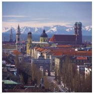
Massive neutrinos in cosmology and the weakly non-linear regime

Mathias Garny (TUM Munich)

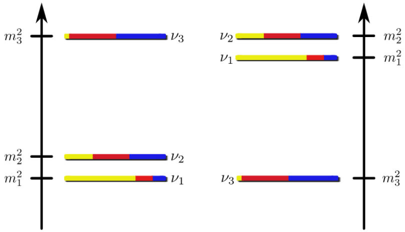


14.10.20

2011.03050, 2008.04943, 1805.12203, 1408.2995

with Thomas Konstandin, Julien Lesgourgues, Laura Sagunski, Petter Taule, Matteo Viel, . . .

Laboratory constraints on neutrino mass scale



Solar, reactor, atmospheric, and accelerator neutrino oscillations

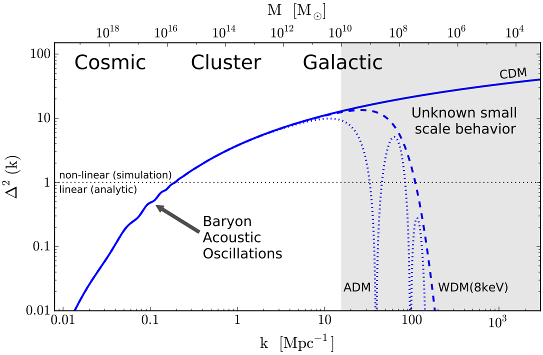
$$\sum m_\nu > \begin{cases} 58\,\text{meV} & \text{normal hierarchy} \\ 108\,\text{meV} & \text{inverted hierarchy} \end{cases}$$

Tritium β-decay endpoint spectroscopy (KATRIN) *PRL123(2019)221802 1909.06048*

$$\sum m_\nu < 3 \times 1.1\text{eV} \quad 90\%\text{C.L.}$$

Exciting interplay with indirect constraints from structure formation

Power spectrum of density perturbations



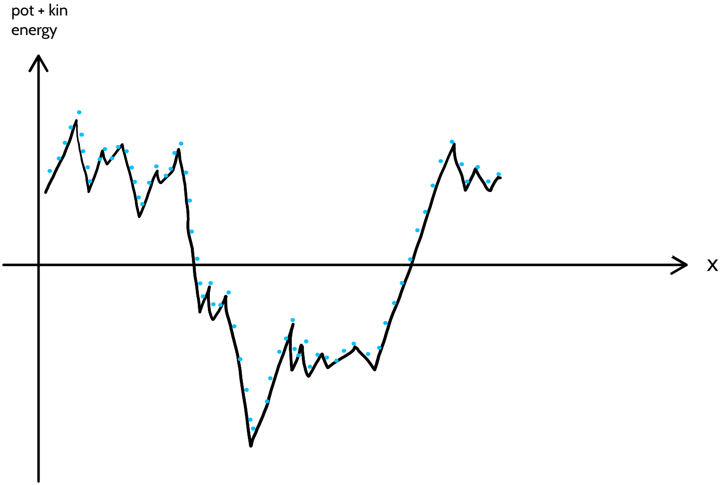
Kuhlen, Vogelsberger, Angulo 1209.5745

$$\delta(\boldsymbol{x}, z) = \rho(\boldsymbol{x}, z)/\bar{\rho}(z) - 1$$

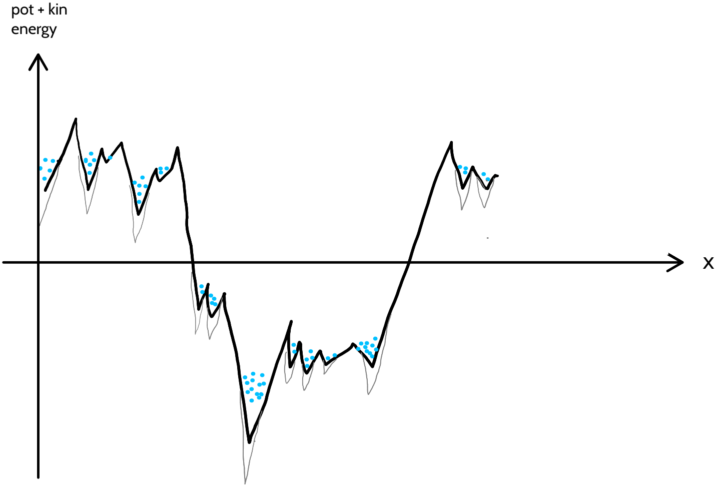
$$\langle \delta(\mathbf{k}, z)\delta(\mathbf{k}', z)\rangle = \delta^{(3)}(\mathbf{k} + \mathbf{k}')P(k, z)$$

$$\Delta^2(k, z) = 4\pi k^3 P(k, z)$$

Cold Dark Matter



Cold Dark Matter



Cosmic neutrino background

- Redshifted relativistic Fermi-Dirac distribution due to neutrino decoupling at $T \sim \text{MeV} \gg m_\nu$

$$f_0(p) = \frac{1}{e^{p/T_\nu(z)} + 1}$$

with

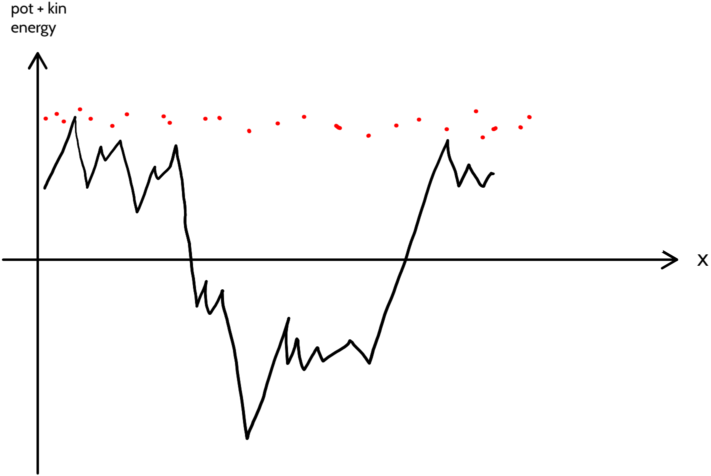
$$T_\nu(z) \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma(z) \simeq 1.96\text{K} \times (1+z) \simeq 0.17 \text{ meV}/k_B \times (1+z)$$

- Average momentum $\langle p \rangle \simeq 3.15\, T_\nu(z)$ becomes smaller than mass m_ν for redshift

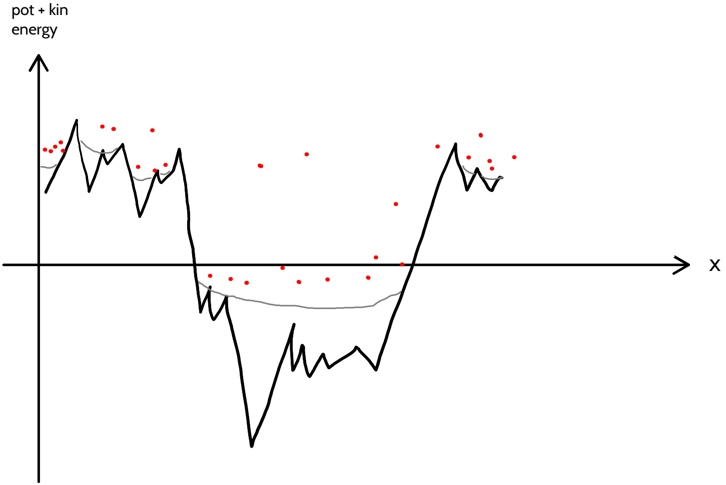
$$z_{\text{nr}} \simeq 189 \frac{m_\nu}{100\,\text{meV}}$$

- Even though CνB neutrinos are non-relativistic today, large thermal velocity compared to cold dark matter (CDM) tends to wash out structures below comoving free-streaming length $\lambda_{\text{FS}} = \int^t \frac{dt'}{a(t')} \langle \frac{p}{E} \rangle$

Hot Dark Matter (cosmic neutrinos)



Hot Dark Matter (cosmic neutrinos)



Cosmic neutrino background

- Neutrino fraction at $z \ll z_{\rm nr}$

$$f_\nu = \frac{\Omega_\nu}{\Omega_m} = \frac{1}{\Omega_m^0 h^2} \times \frac{\sum m_\nu}{93.14\mathrm{eV}}$$

$\sum m_\nu$ [eV]	0.06	0.15	0.21	0.3
f_ν	0.0045	0.0112	0.0156	0.0221

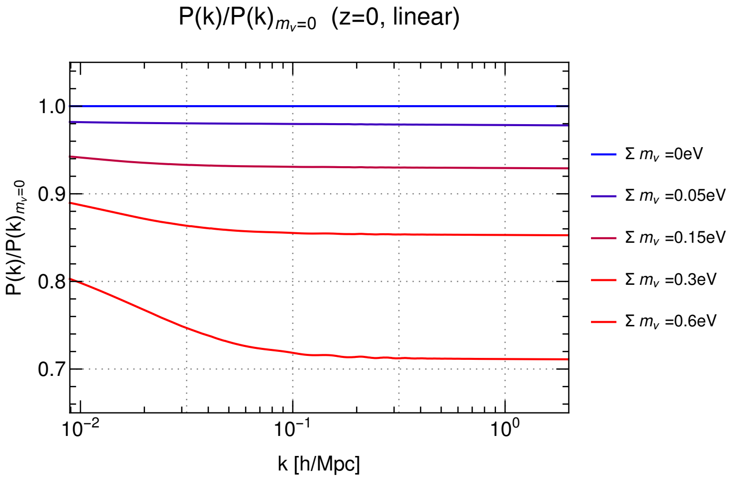
- Growth of linear density perturbations during matter domination

$$\delta(k,t) \propto \begin{cases} a(t) & 2\pi/k \gg \lambda_{\rm FS} \\ a(t)^{1-3f_\nu/5} & 2\pi/k \ll \lambda_{\rm FS} \end{cases}$$

with

$$\lambda_{FS}(z) \simeq 350\mathrm{Mpc}\sqrt{\frac{1+z}{100}}\sqrt{\frac{0.1}{\Omega_m^0 h^2}\frac{0.1\,\mathrm{eV}}{m_\nu}}$$

Power spectrum suppression for massive neutrinos

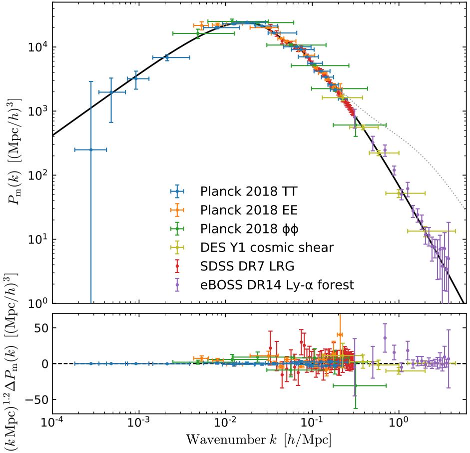


$\Delta P/P \simeq -8 f_\nu$ for $k \gg 2\pi/\lambda_{\rm FS}$ and $z = 0$

cf. e.g. Hannestad 2003; Crotty, Lesgourgues, Pastor 2004; Hannestad, Raffelt 2004; Hannestad, Tu, Wong 2006

Power spectrum of density perturbations: large scales $\gtrsim$ Mpc

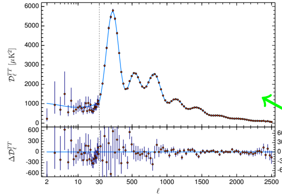
$$P(k) \sim |\delta_k|^2$$



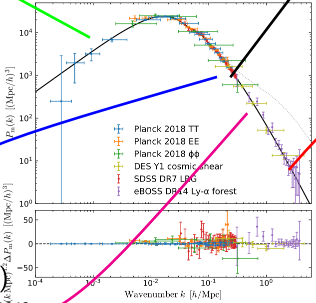
Projection on linear power spectrum at $z = 0$ (model dep.!) *Palanque-Delabrouille et al 1905.08103*

Power spectrum of density perturbations: large scales $\gtrsim$ Mpc

CMB ($z \sim 10^3$)

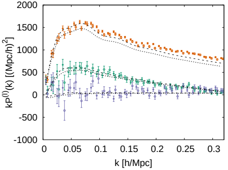


Planck 1807.06209, ACT 2007.07288,...



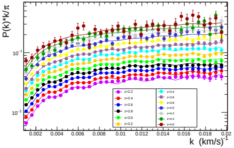
1905.08103

Galaxy surveys ($z \lesssim 2$)



BOSS DR16 2007.08994

Lyα forest ($z \sim 2-4$)



BOSS Lyα 1812.03554

Galaxy cluster counts ($z \lesssim 2$)

Planck/SPT/ACT/DES SZ 2009.08822,2009.11043

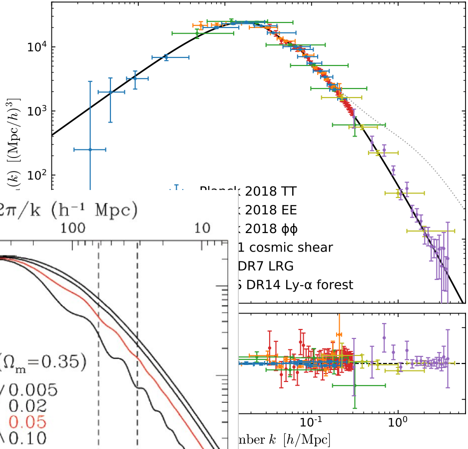
Weak lensing ($z \lesssim 2$)

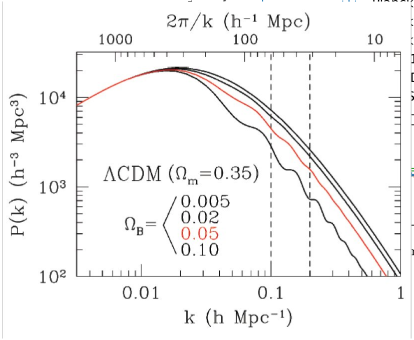
DES 2010.01138

KiDS-1000 2007.15632

Baryon Acoustic Oscillations (BAO)

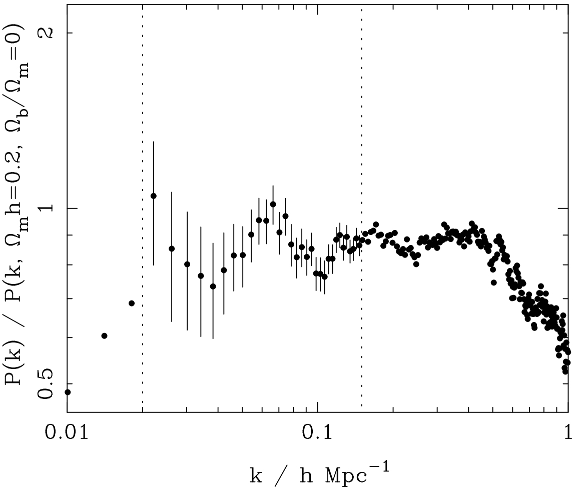
Compilation of LSS measurements $\qquad P(k) \sim |\delta_k|^2,\ \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$





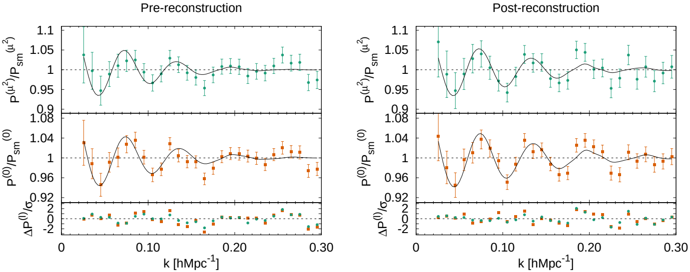
$z = 0$ (model dep.!) *1905.08103*

Baryon Acoustic Oscillations (BAO)



2dFGRS (Percival et al) MNRAS 327:1297,2001 astro-ph/0105252

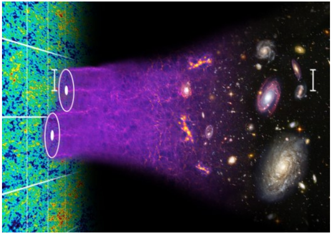
Baryon Acoustic Oscillations (BAO)

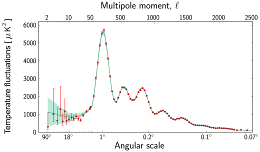


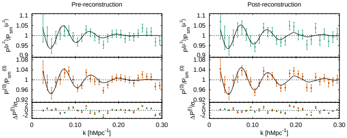
BOSS DR16 2007.08994

Future: Vera C. Rubin Observatory; Euclid, DESI, ...:
(sub-)percent at BAO scales

Baryon Acoustic Oscillations (BAO)

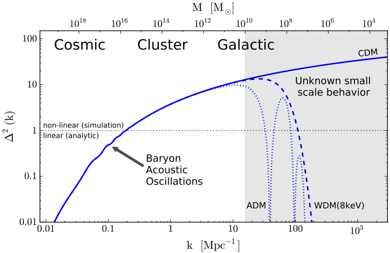






Cosmic microwave background $z \sim 10^3$ Large-scale structure $z \sim 0 ... 1$

Can we understand the (weakly) non-linear regime?

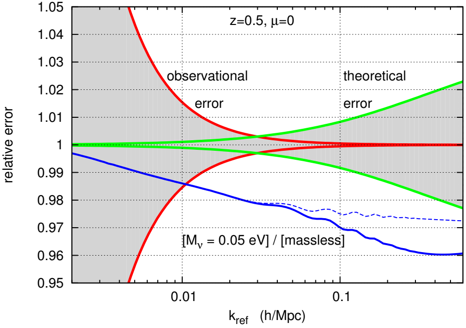


Kuhlen, Vogelsberger, Angulo 1209.5745

- Motivation: want efficient but precise method to obtain predictions for a large set of cos. parameters ⇒ perturbative methods complemented by EFT techniques
- Goal 1: Scrutinize accuracy of (perturbative) approaches for massive ν cosmologies
- Goal 2: Demonstrate worked example for Lyman-α data in BOSS range

Neutrino mass vs theoretical error

Euclid forecast vs theoretical errors *Audren, Lesgourgues, Bird et. al. 1210.2194*



theoretical uncertainties from biased tracers, redshift-space distortions, relativistic effects, baryonic effects, non-linear clustering, . . .

$$\sigma(M_\nu) \simeq \begin{cases} \text{25meV} & \text{fiducial (2\%th. err. at } k = 0.4h/Mpc,\ z = 0.5) \\ \text{14meV} & \text{th. err. } / = 10,\ k_{max} = 0.6h/Mpc \end{cases}$$

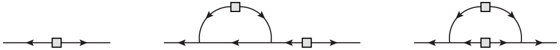
Perturbation theory for large-scale structure

$$\delta(\boldsymbol{k}, z) = \sum_n \delta^{(n)}(\boldsymbol{k}, z)$$
$$= \sum_n \int_{k_1,\ldots,k_n} \delta_D(\boldsymbol{k} - \sum \boldsymbol{k}_i) F_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n; z)\, \delta(\boldsymbol{k}_1, z_{ini}) \cdots \delta(\boldsymbol{k}_n, z_{ini})$$

Power spectrum

$$P(k,z) = \overbrace{P_{11}(k)}^{P_{lin}} + \overbrace{(2P_{13} + P_{22})}^{P_{1-loop}\ \text{(NLO)}} + \underbrace{(2P_{15} + 2P_{24} + P_{33})}_{P_{2-loop}\ \text{(NNLO)}} + \ldots$$

where $P_{nm} = \langle \delta^{(n)} \delta^{(m)} \rangle$



e.g. $P_{22} = 2 \int_q d^3q\, F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}; z)^2\, P_{ini}(q) P_{ini}(|\mathbf{k} - \mathbf{q}|)$

Vlasov-Poisson

- Number of particles $dN = f(\tau, \boldsymbol{x}, \boldsymbol{p}) d^3x d^3p$, conf. time $d\tau = dt/a$
- Vlasov eq

$$0 = \frac{df}{d\tau} = \left(\frac{\partial}{\partial \tau} + \frac{\boldsymbol{p}}{am} \frac{\partial}{\partial \boldsymbol{x}} - am \nabla \psi \frac{\partial}{\partial \boldsymbol{p}} \right) f(\tau, \boldsymbol{x}, \boldsymbol{p})$$

- Poisson eq

$$\nabla^2 \psi(\mathbf{x}, \tau) \quad = \quad \frac{3}{2} \Omega_m \mathcal{H}^2 \delta(\mathbf{x}, \tau)$$

- Moments

$$m \int d^3p\, f = \rho = \bar{\rho}(1 + \delta), \qquad m \int d^3p \frac{\boldsymbol{p}}{am} f = \rho \boldsymbol{u}$$

Fluid description

- Moments of Vlasov eq.

$$\frac{\partial \delta(\mathbf{x},\tau)}{\partial \tau} + \nabla \cdot \{(1+\delta(\mathbf{x},\tau))\mathbf{u}(\mathbf{x},\tau)\} = 0 \quad \text{(continuity)}$$

$$\frac{\partial \mathbf{u}(\mathbf{x},\tau)}{\partial \tau} + \mathcal{H}\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla\psi - \frac{1}{\rho}\nabla_j(\sigma_{ij}\rho) \quad \text{(Euler)}$$

- stress tensor

$$\sigma_{ij} = \frac{1}{\rho} m \int d^3p \frac{p_i p_j}{a^2 m^2} f(\boldsymbol{x},\boldsymbol{p},t) - u_i u_j$$

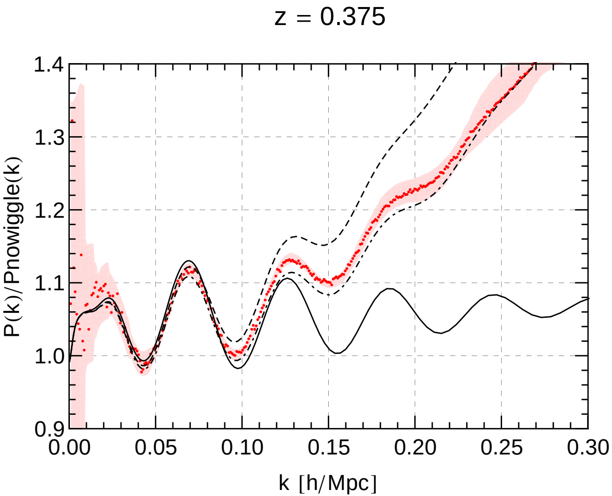
- Standard Perturbation Theory (SPT)

  $\rightarrow$ Pressureless perfect fluid: $\sigma_{ij} = 0$

$\Rightarrow$ rec. relation for kernels $F_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n;z) \approx D(z)^n F_n^s(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$

$$F_1^s = 1, \qquad F_2^s(\boldsymbol{p},\boldsymbol{q}) = \frac{5}{7} + \frac{1}{2}\frac{\boldsymbol{p}\cdot\boldsymbol{q}}{pq}\left(\frac{p}{q}+\frac{q}{p}\right) + \frac{2}{7}\frac{(\boldsymbol{p}\cdot\boldsymbol{q})^2}{p^2q^2}$$

Status of SPT

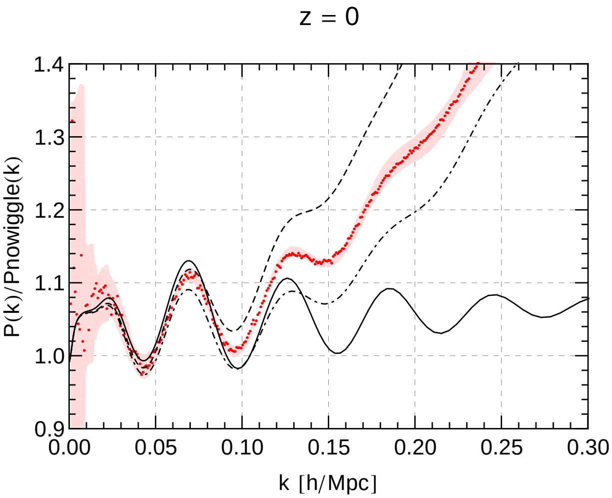


black=linear, NLO=1loop(dashed), NNLO=2loop(dotdashed),

red=Horizon Run 2

Diego Blas, MG, Thomas Konstandin 1309.3308

Status of SPT



black=linear, NLO=1loop(dashed), NNLO=2loop(dotdashed),

red=Horizon Run 2

Diego Blas, MG, Thomas Konstandin 1309.3308

Viscous fluid

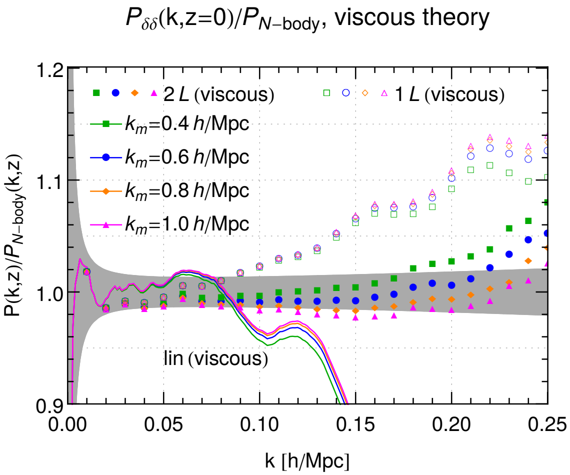
stress tensor

$$\sigma_{ij} = -p\delta_{ij} + \eta(\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\nabla \cdot \boldsymbol{u}\,\delta_{ij}) + \zeta\nabla \cdot \boldsymbol{u}\,\delta_{ij}$$

- p = pressure
- η = shear viscosity
- ζ = bulk viscosity

cf. e.g. Baumann, Nicolis, Senatore, Zaldarriaga 1004.2488

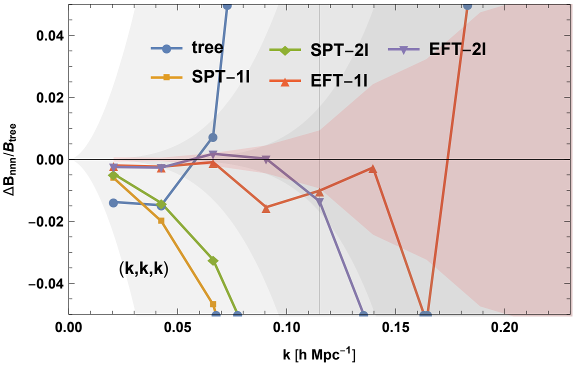
NNLO (2-loop) power spectrum in viscous theory ($z = 0$)



Residual dependence on k_m is an estimate for the theoretical error

Blas, Floerchinger, MG, Tetradis, Wiedemann 1507.06665

NNLO (2-loop) bispectrum in EFT ($z = 0$)



Baldauf, MG, Steele, Taule 21xx.yyyy

Application of (effective) field theory and resummation methods

- $k \ll 0.1h/\text{Mpc}$ bulk flows (IR)

$\rightarrow$ IR resummation $\leftrightarrow$ BAO broadening

Baldauf, Mirbabayi, Simonovic, Zaldarriaga, 1504.04366; Blas, MG, Ivanov, Sibiryakov 1605.02149

- $k \sim 1h/\text{Mpc} \sim$ virial scale (UV)

$\rightarrow$ EFT description (eff. viscosity, more contributions at higher order and in redshift space) *e.g. Foreman, Perrier, Senatore 1507.05326; Abolhasani, Mirbabayi, Pajer 1509.07886; Floerchinger, MG, Tetradis, Wiedemann 1607.03453*

$\rightarrow$ bias expansion $\delta_g = b_1\delta + b_2\delta^2 + \ldots$ *e.g. Desjacques, Jeong, Schmidt 1611.09787*

$\rightarrow$ strategy: treat EFT and bias parameters as free nuisance parameters in fit to data, applied e.g. to BOSS LRG sample ("full-shape analysis")

Ivanov, Simonovic, Zaldarriaga 1909.05277; d'Amico et al 1909.05271

$\rightarrow$ This talk: application to BOSS Lyman-α data for massive neutrinos

MG, Konstandin, Sagunski, Tulin 1805.12203; MG, Konstandin, Sagunski, Viel 2011.03050

Massive neutrino perturbations: linear theory

- Perturbed neutrino distribution function

$$f(\mathbf{x}, \mathbf{q}, \tau) = f_0(q) \times (1 + \Psi(\mathbf{x}, \mathbf{q}, \tau))$$

- Boltzmann equation $0 = df/d\tau \Rightarrow$

$$\left(\partial_\tau + \frac{\mathbf{q} \cdot \nabla}{\epsilon(q, \tau)}\right) \Psi(\mathbf{x}, \mathbf{q}, \tau) + \frac{\mathrm{d} \ln f_0}{\mathrm{d} \ln q} \left(\partial_\tau \phi(\mathbf{x}, \tau) - \frac{\epsilon(q, \tau) \mathbf{q} \cdot \nabla}{q^2} \psi(\mathbf{x}, \tau)\right) = 0$$

- Comoving momentum $q = a(\tau) p = p/(1 + z)$
- Comoving energy $\epsilon(q, \tau) = \sqrt{q^2 + a(\tau)^2 m_\nu^2}$
- Conformal Newtonian gauge

$$ds^2 = -a^2 d\tau^2 (1 + 2\psi(\mathbf{x}, \tau)) + a^2 d\mathbf{x}^2 (1 - 2\phi(\mathbf{x}, \tau)))$$

Ma, Bertschinger AJ455(1995) astro-ph/9401007

Massive neutrino perturbations: linear theory

- Multipole moments

$$\Psi_\ell(k,q,\tau) = i^\ell \int \frac{d\Omega}{4\pi} P_\ell(\hat{k}\cdot\hat{q}) \int d^3x \, e^{i\mathbf{k}\cdot\mathbf{x}} \, \Psi(\mathbf{x},\mathbf{q},\tau)$$

- Coupled hierarchy

$$\begin{aligned}
\Psi_0' &= -\frac{q}{\epsilon}\Psi_1 - \phi' \\
\Psi_1' &= \frac{q}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon}{2q}\psi \\
\Psi_\ell' &= \frac{q}{(2\ell+1)\epsilon}[\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}], \quad \ell \geq 2
\end{aligned}$$

Ψ rescaled by $d\ln f_0/d\ln q$ and $' = d/dx$, $x = k\tau$

- Truncation at some $\ell_{\text{max}} \sim \mathcal{O}(20)$
- For $z \ll z_{\text{nr}}$ typical ν momentum $q \sim \mathcal{O}(T_\nu) \ll \epsilon \simeq a^2 m_\nu^2 \Rightarrow$ higher multipoles become more and more suppressed *e.g. Shoji, Komatsu 1003.0942*

Massive neutrino perturbations: linear theory

- Density contrast $\delta = \delta\rho/\bar{\rho} \propto \int_{\mathbf{q}} \epsilon \times f_0 \Psi_0$
- Divergence of peculiar velocity $\theta = \nabla \cdot \mathbf{u} \propto \int_{\mathbf{q}} q \times f_0 \Psi_1$
- Pressure pert. $\delta P \propto \int_{\mathbf{q}} \frac{q^2}{\epsilon} \times f_0 \Psi_0$
- Anisotropic stress $\sigma \propto \int_{\mathbf{q}} \frac{q^2}{\epsilon} \times f_0 \Psi_2$
- $\int_{\mathbf{q}}$ (equations for Ψ_0 and Ψ_1) $\Rightarrow$

$$\dot{\delta} = -\theta + 3\dot{\phi} \qquad \text{(continuity)}$$
$$\dot{\theta} = -\mathcal{H}\theta + k^2\psi + k^2 c_s^2 \delta - k^2 \sigma \qquad \text{(Euler w pressure+stress)}$$

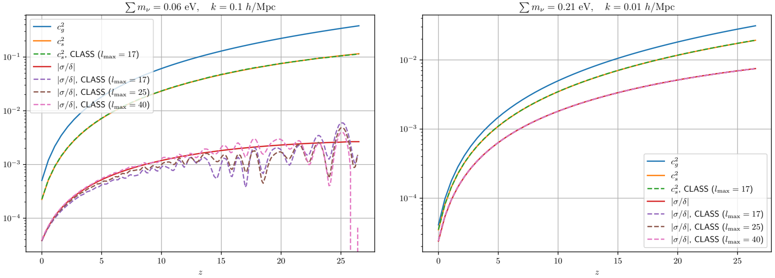
with sound velocity

$$c_s^2 = \delta P/\delta\rho$$

and $k^2\psi = -4\pi G a^2 \sum_i \delta\rho_i$ (Poisson eq.)

$\mathcal{H} = \dot{a}/a$, assuming $w = \bar{P}/\bar{\rho} \ll 1$

Massive neutrino perturbations: linear theory



MG, Taule 2008.00013

- Adiabatic approximation $c_g^2(z) \equiv \dot{\bar{P}}/\dot{\bar{\rho}} = \frac{25}{3}\frac{\zeta(5)}{\zeta(3)}\left(\frac{T_\nu(z)}{m_\nu}\right)^2$
- Full sound velocity $c_s^2(k,z) = \delta P/\delta\rho$
- Anisotropic stress rel. to density constrast σ/δ

Strategy for going beyond linear theory

- (1) $z > z_{\mathrm{match}} \equiv 25$: full Boltzmann hierarchy with $\ell_{\mathrm{max}} = 17$ (from CLASS)
- (2) $z \leq z_{\mathrm{match}} \equiv 25$: two-fluid description for neutrinos and CDM/baryons, with effective sound velocity

$$
\begin{aligned}
\dot{\delta}_{\mathrm{cb}} &= -\theta_{\mathrm{cb}} \\
\dot{\theta}_{\mathrm{cb}} &= -\mathcal{H}\theta_{\mathrm{cb}} + k^2\psi \\
\dot{\delta}_{\nu} &= -\theta_{\nu} \\
\dot{\theta}_{\nu} &= -\mathcal{H}\theta_{\nu} + k^2\psi + k^2 \underbrace{\left(c_s^2 - \sigma/\delta_\nu\right)}_{\equiv c_{\mathrm{eff}}^2} \times \delta_\nu
\end{aligned}
$$

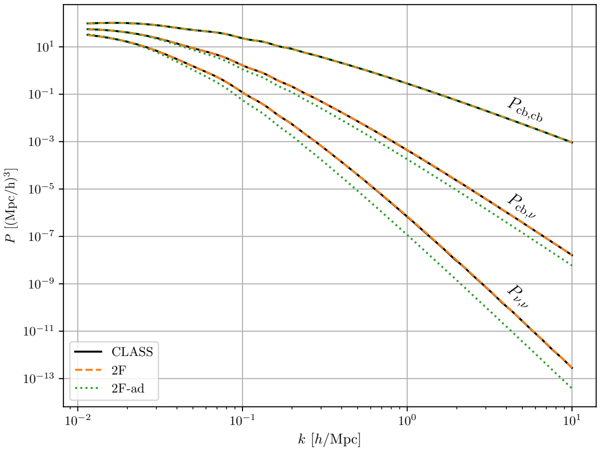
- We consider two slightly different approximations:

  2F-ad Adiabatic approximation $c_{\mathrm{eff}}^2 = c_g^2$

  2F Full linear pressure+shear $c_{\mathrm{eff}}^2 = (c_s^2 - \sigma/\delta_\nu)_{\mathrm{lin}}$ (no truncation in multipoles)

Blas, MG, Konstandin, Lesgourgues 1408.2995; MG, Taule 2008.00013

Massive neutrino perturbations: linear theory

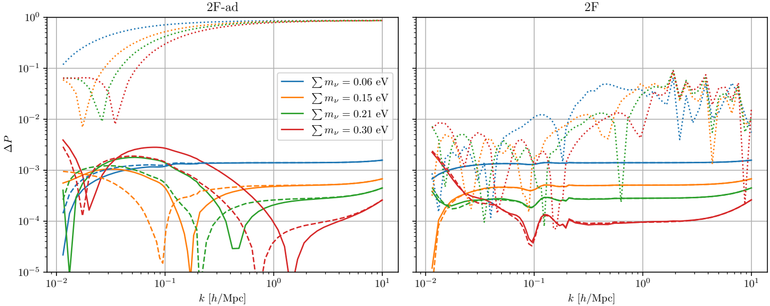


$$P_{mm} = (1-f_\nu)^2 P_{\rm cb,cb} + 2(1-f_\nu) f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_{\nu\nu}$$

Free-streaming scale $k_{\rm FS}^2 = 3\Omega_m \mathcal{H}^2 / 2c_{\rm eff}^2$

$\sum m_\nu = 0.21$*eV MG, Taule 2008.00013*

Massive neutrino perturbations: linear theory



Rel. deviation from full Boltzmann hierarchy ($z = 0$) *MG, Taule 2008.00013*

solid=P_{mm}, dashed=$P_{\text{cb,cb}}$, dotted=$P_{\nu,\nu}$

Non-linear equations

$$
\begin{aligned}
\dot{\delta}_{\text{cb}} + \theta_{\text{cb}} &= -\int_{\mathbf{k}_i} \alpha\theta_{\text{cb}}\delta_{\text{cb}} \\
\dot{\theta}_{\text{cb}} + \mathcal{H}\theta_{\text{cb}} - k^2\psi &= -\int_{\mathbf{k}_i} \beta\theta_{\text{cb}}\theta_{\text{cb}} \\
\dot{\delta}_{\nu} + \theta_{\nu} &= -\int_{\mathbf{k}_i} \alpha\theta_{\nu}\delta_{\nu} \\
\dot{\theta}_{\nu} + \mathcal{H}\theta_{\nu} - k^2\psi - k^2 c_{\text{eff}}^2\delta_{\nu} &= -\int_{\mathbf{k}_i} \beta\theta_{\nu}\theta_{\nu}
\end{aligned}
$$

- $c_{\text{eff}}^2(k,z)$ takes complete impact of linear pressure+anisotropic stress on propagation of *all* non-linear modes into account (2F scheme)
- Neglect non-linearities in $\Psi_\ell, \ell \geq 2$ (check: impact of stress on $P_{mm}^{\geq 1-\text{loop}}$ strongly suppressed for $z \sim \mathcal{O}(1) \ll z_{\text{nr}}$)

  cf. e.g. Chen, Uphadhye, Wong 2011.12503; Dupuy, Bernardeau 1503.05707; Führer, Wong 1412.2764 for alt. approaches, see also Wong 0809.0693 for prev. work

Notation $\int_{\mathbf{k}_i} \alpha\theta_{\text{cb}}\delta_{\text{cb}} = \int d^3k_1 d^3k_2 \delta^{(3)}(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\alpha(\mathbf{k}_1, \mathbf{k}_2)\theta_{\text{cb}}(\mathbf{k}_1)\delta_{\text{cb}}(\mathbf{k}_2)$ $\alpha(\mathbf{k}_1, \mathbf{k}_2) = \frac{\mathbf{k}\cdot\mathbf{k}_1}{\mathbf{k}_1^2}$, $\beta = \frac{\mathbf{k}^2\, \mathbf{k}_1\cdot\mathbf{k}_2}{2\mathbf{k}_1^2\mathbf{k}_2^2}$

Non-linear equations

- Vector notation

$$\psi_a \equiv (\delta_{\rm cb}, -\theta_{\rm cb}/\mathcal{H}f, \delta_\nu, -\theta_\nu/\mathcal{H}f)$$

- Rescaled time variable $\eta \equiv \ln D$,
  $D =$ linear growth rate, $f = d \ln D / d \ln a$
- Non-linear eq. of motion takes the form

$$\partial_\eta \psi_a + \Omega_{ab}(k,\eta)\psi_b = \int_{\mathbf{k}_i} \gamma_{abc}\psi_b(k_1,\eta)\psi_c(k_2,\eta)$$

where

$$\Omega(k,\eta) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -\frac{3}{2}\frac{\Omega_m}{f^2}(1-f_\nu) & \frac{3}{2}\frac{\Omega_m}{f^2}-1 & -\frac{3}{2}\frac{\Omega_m}{f^2}f_\nu & 0 \\ 0 & 0 & 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2}(1-f_\nu) & 0 & -\frac{3}{2}\frac{\Omega_m}{f^2}[f_\nu - k^2 c_{\rm eff}^2(k,\eta)] & \frac{3}{2}\frac{\Omega_m}{f^2}-1 \end{pmatrix}$$

depends on scale and time, and $\gamma_{121} = \alpha, \gamma_{222} = \beta, \gamma_{abc} = 0$ else

Non-linear equations

Perturbative expansion in terms of initial (linear) density contrast δ_0

$$\psi_a(\mathbf{k},\eta) = \sum_{n=1}^{\infty} \int_{\mathbf{q}_1,\ldots,\mathbf{q}_n} \delta^{(3)}(\mathbf{k}-\mathbf{q}_{1\cdots n})\, F_a^{(n)}(\mathbf{q}_1,\ldots,\mathbf{q}_n;\eta)\, \delta_{ini}(\mathbf{q}_1)\cdots\delta_{ini}(\mathbf{q}_n)$$

Equation of motion for the non-linear kernels $F_a^{(n)}$

$$\partial_\eta F_a^{(n)}(\mathbf{q}_1,\ldots,\mathbf{q}_n;\eta) + \Omega_{ab}(k,\eta) F_b^{(n)}(\mathbf{q}_1,\ldots,\mathbf{q}_n;\eta)$$
$$= \sum_{m=1}^{n-1} \gamma_{abc}(\mathbf{k},\mathbf{q}_{1\cdots m},\mathbf{q}_{m+1\cdots n}) F_b^{(m)}(\mathbf{q}_1,\ldots,\mathbf{q}_m;\eta) F_c^{(n-m)}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n;\eta)$$

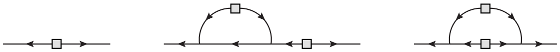
We solve this ODE numerically in a recursive way for all kernels required for the cb and ν power spectra

Non-linear equations

Power spectrum computed with numerically evolved kernels $F_a^{(n)}$

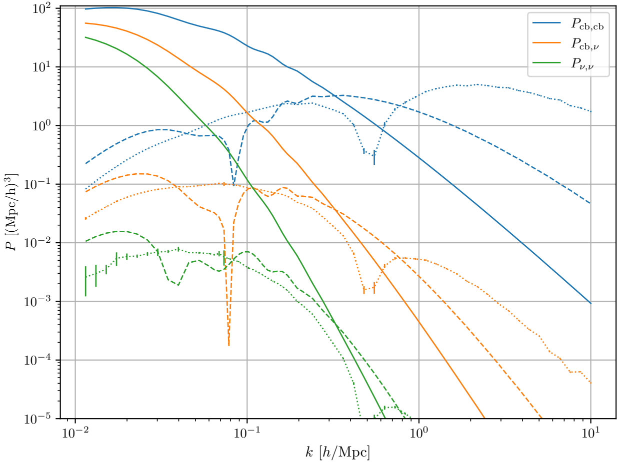
$$P_{ab}(k,z) = \overbrace{P_{11,ab}(k)}^{P_{lin}} + \overbrace{(2P_{13}+P_{22})_{ab}}^{P_{1-loop}\text{ (NLO)}} + \underbrace{(2P_{15}+2P_{24}+P_{33})_{ab}}_{P_{2-loop}\text{ (NNLO)}} + \dots$$

where $P_{nm,ab} = \langle \psi_a^{(n)} \psi_b^{(m)} \rangle$



e.g. $P_{22,\mathrm{cbcb}}[P_{ini}] = 2\int_q d^3q\, F_{\mathrm{cb}}^{(2)}(\mathbf{q},\mathbf{k}-\mathbf{q};z)^2\, P_{ini}(q) P_{ini}(|\mathbf{k}-\mathbf{q}|)$

Massive neutrino perturbations: non-linear theory



Solid=lin,Dashed=1-loop(NLO),Dotted=2-loop(NNLO) *MG, Taule 2008.00013*

Massive neutrino perturbations: non-linear theory

Comparison to Standard Pert. Theory (SPT):

- (i) SPT treats all matter as cold and pressureless

$$\delta_{\mathsf{SPT}} = f_{\mathsf{cdm}}\delta_{\mathsf{cdm}} + f_b\delta_b + f_\nu\delta_\nu$$

- (ii) Is based on a single fluid component $\psi_{\mathsf{SPT}} = (\delta_{\mathsf{SPT}}, -\theta_{\mathsf{SPT}}/\mathcal{H}f)$
- (iii) Uses the so-called EdS approximation for the non-linear kernels

$$\Omega_{\text{1-fluid}} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2} & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 \end{pmatrix} \mapsto \Omega_{\mathsf{EdS}} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

Massive neutrino perturbations: non-linear theory

- The EdS approx. implies that the time- and scale-dependence factorizes

$$
\begin{aligned}
F^{(n)}_{\delta_{\rm SPT}} &= D(z)^n \times F_n(\mathbf{k}_1, \ldots, \mathbf{k}_n) \\
F^{(n)}_{\theta_{\rm SPT}} &= D(z)^n \times G_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)
\end{aligned}
$$

  such that the eq. of motion can be solved by an algebraic recursion relation.

- Here we use instead a coupled system for two fluid components (2F)

$$
\delta_{\rm cb} = \frac{f_{\rm cdm}\delta_{\rm cdm} + f_b\delta_b}{f_{\rm cdm} + f_b}, \qquad \delta_\nu
$$

  and solve ODEs for the appropriate kernels

$$
F^{(n)}_a(\mathbf{k}_1, \ldots, \mathbf{k}_n, z), \qquad a = \delta_{\rm cb}, \theta_{\rm cb}, \delta_\nu, \theta_\nu
$$

  including the $\mathbf{k}_i-$ and z-dependent growth suppression due to ν free-streaming with a 4×4 matrix $\Omega_{ab}(\mathbf{k}, z)$ that propagates all non-linear modes

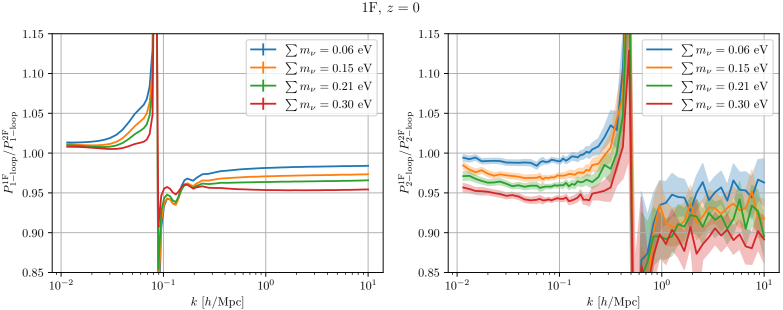
Massive neutrino perturbations: non-linear theory

Comparison to common approaches:

1F : "Naive" scheme using linear input power incl. neutrinos, but non-linear kernels from EdS-SPT

$$P_{mm} = P_{mm}^{\text{lin}} + P_{1+2-\text{loop}}^{\text{EdS-SPT}}[P_{mm}^{\text{lin}}]$$

Massive neutrino perturbations: non-linear theory



(1-loop)$_{1F}$/(1-loop)$_{2F}$ (2-loop)$_{1F}$/(2-loop)$_{2F}$

Ratio of loop corr. to $P_{mm}(k)$ compared to full 2F scheme at $z = 0$ (left=NLO(1-loop),right=NNLO(2-loop))

Sign change leads to spike (negative on the left)

MG, Taule 2008.00013

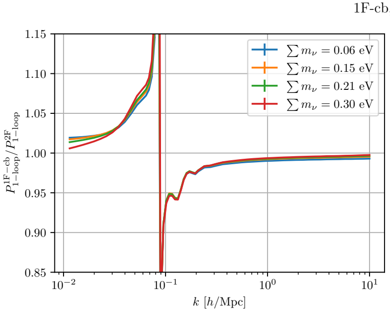
Massive neutrino perturbations: non-linear theory

Comparison to common approaches:

1F-cb : Non-linear kernels from EdS-SPT, but use only cb power as linear input for loops *Saito et al 0801.0607., Castorina et al 1505.07148*

$$P_{mm} = P_{mm}^{\text{lin}} + (1 - f_\nu)^2 P_{1+2-\text{loop}}^{\text{EdS-SPT}}[P_{\text{cb,cb}}^{\text{lin}}]$$

Massive neutrino perturbations: non-linear theory



$(1\text{-loop})_{1F-cb}/(1\text{-loop})_{2F}$ $(2\text{-loop})_{1F-cb}/(2\text{-loop})_{2F}$

Ratio of loop corr. to $P_{mm}(k)$ compared to full 2F scheme at $z = 0$ (left=NLO(1-loop),right=NNLO(2-loop))

Sign change leads to spike (negative on the left) *MG, Taule 2008.00013*

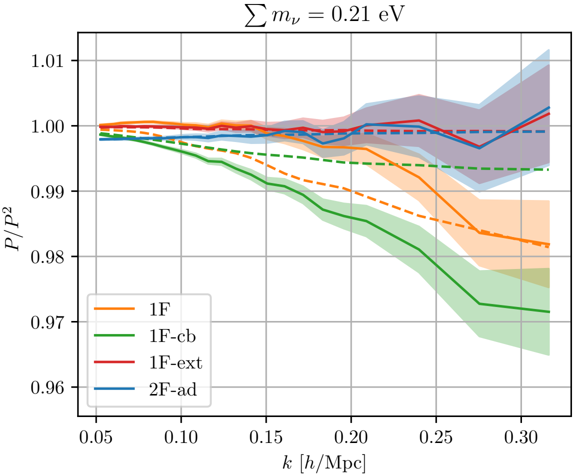
Massive neutrino perturbations: non-linear theory

1F-ext : Numerical kernels solved via ODE for an eff. single cb fluid system with *Lesgourgues, Matarrese, Pietroni, Riotto 0901.4550*

$$\Omega_{\text{1F-ext}}(k,\eta) = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_m}{f^2}\xi(k,\eta) & \frac{3}{2}\frac{\Omega_m}{f^2} - 1 \end{pmatrix}$$

where $\xi(k,\eta) = 1 - f_\nu + f_\nu \left(\frac{\delta_\nu(k,\eta)}{\delta_{\rm cb}(k,\eta)}\right)_{\rm lin.}$

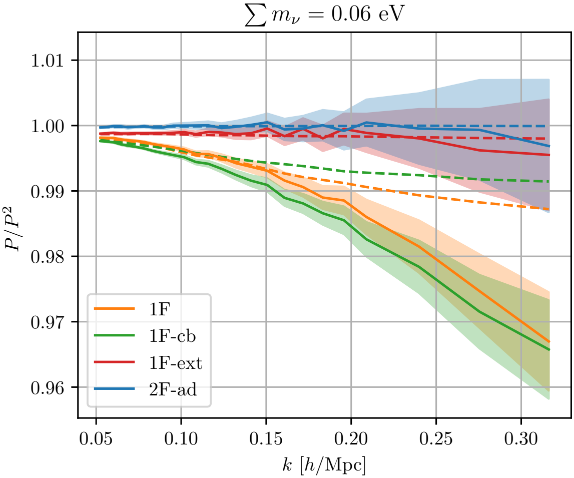
Massive neutrino perturbations: non-linear theory



Ratio of $P_{mm}(k)$ compared to full 2F scheme at $z = 0$
(dashed=NLO(1-loop),solid=NNLO(2-loop))

MG, Taule 2008.00013

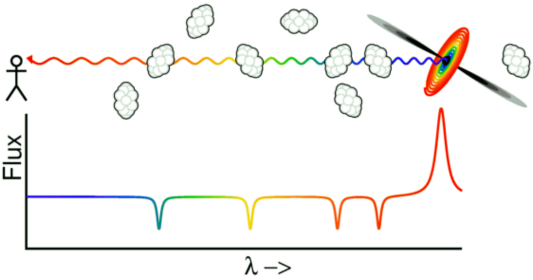
Massive neutrino perturbations: non-linear theory



Ratio of $P_{mm}(k)$ compared to full 2F scheme at $z = 0$
(dashed=NLO(1-loop),solid=NNLO(2-loop))

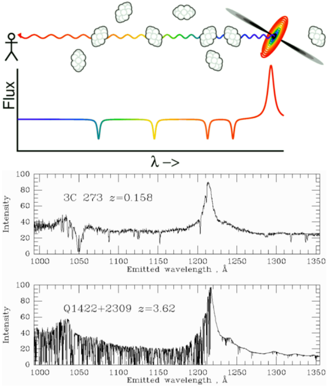
MG, Taule 2008.00013

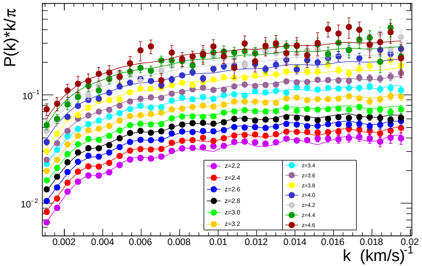
Lyman α forest



http://www.astro.ucla.edu/˜wright/Lyman-alpha-forest.html

Lyman α forest

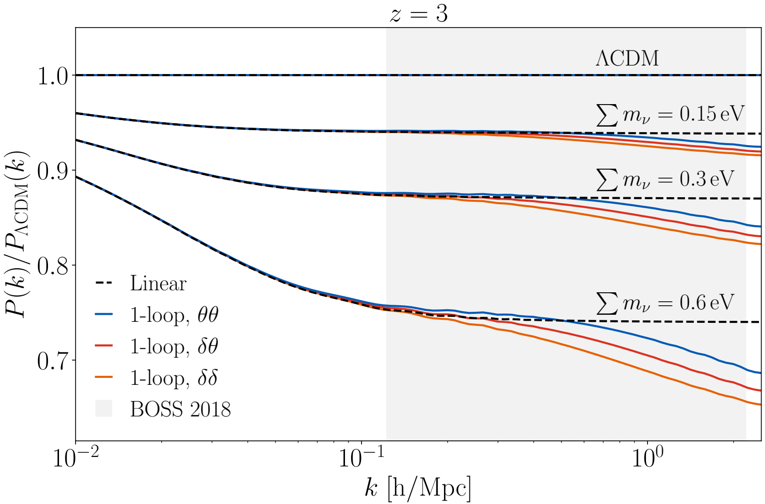




BOSS DR14 1812.03554

(total $\sim 180,413$ quasar spectra)

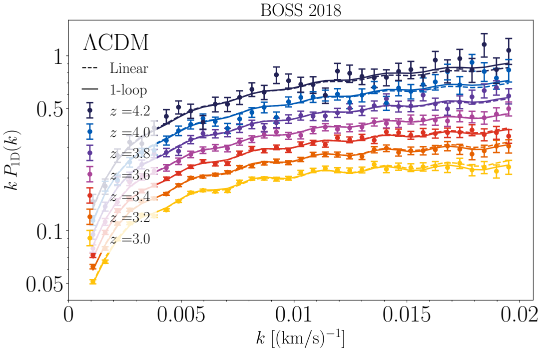
Lyman α forest and massive neutrinos



MG, Konstandin, Sagunski, Tulin 1805.12203, Pedersen , Font-Ribera , Kitching , McDonald , Bird 1911.09596,

MG, Konstandin, Sagunski, Viel 2011.03050, cf also Hannestad, Wong 2006.04995

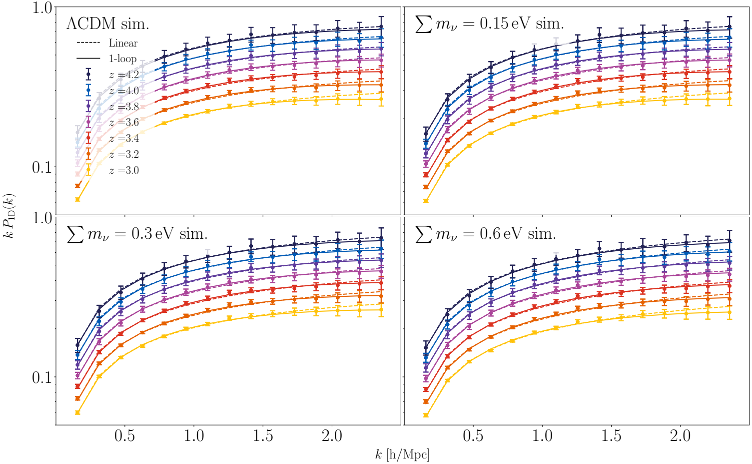
Application of effective model to 1D BOSS DR14 Lyα



- Perturbative input: 1/2-loop $P_{\delta\delta}, P_{\delta\theta}, P_{\theta\theta}$
- 6 EFT parameters (δ, θ bias(z), thermal broadening, baryon Jeans scale following Gnedin/Hui astro-ph/9706219, +UV counterterm) varied in fit to BOSS DR14 1Dlyα data 1812.03554 for $z = 3 - 4.2$ w/o prior
- ΛCDM χ^2/dof$\sim$ 231/210, obtained for reasonable best-fit values

MG, Konstandin, Sagunski, Tulin 1805.12203, MG, Konstandin, Sagunski, Viel 2011.03050

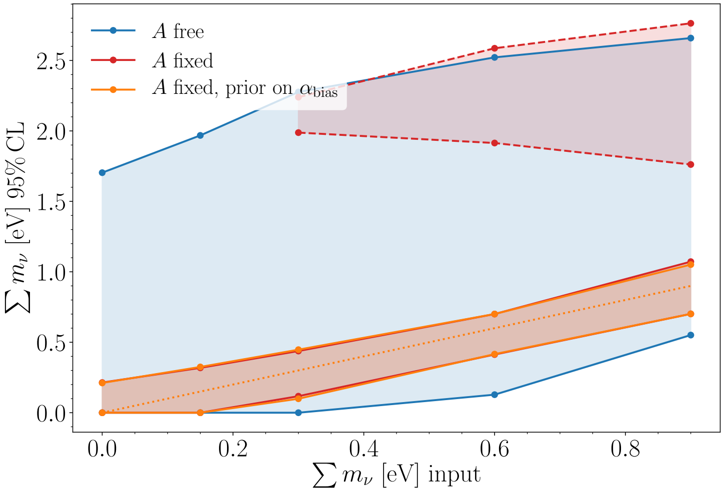
Validation with hydro sim



Validation with hydro sim

MG, Konstandin, Sagunski, Viel 2011.03050

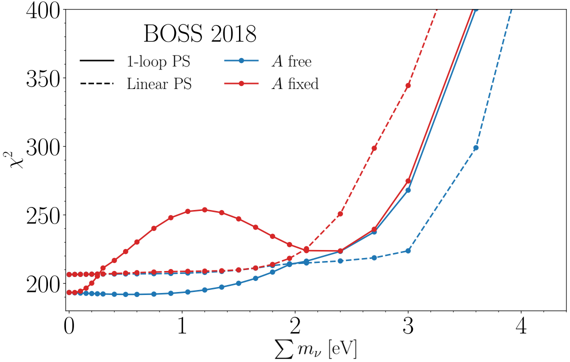
Validation with hydro sim (ν mass estimation)



Validation/calibration with hydro sim

MG, Konstandin, Sagunski, Viel 2011.03050

BOSS 2018 Lyα data: results

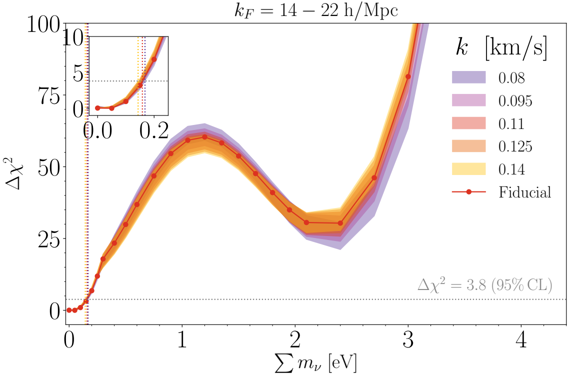


$$\sum m_\nu \leq \begin{cases} 0.16\,\mathrm{eV}\ 95\%\mathrm{C.L.} & A \text{ fixed (Planck18)} \\ 1.24\,\mathrm{eV}\ 95\%\mathrm{C.L.} & A \text{ free} \end{cases}$$

MG, Konstandin, Sagunski, Viel 2011.03050

(compare to result from grid of hydro-sim incl. lower z from 1911.09073: 0.10eV w P18, 0.71eV w/o Planck)

BOSS 2018 Lyα data: discussion

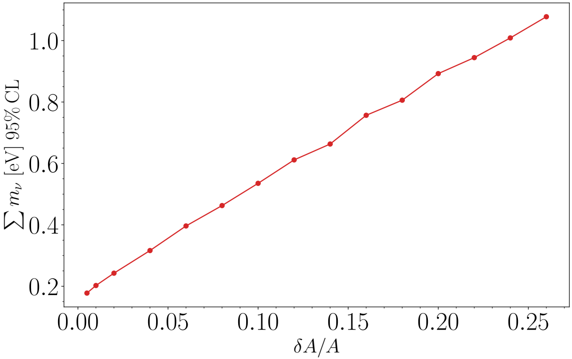


Jeans scale k_F and thermal broadening k_s (note that velocity bias and counterterms are marginalized over in all cases)

MG, Konstandin, Sagunski, Viel 2011.03050

BOSS 2018 Lyα data: discussion

Sensitivity to density bias + Planck prior related to overall amplitude A



$$\delta A/A \lesssim \begin{cases} 0.1 & 1.1 \cdot 10^4\mathrm{K} \leq T(z=3) \leq 2.3 \cdot 10^4\mathrm{K} \quad \text{(IGM temp.)} \\ 0.025 & 1.0 \leq \gamma \leq 1.6 \quad \text{(IGM adiabat. index)} \\ 0.002 & 5.4 \leq z_r \leq 7.4 \quad \text{(reionization)} \end{cases}$$

MG, Konstandin, Sagunski, Viel 2011.03050

Prospects

Estimate of sensitivity of a DESI-like survey ($\sim 700,000$ quasar spectra at higher resolution, Walther et al 2012.04008) with $\lesssim 1\%$ rel. error for the same k and z range as BOSS, and fixed A

- Mass determination for true value $M_\nu \equiv \sum m_\nu \geq 0.15$eV

$$\frac{\Delta M_\nu}{M_\nu} \simeq 17\% - 23\% \text{ @ } 95\%\text{C.L.}$$

- Upper bound assuming hypothetical "true" value $M_\nu = 0$

$$M_\nu \equiv \sum m_\nu \leq 0.056\text{eV @ } 95\%\text{C.L.}$$

MG, Konstandin, Sagunski, Viel 2011.03050

Note: further improvements by comb. with other datasets possible; numbers assume marginalization over velocity bias and counterterms, with fixed amplitude (Planck18)

Conclusion

- Precision comparison of non-linear corrections to the power spectrum in presence of massive neutrinos

  taking the impact of linear neutrino shear and pressure on the propagation of non-linear modes into account in a hybrid Boltzmann/two-fluid model with numerically evolved non-linear kernels

  $\Rightarrow$ We find 1-2% difference compared to conventional, simplified approaches at $z = 0$

  $\Rightarrow$ First time this difference has been assessed at NNLO; above nominal projected accuracy of future galaxy surveys

Conclusion

- EFT approach to BOSS (low-res) Lyman-α forest observations

  taking advantage of scale separation between BOSS observations and non-linear as well as IGM scales at relevant redshifts

  $\Rightarrow$ more efficient evaluation of likelihood

  does *not* require to run hydro sim for each point in parameter space

  $\Rightarrow$ marginalization over IGM parameters (velocity bias,...)

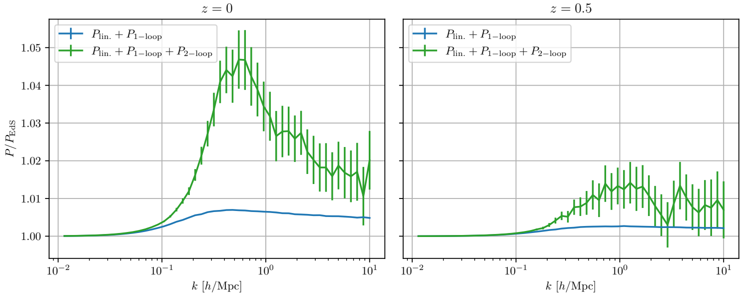
  $\Rightarrow$ can be easily adapted to non-standard DM models

  e.g. strongly self-interacting "cannibal" DM, *MG, Konstandin, Sagunski, Tulin 2018*

thank you

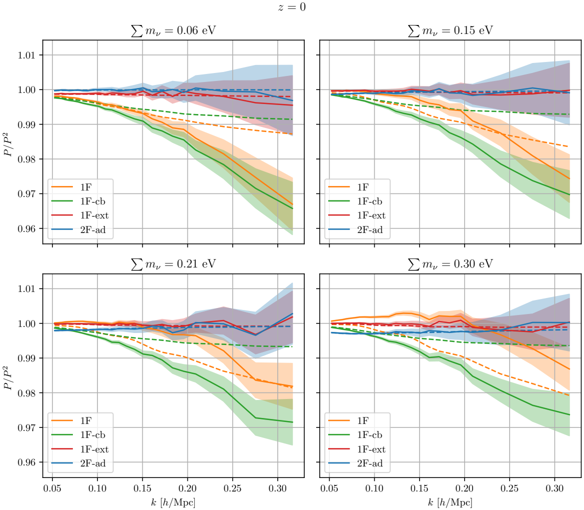
Backup

EdS vs exact time dep. for ΛCDM with $m_\nu = 0$



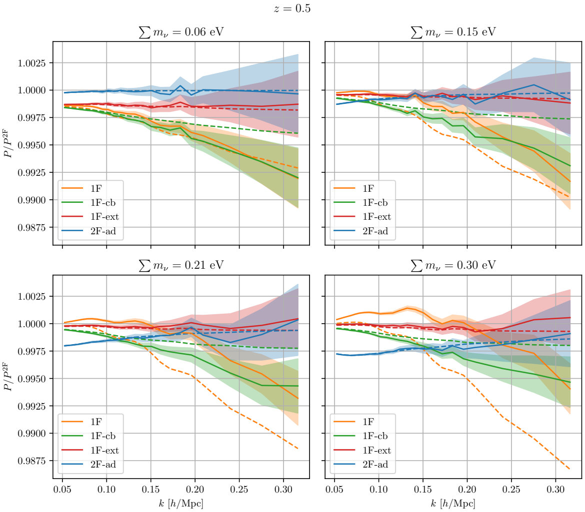
MG, Taule 2008.00013

2F vs approx. schemes



MG, Taule 2008.00013

2F vs approx. schemes ($z = 0.5$)



MG, Taule 2008.00013

Large-scale Lyα forest power spectrum

- Scale of interest (BOSS Lyα data, $z = 2.2 - 4.4$)

$$k = 0.001 - 0.02(\mathrm{km/s})^{-1} \sim 0.1h/\mathrm{Mpc} - 2h/\mathrm{Mpc}$$

- Non-linear scale for mass density

$$k_{nl} \sim 0.03 - 0.05(\mathrm{km/s})^{-1} \text{ at } z \sim 2 - 4$$

- Thermal broadening along line of sight

$$k_s \simeq \sqrt{m_p/T} \sim 0.1(\ \mathrm{km/s})^{-1}$$

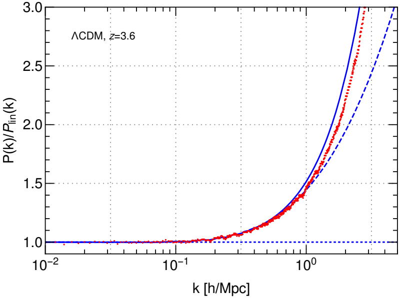
- Jeans scale $k_J = \mathcal{H}/c_s$, $c_s^2 = T\gamma/(\mu_p m_p)$, filtering scale $\propto \exp(-(k/k_F)^2)$

e.g. Gnedin, Hui astro-ph/9706219

$$k_F \sim 0.2(\mathrm{km/s})^{-1} \gg k_{nl}$$

3D matter power spectrum at $z = 3$

Non-linear enhancement of mass density can be computed with percent-level accuracy for $k \lesssim 0.02(\mathrm{km/s})^{-1}$, $z > 3$



dotted = LO, dashed = NLO, solid = NNLO, red=N-body

MG, Konstandin, Sagunski, Tulin 1805.12203; MG, Konstandin, Sagunski, Viel 2011.03050

Effective model for 1D Lyα flux power spectrum

- Redshift-dependent density and velocity bias $[A, \beta]$
- Gaussian Jeans smoothing and thermal broadening [k_s, k_F, not essential on BOSS scales]
  Gnedin, Hui astro-ph/9706219
- Non-linear density, velocity and cross power spectra up to 2-loop
- UV counterterm taking the dominant sensitivity to non-linear scales from line-of-sight integration into account $[\bar{I}_0]$

$$
\begin{aligned}
P_{1\mathrm{D}}(k_\parallel; A, \beta, \bar{I}_0, k_s, k_F) &= \frac{1}{2\pi}\int_{k_\parallel} P_F(k, \mu = k_\parallel/k)\, k\, dk \\
&= A(z)\exp(-(k_\parallel/k_s(z))^2)\,(I_0 + 2\beta(z) I_2 + \beta(z)^2 I_4) \\
I_0(k_\parallel, z) &= \int_{k_\parallel} dk\, k\, \exp(-(k/k_F)^2)\, P_{\delta\delta}(k, z) + \bar{I}_0(z)\,, \\
I_2(k_\parallel, z) &= \int_{k_\parallel} \frac{dk\, k_\parallel^2}{k} \exp(-(k/k_F)^2)\, P_{\delta\theta}(k, z)\,, \\
I_4(k_\parallel, z) &= \int_{k_\parallel} \frac{dk\, k_\parallel^4}{k^3} \exp(-(k/k_F)^2)\, P_{\theta\theta}(k, z)\,,
\end{aligned}
$$

MG, Konstandin, Sagunski, Tulin 1805.12203; MG, Konstandin, Sagunski, Viel 2011.03050

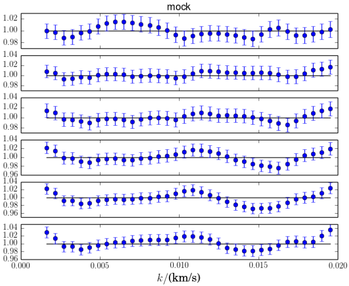
Effective model for 1D Lyα flux power spectrum

- 11 free nuisance/EFT parameters (z-dep density and velocity bias of intergalactic medium, extra ‘counterterm’ due to integration across line-of-sight)
- 5 are almost degenerate or marginally relevant
- remaining 6 varied in fit to Lyα forest data for $z = 3 - 4.2$ w/o prior

MG, Konstandin, Sagunski, Tulin 1805.12203; MG, Konstandin, Sagunski, Viel 2011.03050

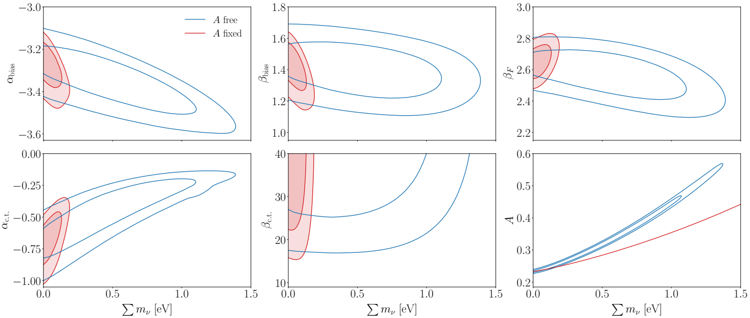
Validation with hydro sim

- Check with high-resolution hydro simulation (average over 5k line-of-sight mock spectra, very good agreement within stat. uncertainty $< 1.5\%$) *Bolton et al 1605.03462*



MG, Konstandin, Sagunski, Tulin 1805.12203

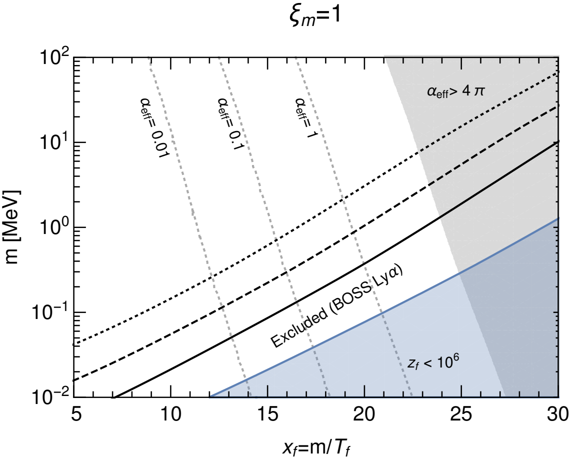
EFT parameters



MG, Konstandin, Sagunski, Viel 2011.03050

Strongly self-interacting Dark Matter

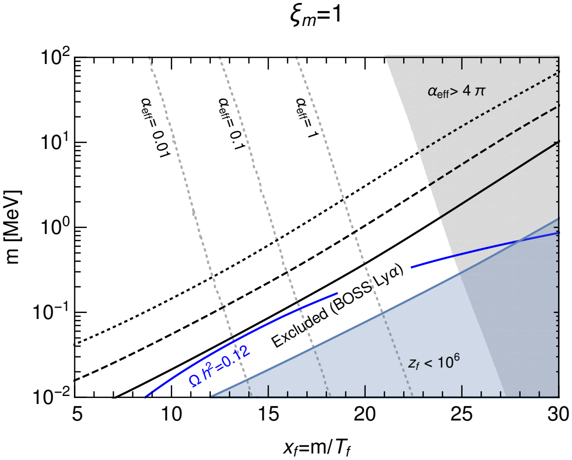
$\sigma_{3\to2} \sim \alpha_{\rm eff}^3/m^5$



MG, Konstandin, Sagunski, Tulin 1805.12203

Strongly self-interacting Dark Matter

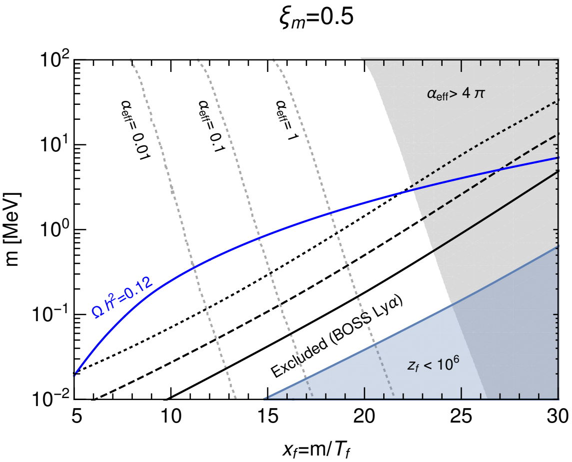
$\sigma_{3\to2} \sim \alpha_{\text{eff}}^3/m^5$



MG, Konstandin, Sagunski, Tulin 1805.12203

Strongly self-interacting Dark Matter

$\sigma_{3\to2} \sim \alpha_{\text{eff}}^3/m^5$



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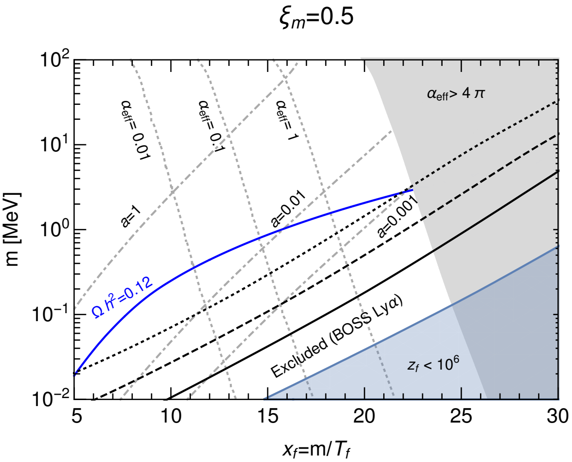
Strongly self-interacting Dark Matter

$$\sigma_{3\to 2} \sim \alpha_{\text{eff}}^3/m^5 \qquad \sigma_{2\to 2} \sim a^2\alpha_{\text{eff}}^2/m^2 = 1\text{cm}^2/\text{g}$$

Y. Hochberg, E. Kuflik, T. Volansky, J. Wacker Phys.Rev.Lett. 113 (2014) 171301, 1402.5143



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