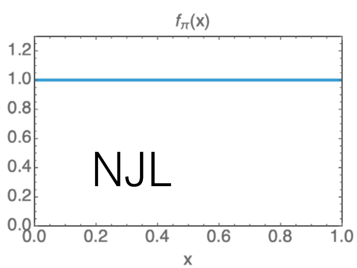
MIT Bag

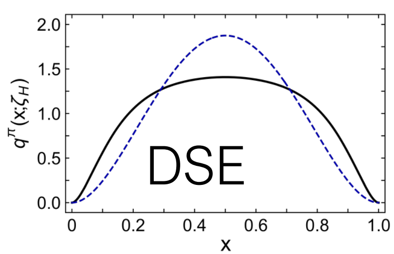


Light-Cone CQM









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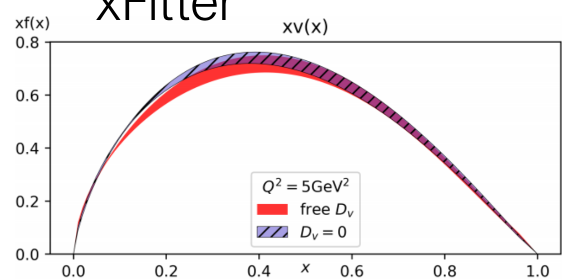


Pion

Proton and pion PDFs

PDFs in nonperturbative QCD

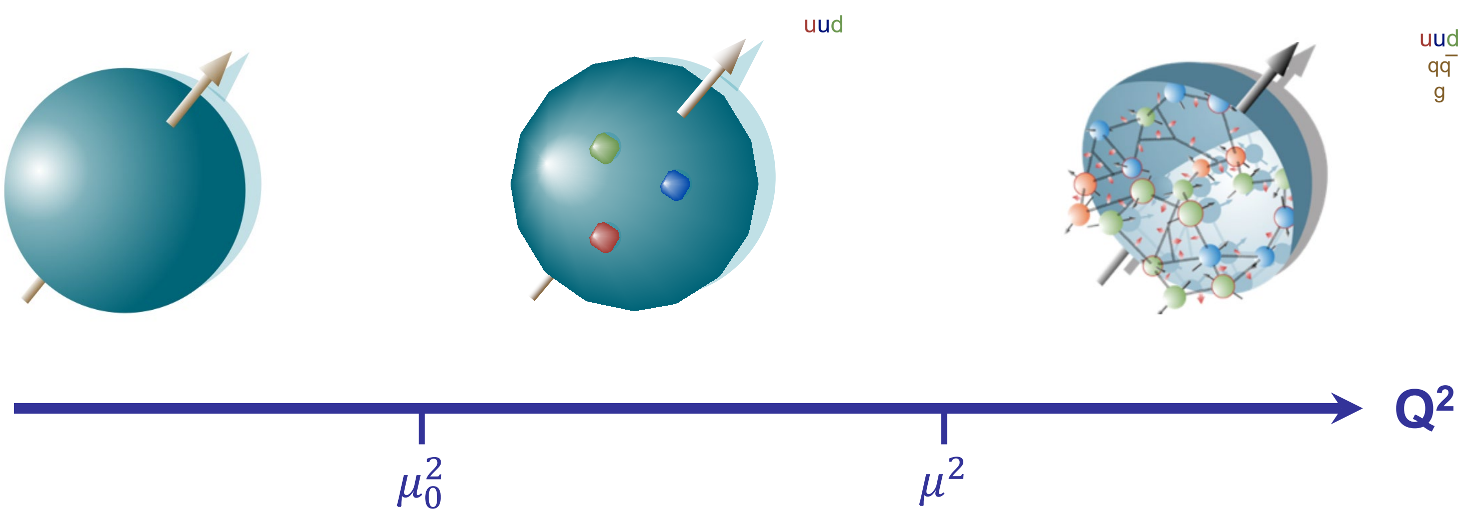
at hadronic scale μ_0^2< 1GeV²

- nonperturbative dynamics
- model's degrees of freedom
- Not factorized

Phenomenological PDFs

at factorization scale μ^2> 1GeV²

- quasi-free partonic degrees of freedom
- defined in the $\overline{MS}$ scheme
- leading-power approximation to full dynamics

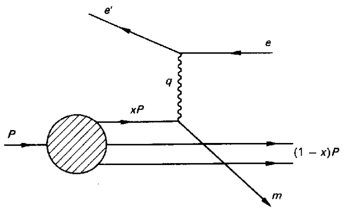


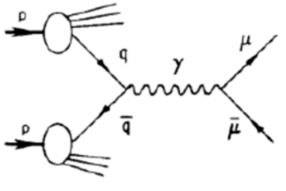
Proton and pion PDFs

PDFs in nonperturbative QCD

Relevant for processes
at $Q^2 \approx 1\ GeV^2$?

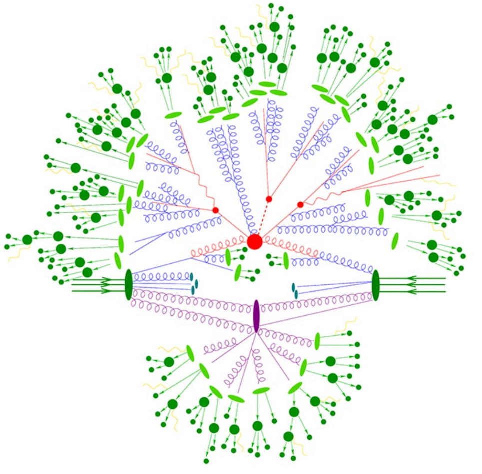
⇨ we can learn about nonperturbative dynamics by comparing predictions to data for the simplest scattering processes (DIS and DY)





Phenomenological PDFs

Determined from processes
at $Q^2 \gg 1\ GeV^2$



⇨ pheno PDFs are determined from analyzing many processes with complex scattering dynamics

How to relate the x dependence of the perturbative and nonperturbative pictures?

Does the evidence from primordial dynamics survive PQCD radiation?

What is the best strategy to confront large-x data with predictions of nonperturbative QCD?

For example, can we test quark counting rules?

- No, if we try to deduce the exact analytical form of $f_a(x,Q)$ from data.
- Yes, if we measure a finite-difference derivative $A_2^{eff}(x_B,Q)$ of $F(x_B,Q)$ or PDFs $f_a(x,Q)$.

Quark counting rules in DIS in the threshold limit $x_B \to 1$

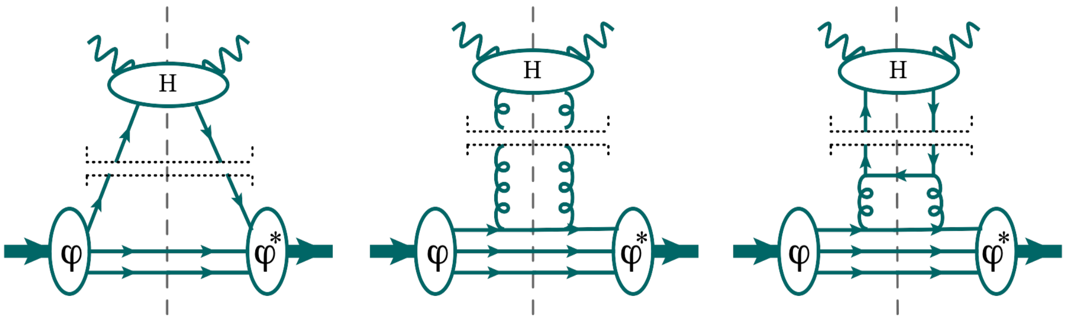
Brodsky and Farrar, PRL31 and PRD11
Ezawa, Nuovo Cim. A23
Berger and Brodsky, PRL42
Soper, PRD15
many others

1. For a structure function F, assuming $p^+ \gg 1\ GeV$:

$$\lim_{x_B \to 1} F(x_B, Q \approx 1\ GeV, \lambda_q) \propto (1 - x_B)^{2\,n_s - 1 + 2|\lambda_q - \lambda_A|}$$

n_s is the number of spectator fermions, λ_q and λ_A are helicities of struck quark and target

2. For unpolarized PDFs: $\lim_{x \to 1} f_a(x, \mu_0 \approx 1\ GeV) \propto (1 - x)^{A_2}$



$f(x, \mu_0) \xrightarrow[x \to 1]{} (1 - x)^{A_2 = 3}$ valence quarks

$(1 - x)^{A_2 > 4}$ gluon

$(1 - x)^{A_2 > 5}$ sea quarks

Assuming the simplest PQCD diagrams dominate at $x_B \to 1$

Predictions:

- A_2 is the same for u and d quarks
- At $\mu > \mu_0$, anomalous dimensions increase the *true* A_2.
- The *apparent* A_2 may increase or decrease.

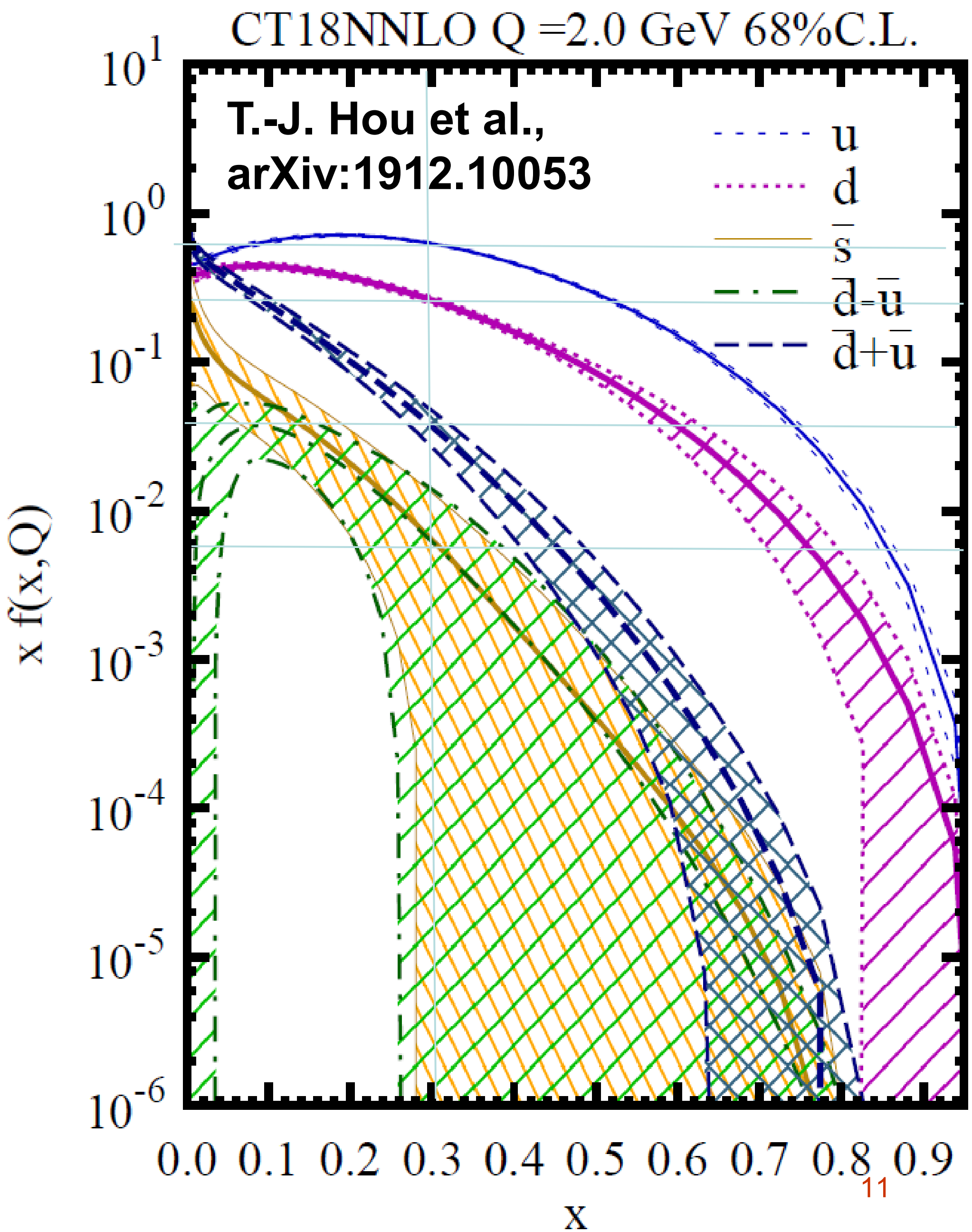
Proton PDFs at $x \to 1$

At $x \to 1$, it is easy to radiate a gluon or a sea quark off a valence quark with a much larger PDF

Mimicry – a fundamental feature of multivariate optimization:

Diverse functional forms of PDFs at the initial scale Q_0 provide equally good description of QCD data from DIS, Drell-Yan pair, jet, and $t\bar{t}$ production

Mathematically, it is not possible to determine the primordial A_2^{true} from discrete, fluctuating data at $x < 1$.

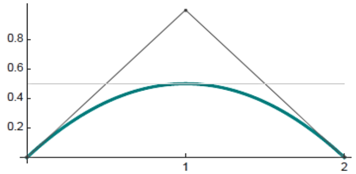


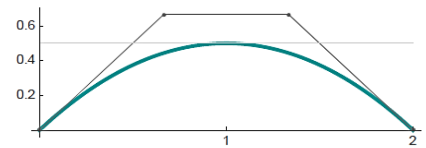
Bézier curves

[AC, Nadolsky, arXiv:2011.10078]

2. Bézier curves give an example of mathematical equivalence of polynomials of different orders

- defined using Bernstein basis polynomials:





$$f(x) = \alpha(1-x)^2 + 2\beta(1-x)x + \gamma x^2 \qquad f(x) = \alpha(1-x)^3 + 3\beta'(1-x)^2 x + 3\beta''(1-x)x^2 + \gamma x^3 \qquad \beta', \beta'' \equiv F[\alpha, \beta, \gamma]$$

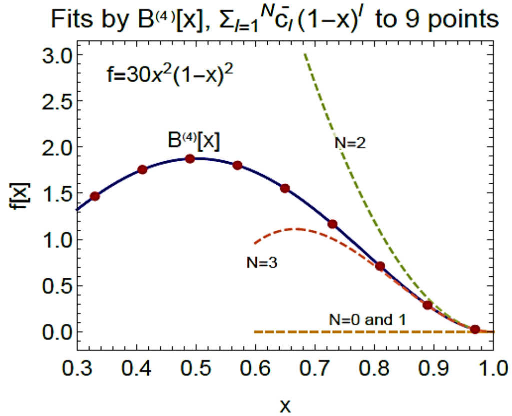
- can be used to interpolate discrete data points

Interpolation by a Bézier curve is unique if the polynomial degree= (# points-1):
there's a closed-form solution to the problem,

$$\mathcal{B}^{(n)}(x) = \sum_{l=0}^{n} c_l \; B_{n,l}(x) \quad \text{with the Bernstein pol.} \quad B_{n,l}(x) \equiv \binom{l}{n} x^l (1-x)^{n-l}.$$

This interpolation can be expanded in monomials of (1-x) about x=1:

$$u_\pi(x \to 1) = \sum_{i=0}^{n} \bar{c}_i \, (1-x)^i$$



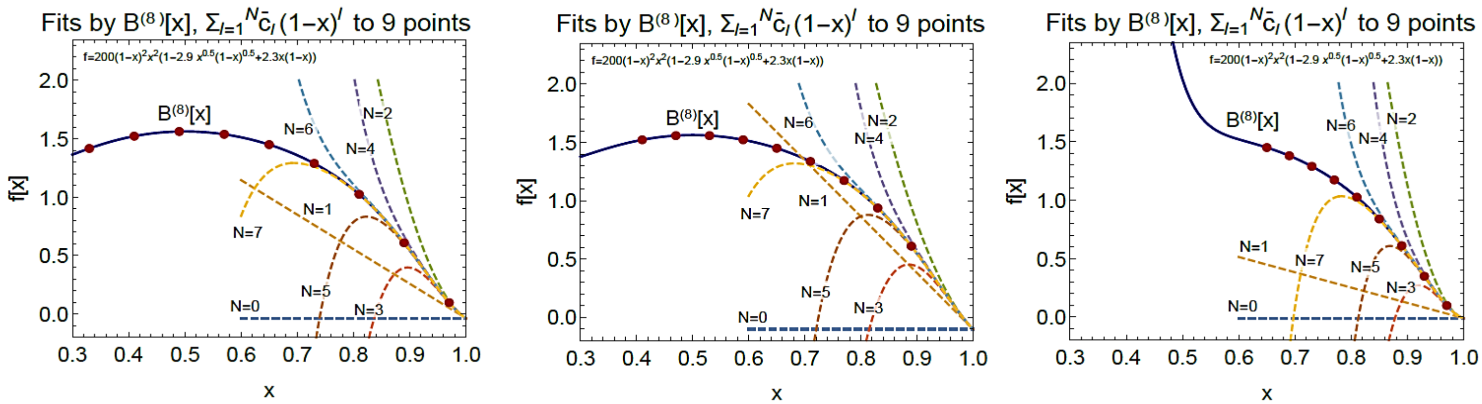
Pinning down the large-x behaviour?

Take a realistic functional form, here $f=200(1-x)^2x^2(1-2.9\,x^{0.5}(1-x)^{0.5}+2.3x(1-x))$

- Sample 9 points with $x < 0.95$ from this function and interpolate by a Bezier curve of order 8. [This interpolation is exact.]
- The lowest coefficients of the monomial expansion (quantified by the truncated solutions with N=1, 2, 3) are spurious and depend on the range and spacing of the sampled data.

⇨ The lowest powers of the monomial expansion $(1-x)^p$ cannot be meaningfully reconstructed.

For each data set sampled from the same f(x) we obtain a different Bézier curve. They differ in derivatives of order > 8.



A_2^{eff}: a finite-difference approximation to A_2^{true}

We will examine 363 CT18 candidate parametrization forms.

If at $x \to 1$:

$$f(x,Q) = \underbrace{(1-x)^{A_2^{true}}}_{fast\ function} \times \underbrace{\Phi(x)}_{slow\ function},$$

then

$$A_{2,eff}(x,Q) \equiv \frac{\partial \ln\big(f(x,Q)\big)}{\partial \ln(1-x)} \approx A_2^{true} + small\ term.$$

Dependence of $A_{2,eff}$ on x and Q indicates how important the higher-order PQCD corrections are.

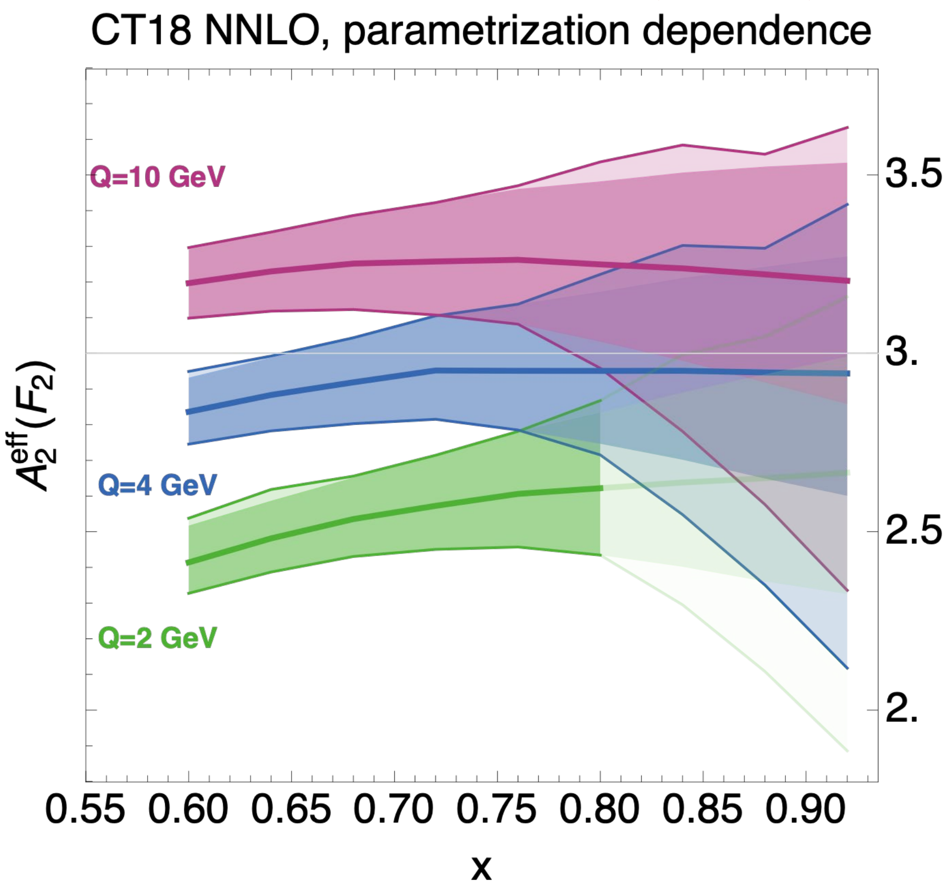
A_2^{eff} for the DIS structure function $F_2(x, Q)$

$F_2(x, Q)$ are computed using CT18 NNLO

Dark bands: 68% CL Hessian errors
Light bands: envelopes of PDF parametrization dependence

- A_2^{eff} agrees with the predicted $A_2 = 3$ within a large PDF uncertainty
- $\delta_{PDF} A_2^{eff} \sim 1 \; at \; x \to 1$
- Non-negligible running with Q^2
- The leading-twist picture is meaningful for $W^2 > m_p^2 + (1-x)/x \, Q^2$,

corresponding to $x < 0.8$ at $Q = 2$ GeV



This prediction can be directly compared to the large-x, large-Q ZEUS DIS data
[I. Abt et al., PRD 101 (2020)112009]

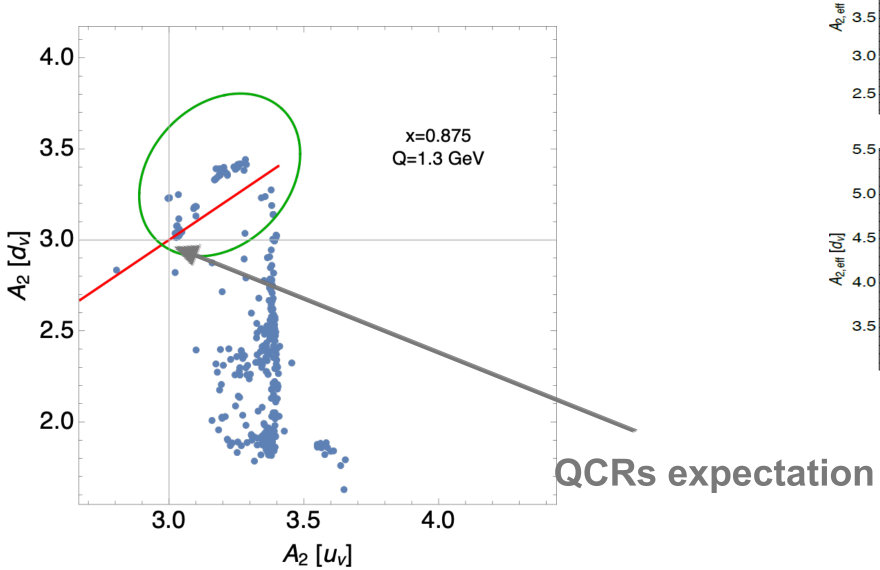
A_2^{eff} for u, d PDFs

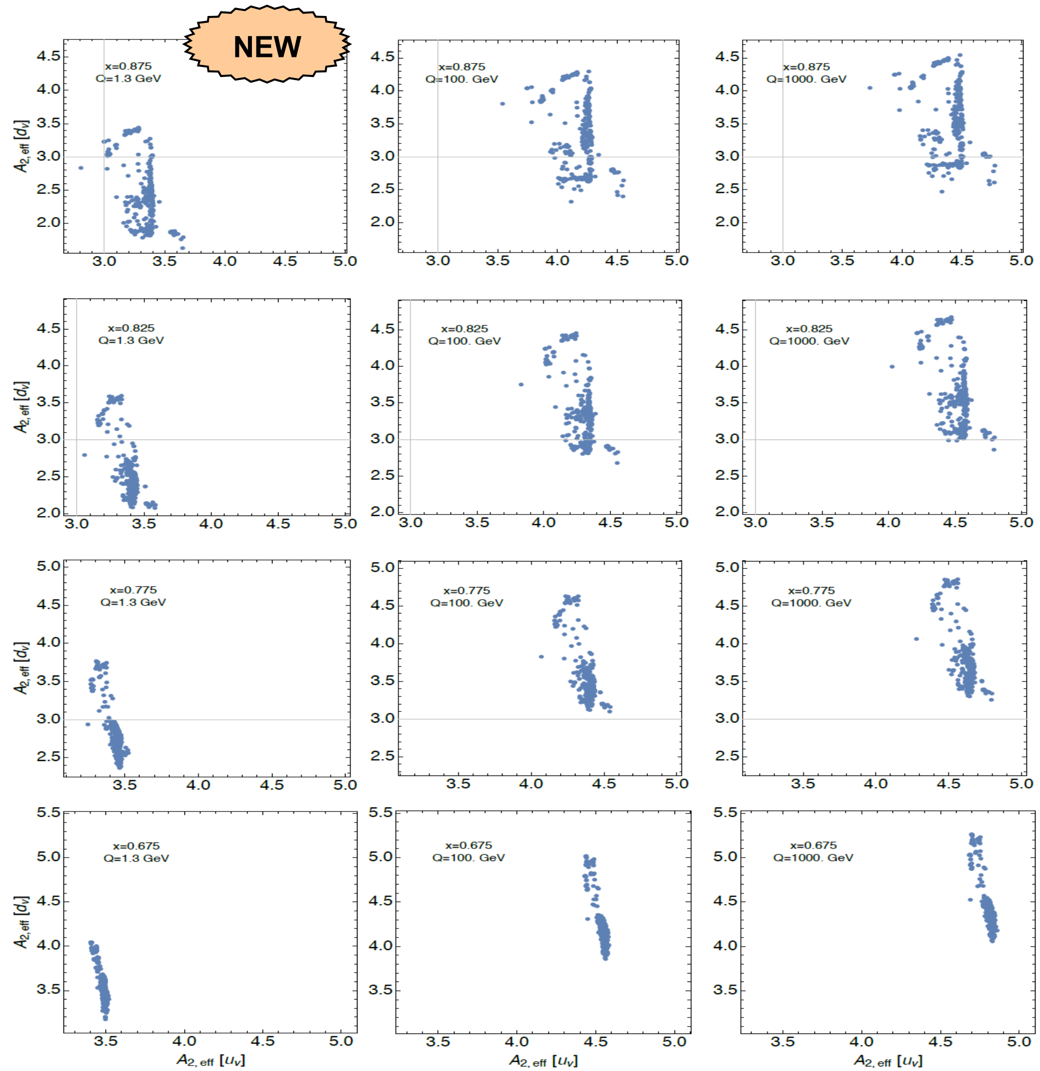
Using CT18 NNLO

Red: Hessian uncertainty on the nominal PDF parameter $A_{2,u} = A_{2,d}$

Green: Hessian error ellipse for $A_{2,u}^{eff}$, $A_{2,d}^{eff}$

Blue: $A_{2,u}^{eff}$, $A_{2,d}^{eff}$ for 363 functional forms with $A_{2,u} \neq A_{2,d}$





Predictions based on 363 functional forms

Substantial dependence on Q and x: higher-order PQCD is important, but param. dependence preserved to high Q

A_2^{eff} for g, $\bar{u}+\bar{d}$ PDFs

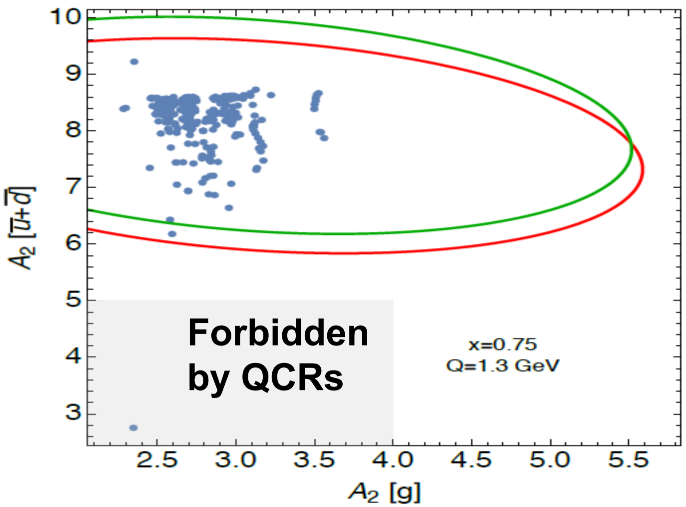
NEW

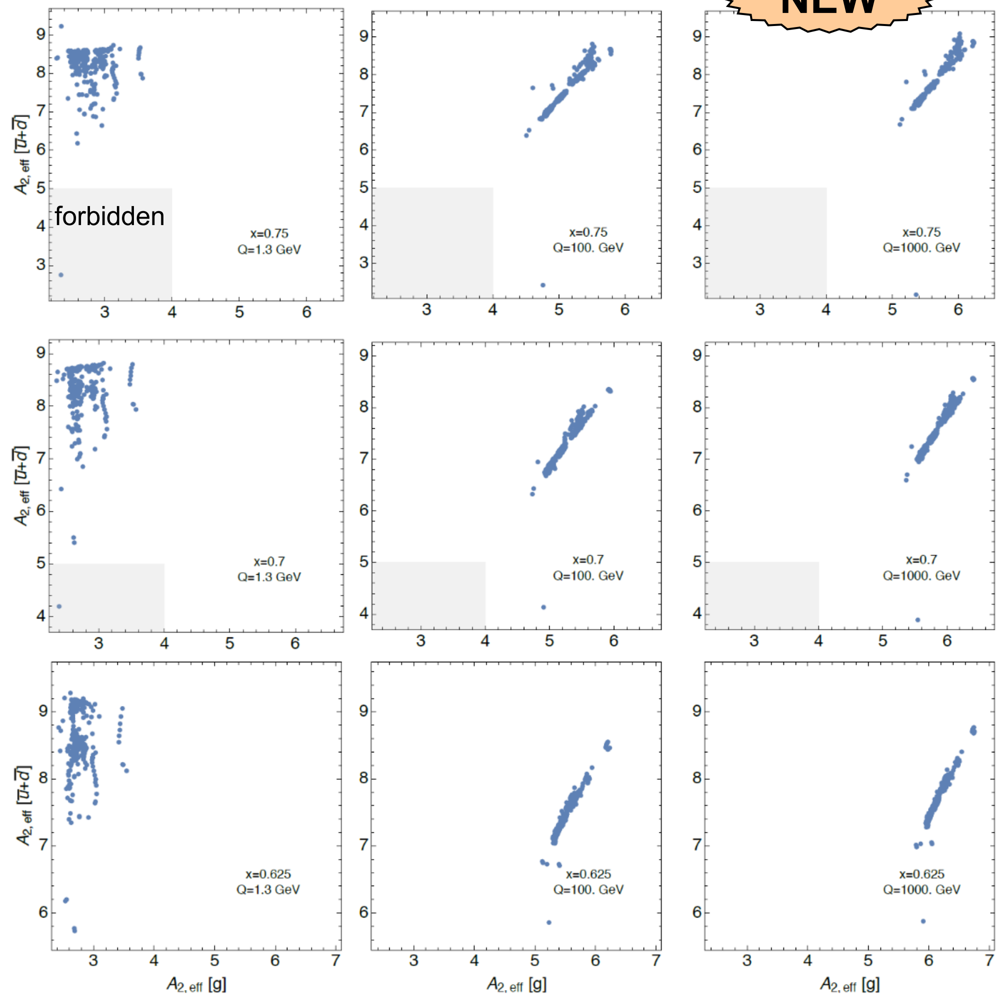
Using CT18 NNLO

Red: Hessian error ellipse for the nominal PDF parameters $A_{2,g}$, $A_{2,\bar{u}+\bar{d}}$

Green: Hessian error ellipse for $A_{2,g}^{eff}$, $A_{2,\bar{u}+\bar{d}}^{eff}$

Blue: $A_{2,g}^{eff}$, $A_{2,\bar{u}+\bar{d}}^{eff}$ for 363 functional forms





Predictions based on 363 functional forms

Substantial dependence on Q and x for g, less for $\bar{u}+\bar{d}$ (in the downward direction)

Are experimental constraints on large-x PDFs mutually consistent?

Effective powers may be non-universal among ep and pp processes

QCRs may work better in processes with low hadron multiplicities

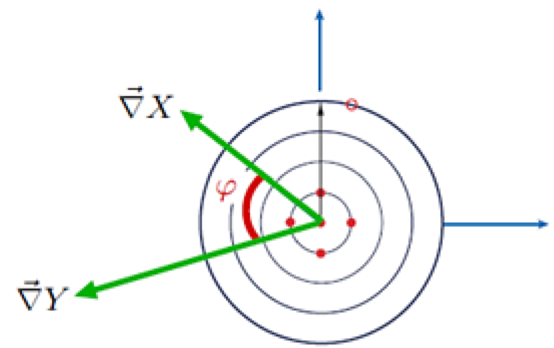
We examine this question in the CT18 NNLO analysis using the method of L_2 sensitivities [**T.Hobbs, B.T. Wang, PN, Olness, 1904.00022**]

L_2 sensitivity, definition

$S_{f,L_2}(E)$ for experiment E is the estimated $\Delta\chi^2_E$ for this experiment when a PDF $f_a(x_i, Q_i)$ increases by the +68% c.l. Hessian PDF uncertainty

Take $X \equiv f_a(x_i, Q_i)$ or $\sigma(f)$; $Y \equiv \chi^2_E$ for experiment E.

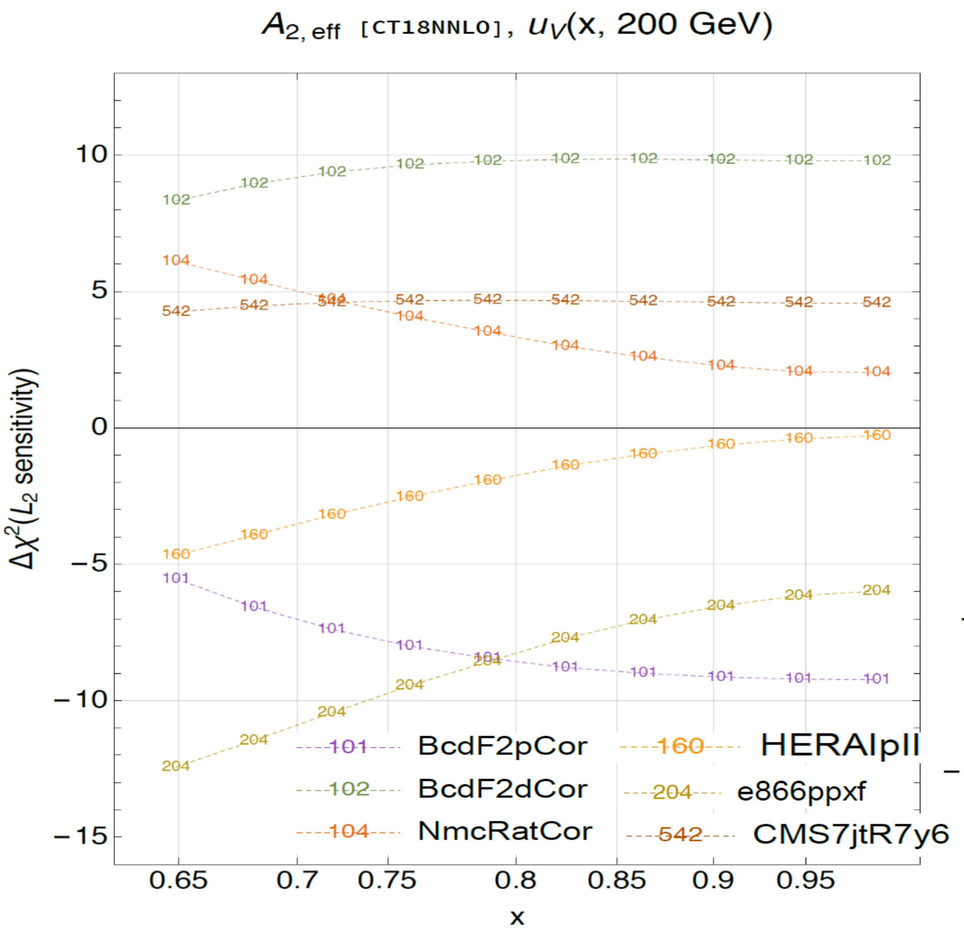
$\hat{z}_X \equiv \nabla X / |\nabla X|$ is the unit vector in direction of the PDF uncertainty of X.

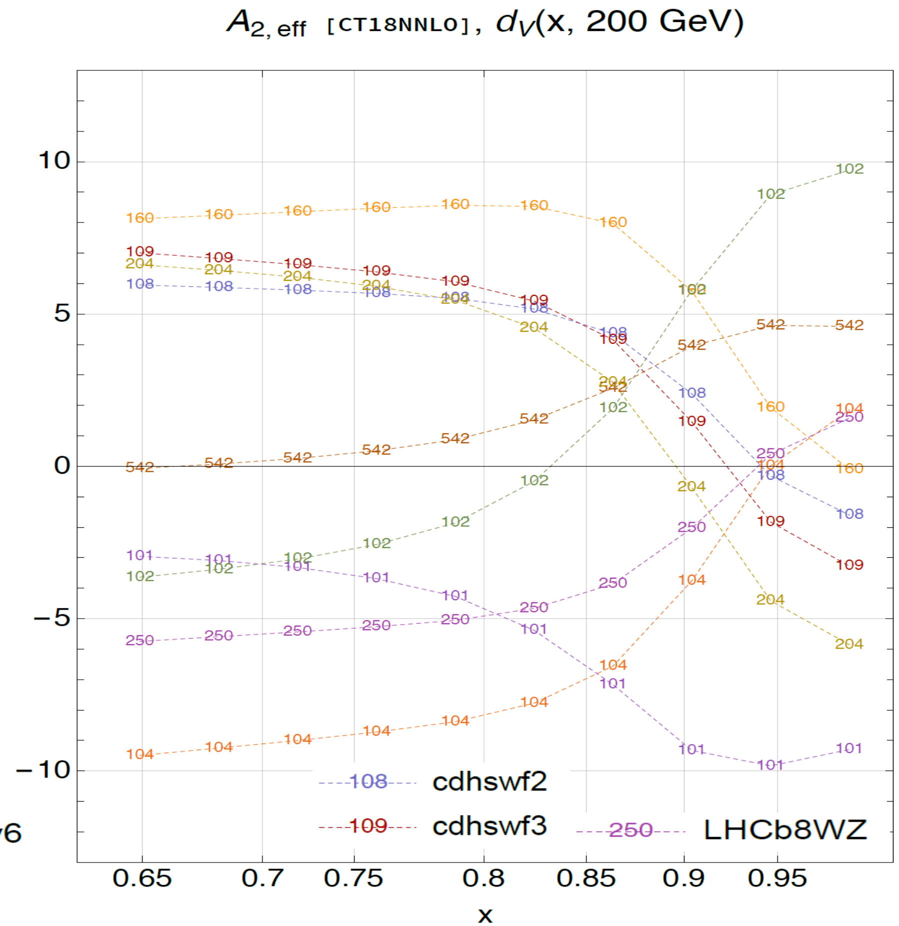


$$S_{X,L_2} \equiv \Delta Y(\hat{z}_X) = \nabla Y \cdot \hat{z}_X = \nabla Y \cdot \frac{\nabla X}{|\nabla X|} = \Delta Y \cos\varphi\,.$$

A fast version of the Lagrange Multiplier scan of χ^2_E along the direction of $f_a(x_i, Q_i)$!

Pulls on A_2^{eff} for u_v and d_v





* Proton BCDMS and DY E866 favor a larger value of $A_{2,eff}[uv]$
* Deuteron BCDMS favors a smaller value of $A_{2,eff}[uv]$
* $A_2^{eff}[u_v]$ largely follows $A_2^{eff}[F_2]$

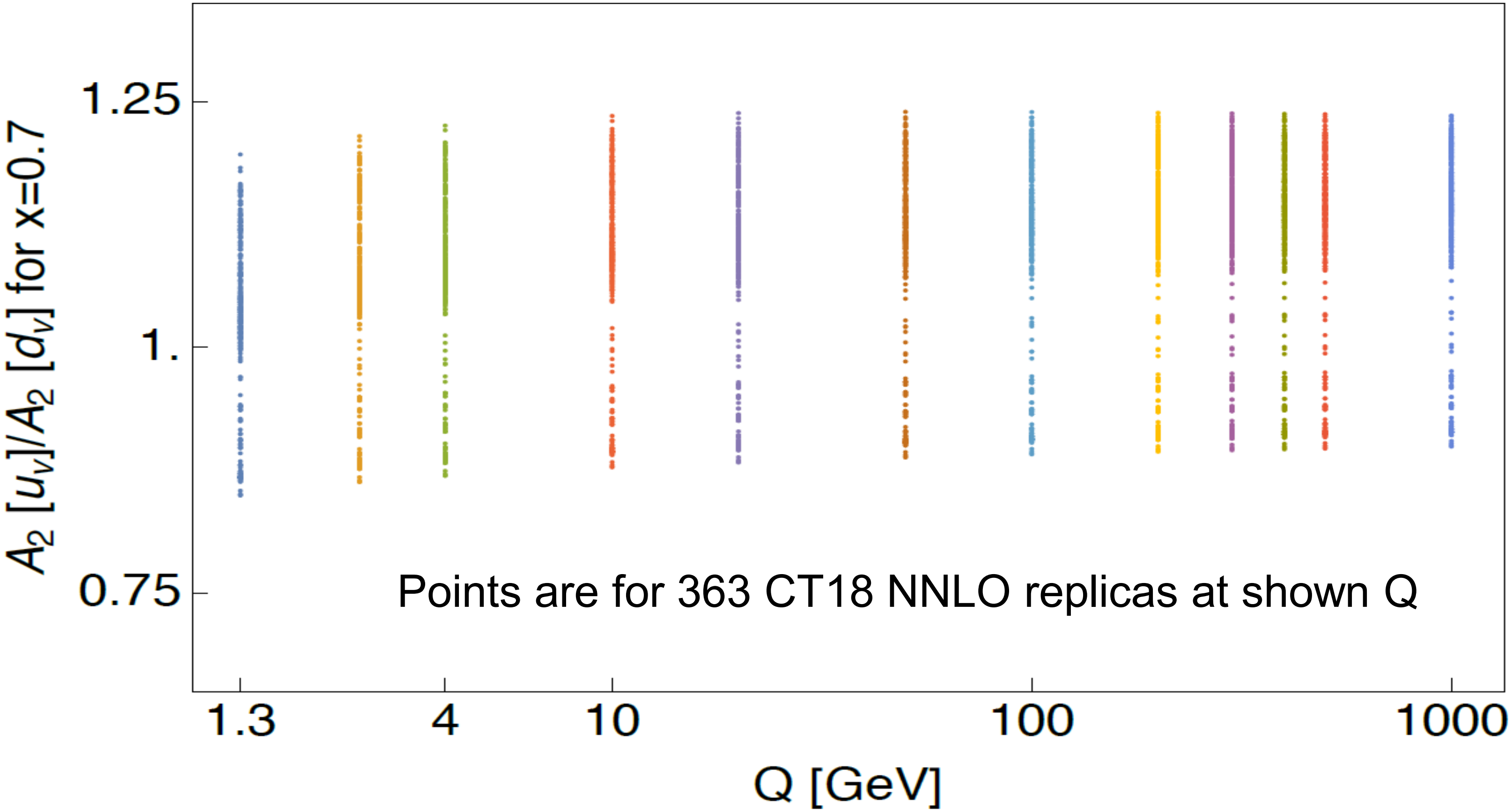
* Tradeoff between opposite pulls of HERA and NMC at $x < 0.85$
* $A_{2,d}^{eff} = A_{2,u}^{eff}$ at $x \to 1$ by construction

How can we access the $x > 0.5$ region at colliders (HERA, LHC, EIC)?

For large-x opportunities at HERA, see A. Caldwell's talk

Examine $A_{2,u}^{eff}/A_{2,d}^{eff}$

NEW

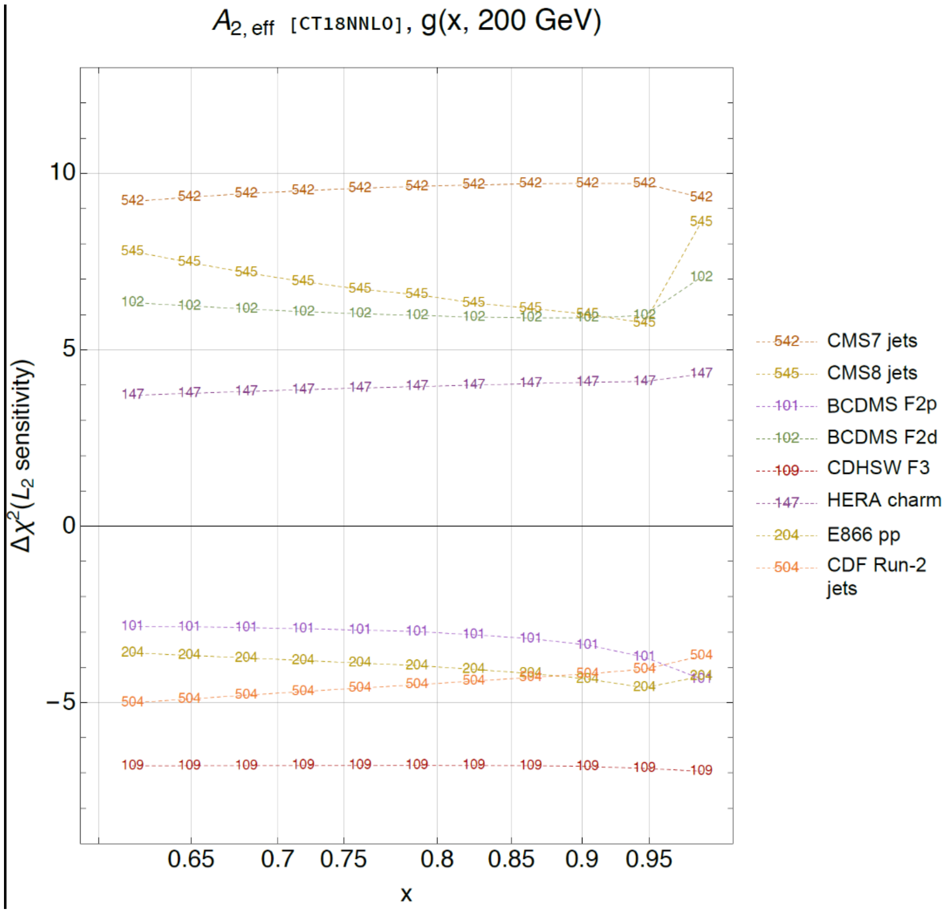


The Q dependences of $A_{2,u}^{eff}$ and $A_{2,d}^{eff}$ cancel out because both obey the non-singlet DGLAP equation

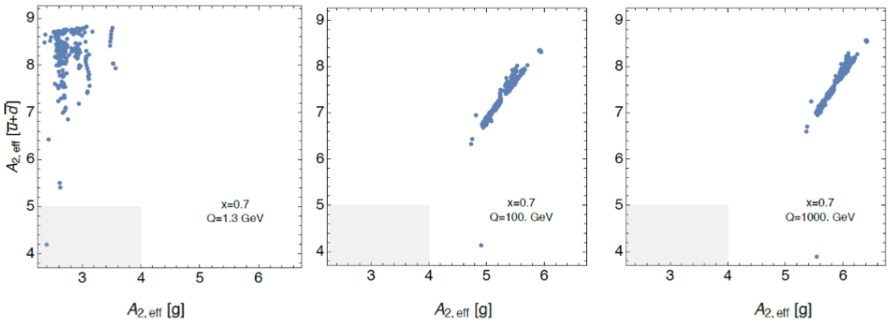
Flavor separation of nonperturbative $A_{2,u}^{eff}$ and $A_{2,d}^{eff}$ can be deduced from high-Q data

Pulls on A_2^{eff} for gluon

NEW



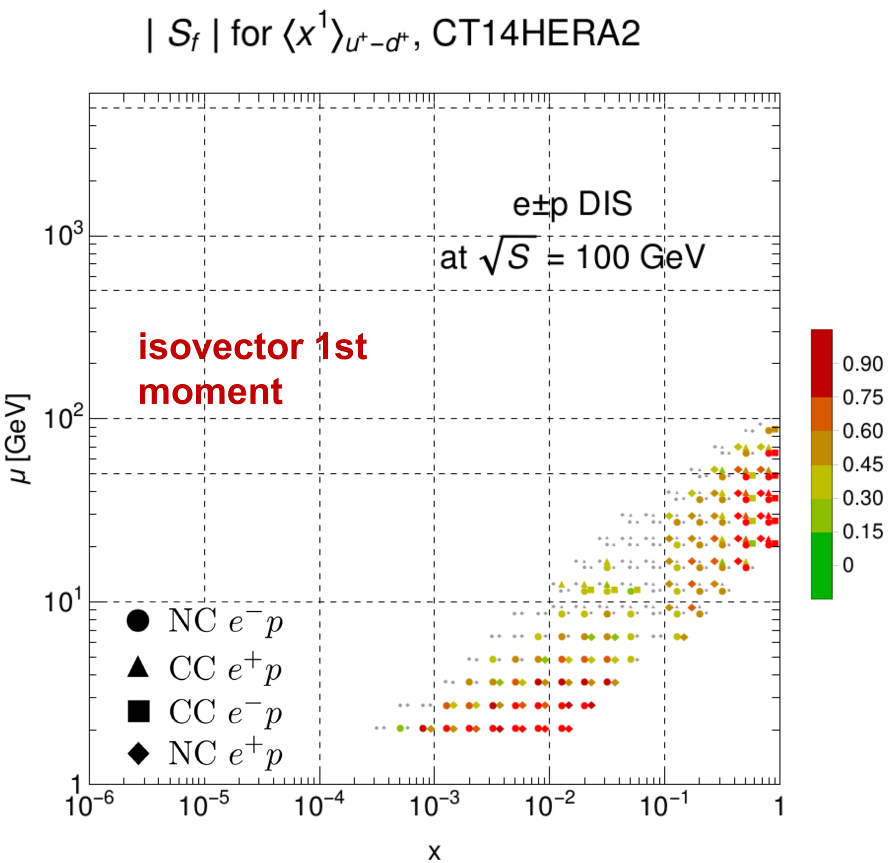
At $x > 0.65$, $A_{2,g}^{eff}$ is determined to a large degree by the tradeoff between jet production experiments [CMS, CDF, …], νA DIS [CDHSW], BCDMS and E866.

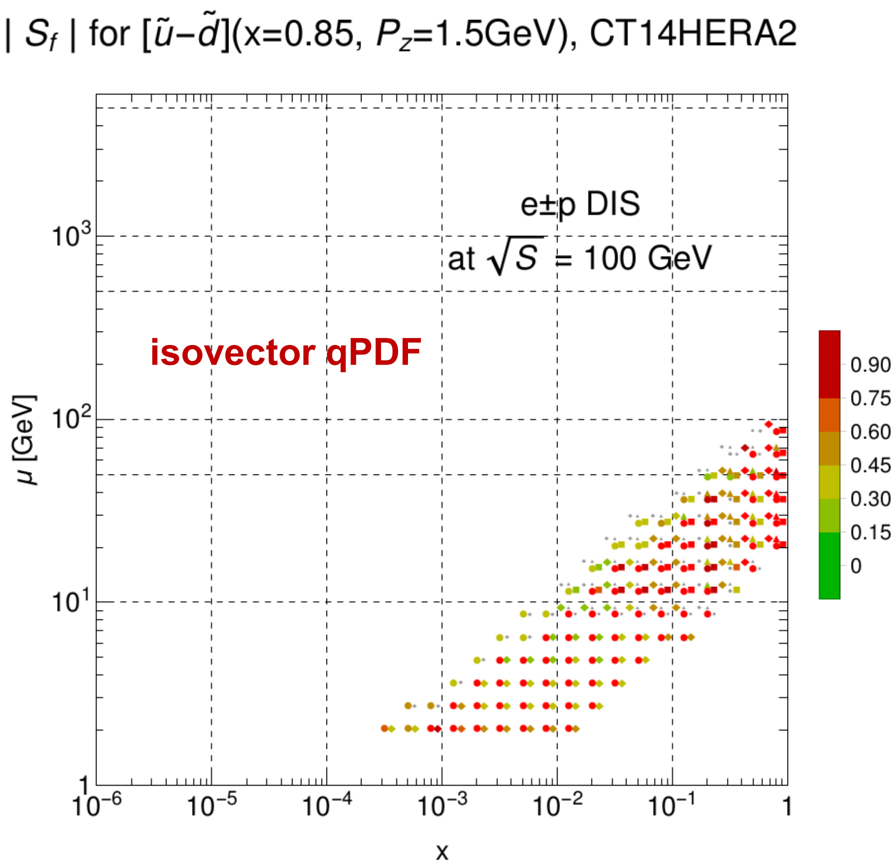


The rate of Q evolution weakly depends on the parametrization form, can be used to reconstruct $A_{2,g}^{eff}$ at $Q \sim 1$ GeV from high-Q measurements.

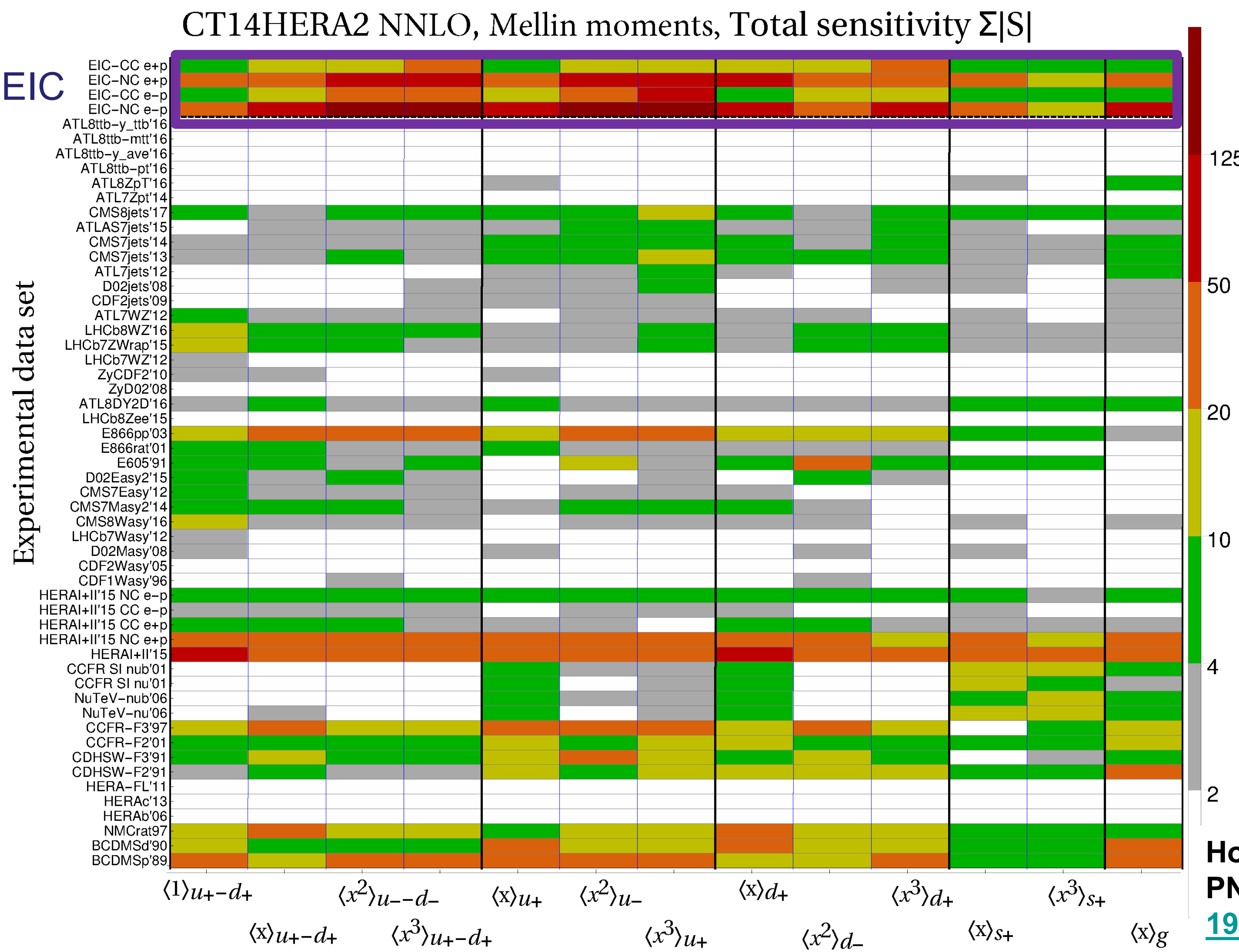
An EIC would drive lattice phenomenology

- A high-luminosity lepton-hadron collider will impose very tight constraints on many lattice observables; below, the isovector first moment and qPDF
- Many of the experiments most sensitive to PDF Mellin moments and qPDFs involve nuclear targets ⟶ **eA data from EIC would sharpen knowledge of nuclear corrections**





Total sensitivity to Mellin moments

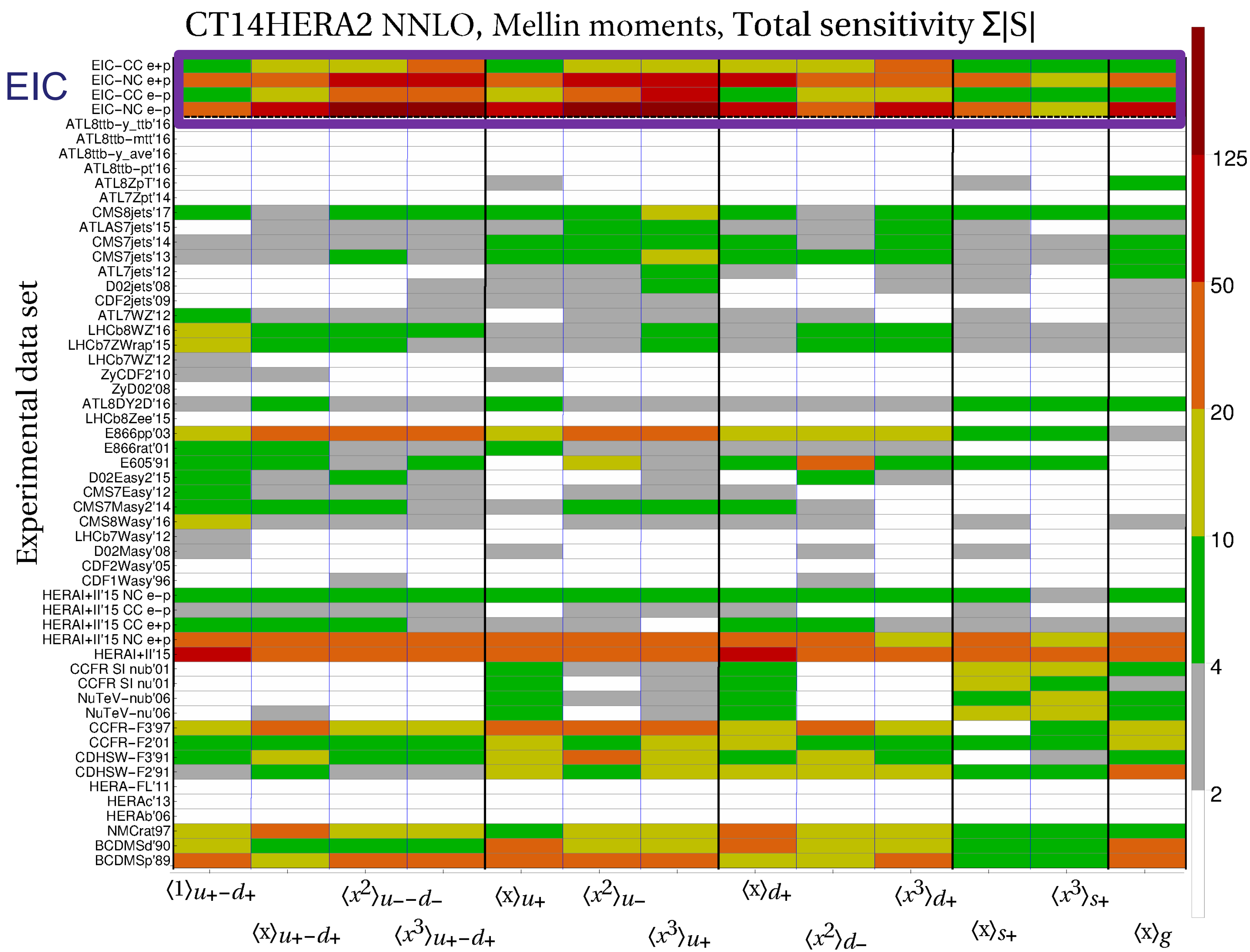


For Mellin moments computable on the lattice at scale 2 GeV

Dark red (white) indicates experiments with highest (lowest) sensitivity of the shown experiment

Hobbs, B. T. Wang, PN, Olness, 1904.00022

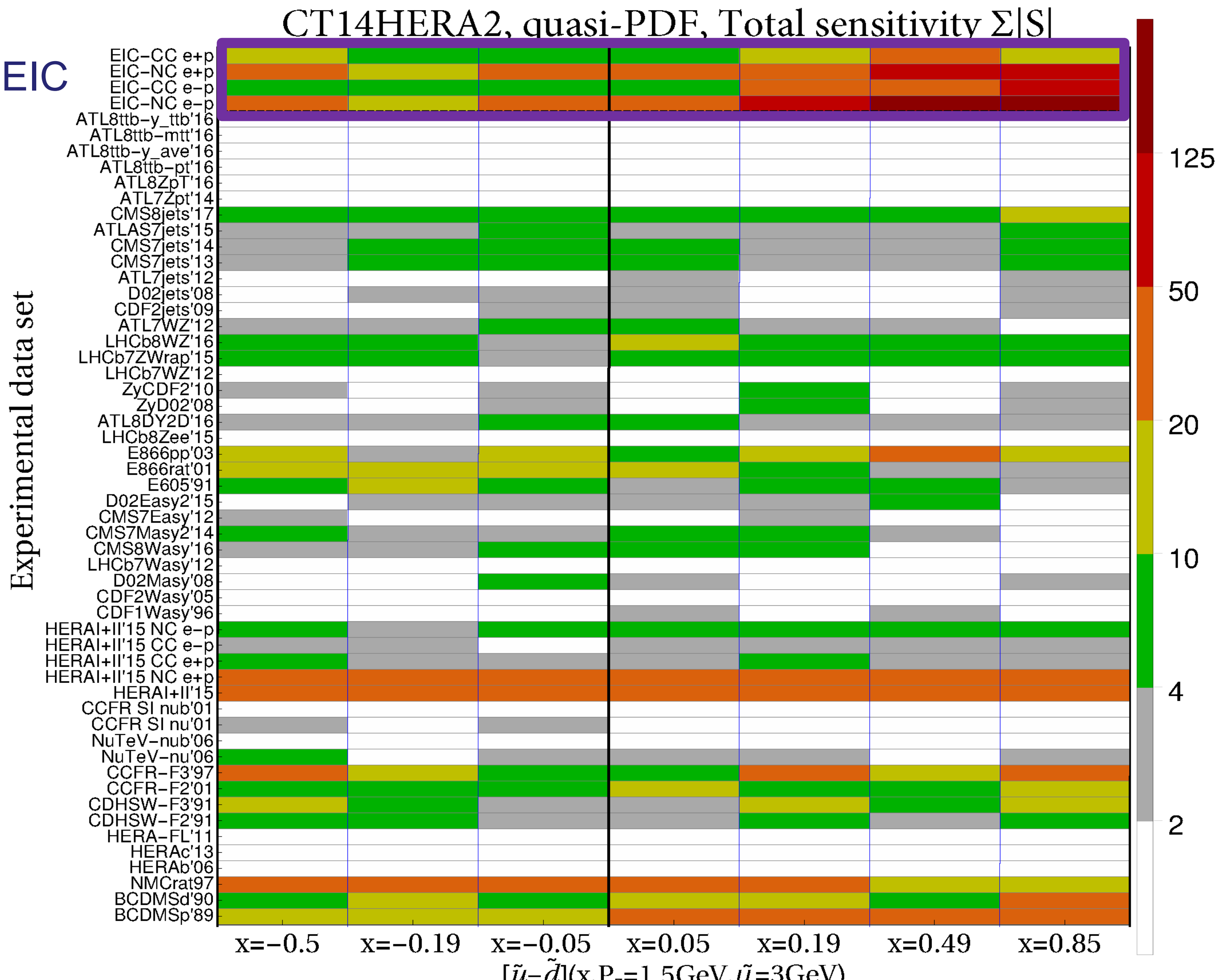
Total sensitivity to Mellin moments



Pre-2019 data: HERA, BCDMS, NMC, E866 DY pair production are most sensitive to the moments

The EIC will dramatically improve the constraints

Total sensitivity to lattice quasi-PDFs



$$\tilde{q}(x) \equiv \begin{cases} q(x), & x > 0; \\ -\bar{q}(|x|), & x < 0 \end{cases}$$

$Q = 3$ GeV,
$P_z = 1.5$ GeV

Conclusions

We have analyzed the **quark counting rules** for the CT18NNLO global fit of proton PDFs.

We examined their **universality** w.r.t. scattering processes and flavors, as well as for structure functions vs. PDFs.

Global analyses rely on complex processes. Dependence on x generally reflects power-suppressed hadronic activity, not only scaling violations or resummation.

Mimicry reconciles many parametrizations of PDFs with measurements.

How do we cast nonperturbative manifestations into measurable observables? We advocate for using **an effective (1-x)-exponent $\boldsymbol{A_2^{eff}}$**.

The Q^2 dependence of A_2^{eff} is not negligible — supported by other global fits and by PQCD. We can solve for Q^2 dependence to obtain A_2^{eff} at $Q \approx 1$ GeV from collider measurements at HERA, LHC, and EIC.

Thank you
for your attention

Multivariate parametric forms

A typical PDF set may depend on tens to several hundreds of free parameters

PDF functional forms must be flexible to accommodate a variety of behaviors
CT18 parametrizations at initial scale Q_0 are given by

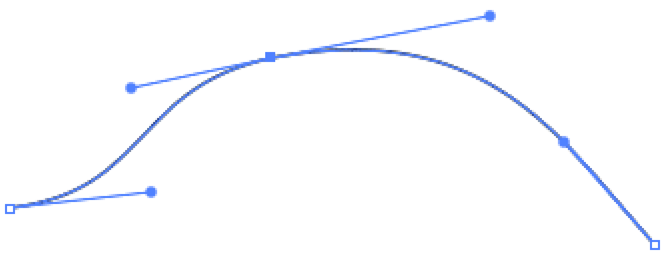
$$f_a(x, Q_0) = A x^{a_1} (1-x)^{a_2} B_a^{(n)}(x; a_3, a_4, \ldots)$$

$$B_a^{(n)}(x) = \sum_{k=0}^{n} a_{k+2} \binom{n}{k} x^k (1-x)^{n-k}$$

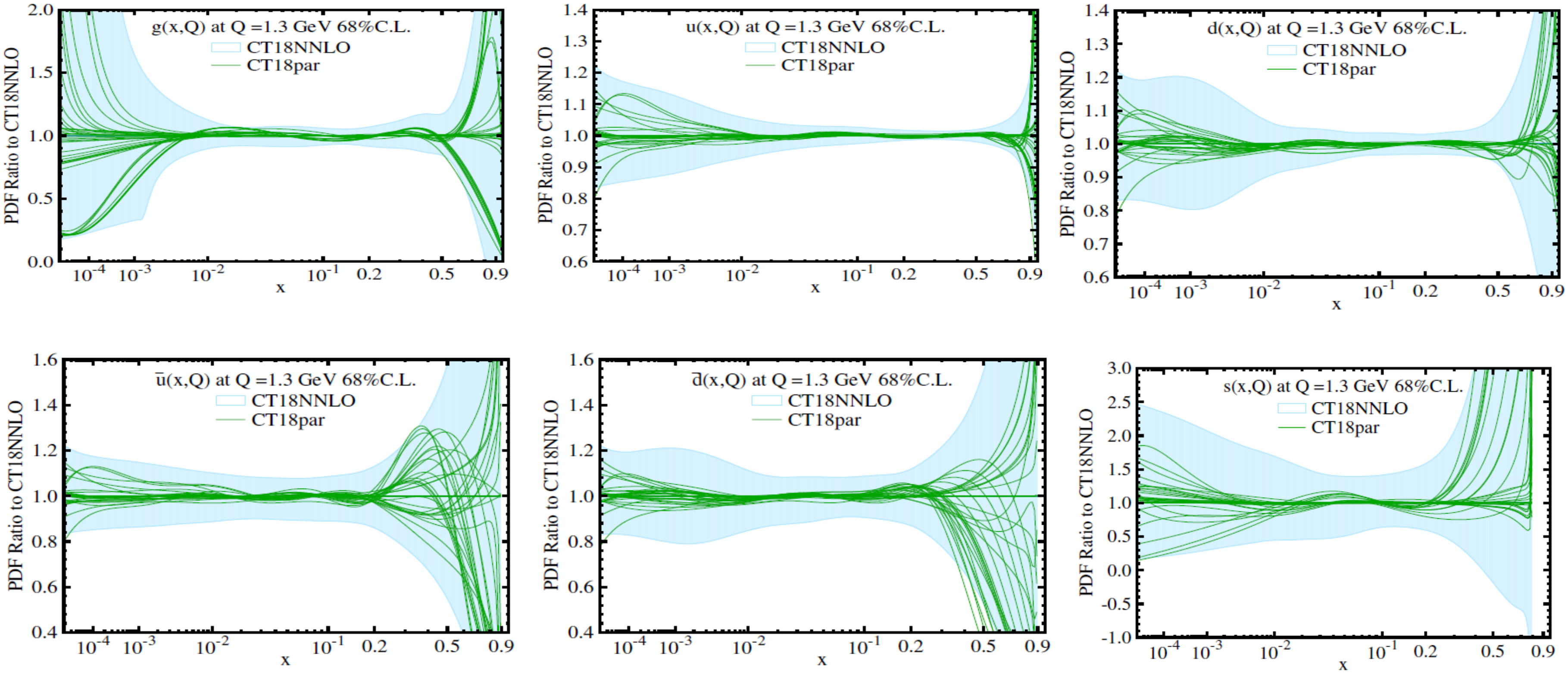
are **Bézier curves** – flexible polynomials familiar from vector graphics programs

Bézier curves can mimic a variety of behaviors of PDFs and their uncertainties. A powerful alternative to neural networks!

[A. Courtoy, P. N., arXiv: 2011.10078, accepted to PRD*]*



250+ candidate nonperturbative parametrization forms of CT18 PDFs



- CT18par – a sample of **some** non-perturbative parametrization forms tried in CT18
- No data constrain very large x or very small x regions.