NLO corrections to dijet production in DIS

Yair Mulian
Jyvaskyla University

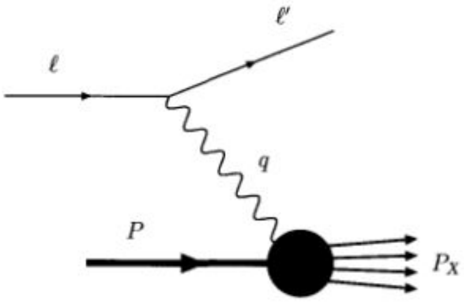
Ongoing collaborations with
E. Iancu for inclusive 3-jet and NLO 2-jet production
G. Beuf, H. Hänninen, T. Lappi, H. Mäntysaari for the extension to the diffractive case

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

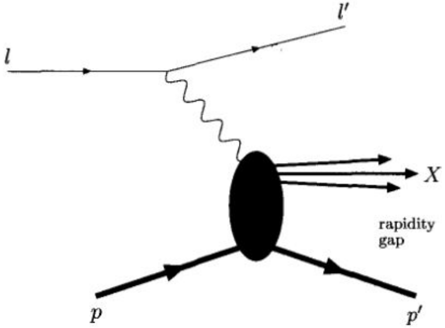
Inclusive vs Diffractive

The characteristic feature of diffractive processes is the presence of a large rapidity gap between the remnants of the beam and target. A certain fraction (about 10% in HERA) of DIS events are diffractive, with target proton which remains nearly intact.

The leading order result for the inclusive and diffractive cross section appears in hep-ph/9903246 (Y. Kovchegov and L. McLerran).



Inclusive DIS



Diffractive DIS

The Outgoing State Formalism

The time evolution of the initial (bare) virtual photon state is given by:

$$|\gamma\rangle_{in} \equiv U(0,\,-\infty)\,|\gamma\rangle_0$$

Where U denotes the unitary operator, defined as

$$U(t,t_0) = \mathrm{T}\exp\left\{-i\int_{t_0}^{t} dt_1\, H_I(t_1)\right\}$$

The virtual photon outgoing state is given by:

$$|\gamma\rangle_{out} \equiv U(\infty,\,0)\,\hat{S}\,U(0,\,-\infty)\,|\gamma\rangle_0$$

This state encodes the information both on the ***time evolution*** and ***interaction with the target nucleus*** of the incoming quark state.

The expectation values of operators are directly related to the outgoing state:

$$\left\langle \hat{O} \right\rangle = \left\langle {}_{out}\langle\gamma|\,\hat{O}\,|\gamma\rangle_{out} \right\rangle_{cgc}$$

The LO Outgoing State

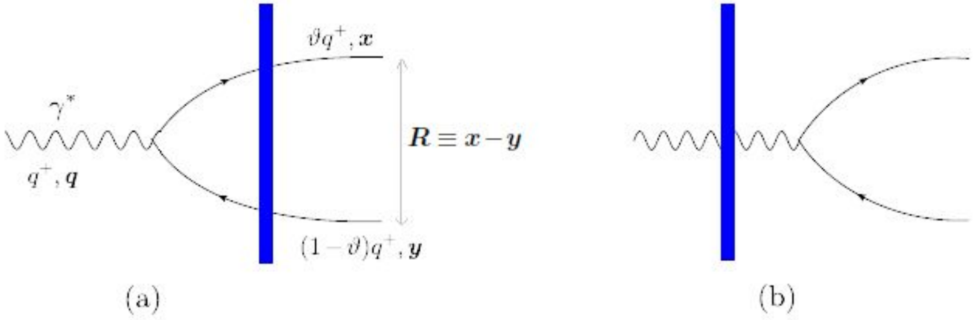
The outgoing state at leading order is given by

$$|\gamma\rangle_{out} \simeq |\gamma\rangle_0 + |\gamma\rangle_{LO}$$

The additional term reads:

$$|\gamma^i(Q, q^+, \boldsymbol{w})\rangle_{\mathrm{LO}} = \int_{\boldsymbol{x},\boldsymbol{y}} \int_0^1 d\vartheta \, \frac{i e \, e_f \sqrt{q^+} \, \varphi^{ij}_{\lambda_1\lambda_2}(\vartheta)}{(2\pi)^2\sqrt{2}} \, \frac{\boldsymbol{R}^j}{|\boldsymbol{R}|} \bar{Q} \, \mathrm{K}_1(\bar{Q}|\boldsymbol{R}|)$$
$$\times \left(V(\boldsymbol{x}) V^\dagger(\boldsymbol{y}) - 1\right)_{\alpha\beta} \delta^{(2)}(\boldsymbol{w} - \boldsymbol{c}) \left| \bar{q}^\beta_{\lambda_2}((1-\vartheta)q^+, \boldsymbol{y}) \, q^\alpha_{\lambda_1}(\vartheta q^+, \boldsymbol{x}) \right\rangle$$

Diagrammatically:



$$\vartheta \equiv k^+/q^+$$

$$\boldsymbol{c} = \vartheta \boldsymbol{x} + (1-\vartheta)\boldsymbol{y}$$

$$V(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig \int dx^+ \, t^a A^-_a(x^+, \boldsymbol{x})\right\}$$

$$U(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig \int dx^+ \, T^a A^-_a(x^+, \boldsymbol{x})\right\}$$

Blue bar denotes a shockwave = interaction with the target.

The LO Inclusive Cross Section

The cross section for measuring an inclusive dijet:

$$\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2}(2\pi)\,\delta(q^+ - k_1^+ - k_2^+)$$
$$= \int d^2\overline{\boldsymbol{w}}\, d^2\boldsymbol{w}\, e^{-i\boldsymbol{q}\cdot(\boldsymbol{w}-\overline{\boldsymbol{w}})}\ {}_{\rm LO}\big\langle \gamma_T^i(Q, q^+, \overline{\boldsymbol{w}})\big|\, \mathcal{N}_{\bar{q}}(k_1)\, \mathcal{N}_q(k_2)\, \big|\gamma_T^i(Q, q^+, \boldsymbol{w})\big\rangle_{\rm LO}$$

With the number density operators $\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\, b_\lambda^{\alpha\dagger}(p)\, b_\lambda^\alpha(p)$

After insertion of the outgoing state:

$$\frac{d\sigma}{dk_1^+ d^2\boldsymbol{k}_1 dk_2^+ d^2\boldsymbol{k}_2} = \frac{2\alpha_{em} N_c}{(2\pi)^6 q^+}\left(\vartheta^2 + (1-\vartheta)^2\right)\left(\sum e_f^2\right)\delta(q^+ - k_1^+ - k_2^+)$$
$$\times \int_{\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{y}} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}}) - i\boldsymbol{k}_2\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}})}\ \frac{\boldsymbol{R}\cdot\overline{\boldsymbol{R}}}{|\boldsymbol{R}|\,|\overline{\boldsymbol{R}}|}\bar{Q}^2\, \mathrm{K}_1\left(\bar{Q}|\boldsymbol{R}|\right)\, \mathrm{K}_1\left(\bar{Q}|\overline{\boldsymbol{R}}|\right)\ \mathcal{W}\left(\boldsymbol{x}, \boldsymbol{y}, \overline{\boldsymbol{y}}, \overline{\boldsymbol{x}}\right)$$

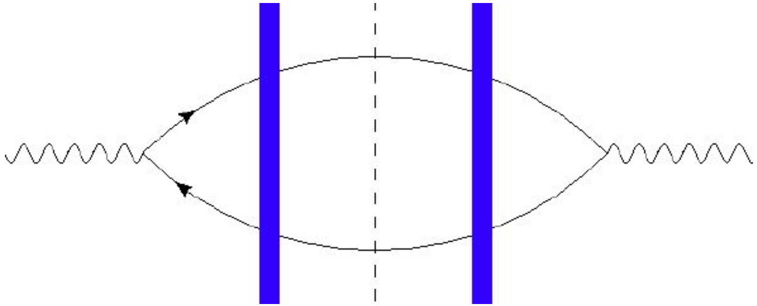
For the color structure:

$$\mathcal{W}(x, y, \overline{y}, \overline{x}) \equiv \mathcal{Q}(x, y, \overline{y}, \overline{x}) - \mathcal{S}(x, y) - \mathcal{S}(\overline{y}, \overline{x}) + 1$$

With the dipole and quadrupole definitions:

$$\mathcal{S}(x, \overline{y}) \equiv \frac{1}{N_c} \left\langle \mathrm{tr}(V(x)\, V^\dagger(\overline{y})) \right\rangle \qquad \mathcal{Q}(x, y, \overline{y}, \overline{x}) \equiv \frac{1}{N_c} \left\langle \mathrm{tr}\big(V(x)\, V^\dagger(y)\, V(\overline{y})\, V^\dagger(\overline{x})\big) \right\rangle$$

Diagrammatically:



The LO Diffractive Cross Section

Since diffractive events involve no color exchange, the differential cross section is given in terms of singlet projections. At leading order:

$$\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2}(2\pi)\,\delta(q^+ - k_1^+ - k_2^+)$$
$$= \int d^2\overline{\boldsymbol{w}}\, d^2\boldsymbol{w}\, e^{-iq\cdot(\boldsymbol{w}-\overline{\boldsymbol{w}})}\; {}_{\text{LO}}^{out}\langle \gamma^i\,(Q,\, q^+,\, \overline{\boldsymbol{w}})|\; \mathbb{P}_{q\bar{q}}(k_1,\, k_2)\; |\gamma^i\,(Q,\, q^+,\, \boldsymbol{w})\rangle_{\text{LO}}^{out}$$

With the singlet projection operator:

$$\mathbb{P}_{q\bar{q}}(k_1,\, k_2) \equiv \delta\left(M_X^2 - (k_1+k_2)^2\right)\, |\bar{q}_{\lambda_1}^{\alpha}(k_1^+,\, \boldsymbol{k}_1)\, q_{\lambda_2}^{\alpha}(k_2^+,\, \boldsymbol{k}_2)\rangle\, \langle \bar{q}_{\lambda_1}^{\beta}(k_1^+,\, \boldsymbol{k}_1)\, q_{\lambda_2}^{\beta}(k_2^+,\, \boldsymbol{k}_2)|$$

After insertion of the outgoing state:

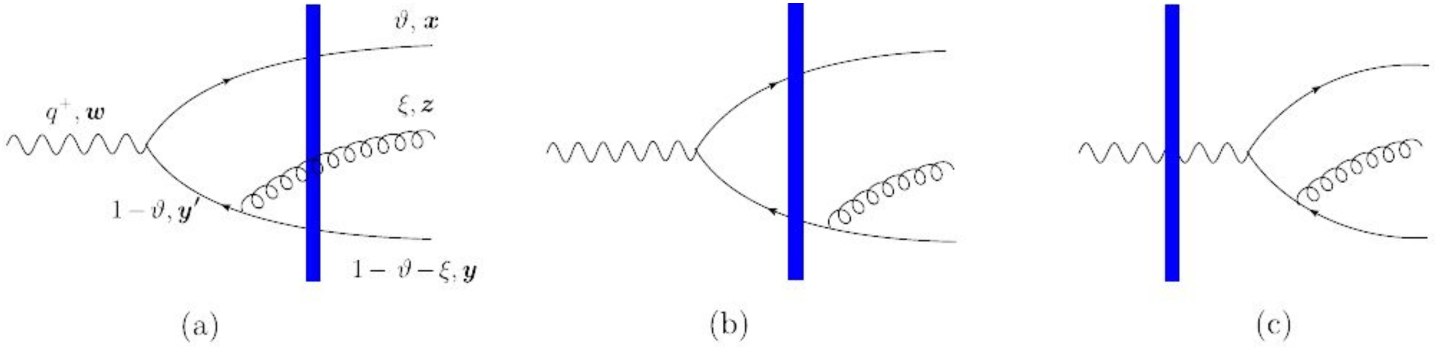
$$\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2} = \frac{\alpha_{em} N_c}{2(2\pi)^6 q^+} \left(1+(1-2\vartheta)^2\right) \left(\sum e_f^2\right)$$
$$\times \int_{\overline{\boldsymbol{x}}, \overline{\boldsymbol{y}}, \boldsymbol{x}, \boldsymbol{y}} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}_2\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}})}\, \delta(q^+ - k_1^+ - k_2^+)\, \delta\left(M_X^2 - \frac{\boldsymbol{l}^2}{\vartheta(1-\vartheta)}\right)$$
$$\times \frac{\boldsymbol{R}\cdot\overline{\boldsymbol{R}}}{|\boldsymbol{R}|\,|\overline{\boldsymbol{R}}|}\, \widetilde{Q}^2\, K_1\left(\widetilde{Q}|\boldsymbol{R}|\right)\, K_1\left(\widetilde{Q}|\overline{\boldsymbol{R}}|\right)\, \mathcal{N}_{q\overline{q}}(\boldsymbol{x}, \boldsymbol{y})\, \mathcal{N}_{q\overline{q}}(\overline{\boldsymbol{y}}, \overline{\boldsymbol{x}})$$

With the following combination of Wilson lines:

$$\mathcal{N}_{q\overline{q}}(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{N_c} tr\left[V(\boldsymbol{x})\, V^\dagger(\boldsymbol{y}) - \boldsymbol{1}\right]$$

The NLO outgoing state

At NLO, we can generate an extra gluon from the anti-quark:



The case of emission from the quark can be obtained from that of the anti-quark by replacing all the quantum numbers, taking the conjugate and flipping the overall sign factor.

In addition to the contributions which are generated via two successive parton splittings in the light-cone formalism, there are also 1->3 instantaneous vertices.

The Trijet Cross Sections

For the inclusive case, the direct contribution reads:

$$\frac{d\sigma}{dk_1^+ d^2\boldsymbol{k}_1 dk_2^+ d^2\boldsymbol{k}_2 dk_3^+ d^2\boldsymbol{k}_3}(2\pi)\,\delta(q^+ - k_1^+ - k_2^+ - k_3^+)$$
$$= \int_{\overline{\boldsymbol{w}},\boldsymbol{w}} e^{-i\boldsymbol{q}\cdot(\boldsymbol{w}-\overline{\boldsymbol{w}})}\ {}_{q\bar{q}g}\langle\gamma^i\,(Q, q^+, \overline{\boldsymbol{w}})|\ \mathcal{N}_q(k_1)\mathcal{N}_{\bar{q}}(k_2)\mathcal{N}_g(k_3)\ |\gamma^i\,(Q, q^+, \boldsymbol{w})\rangle_{q\bar{q}g}$$

For the diffractive case, we have to project the quark, anti-quark, gluon state into a singlet:

$$\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2\, dk_3^+\, d^2\boldsymbol{k}_3}(2\pi)\,\delta(q^+ - k_1^+ - k_2^+ - k_3^+)$$
$$= \int_{\overline{\boldsymbol{w}},\,\boldsymbol{w}} e^{-i\boldsymbol{q}\cdot(\boldsymbol{w}-\overline{\boldsymbol{w}})}\ {}_{q\,\bar{q}\,g}\langle\gamma^i(Q,\, q^+,\, \overline{\boldsymbol{w}})|\ \mathbb{P}_{q\,\bar{q}\,g}(k_1,\, k_2,\, k_3)\ |\gamma^i(Q,\, q^+,\, \boldsymbol{w})\rangle_{q\,\bar{q}\,g}$$

With the projection operator:

$$\mathbb{P}_{q\,\bar{q}\,g}(k_1,\, k_2,\, k_3) \equiv \frac{t^a_{\alpha\beta}\, t^b_{\delta\gamma}}{N_c\, C_F}\int_{\overline{\boldsymbol{x}},\,\overline{\boldsymbol{y}},\,\overline{\boldsymbol{z}},\,\boldsymbol{x},\,\boldsymbol{y},\,\boldsymbol{z}} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}_2\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}})-i\boldsymbol{k}_3\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\,\delta\left(M_X^2 - (k_1+k_2+k_3)^2\right)$$
$$\times\left|\bar{q}^{\alpha}_{\lambda_1}(k_1^+,\, \boldsymbol{x})\, g^a_i(k_2^+,\, \boldsymbol{y})\, q^{\beta}_{\lambda_2}(k_3^+,\, \boldsymbol{z})\right\rangle\left\langle\bar{q}^{\gamma}_{\lambda_1}(\overline{k_1^+},\, \overline{\boldsymbol{x}})\, g^b_i(\overline{k_2^+},\, \overline{\boldsymbol{y}})\, q^{\delta}_{\lambda_2}(\overline{k_3^+},\, \overline{\boldsymbol{z}})\right|$$

The Direct Contribution

$$\frac{d\sigma^{(a),\text{direct}}}{dk_1^+ d^2\boldsymbol{k}_1 dk_2^+ d^2\boldsymbol{k}_2 dk_3^+ d^2\boldsymbol{k}_3} = \frac{\alpha_s \alpha_{em} C_F N_c \bar{Q}^2}{2(2\pi)^{10}(q^+)^2 \xi (1-\vartheta)^2} \left(\sum e_f^2\right)$$

$$\times \delta(q^+ - k_1^+ - k_2^+ - k_3^+) \; \delta\left(M_X^2 - (k_1 + k_2 + k_3)^2\right)$$

$$\times \int_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}}} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}}) - i\boldsymbol{k}_2\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}}) - i\boldsymbol{k}_3\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})} \frac{\boldsymbol{R}^i \boldsymbol{Y}^m \overline{\boldsymbol{R}}^j \overline{\boldsymbol{Y}}^n}{\boldsymbol{Y}^2 \overline{\boldsymbol{Y}}^2}$$

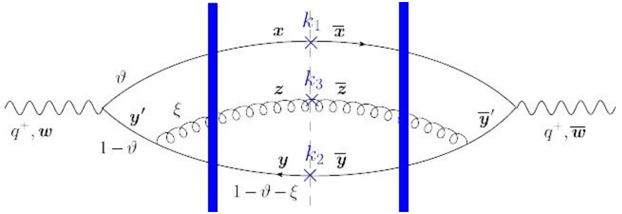
$$\times \Big[\mathcal{K}_1^{imjn}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\vartheta,\xi,\bar{Q}) \;\; \text{Trijet Color structure}$$

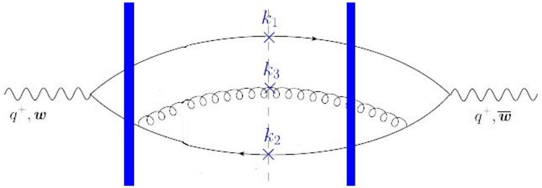
$$- (\boldsymbol{y},\boldsymbol{z}\to\boldsymbol{y}') - (\overline{\boldsymbol{y}},\overline{\boldsymbol{z}}\to\overline{\boldsymbol{y}}') + (\boldsymbol{y},\,\boldsymbol{z}\to\boldsymbol{y}' \;\&\; \overline{\boldsymbol{y}},\,\overline{\boldsymbol{z}}\to\overline{\boldsymbol{y}}') \Big]$$

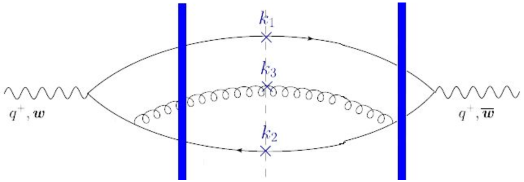
Trijet color structure

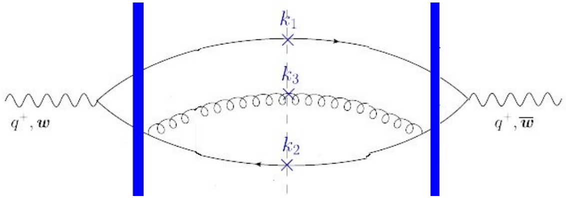
Inclusive: $\mathcal{W}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}})$

Diffractive: $\mathcal{N}_{q\bar{q}g}(\boldsymbol{x},\,\boldsymbol{y},\,\boldsymbol{z})\,\mathcal{N}_{q\bar{q}g}(\overline{\boldsymbol{y}},\,\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}})$









The kernel defined by:

$$\mathcal{K}_1^{imjn}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\vartheta,\xi,Q) \equiv \Phi_{\lambda_1\lambda}^{lirm}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\vartheta,\xi)\,\Phi_{\lambda_1\lambda}^{ljrn\,*}(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\vartheta,\xi) \times \frac{\mathrm{K}_1\left(\bar{Q}D(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\vartheta,\xi)\right)\,\mathrm{K}_1\left(\bar{Q}D(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\vartheta,\xi)\right)}{D(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\vartheta,\xi)\,D(\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}},\vartheta,\xi)}$$

With effective vertex, combining the regular and instantaneous emissions:

$$\Phi_{\lambda_1\lambda_2}^{ijmn}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\vartheta,\xi) \equiv \boxed{\varphi_{\lambda_1\lambda}^{ij}(\vartheta)\,\tau_{\lambda\lambda_2}^{mn}(\xi,1-\vartheta-\xi)} - \boxed{\delta^{nj}\sqrt{\xi}(1-\vartheta)\left(\chi_{\lambda_1}^{\dagger}\sigma^i\sigma^m\chi_{\lambda_2}\right)\frac{\boldsymbol{Y}^2}{\boldsymbol{R}\cdot\boldsymbol{Y}}}$$

And: $\vartheta = \frac{k_1^+}{q^+}$, $\xi = \frac{k_3^+}{q^+}$ $\quad D(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\vartheta,\xi) \equiv \sqrt{\boldsymbol{R}^2 + \frac{\xi(1-\vartheta-\xi)}{\vartheta(1-\vartheta)^2}\boldsymbol{Y}^2}$

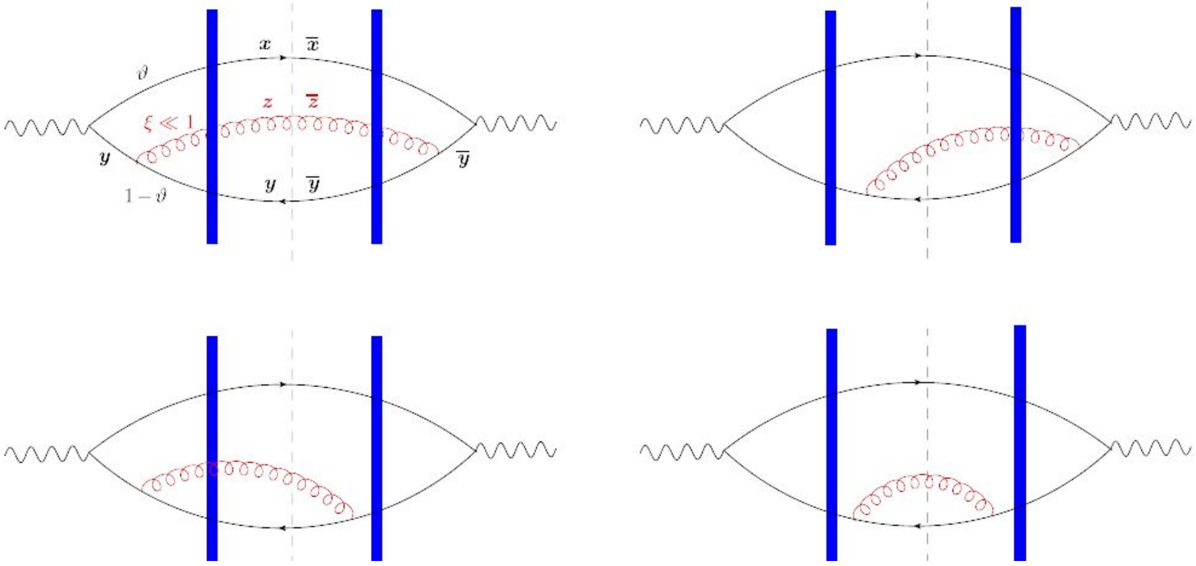
For the combination of Wilson lines:

$$\mathcal{W}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}}) \simeq \mathcal{Q}(\boldsymbol{x},\boldsymbol{z},\overline{\boldsymbol{z}},\overline{\boldsymbol{x}})\,\mathcal{Q}(\boldsymbol{z},\boldsymbol{y},\overline{\boldsymbol{y}},\overline{\boldsymbol{z}}) - \mathcal{S}(\boldsymbol{x},\boldsymbol{z})\,\mathcal{S}(\boldsymbol{z},\boldsymbol{y}) - \mathcal{S}(\overline{\boldsymbol{z}},\overline{\boldsymbol{x}})\,\mathcal{S}(\overline{\boldsymbol{y}},\overline{\boldsymbol{z}}) + \mathbf{1}$$

$$\mathcal{N}_{q\bar{q}g}(\boldsymbol{x},\,\boldsymbol{y},\,\boldsymbol{z}) \equiv \frac{1}{C_F\,N_c}U^{ba}(\boldsymbol{z})\,tr\left[t^b\,V(\boldsymbol{x})\,t^a\,V^{\dagger}(\boldsymbol{y})\right] - \mathbf{1}$$

Recovering the JIMWLK Evolution

In the limit when one of the gluons become soft (eikonal emission vertex = no recoil of the emitter), the general NLO result has to reduce to one step in the real part of JIMWLK evolution of the leading order result. We managed to show that this is indeed the case in our result.



JIMWLK for the Inclusive Case

$$\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2\, dk_3^+\, d^2\boldsymbol{k}_3}\bigg|_{\xi\to 0} = \frac{2\alpha_{em} N_c}{(2\pi)^6 q^+}\left(\vartheta^2 + (1-\vartheta)^2\right)\left(\sum e_f^2\right)\delta(q^+ - k_1^+ - k_2^+)$$

$$\times \int_{\overline{\boldsymbol{x}},\overline{\boldsymbol{y}},\boldsymbol{x},\boldsymbol{y}} e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}_2\cdot(\boldsymbol{y}-\overline{\boldsymbol{y}})}\frac{\boldsymbol{R}\cdot\overline{\boldsymbol{R}}}{R\overline{R}}\bar{Q}^2\,\mathrm{K}_1(\bar{Q}R)\mathrm{K}_1(\bar{Q}\overline{R})$$

$$\times \frac{4}{(2\pi)^4}\frac{\alpha_s C_F}{\xi q^+}\int_{\overline{\boldsymbol{z}},\boldsymbol{z}} e^{-i\boldsymbol{k}_3\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\,\Delta_{\text{real}}\mathcal{Q}(\boldsymbol{x},\boldsymbol{y},\overline{\boldsymbol{y}},\overline{\boldsymbol{x}};\,\boldsymbol{z},\overline{\boldsymbol{z}})$$

→ LO result

The action of the production Hamiltonian on the LO color structure

The production Hamiltonian:

$$H_{\text{prod}}(\boldsymbol{k}) = \frac{1}{4\pi^3}\int_{\boldsymbol{z},\overline{\boldsymbol{z}},\boldsymbol{z}',\overline{\boldsymbol{z}}'} e^{-i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\frac{\boldsymbol{Z}\cdot\overline{\boldsymbol{Z}}}{Z^2\,\overline{Z}^2}\left(J_L^a(\boldsymbol{z}') - U^{\dagger ab}(\boldsymbol{z})J_R^b(\boldsymbol{z}')\right)\left(J_L^a(\overline{\boldsymbol{z}}') - U^{\dagger ac}(\overline{\boldsymbol{z}})J_R^c(\overline{\boldsymbol{z}}')\right)$$

After integration over k_3 we recover the action of the real part of JIMWLK equation.

JIMWLK for the Diffractive Case

$$
\begin{aligned}
\left.\frac{d\sigma}{dk_1^+\, d^2\boldsymbol{k}_1\, dk_2^+\, d^2\boldsymbol{k}_2\, dk_3^+\, d^2\boldsymbol{k}_3}\right|_{\xi \longrightarrow \vartheta} = & \frac{\alpha_s\, \alpha_{em}\, C_F\, N_f}{2(2\pi)^{10}(q^+)^2\varepsilon} \left(\sum e_f^2\right) \delta(q^+ - k_1^+ - k_2^+ - k_3^+) \\
& \times \int_{x,\, z,\, z',\, \overline{x},\, \overline{z},\, \overline{z}'} e^{-ik_1\cdot(x-\overline{x}) - ik_2\cdot(z-\overline{z}) - ik_3\cdot(z'-\overline{z}')}\, \delta\left(M_X^2 - (k_1 + k_2 + k_3)^2\right) \\
& \times \left(1 + (1-2\vartheta)^2\right) \frac{\boldsymbol{R}\cdot\overline{\boldsymbol{R}}}{\sqrt{\boldsymbol{R}^2\, \overline{\boldsymbol{R}}^2}}\ \tilde{Q}^2\, K_1\left(\tilde{Q}\sqrt{\boldsymbol{R}^2}\right)\, K_1\left(\tilde{Q}\sqrt{\overline{\boldsymbol{R}}^2}\right) \\
& \times \frac{\boldsymbol{Z}\cdot\overline{\boldsymbol{Z}}}{\boldsymbol{Z}^2\, \overline{\boldsymbol{Z}}^2} \left[\mathcal{N}_{q\overline{q}g}(x, z, z')\, \mathcal{N}_{q\overline{q}g}(\overline{z}, \overline{x}, \overline{z}') - \mathcal{N}_{q\overline{q}g}(x, y)\, \mathcal{N}_{q\overline{q}g}(\overline{z}, \overline{x}, \overline{z}') - \mathcal{N}_{q\overline{q}g}(x, z, z')\, \mathcal{N}_{q\overline{q}g}(\overline{y}, \overline{x}) + \mathcal{N}_{q\overline{q}}(x, y)\, \mathcal{N}_{q\overline{q}}(\overline{y}, \overline{x})\right]
\end{aligned}
$$

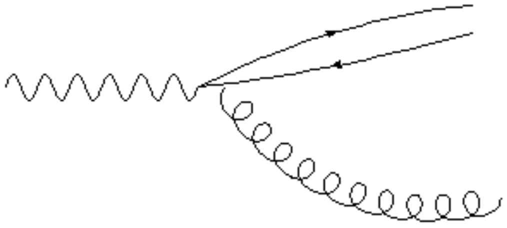
→ LO result (blue box: first three lines)

→ The action of the production Hamiltonian (red box: last line)

This result known as the Kovchegov-Levin equation, hep-ph/9911523.

Further Tests

Wusthoff limit: When the transverse separation between the quark and the anti-quark is much smaller than the distance between the quark and the gluon. Thus, the quark and anti-quark pair and the gluon form an effective color dipole.



Munier-Shoshi limit: Eikonal together with large Nc approximation.

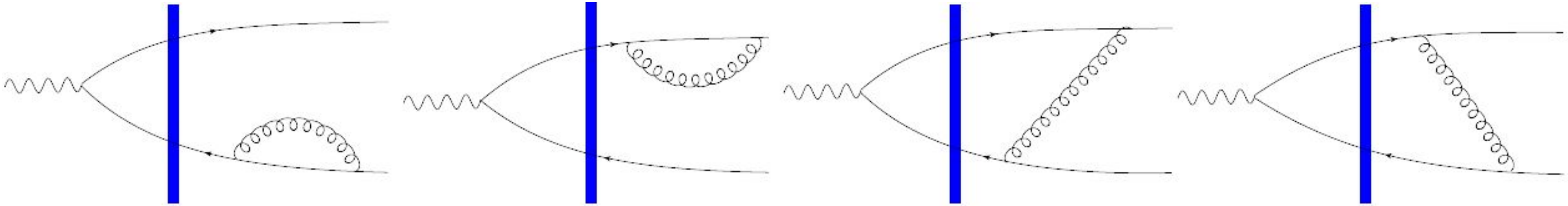
$$\mathcal{N}_{q\bar{q}g}(x, z, z') \approx \mathcal{N}_{q\bar{q}}(z', z)\mathcal{N}_{q\bar{q}}(x, z') - \mathcal{N}_{q\bar{q}}(z', z) - \mathcal{N}_{q\bar{q}}(x, z')$$

The color structure simplifies:

$$\mathcal{N}_{q\bar{q}g}(x, z, z')\,\mathcal{N}_{q\bar{q}g}(\bar{z}, \bar{x}, \bar{z}') - (z, z' \to y) - (\bar{z}, \bar{z}' \to \bar{y}) + (z, z' \to y \;\&\; \bar{z}, \bar{z}' \to \bar{y})$$
$$\approx \left(\mathcal{N}_{q\bar{q}}(z', z)\mathcal{N}_{q\bar{q}}(x, z') - \mathcal{N}_{q\bar{q}}(z', z) - \mathcal{N}_{q\bar{q}}(x, z') + \mathcal{N}_{q\bar{q}}(x, y)\right)$$
$$\times \left(\mathcal{N}_{q\bar{q}}(\bar{z}', \bar{z})\mathcal{N}_{q\bar{q}}(\bar{x}, \bar{z}') - \mathcal{N}_{q\bar{q}}(\bar{z}', \bar{z}) - \mathcal{N}_{q\bar{q}}(\bar{x}, \bar{z}') + \mathcal{N}_{q\bar{q}}(\bar{x}, \bar{y})\right)$$

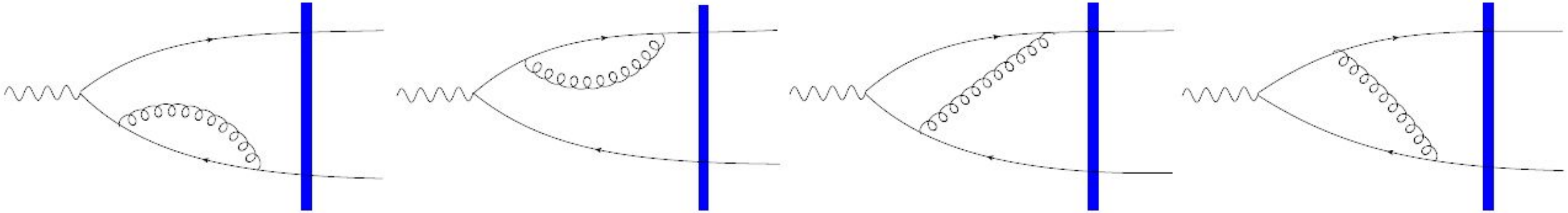
Treating the Virtual Contributions

The virtual contributions are grouped according to the time of the gluon emission:



Type A = gluon emission After the shockwave

The naive perturbation theory yields branch cuts with multiple poles. In our approach, we treated these contributions by demanding unitarity rather than a direct computation. We differ from hep-ph/2108.06347 (P. Caucal, F. Salazar, R. Venugopalan).



Type B = gluon emission Before the shockwave

The divergences are treated using dimensional regularization and sharp cutoff, they cancel when combining together the four contributions. With agreement with hep-ph/1708.06557 (B. Guillaume) and hep-ph/1711.08207 (H. Hänninen, T. Lappi, R. Paatelainen).

Summary

1) We computed the real NLO outgoing state of the virtual photon.

2) The corresponding real inclusive cross section was obtained by acting with two number density operators. The diffractive cross sections was obtained by combining the two possible singlet projections of the outgoing state.

3) Match with Wusthoff result and Munier-Shoshi limit.

4) Match has been established between the eikonal limit of the result and the JIMWLK evolution of the leading order cross section.