From small to large x:

(toward a unified formalism for particle production in high energy collisions)

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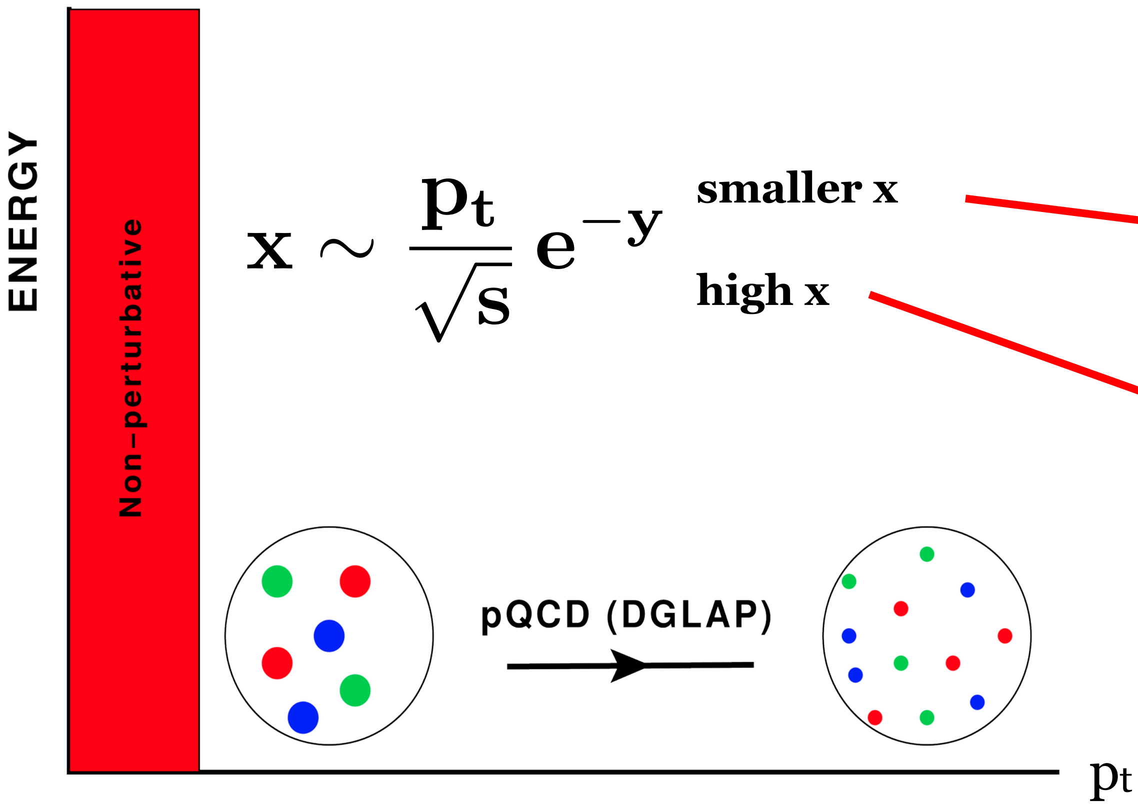
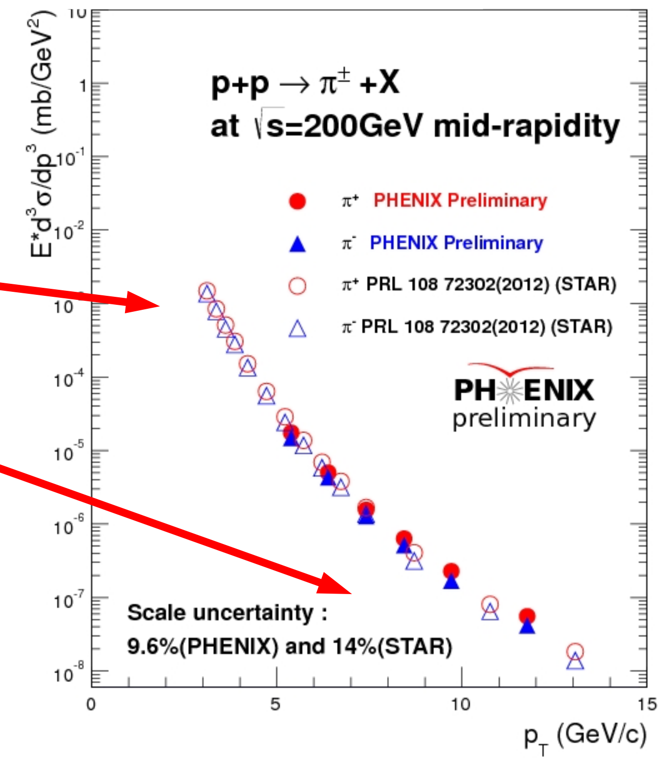
based on:
PRD102 (2020) 1, 014008
PRD99 (2019) 1, 014043
and work in progress

pQCD: the standard paradigm

$$E\frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2) + \cdots\cdots$$

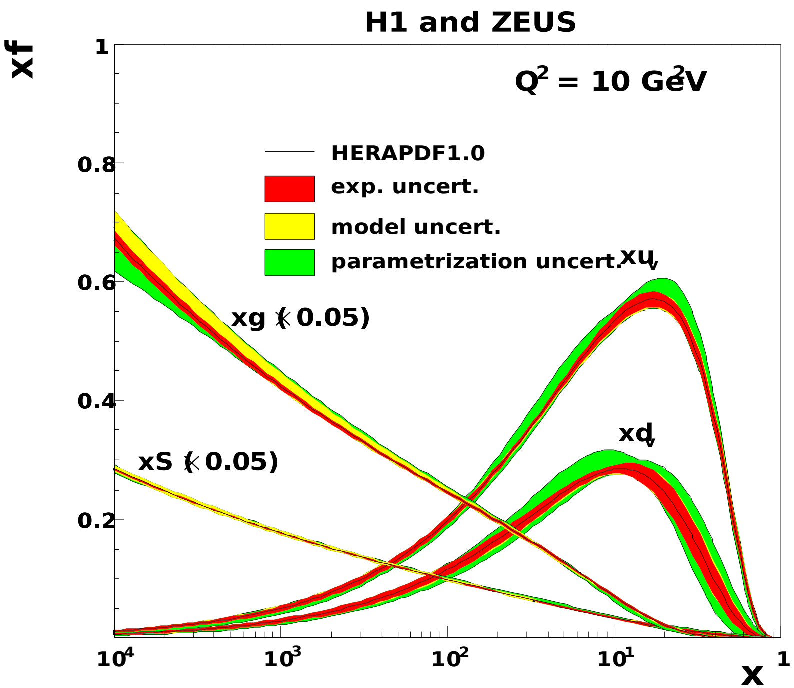
$$x \sim \frac{p_t}{\sqrt{s}} e^{-y}$$

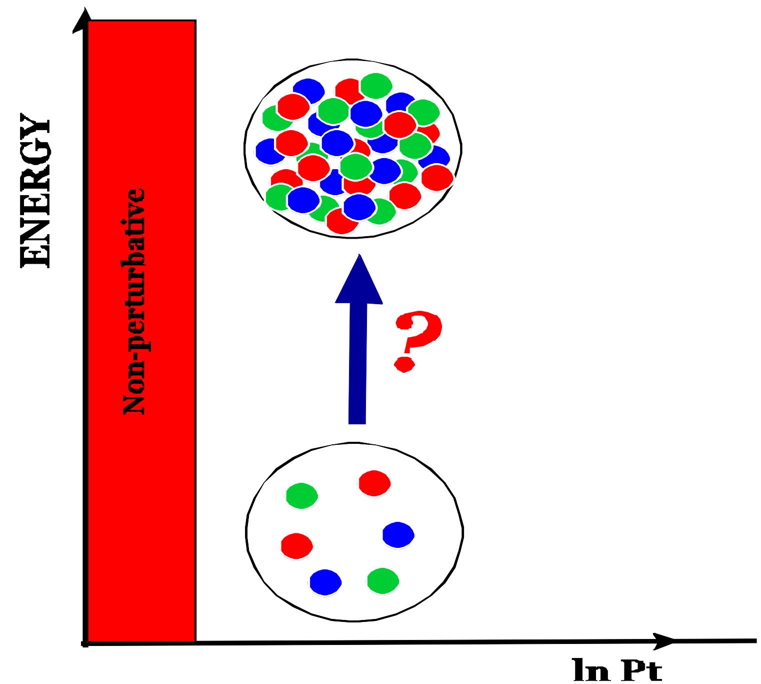




bulk of QCD phenomena happens at low pt (small x)

dynamics of *universal gluonic matter:*
gluon saturation





$$P_{gg} \sim P_{gq} \sim \frac{1}{x}$$

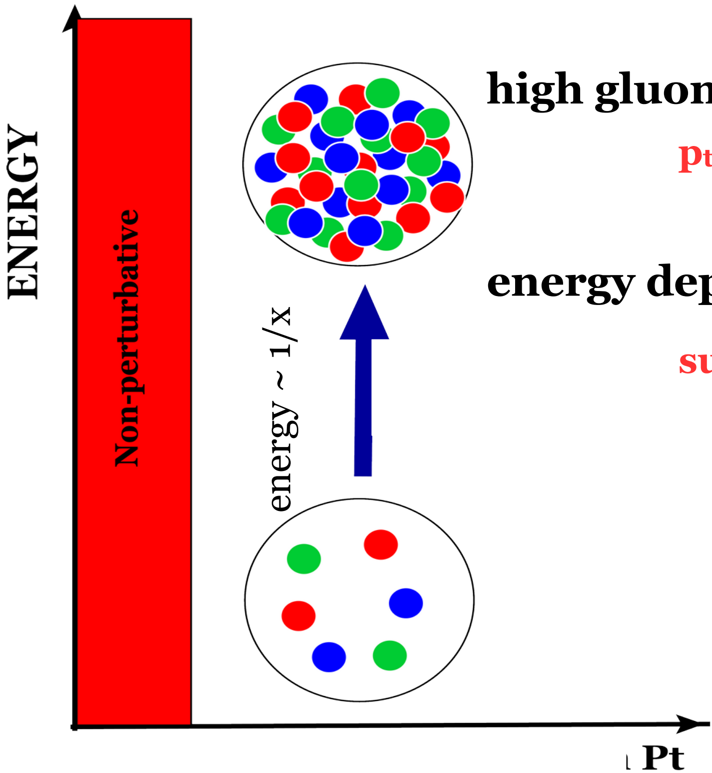
How does this happen ?

How do correlation functions evolve ?

Is there a universal fixed point for the evolution ?

Are there scaling laws ?

QCD at high energy/small x: gluon saturation



high gluon density: <u>Eikonal multiple scattering</u>

p_t broadening (generic to multiple scattering)

energy dependence: <u>x-evolution via JIMWLK/BK</u>

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

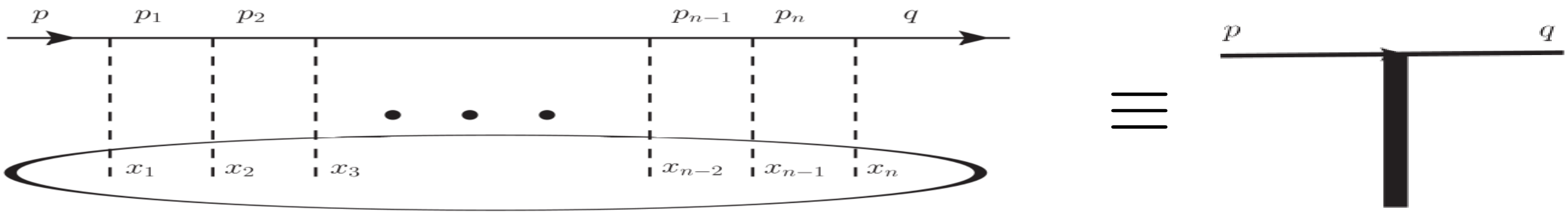
a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
<u>Azimuthal angular correlations (photon-hadron,...)</u>
Long range rapidity correlations (ridge,...)
Initial conditions for hydro
Thermalization (?)

$$\mathbf{x \leq 0.01}$$

$$\alpha_s \ln(x_v/x) \sim 1$$

CGC: eikonal approximation (tree level)



$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\bar{u}(q)\not{n} \int d^2x_t \, e^{-i(q_t - p_t)\cdot x_t} \, [V(x_t) - 1] \, u(p)$$

with $V(x_t) \equiv \hat{P} \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+ S_a^-(x^+, x_t) t_a \right\}$

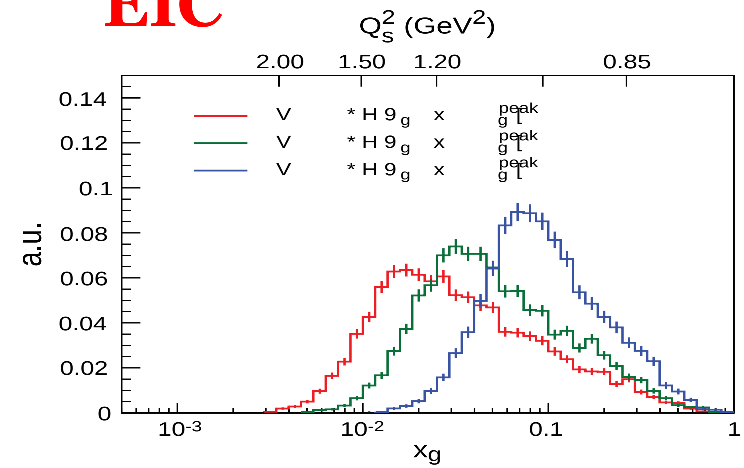
scattering from small x gluons of the target can cause only a *<u>small angle deflection</u>*

Dipole: DIS, proton-nucleus collisions

x dependence from JIMWLK/BK evolution equation

$$< Tr\, V(x_\perp)\, V^\dagger(y_\perp) >$$

EIC



Aschenauer et al. ArXiv:1708.01527

<u>toward precision at small x</u>:

NLO corrections:

Chirilli+Xiao+Yuan, PRL (2012)

Balitsky+Chirilli, PRD88 (2013)

....

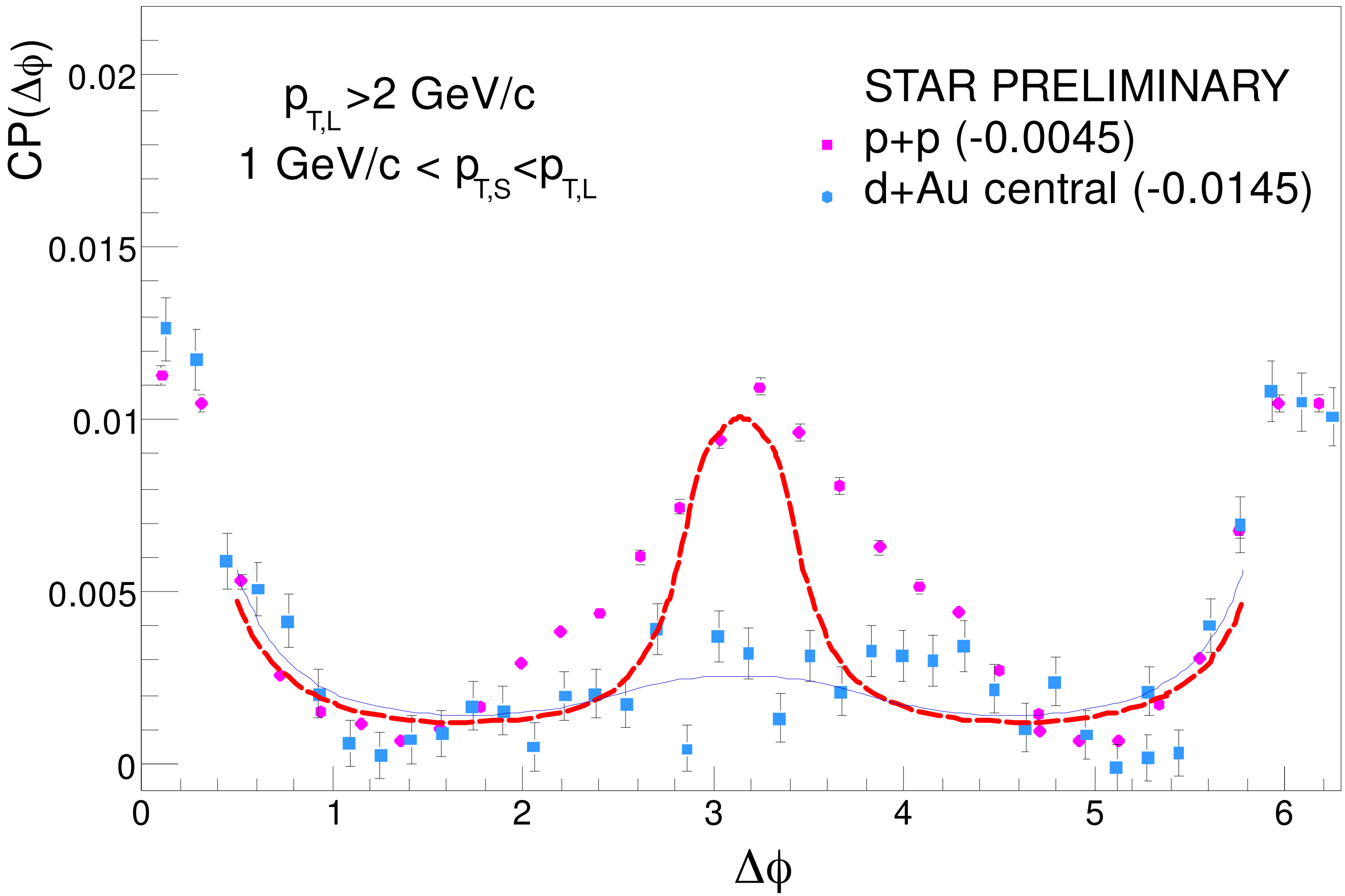
sub-eikonal corrections:

Kovchegov+Pitonyak+Sievert, JHEP (2017)

Agostini+Altinoluk+Armesto, EPJC (2019)

....

Forward-forward di-hadron correlations at RHIC

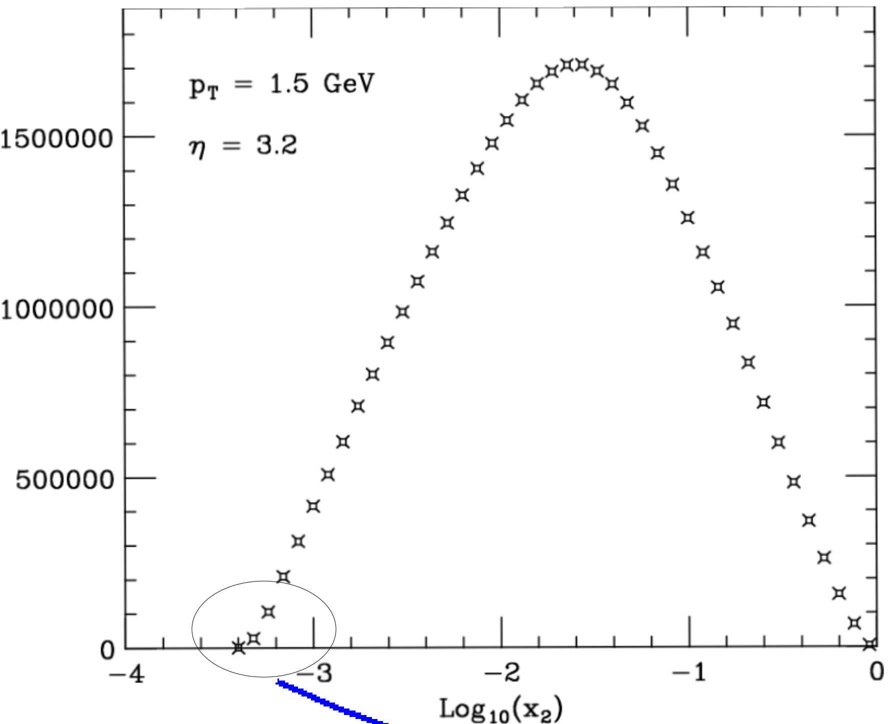


Albacete+Marquet, PRL105 (2010) 162301

Single inclusive pion production in pp at RHIC

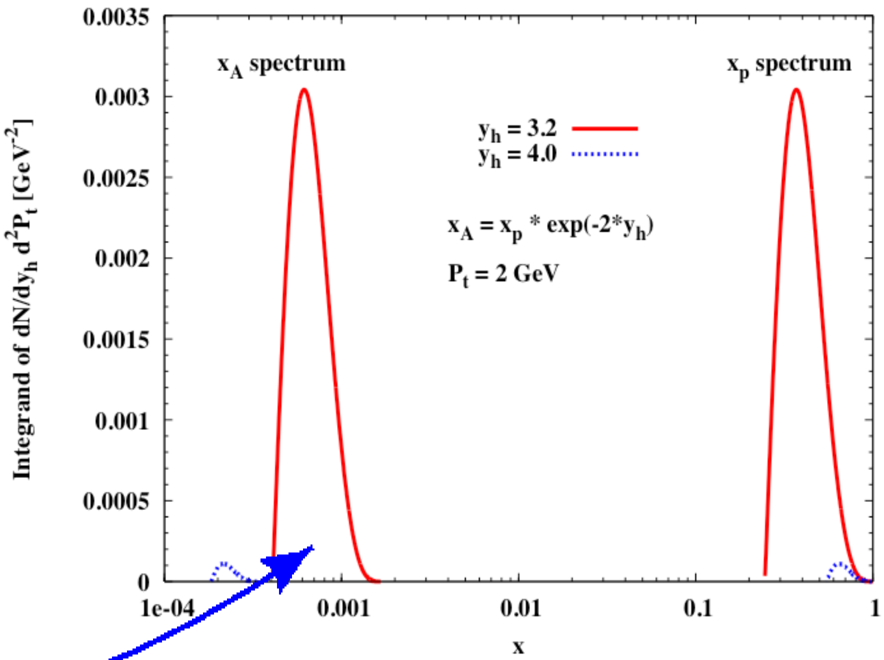
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

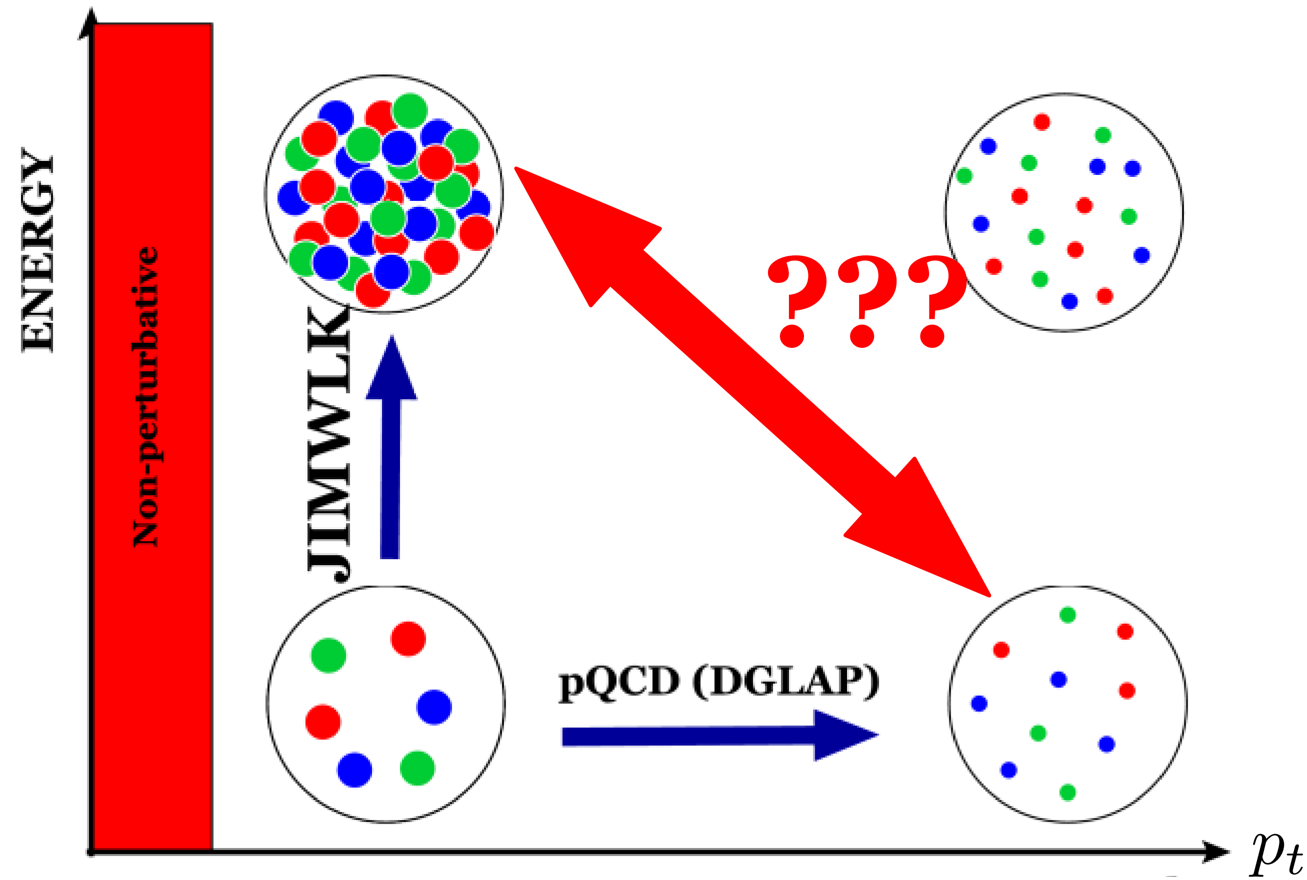
DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x,Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min},Q^2)\cdots$$

which kinematics are we in?

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

<u>kinematics of saturation: *where is saturation applicable?*</u>

jet physics, high p_t and forward-backward correlations,

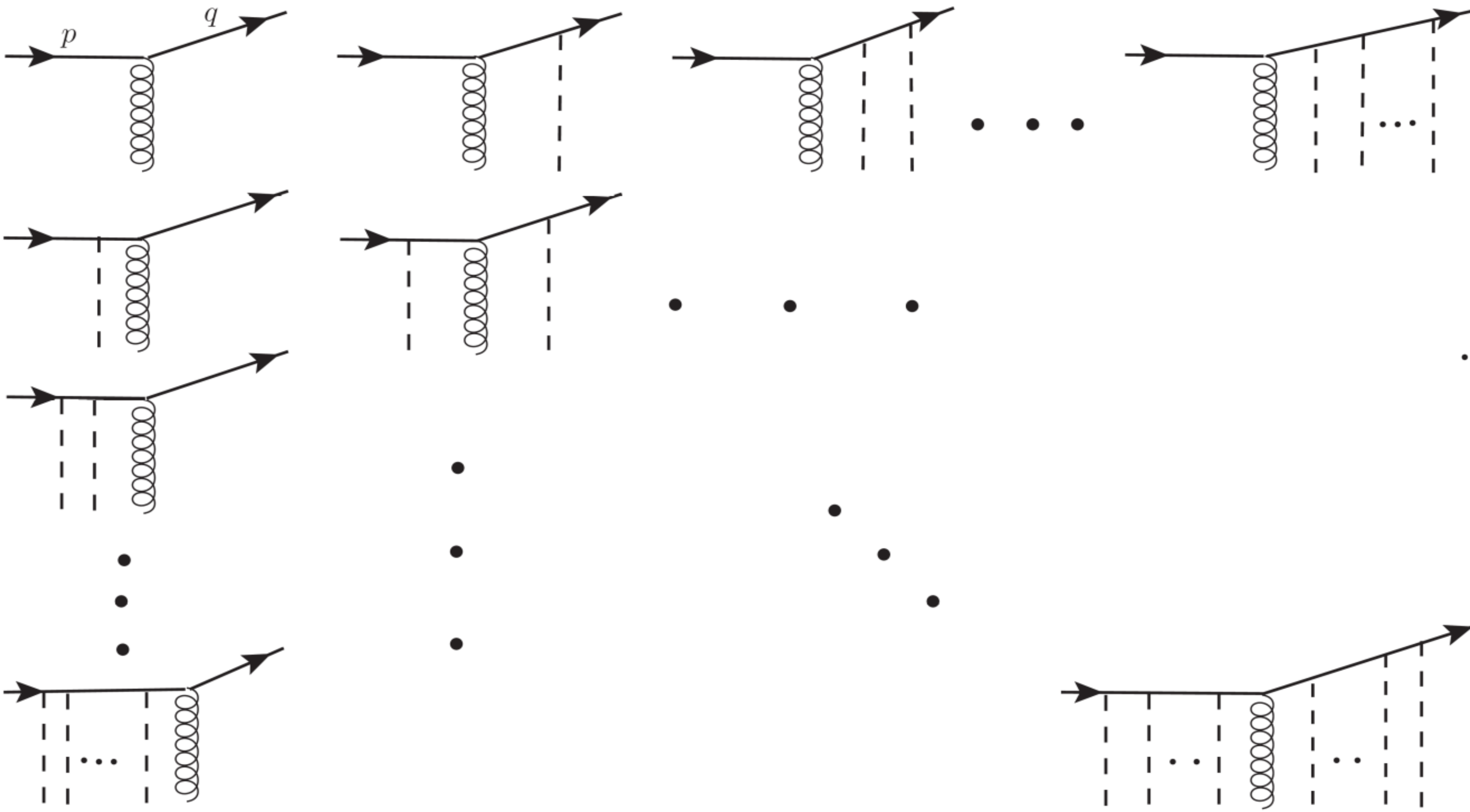
spin physics, early time e-loss in heavy ion collisions, ...

Beyond eikonal approximation: longitudinal momentum exchange

$\mathcal{A}^\mu = \mathbf{S}^\mu + \mathbf{A}^\mu$

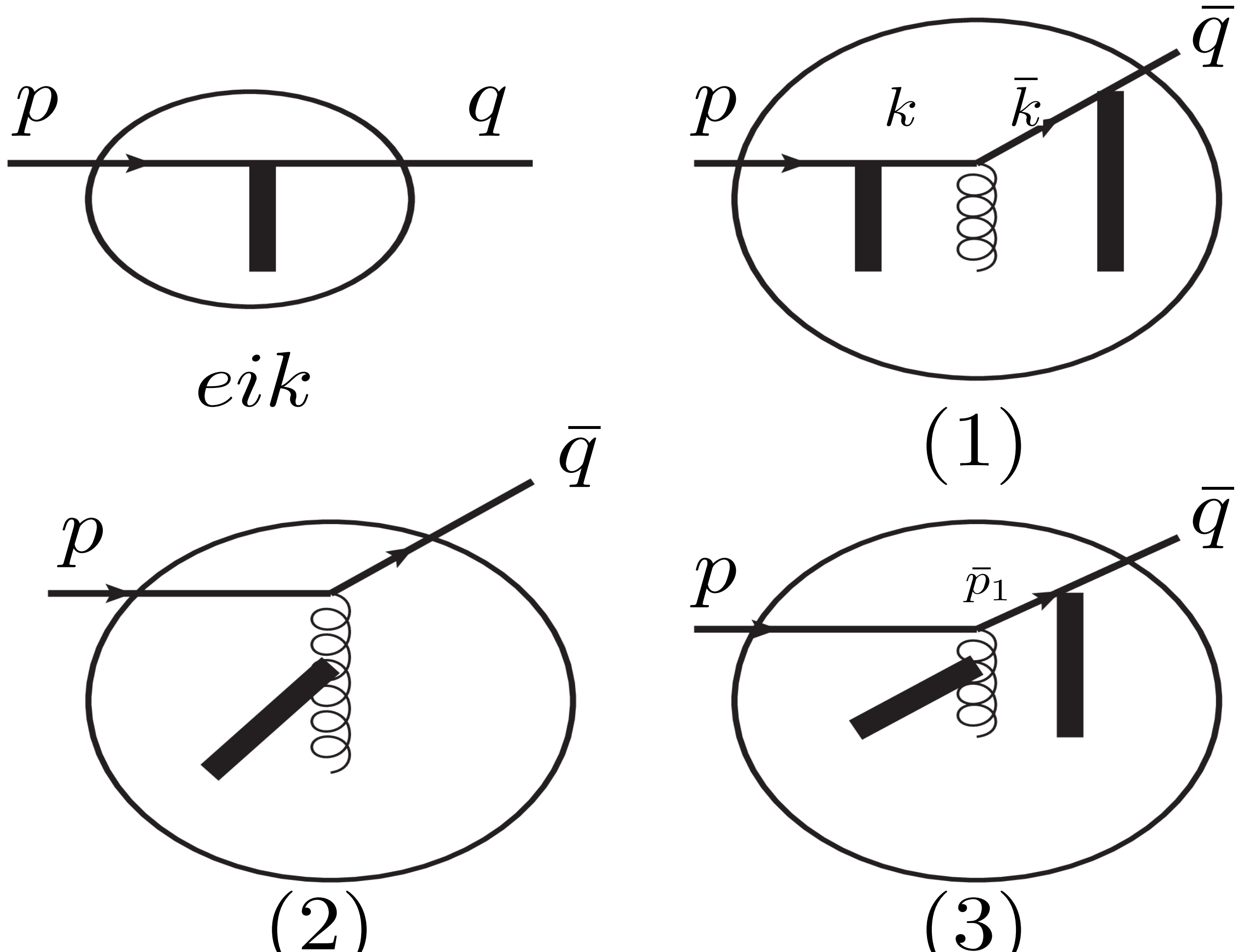
single scattering from large x gluons of target $\mathbf{A}^\mu = (\mathcal{A}^\mu - \mathbf{S}^\mu)$

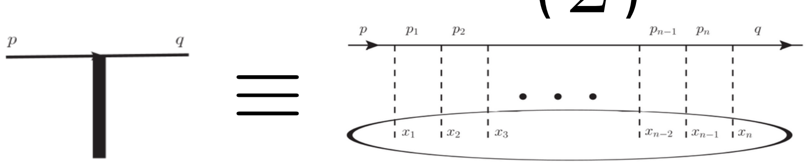
multiple scatterings from small x gluons of target $\mathbf{S}^\mu$



Quark scattering: beyond small x approximation

large x partons of target can cause a <u>large-angle deflection</u> of the projectile





soft (eikonal) limit: $i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$

use spinor helicity formalism: helicity amplitudes

Including large x gluons of the target leads to:

longitudinal double spin asymmetries (A_{LL})

baryon transport (beam rapidity loss), ……

one-loop corrections: factorized cross section at all x (p_t)

gluon radiation

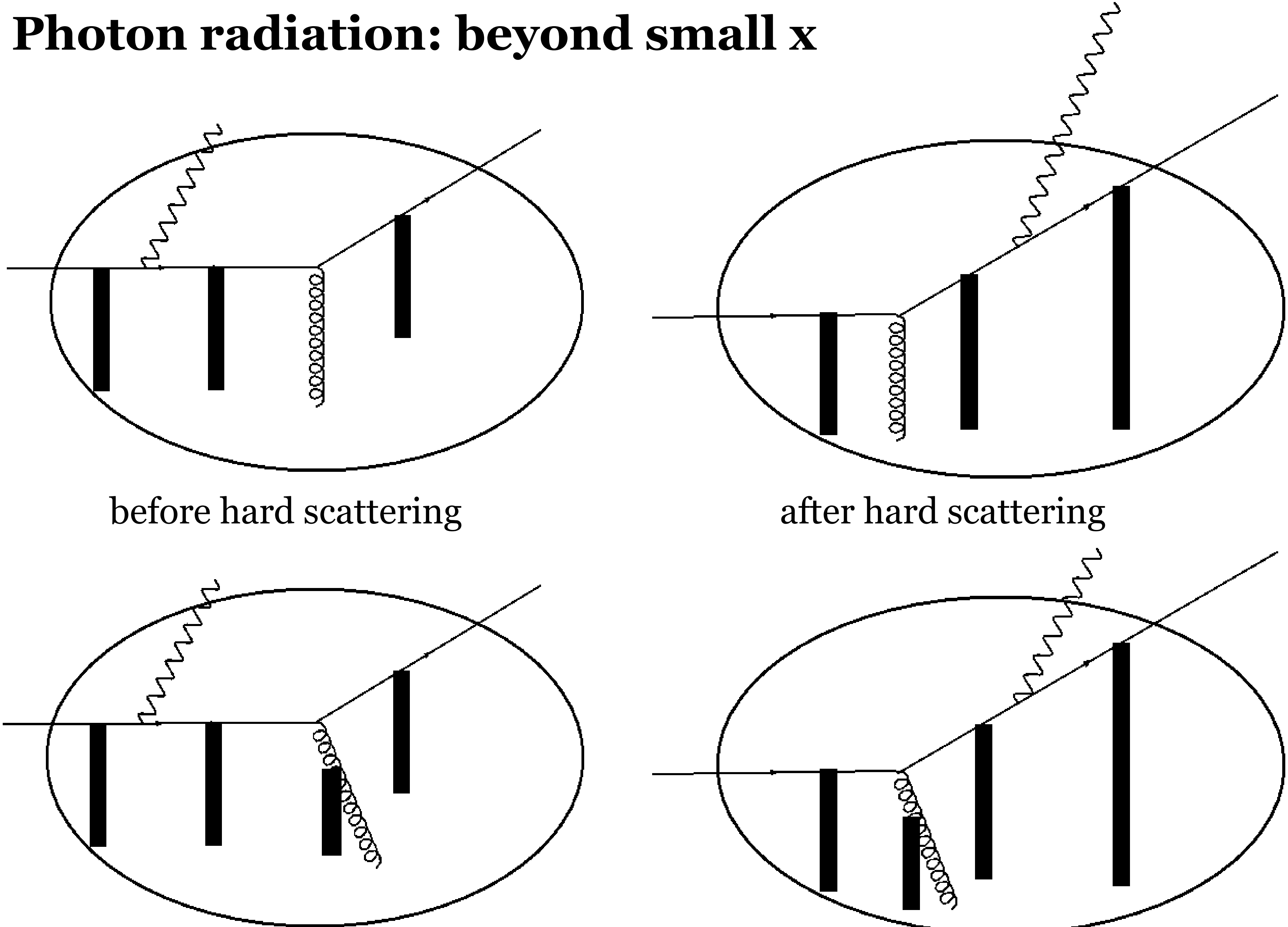
related problem: photon radiation

photon-hadron correlations:

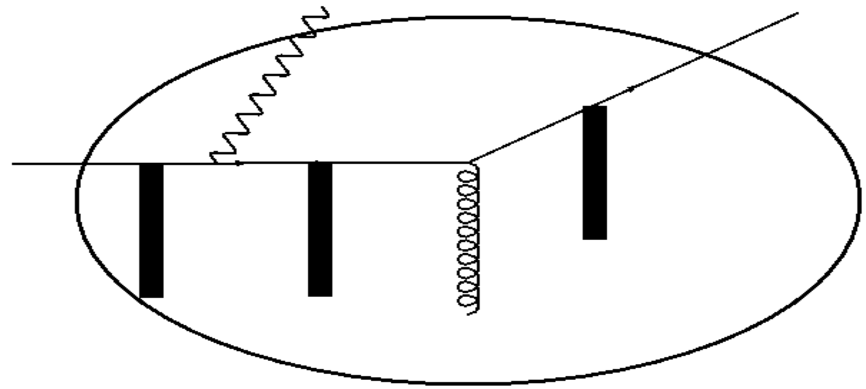
azimuthal angular correlations from low to high p_t

forward-backward rapidity correlations

Photon radiation: beyond small x

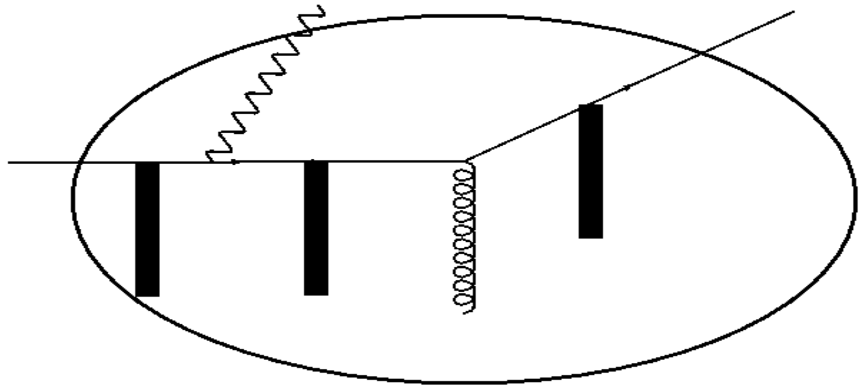


photon radiation: helicity amplitudes



$$
\begin{aligned}
& i\mathcal{M}_1(p,q,l) = \\
& eg \int \frac{d^2k_{2t}}{(2\pi)^2}\,\frac{d^2k_{3t}}{(2\pi)^2}\,\frac{d^2\bar{k}_{1t}}{(2\pi)^2} \int d^4x\, d^2y_{1t}\, d^2y_{2t}\, d^2\bar{y}_{1t}\, dz^+\, \theta(x^+ - z^+)\, e^{i(l^+ + \bar{q}^+ - p^+)x^-} \\
& e^{-i(\bar{q}_t - \bar{k}_{1t})\cdot \bar{y}_{1t}}\, e^{-i(\bar{k}_{1t} - k_{3t})\cdot x_t}\, e^{-i(k_{3t} - k_{2t})\cdot y_{2t}}\, e^{-i(l_t + k_{2t} - p_t)\cdot y_{1t}}\, \bar{u}(\bar{q})\, \overline{V}(\bar{y}_{1t}; x^+, \infty)\, \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}} \\
& \not{A}(x)\, \left[\frac{\not{k}_3}{2n\cdot(p-l)}\, V(y_{2t}; z^+, x^+)\, \frac{\not{n}\,\not{k}_2}{2n\cdot(p-l)} + i\,\frac{\delta(x^+ - z^+)}{2n\cdot(p-l)}\not{n} \right] \\
& \not{\epsilon}(l)\, \frac{\not{k}_1}{2n\cdot p}\, V(y_{1t}; -\infty, z^+)\, \not{n}\, u(p)
\end{aligned}
$$

photon radiation: helicity amplitudes



$$\mathcal{N}_{1-1} = \bar{u}(\bar{q})\,\frac{\bar{\not{n}}\,\bar{\not{k}}_1}{2\bar{n}\cdot\bar{q}}\,\not{A}(x)\,\frac{\not{k}_3\,\not{n}\,\not{k}_2\not{\epsilon}(l)\,\not{k}_1\,\not{n}}{2n\cdot p\,2n\cdot(p-l)\,2n\cdot(p-l)}\,u(p)$$

$$\mathcal{N}_{1-2} = \bar{u}(\bar{q})\,\frac{\bar{\not{n}}\,\bar{\not{k}}_1}{2\bar{n}\cdot\bar{q}}\,\not{A}(x)\,\frac{\not{n}\,\not{\epsilon}(l)\,\not{k}_1\,\not{n}}{2n\cdot p\,2n\cdot(p-l)}\,u(p)$$

$$\mathcal{N}_{1-1}^{++} = \left(\mathcal{N}_{1-1}^{--}\right)^{\star} = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\frac{[n\cdot l\,k_{2\perp}\cdot\epsilon_{\perp}^{\star} - n\cdot(p-l)\,l_{\perp}\cdot\epsilon_{\perp}^{\star}]}{n\cdot l\,n\cdot(p-l)}\,\langle\bar{k}_1^{+}|\not{A}(x)|k_3^{+}\rangle$$

$$\mathcal{N}_{1-2}^{++} = \left(\mathcal{N}_{1-2}^{--}\right)^{\star} = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\langle\bar{k}_1^{+}|\not{A}(x)|n^{+}\rangle$$

$$\mathcal{N}_{1-1}^{+-} = \left(\mathcal{N}_{1-1}^{-+}\right)^{\star} = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\frac{[n\cdot p\,l_{\perp}\cdot\epsilon_{\perp} - n\cdot l\,k_{1\perp}\cdot\epsilon_{\perp}]}{n\cdot p\,n\cdot l}\,\langle\bar{k}_1^{+}|\not{A}(x)|k_3^{+}\rangle$$

$$\mathcal{N}_{1-2}^{+-} = \mathcal{N}_{1-2}^{-+} = 0$$

So far

Classical CGC is generalized by including large angle scattering from the target

beam rapidity loss

Helicity amplitudes for quark and photon production are evaluated

spin asymmetries

Relevant operators are identified

products of Wilson lines and large x gluon field
computing expectation values?

Need to classify/regulate the divergences

Toward a factorized cross section at all x

gluon radiation

Combining with small x

sharp boundary ($x = 0.01$)?
matching field strengths?

SUMMARY

CGC is a systematic approach to high energy collisions

strong hints from RHIC, LHC,...

toward precision: NLO, sub-eikonal corrections, ...

CGC breaks down at large x (high p_t)

a significant part of EIC/RHIC/LHC phase space is at large x

transition from large x physics to CGC (kinematics?)

Toward inclusion of large x physics:

spin asymmetries

beam rapidity loss

particle production in both small and large p_t kinematics

two-particle correlations: from forward-forward to forward-backward

one-loop correction: both collinear and CGC factorization limits

need to clarify/understand: gauge invariance, initial conditions, ...