Is BFKL factorization valid for Mueller-Tang jets?

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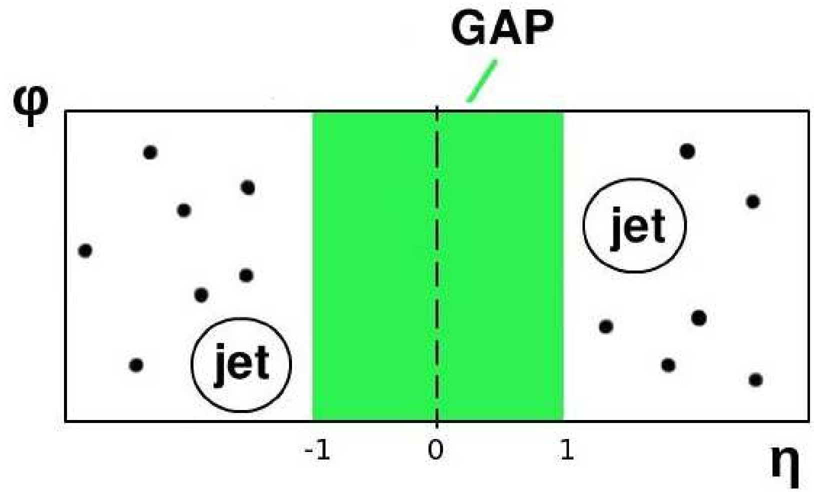
Mueller-Tang jets

An important process for studying PT high-energy QCD and the Pomeron at hadron colliders *[Mueller, Tang '87]*



Final state: • two jets with similar p_T
• large rapidity distance $Y \simeq \log(s/p_T^2)$;
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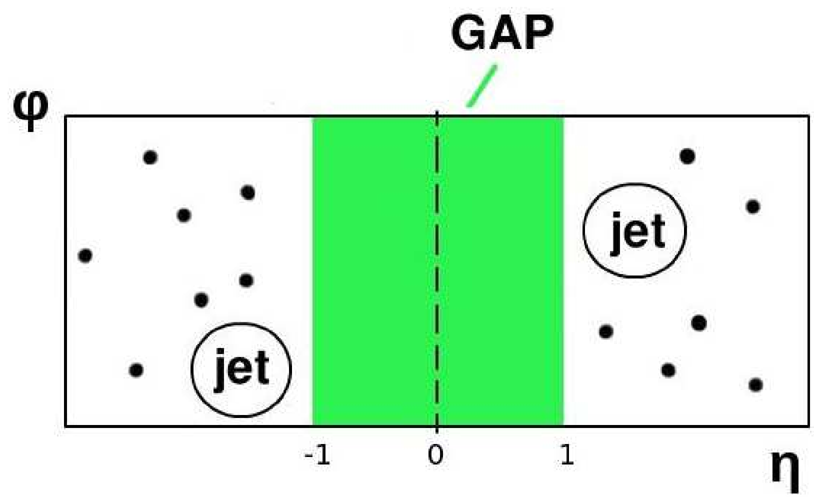

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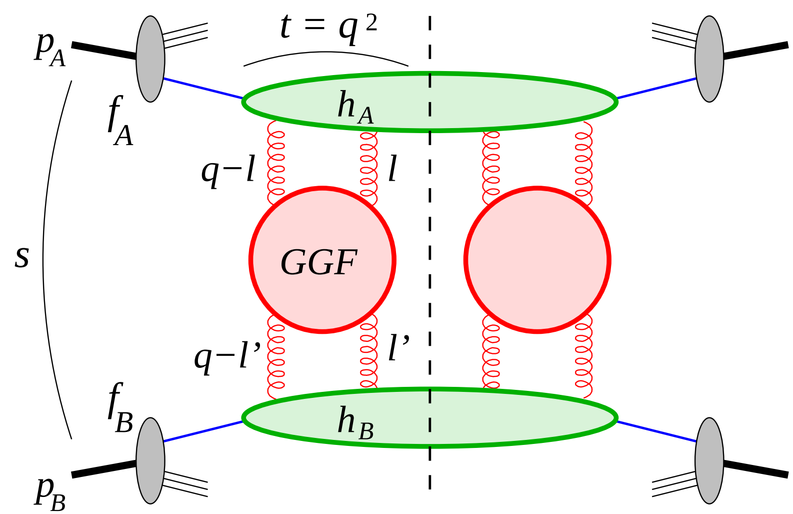
• large rapidity distance $Y \simeq \log(s/p_T^2)$;
• absence of any additional emission in central rapidity region (gap)

- Gap ⇒ mostly colour-singlet exchanges contribute to cross section
- $Y \gg 1 \Rightarrow$ enhanced PT series $(\alpha_s Y)^n$ resummed into singlet BFKL GGF
- In LLA factorization formula holds

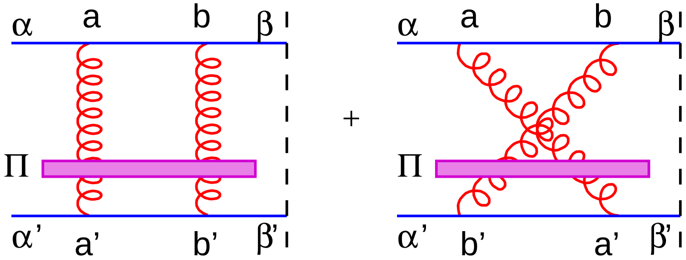


Outline

- Introduction:
  - Review of Mueller-Tang jets in LO and leading $\log s$ (LL)
  - LL factorization formula
- Beyond LL approximation
  - Phenomenology with LL and NLL GGF
  - $\rightarrow$ need of a full NLL calculation?
- NLL impact factors
  - Structure of NLL impact factor calculation
  - Implementation of NLL impact factors: numerical and conceptual issues
  - Breaking of factorization at NLL level
- Other PT contributions to Mueller-Tang jets
  - Colour singlet VS non-singlet exchange
- Conclusions and outlook

Mueller-Tang jets at LO and LL

- LO amplitude: box + crossed diagrams projected onto colour-singlet $\Pi^{ab,a'b'} = \delta^{ab}\delta^{a'b'}/(N_c^2-1)$



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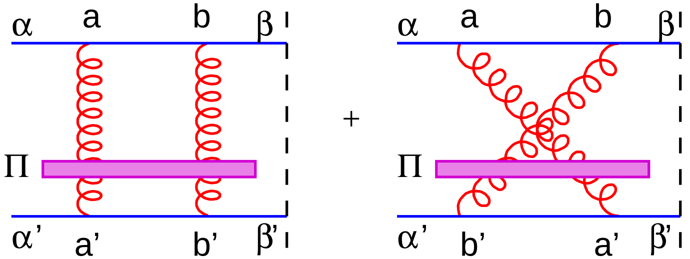
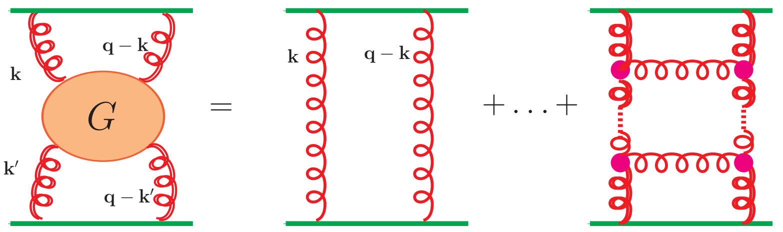


- Elastic amplitude at higher orders: affected by large $\log^n s$ due to gluon-ladder diagrams (UV and IR finite)
- All LL resummed in (colour-singlet) gluon Green function (GGF)



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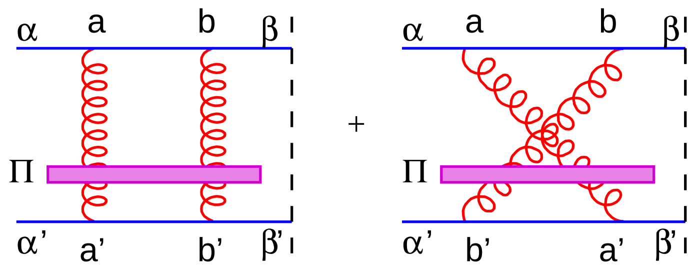

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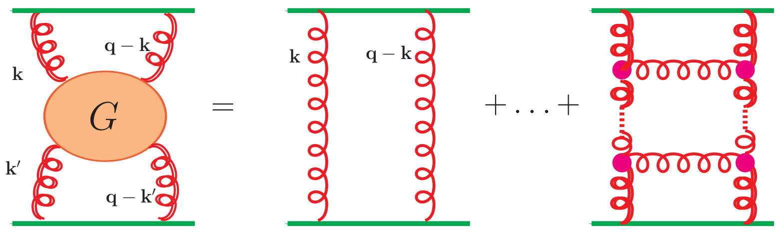
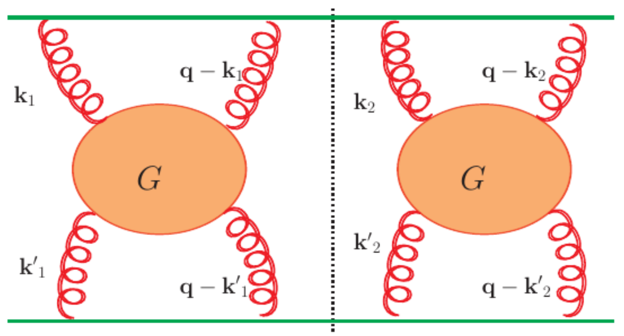
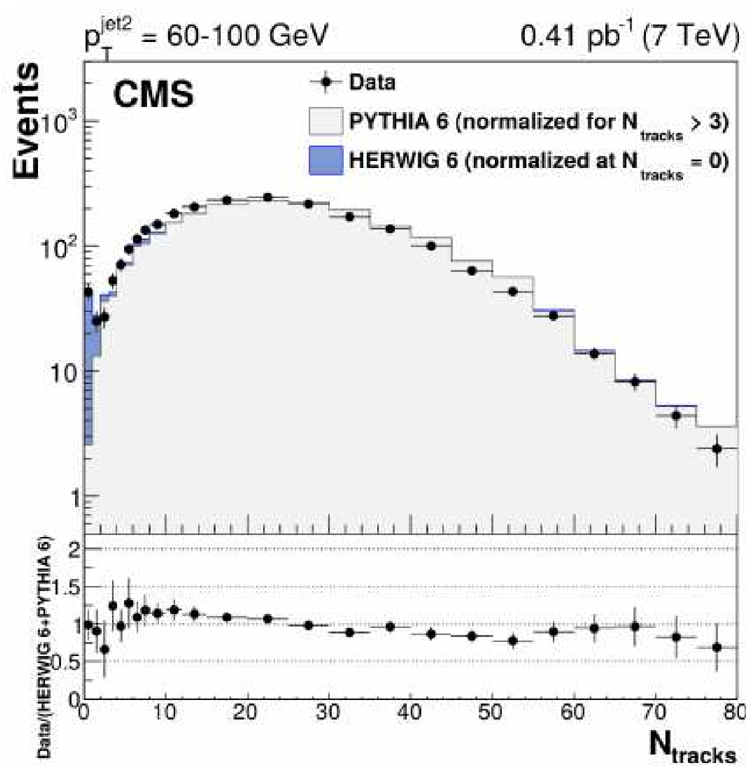


- LL partonic cross section: 2 GGF $*$ 2 (trivial) impact factors
- Two outgoing partons to be identified with the (back-to-back) jets

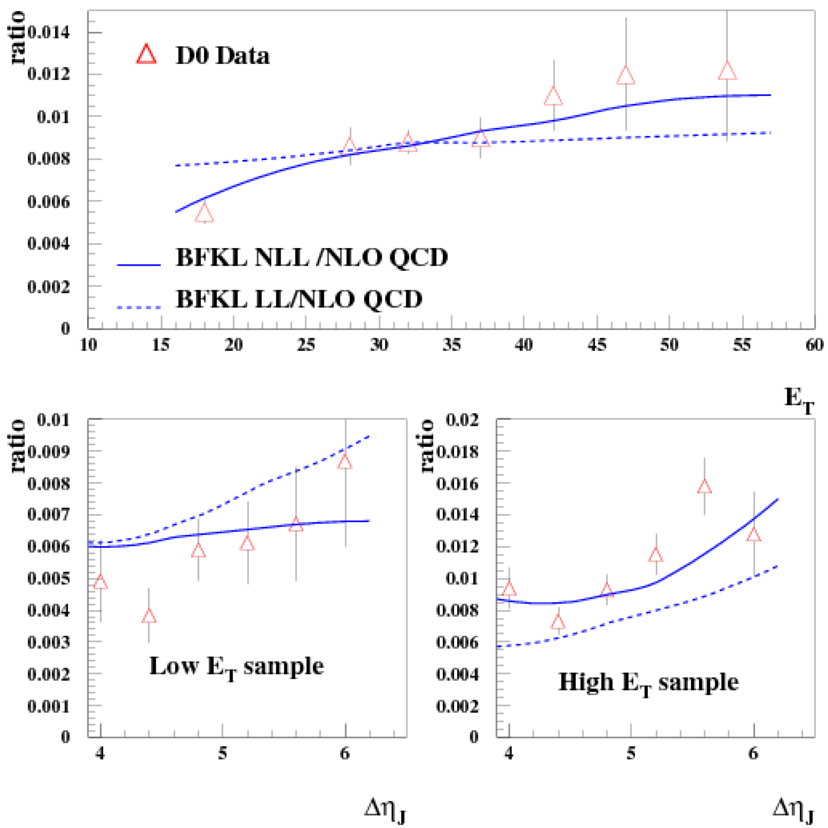


CMS analysis at 7 TeV



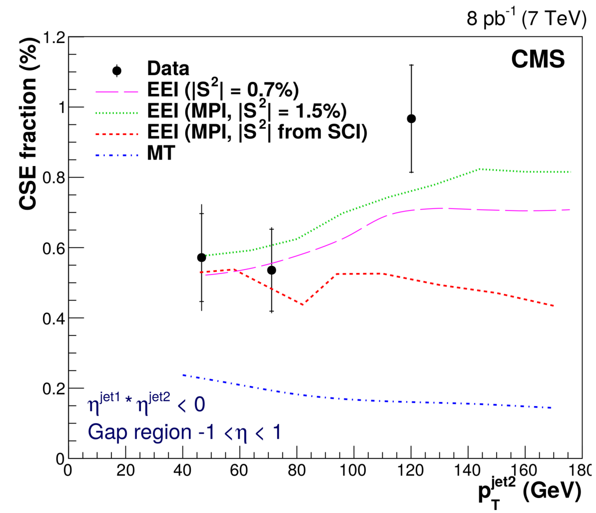
- Charged-particle multiplicity in the gap region between the tagged jets compared to PYTHIA and HERWIG predictions.
- HERWIG 6: include contributions from color singlet exchange (CSE), based on BFKL at LL.
- PYTHIA 6: inclusive dijets (tune Z2*), no-CSE.

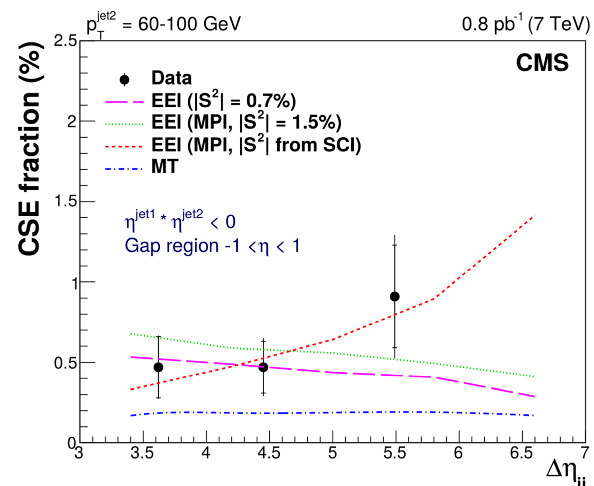
D0 and CMS analysis at 7 TeV



Left: LL & NLL BFKL at Tevatron [hep-ph/1012.3849].

- Ratio $R = \frac{NLL^* BFKL}{NLOQCD}$ of jet-gap-jet events to inclusive dijet events as a function of p_t and the rapidity gap Y.

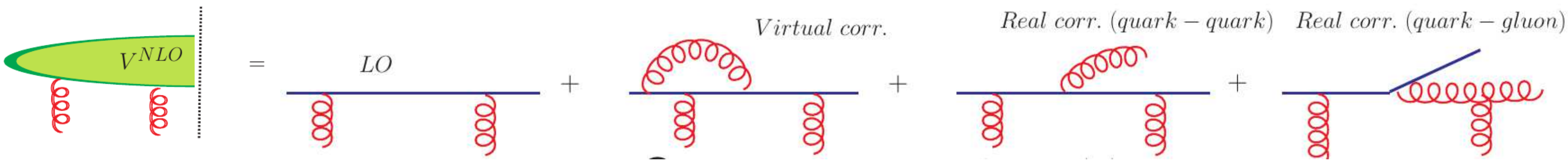




NLL* BFKL calculations different implementations of the soft rescattering processes (EEI models), describe many features of the data, but none of the implementations is able to simultaneously describe all the features of the measurement. Ekstedt, Enberg, Ingelman, [1703.10919]

NL impact factors

- Compelling to include all NLL corrections into the game



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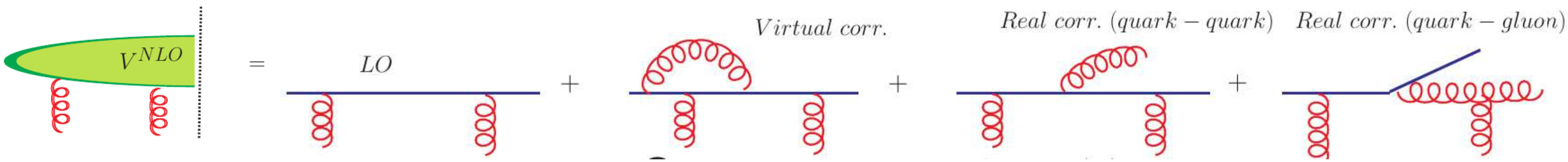
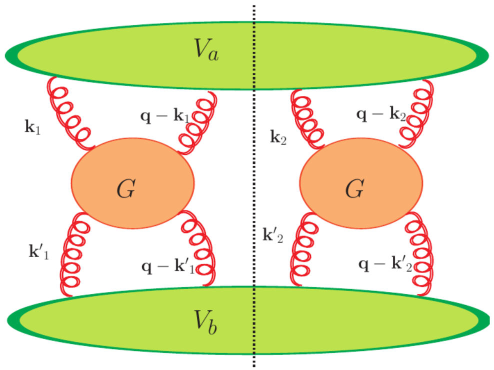


- Idea: generalize MT factorization formula at NLL
- NL impact factors determined by NLO calculation, with IR (soft and collinear) divergencies



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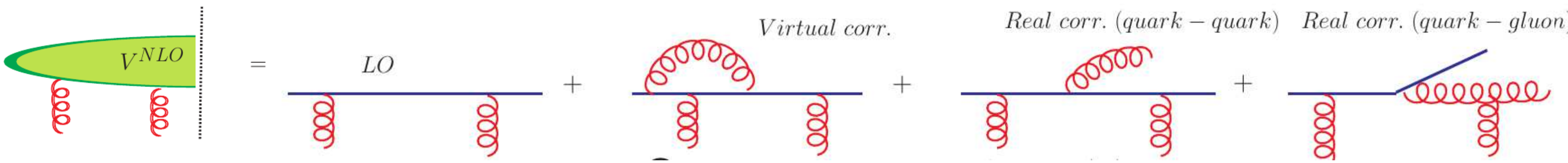

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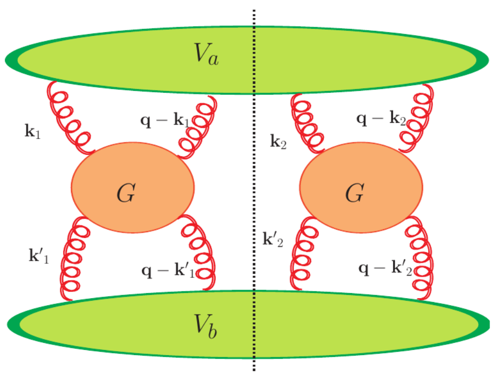
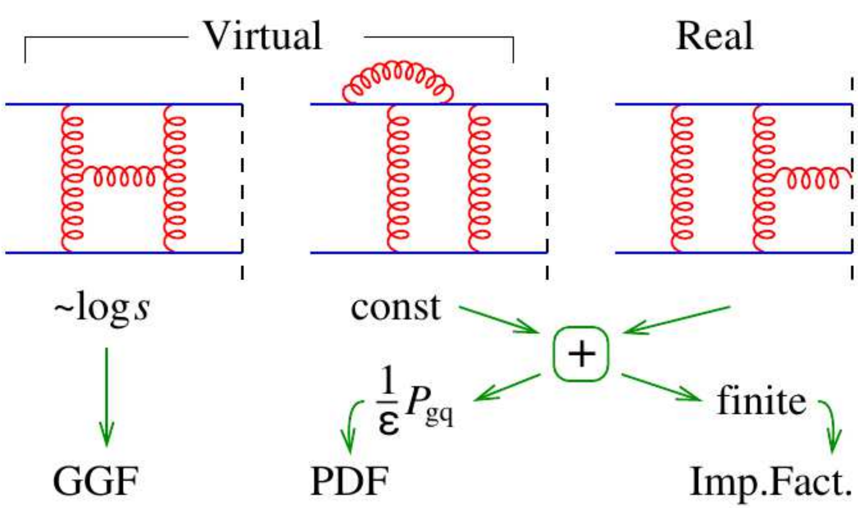


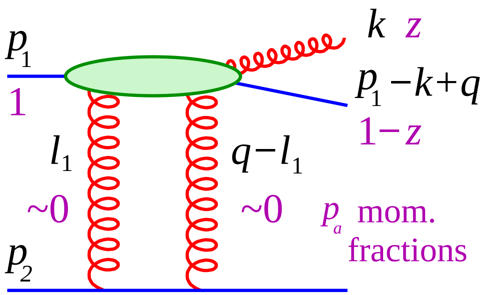


Not a trivial statement:

- all $\log(s)$ terms must reproduce LL kernel (GGF at 1st order)
- all IR singularities (taken away collinear ones proportional to splitting functions) must cancel

NL impact factors

$$\Phi(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{q}) = \frac{\alpha_s^3}{2\pi(N_c^2-1)}\int_0^1 \mathrm{d}z \int \mathrm{d}\boldsymbol{k}$$



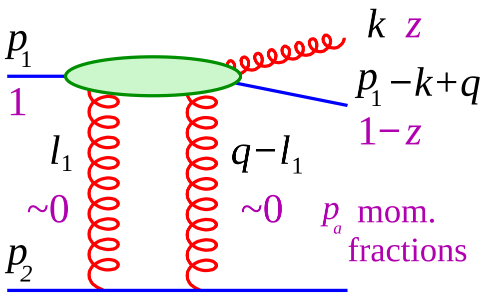
$\times$ h.c. $(\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2)$

$$\times\, \boldsymbol{S_J}(\boldsymbol{k},\boldsymbol{q},z)\, C_F \frac{1+(1-z)^2}{z}$$

$$\times \left\{ C_F^2 \frac{z^2\boldsymbol{q}^2}{\boldsymbol{k}^2(\boldsymbol{k}-z\boldsymbol{q})^2} + C_F C_A\, f_1(\boldsymbol{l}_{1,2},\boldsymbol{k},\boldsymbol{q},z) + C_A^2\, f_2(\boldsymbol{l}_{1,2},\boldsymbol{k},\boldsymbol{q}) \right\}$$

- The calculation of NL impact factors for Mueller-Tang jets was performed by *[Hentchinski, Madrigal Martinez, Murdaca, Sabio Vera, '14]* using Lipatov's effective action.

NL impact factors



$$\Phi(\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{q}) = \frac{\alpha_s^3}{2\pi(N_c^2 - 1)} \int_0^1 \mathrm{d}z \int \mathrm{d}\boldsymbol{k} \quad \times \quad \begin{matrix}\text{h.c.} \\ (\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2)\end{matrix}$$

$$\times\, S_J(\boldsymbol{k}, \boldsymbol{q}, z)\, C_F \frac{1 + (1-z)^2}{z}$$

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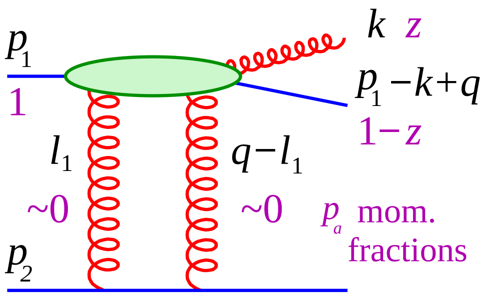
- Phase-space integration restricted by IR-safe jet algorithm (e.g., kt $\simeq$ cone)
- In these diffractive processes, "lower" quark p_2 is the "backward" jet
  Other two partons are in the forward hemisphere and form (at least) one jet:
  - $\Delta\Omega \equiv \sqrt{\Delta y^2 + \Delta\phi^2} < R \quad \Rightarrow J = \{qg\}$ composite jet
  - $\Delta\Omega > R \Rightarrow J = \{g\}$ and q outside jet cone or $J = \{q\}$ and g outside

NL impact factors



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- F.Deganutti and I checked their calculation with standard QCD Feynman rules.

Problem with NL impact factor

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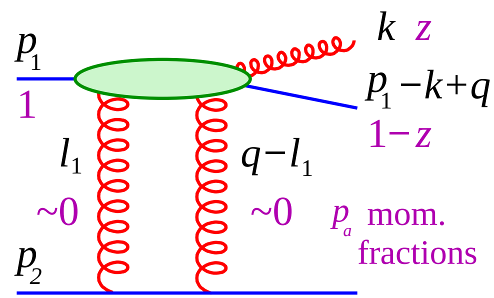

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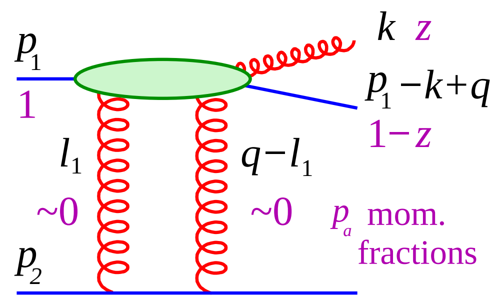

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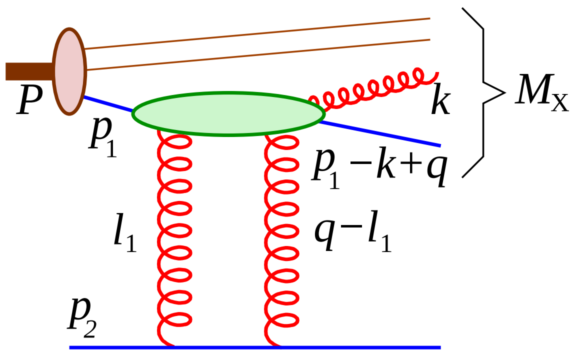
- There is a problem in the C_A^2 term, due to $\int_0^1 \mathrm{d}z/z$ integration
- If integration is not constrained, we have a divergence
- Such region $z \to 0$ corresponds to gluon in central (and backward) region, where the emission probability of the gluon turns out to be flat in rapidity: $\int_0^1 \mathrm{d}z/z = \int_{-\infty}^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y$
- If we believe the IF calculation to be reliable at least in the forward hemisphere $(y > 0) \;\Rightarrow\; \int_0^{\log\sqrt{s}/\boldsymbol{k}} \mathrm{d}y = \int_{\boldsymbol{k}/\sqrt{s}}^1 \mathrm{d}z/z = \frac{1}{2}\log(s/\boldsymbol{k}^2)$
- But a $\log(s)$ in IFs is not acceptable within the spirit of BFKL factorization

Constraint on diffractive invariant mass

In order to solve this problem, *[HMMS]* constrain mass of diffractive system $M_X^2 \equiv (P+q)^2 < M_{\max}^2$

- In this case $z \gtrsim \boldsymbol{k}^2/M_{\max}^2$

$$\Rightarrow \quad \text{finite } z\text{-integral} \sim \log(M_{\max}^2/\boldsymbol{k}^2)$$



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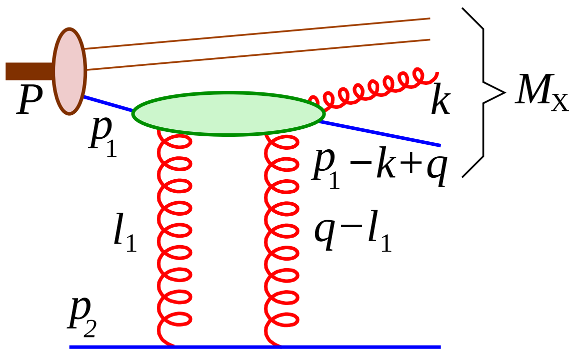

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  $\Rightarrow$ finite z-integral $\sim \log(M_{\max}^2 / \boldsymbol{k}^2)$

Crucial question: do we really need to / can we impose a cut on diffractive mass?

Arguments in favour

- Avoid $\log s$ in IF

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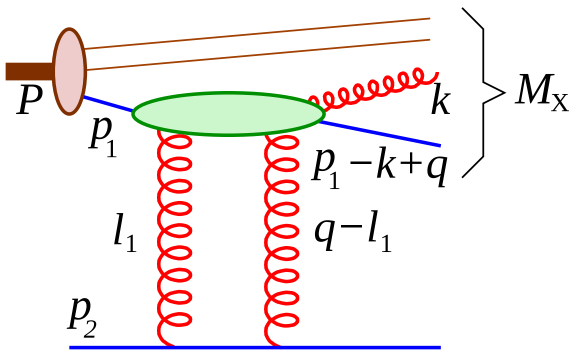

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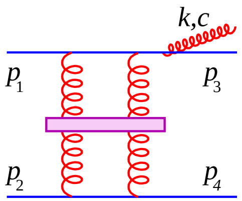
Arguments in favour

- Avoid $\log s$ in IF

Arguments against

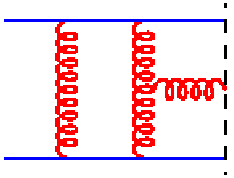
- Singlet exchange should suppress gluons in central region, no $\log s$ (wrong!)
- Diffractive mass requires measuring outgoing proton or its remnants
- Diffractive mass cut effective if able to measure arbitrarily soft particle energies

Violation of BFKL factorization



The theoretical argument is wrong:

- colour-singlet momentum transfer, no $\log s$
- colour-singlet either below or above. $\log s$ unavoidable



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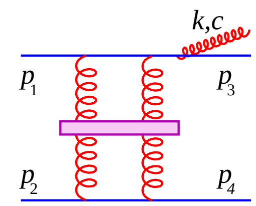

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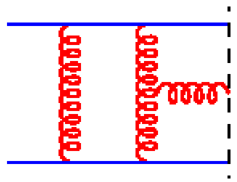


But we cannot select diagrams!
We can only select final states.

- Given that we cannot measure particles (partons, hadrons) below energy threshold $E_{\rm th}$, we can at most require no activity <u>above threshold</u> within the rapidity gap
- This prescription is IR safe because inclusive for $E_g < E_{\rm th}$

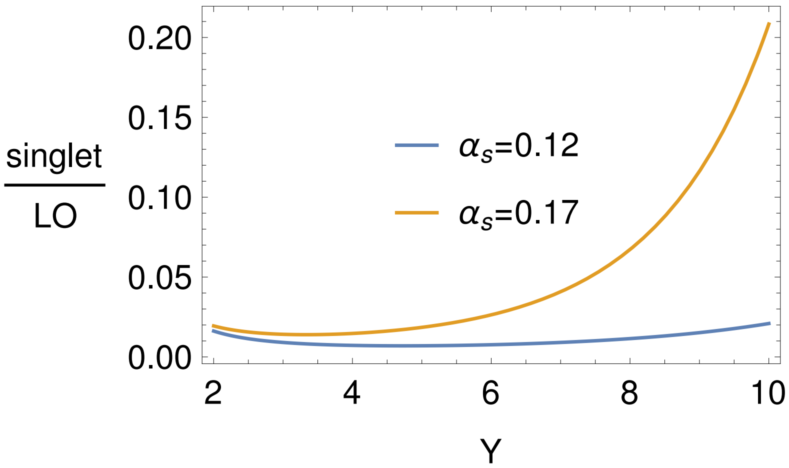
  Here the gluon can have any rapidity $\Rightarrow \sigma \ni C_A^2 \frac{E_{\rm th}^2}{E_J^2} \log \frac{s}{E_J^2}$

The experimental argument is valid, therefore BFKL factorization is violated (impact factors depend on s). However violation is expected to be small.

Singlet VS non-singlet colour exchanges

Since it is not clear how to define a pure colour-singlet dynamics

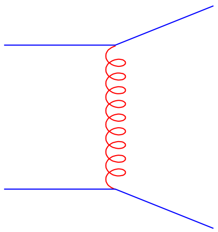
reconsider non-singlet exchanges



Simplest contribution:

Born cross section

α_s^2 VS $\alpha_s^{4+n} \log^n s$



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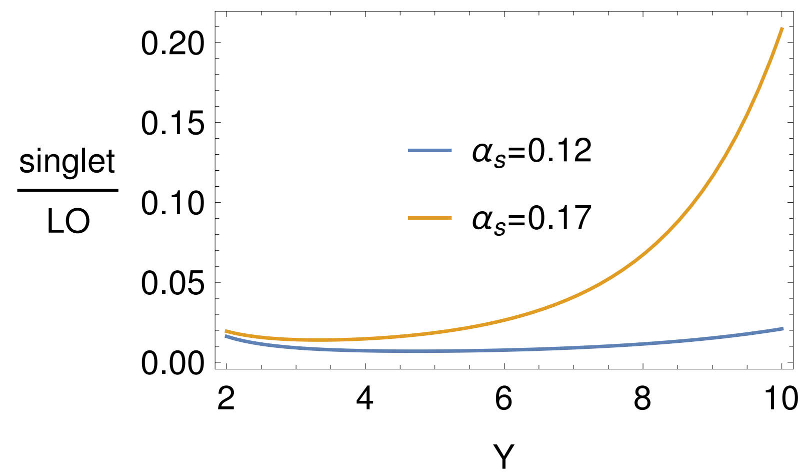

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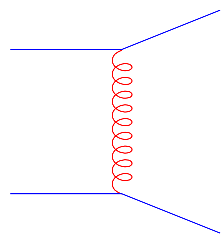
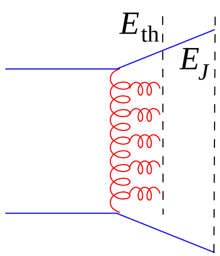


$\alpha_{\rm s}^2$ VS $\alpha_{\rm s}^{4+n}\log^n s$



Higher-order LL corrections:

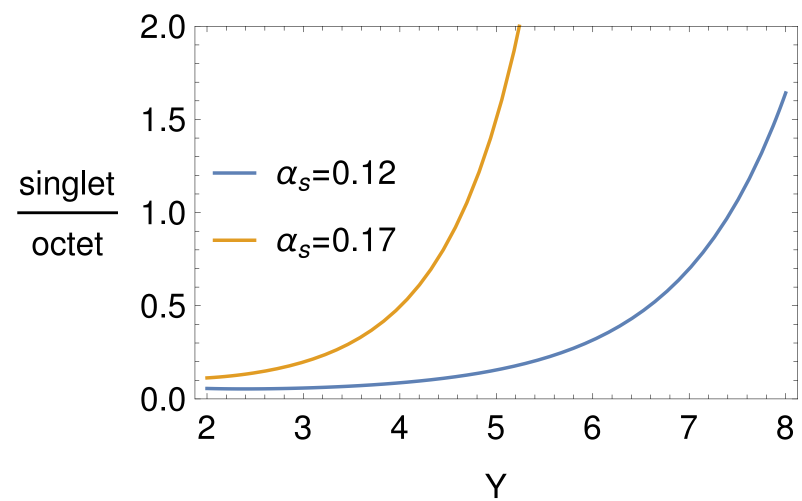
gluons below threshold in central region

$\alpha_{\rm s}^{2+n}\log^n s$ VS $\alpha_{\rm s}^{4+n}\log^n s$

If $E_{\rm th} \ll E_J$, real corrections

don't compensate virtual contribution

$$\frac{d\sigma_8}{dt} \sim e^{2\omega(t)Y} = \left(\frac{E_{\rm th}^2}{E_J^2}\right)^{\alpha_{\rm s} Y}$$



$E_{\rm th}$ suppression but less powers of $\alpha_{\rm s}$

Conclusions and outlook

- Theoretical determination of MT jets at LHC in NLL is feasible and close to completion *[see F.Deganutti's talk]*

- Strictly speaking jet-gap-jet observable violates BFKL factorization in NLLA

- Nevertheless the violation is small and factorization formula is expected to work well for LHC (non-asymptotic) kinematics.
- Colour non-singlet contributions are expected to be non-negligible at LHC, in particular for small value of rapidity distance between jets. Mueller-Navelet contrubution below threshold should be included