Jet quenching and transverse momentum of quarks and gluons

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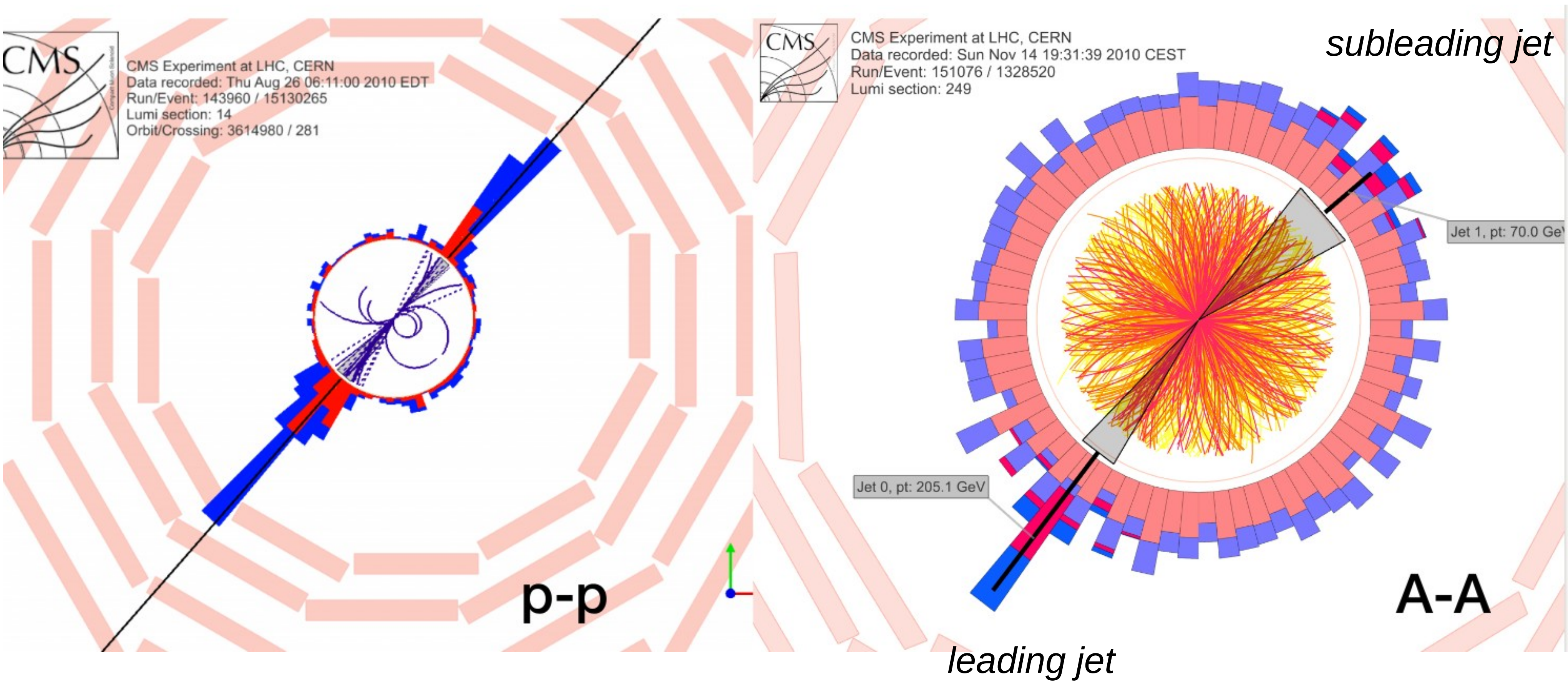
HORIZON 2020

Jets in vacuum and in medium



Kinetic equation

Maxwell-Boltzmann equation

Not easy to derive in QCD. Impossible? - Kovchegov, Wu 17
Nonuniform plasma

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}\right) f(t, \boldsymbol{x}, \boldsymbol{p}) \ = \ C_{\rm el}[f] + C_{\rm br}[f]$$

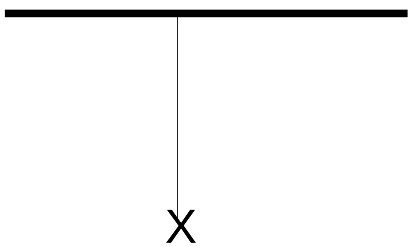
$x = \omega / E$

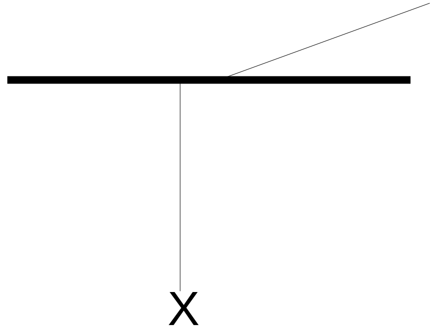
energy of observed gluon (ω)

energy of leading particle (E)

p *– transverse momentum of observed gluon*

V* = *p*/*p





Baier, Mueller, Schiff, Son '00
Iancu, Wu '15
Kurkela….

Jet quenching formalism

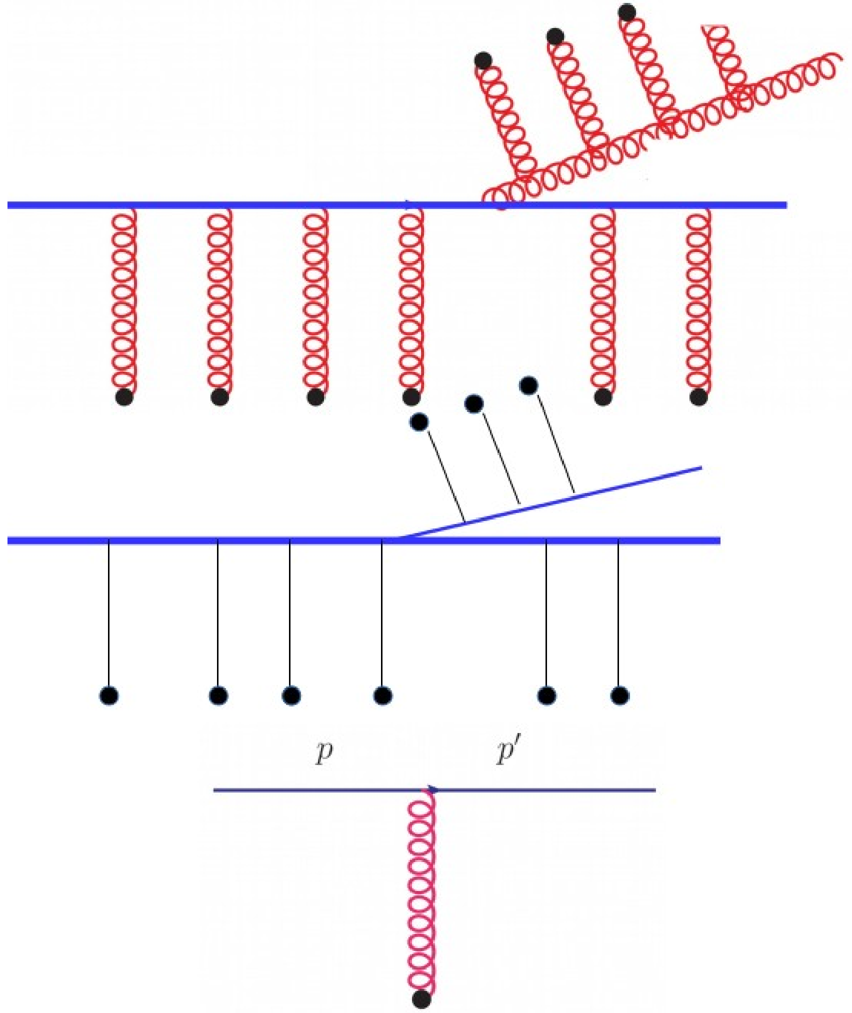
General formalism established
by BDMPS-Z (Baier, Dokshitzer,
Mueller, Peigné, Schiff; Zakharov).
Later, AMY (Arnold, Moore, Yaffe).

Solved for gluons without approximations
Caron-Huot, Gale '10

Additionally, there exists two approximation
schemes to obtain an analytical form
of the spectrum:
soft gluon approximation

- soft gluon approximation (BDMPS-Z; AMY; Wiedemann, Salgado, …)

- Single scattering approximation valid for thin media (Gyulassy, Levai, Vitev; Wiedemann)



BDMPS-Z

Multiple soft scattering resummed to all orders. It is expected to be important for short mean free-path

Because medium-induced radiation can occur anywhere along the medium with equal probability, the radiation spectrum is expected to scale linearly with L.

Many scattering centers act coherently

during the radiation over time $t_{coh} << t_{mfp}$.

Radiation spectrum

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{t_{\mathrm{coh}}} = \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

maximal energy that can be taken by single gluon

energy of observed gluon

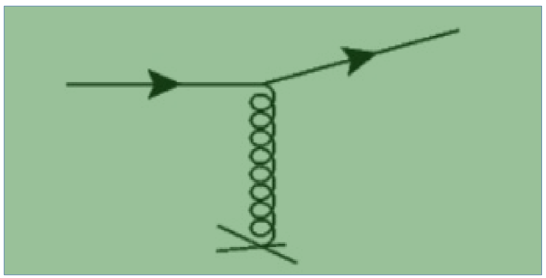
$$\omega \frac{\mathrm{d}I_{\mathrm{BH}}}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\ell_{\mathrm{mfp}}} = \alpha_s N_{\mathrm{scatt}}$$

if $t_{coh} \sim t_{mfp}$ only one scattering is involved in radiation

Look at range $\omega_{BH} < \omega < \omega_{C}$...

BDIM equation

scattering...



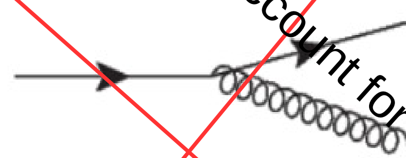
Transverse momentum transfer!

$p \to p + k_T$

Scattering Kernel: $C(k_T)$

Average transfer: $\hat{q}$

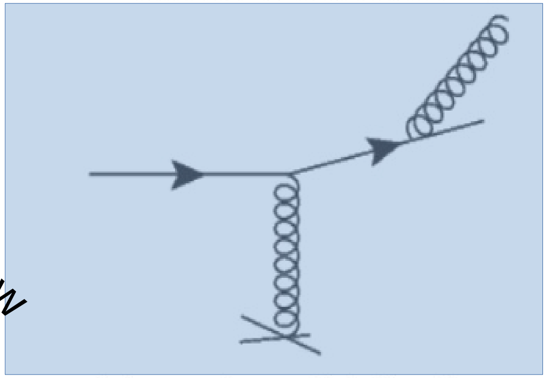
...splitting...



we do not account for this now

Bremsstrahlung as in vacuum.

...induced radiation



Momentum distribution:

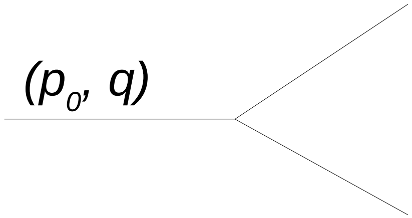
$p \to zp$

Blaizot, Dominguez, Iancu, Mehtar-Tani '12

$$\frac{\partial}{\partial t} D_g(x, \boldsymbol{k}, t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2 \boldsymbol{q}}{(2\pi)^2} \alpha_s \left[2\mathcal{K}_{gg}\left(\boldsymbol{Q}, z, \frac{x}{z} p_0^+\right) D_g\left(\frac{x}{z}, \boldsymbol{q}, t\right) - \mathcal{K}_{gg}(\boldsymbol{q}, z, x p_0^+) D_g(x, \boldsymbol{k}, t) \right]$$

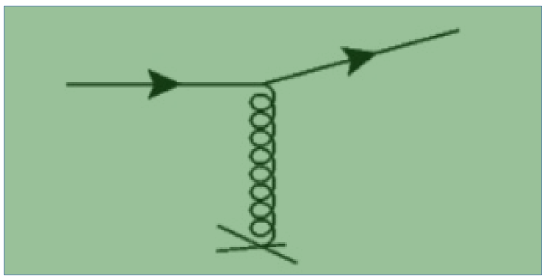
$$+ \int \frac{\mathrm{d}^2 \boldsymbol{l}}{(2\pi)^2} C_g(\boldsymbol{l}) D_g(x, \boldsymbol{k} - \boldsymbol{l}, t)$$

$$\boldsymbol{Q} \equiv \boldsymbol{k} - z\boldsymbol{q}$$



BDIM equation

scattering...



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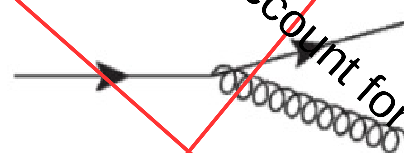

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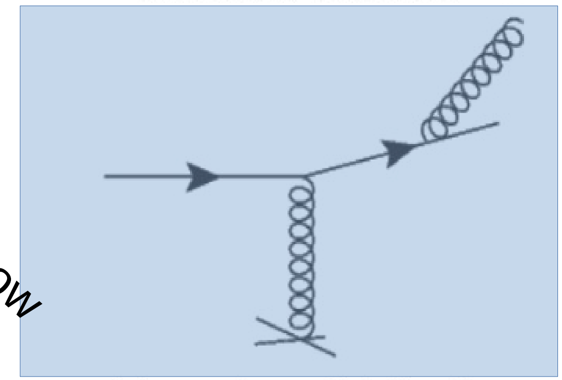
...induced radiation



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$$+ \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_g(\boldsymbol{l})\, D_g(x, \boldsymbol{k} - \boldsymbol{l}, t)$$

Equation describes interplay of rescatterings and branching.
This particular equation has k_t independent kernel.
This is an approximation. The whole broadening comes from rescattering. Energy of emitted gluon is much larger than its transverse momentum

Equation for energy distribution

Integral of the former equation over kt

$$\frac{\partial}{\partial t}D(x,t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\sqrt{\frac{z}{x}}D\left(\frac{x}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,t)\right]$$

In simplified version of the kernel - f(z)=1 - case analytical solution is possible.

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\exp\left(-\pi\frac{\tau^2}{1-x}\right)$$

Blaizot, Iancu, Mehtar-Tani'13

The solution features so called turbulent behavior. Here that means that at low x the solution factorizes into x and t dependent distributions.

The fact that the spectrum keeps the same x-dependence when t keeps increasing reflects the fact that the energy flows to x = 0 without accumulating at any finite value of x
→ *wave turbulence*

Rearrangement of the equation for gluon density

$$\frac{\partial}{\partial t}D(x,\mathbf{k},t) = \frac{1}{t^*}\int_0^1 dz\,\mathcal{K}(z)\left[\frac{1}{z^2}\sqrt{\frac{z}{x}}D\left(\frac{x}{z},\frac{\mathbf{k}}{z},t\right)\theta(z-x) - \frac{z}{\sqrt{x}}D(x,\mathbf{k},t)\right]$$
$$+\int\frac{d^2\mathbf{q}}{(2\pi)^2}C(\mathbf{q})\,D(x,\mathbf{k}-\mathbf{q},t),$$

Kutak, Płaczek, Straka Eur.Phys.J.C 79 (2019) 4, 317

mathematics: transformation of differential equation to integral equation

physics: resummation of virtual and unresolved real emissions

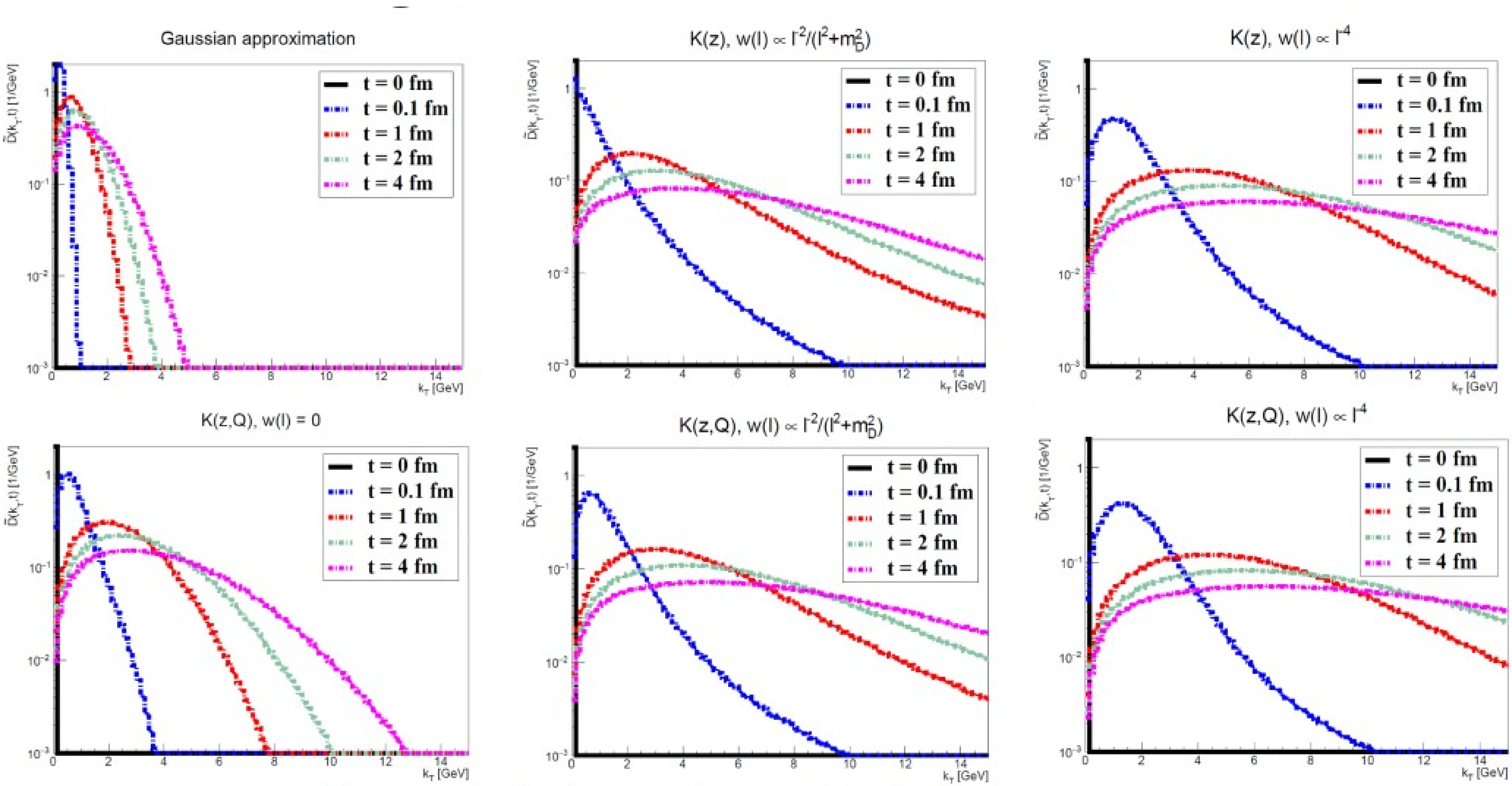
Sudakov form factor resumes virtual and unresolved real emissions

$$D(x,\mathbf{k},\tau) = e^{-\Psi(x)(\tau-\tau_0)}D(x,\mathbf{k},\tau_0)$$

(in this example Q is integrated over)

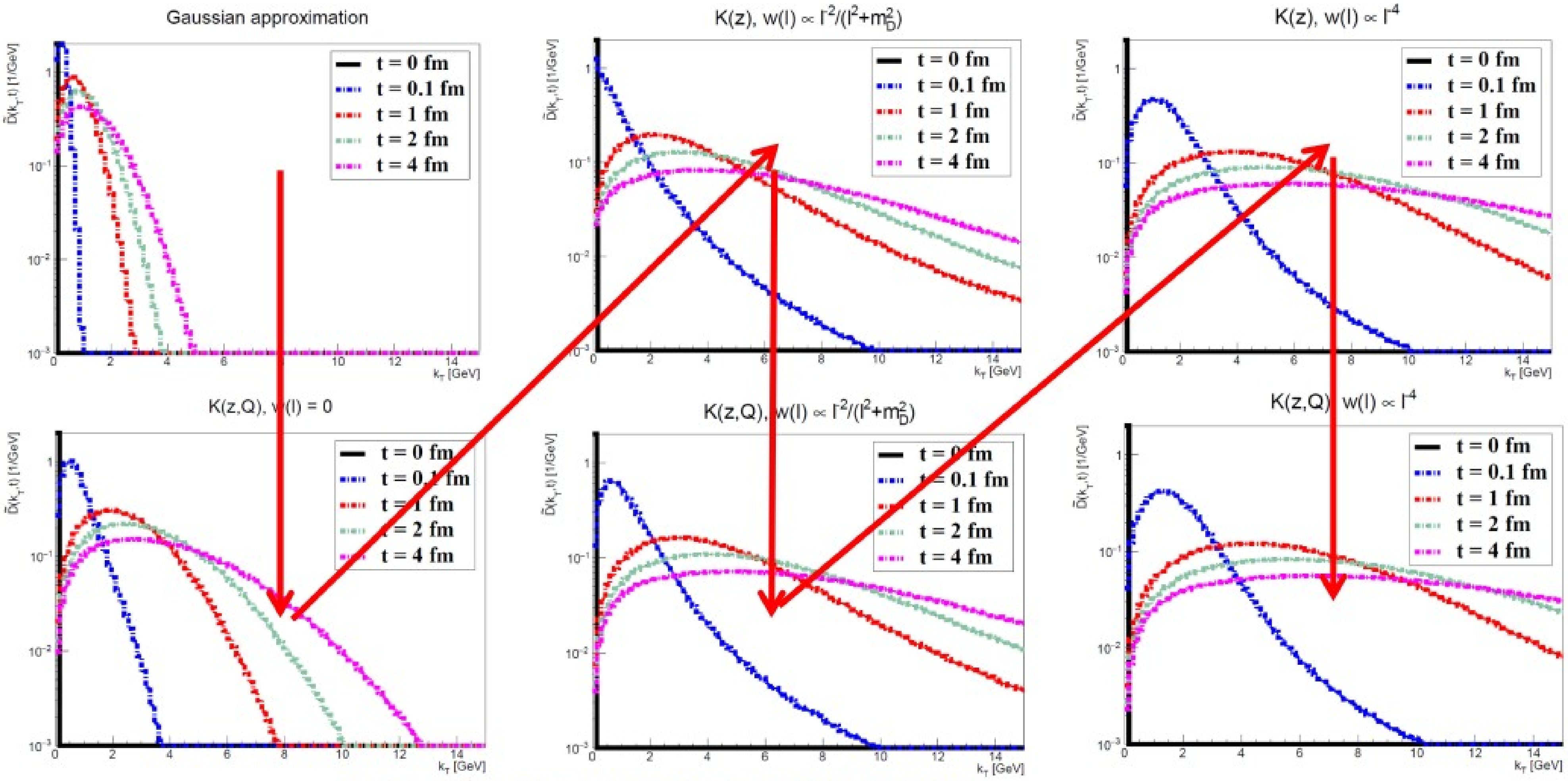
$$+\int_{\tau_0}^{\tau} d\tau'\int_0^1 dz\int_0^1 dy\int d^2\mathbf{k}'\int d^2\mathbf{q}\;\mathcal{G}(z,\mathbf{q})$$
$$\times\,\delta(x - zy)\,\delta(\mathbf{k}-\mathbf{q}-z\mathbf{k}')\,e^{-\Psi(x)(\tau-\tau')}D(y,\mathbf{k}',\tau')$$

k_T distributions and broadening



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

k_T distributions and broadening



[Blanco, Kutak, Płaczek, MR, Straka, JHEP 04(2021)014]

System of equations for quarks and gluons

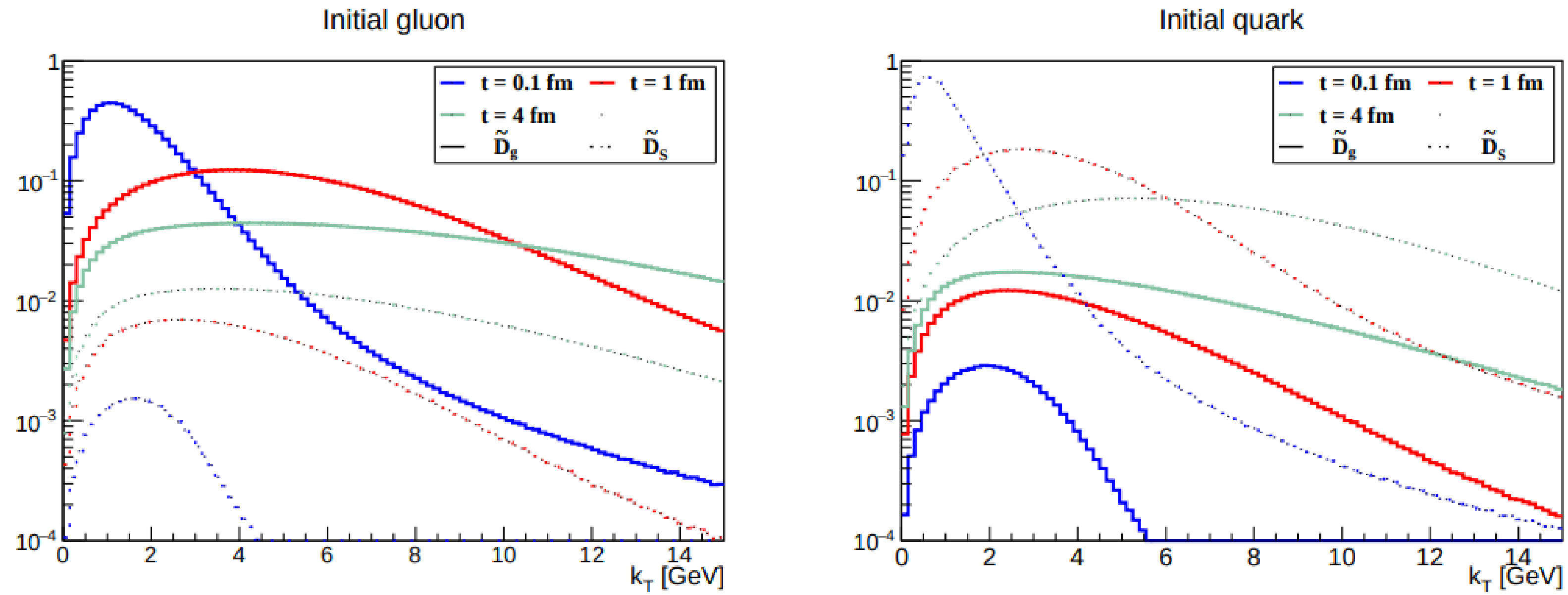
E. Blanco, K. Kutak, W. Placzek, M. Rohrmoser, K. Tywoniuk '21

$$\frac{\partial}{\partial t} D_g(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \alpha_s \Big\{ 2\mathcal{K}_{gg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z}, \boldsymbol{q}, t\right) + \mathcal{K}_{gq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) \sum_i D_{q_i}\left(\frac{x}{z}, \boldsymbol{q}, t\right)$$

$$- \left[\mathcal{K}_{gg}(\boldsymbol{q}, z, xp_0^+) + \mathcal{K}_{qg}(\boldsymbol{q}, z, xp_0^+)\right] D_g(x,\boldsymbol{k},t) \Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_g(\boldsymbol{l})\, D_g(x, \boldsymbol{k}-\boldsymbol{l}, t),$$

$$\frac{\partial}{\partial t} D_{q_i}(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \alpha_s \Big\{ \mathcal{K}_{qq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_{q_i}\left(\frac{x}{z}, \boldsymbol{q}, t\right) + \frac{1}{N_F}\mathcal{K}_{qg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z}, \boldsymbol{q}, t\right)$$

$$- \mathcal{K}_{qq}(\boldsymbol{q}, z, xp_0^+)\, D_{q_i}(x,\boldsymbol{k},t) \Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_q(\boldsymbol{l})\, D_{q_i}(x, \boldsymbol{k}-\boldsymbol{l}, t)\,,$$



Gluons broader more then quarks

System of equations for quarks and gluons

$$\frac{\partial}{\partial t} D_g(x,\boldsymbol{k},t) = \int_0^1 \mathrm{d}z \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \alpha_s \Big\{ 2\mathcal{K}_{gg}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) D_g\left(\frac{x}{z},\boldsymbol{q},t\right) + \mathcal{K}_{gq}\left(\boldsymbol{Q}, z, \frac{x}{z}p_0^+\right) \sum_i D_{q_i}\left(\frac{x}{z},\boldsymbol{q},t\right)$$
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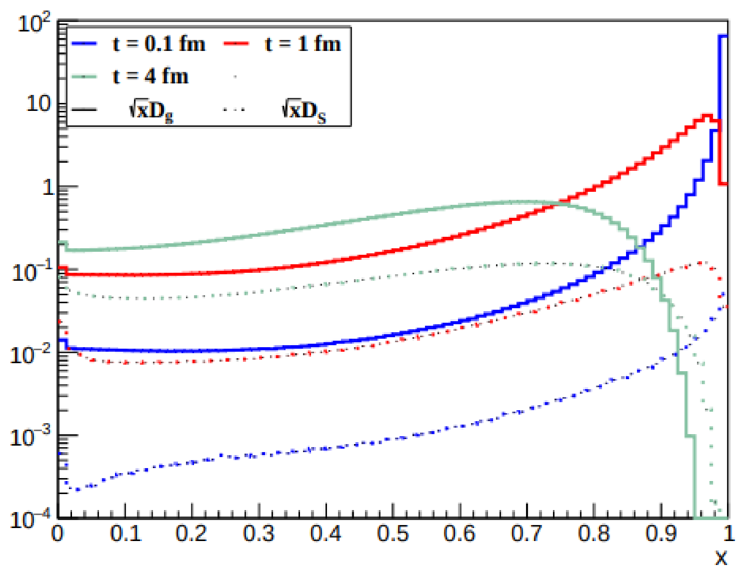
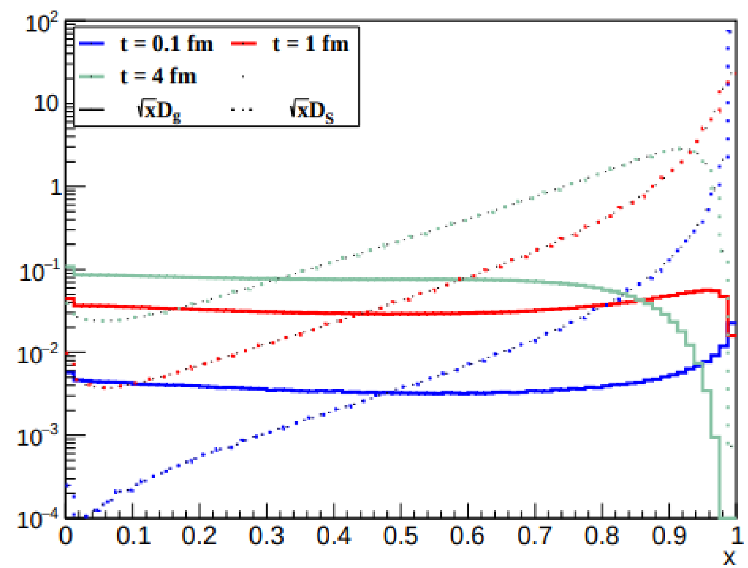
$$- \mathcal{K}_{qq}(\boldsymbol{q}, z, xp_0^+)\, D_{q_i}(x,\boldsymbol{k},t) \Big\} + \int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2} C_q(\boldsymbol{l})\, D_{q_i}(x,\boldsymbol{k}-\boldsymbol{l},t)\,,$$

Initial gluon



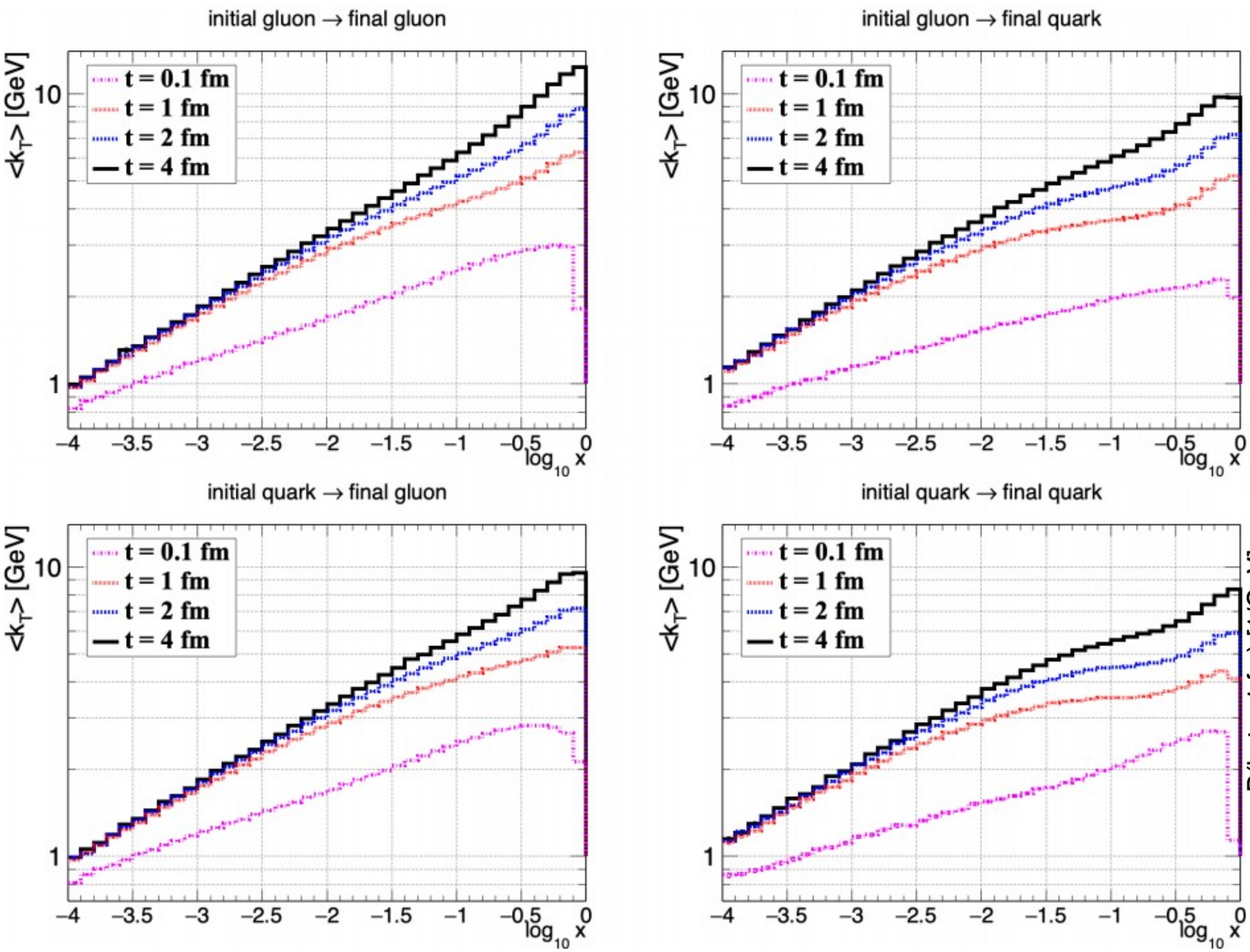
Initial quark



After integrating over transverse momentum we reproduce equations from Mehtar-Tani Schlichting JHEP 09 (2018) 144

Quenching line

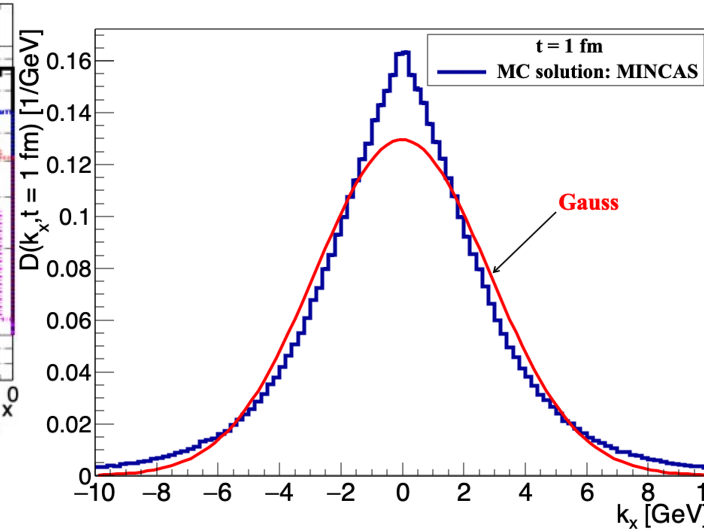
Kutak, Płaczek, Straka Eur.Phys.J.C 79 (2019) 4, 317



$$\langle k_T \rangle = \frac{\int \mathrm{d}^2\boldsymbol{k}\, |\boldsymbol{k}| D(x, \boldsymbol{k}, t)}{\int \mathrm{d}^2\boldsymbol{k}\, D(x, \boldsymbol{k}, t)}$$

Similar average for kT2 does not converge. The integral above diverges

This is not the case in Gaussian approximation



Thermalized medium suppresses jets stronger

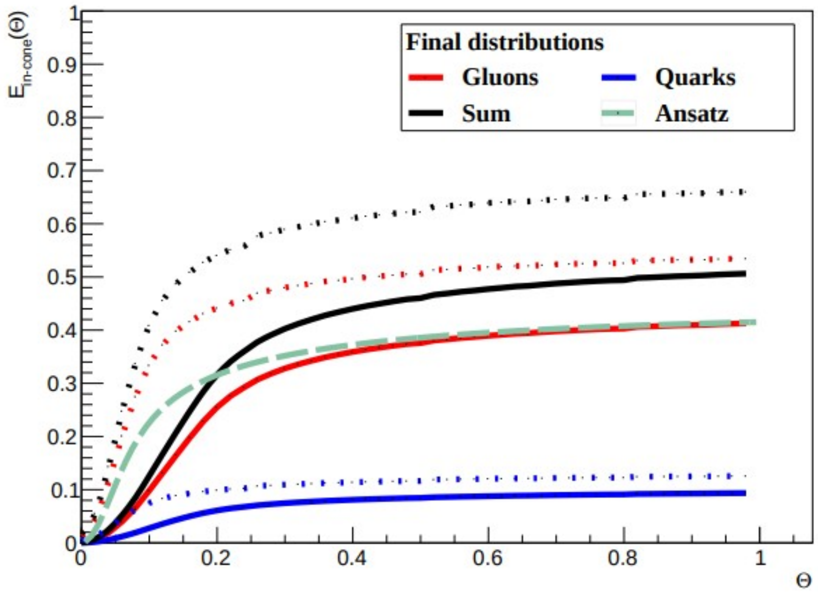
Universal behavior at larger times

Similar line obtained in from analitical approximated solution by Blaizot, Torres, Mehtar-Tani

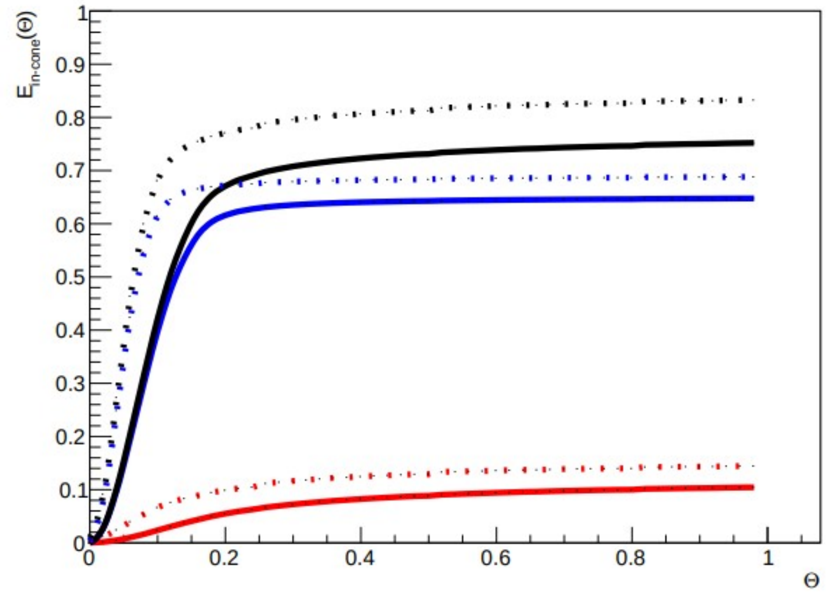
Energy in a cone – angle dependence

$$E_{\text{in-cone}}(\Theta) = \int_0^1 \mathrm{d}x \int_0^{xE\sin\Theta} \mathrm{d}k_T\, \tilde{D}(x, k_T, t)$$

Initial gluon



Initial quark



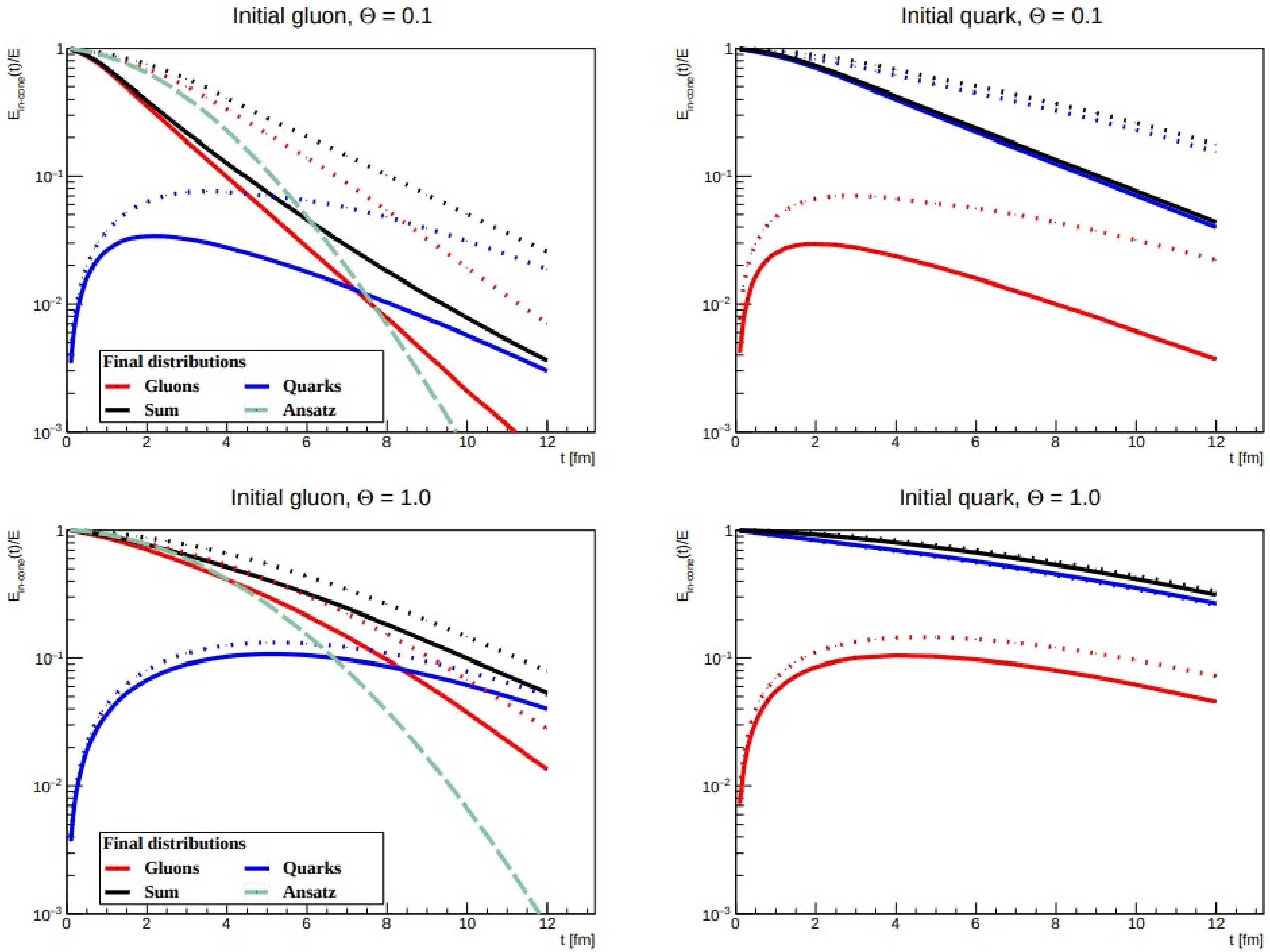
Quarks are more colimated
more energy in the cone

For the Ansatz curve and dotted lines we use

$$D_{\mathrm{G}}(x, \boldsymbol{k}, t) = D(x, t)\, P(\boldsymbol{k}, t) \qquad P(\boldsymbol{k}, t) = \frac{4\pi}{\hat{q}t} \exp\left(-\frac{\boldsymbol{k}^2}{\hat{q}t}\right)$$

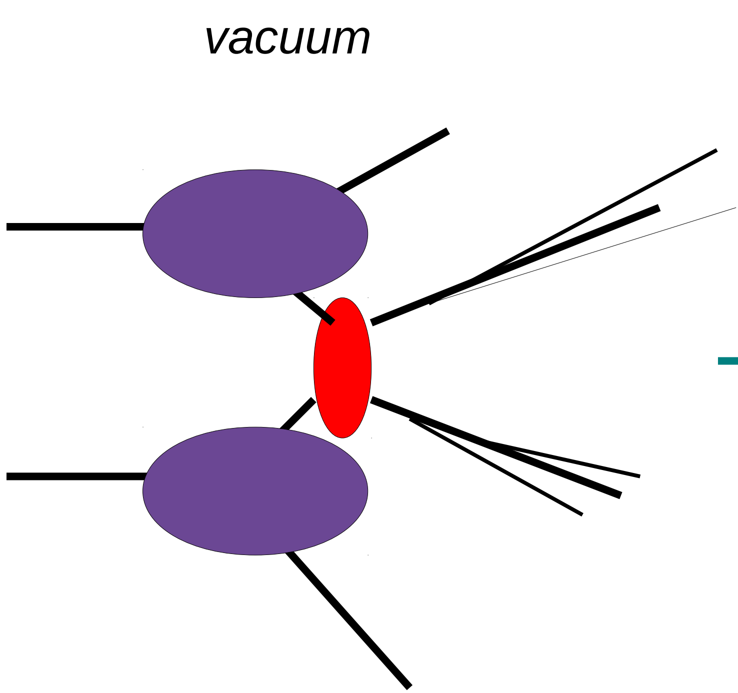
And the x distributions are obtained either from approx analitycal solution or solution of eq
Integrated over transverse momentum

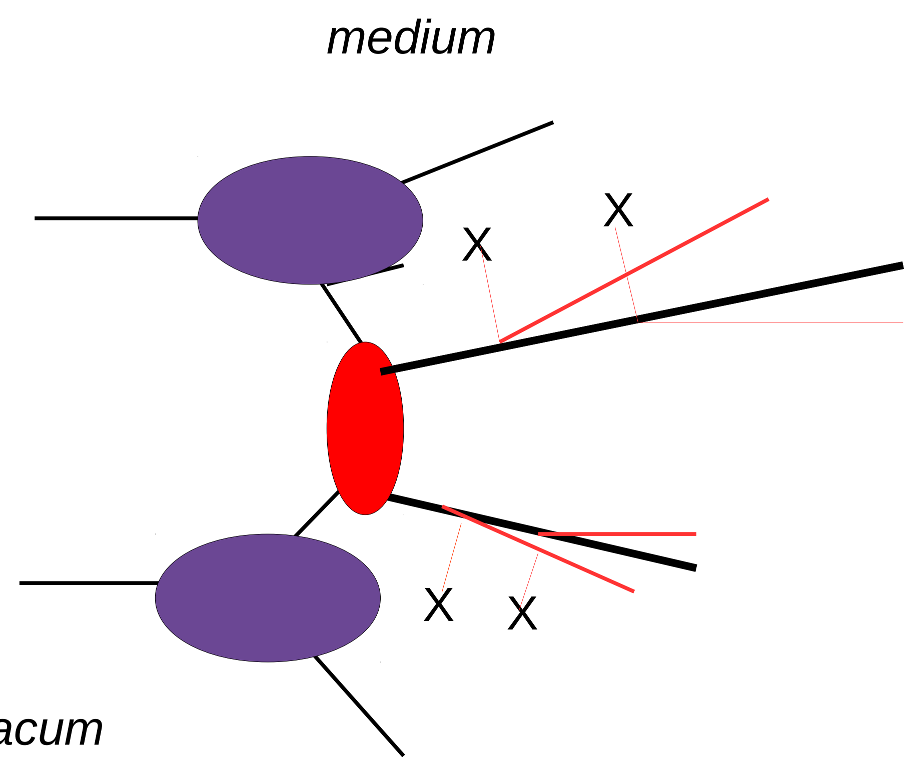
Energy in a cone – time dependence



Quarks dominate at large times

From vacuum to medium





*vacuum x-section = ME * pdf * fragmentaion in vacum*

*complete x-section = ME * pdf * fragmentaion in medium +*
*ME * pdf * fragmentation in vacum*

From vacuum to medium

1911.05463
Van Hameren, Kutak,Placzek, Rohrmoser, Tywoniuk'19

$$\frac{d\sigma_{pp}}{dy_1 dy_2 d^2q_{1T} d^2q_{2T}} = \int \frac{d^2k_{1T}}{\pi}\frac{d^2k_{2T}}{\pi}\frac{1}{16\pi^2(x_1x_2s)^2}\overline{|\mathcal{M}^{\text{off-shell}}_{g^*g^*\to gg}|^2} \times \delta^2\left(\vec{k}_{1T}+\vec{k}_{2T}-\vec{q}_{1T}-\vec{q}_{2T}\right)\mathcal{F}_g(x_1,k_{1T}^2,\mu_F^2)\,\mathcal{F}_g(x_2,k_{2T}^2,\mu_F^2)$$

$$\frac{d\sigma_{AA}}{d\Omega_{p_1}\Omega_{p_2}} = \int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \int_0^1 \frac{d\tilde{x}_1}{\tilde{x}_1^2}\int_0^1 \frac{d\tilde{x}_2}{\tilde{x}_2^2} D(\tilde{x}_1,\boldsymbol{p}_1-\boldsymbol{q}_1,\tau(p_1^+/\tilde{x}_1))D(\tilde{x}_2,\boldsymbol{p}_2-\boldsymbol{q}_2,\tau(p_2^+/\tilde{x}_2)) \left.\frac{d\sigma_{pp}}{dq_1^+dq_2^+d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2}\right|_{q_1^+=p_1^+/\tilde{x}_1,\,q_2^+=p_2^+/\tilde{x}_2}$$

Our assumptions:

- *only gluonic jets*
- *uniform plasma*
- *we neglect shower outside of plasma*
- *we neglect vacuum like emissions in plasma*
- *we assume Bjorken model to tune the temperature to describe R_{AA}*

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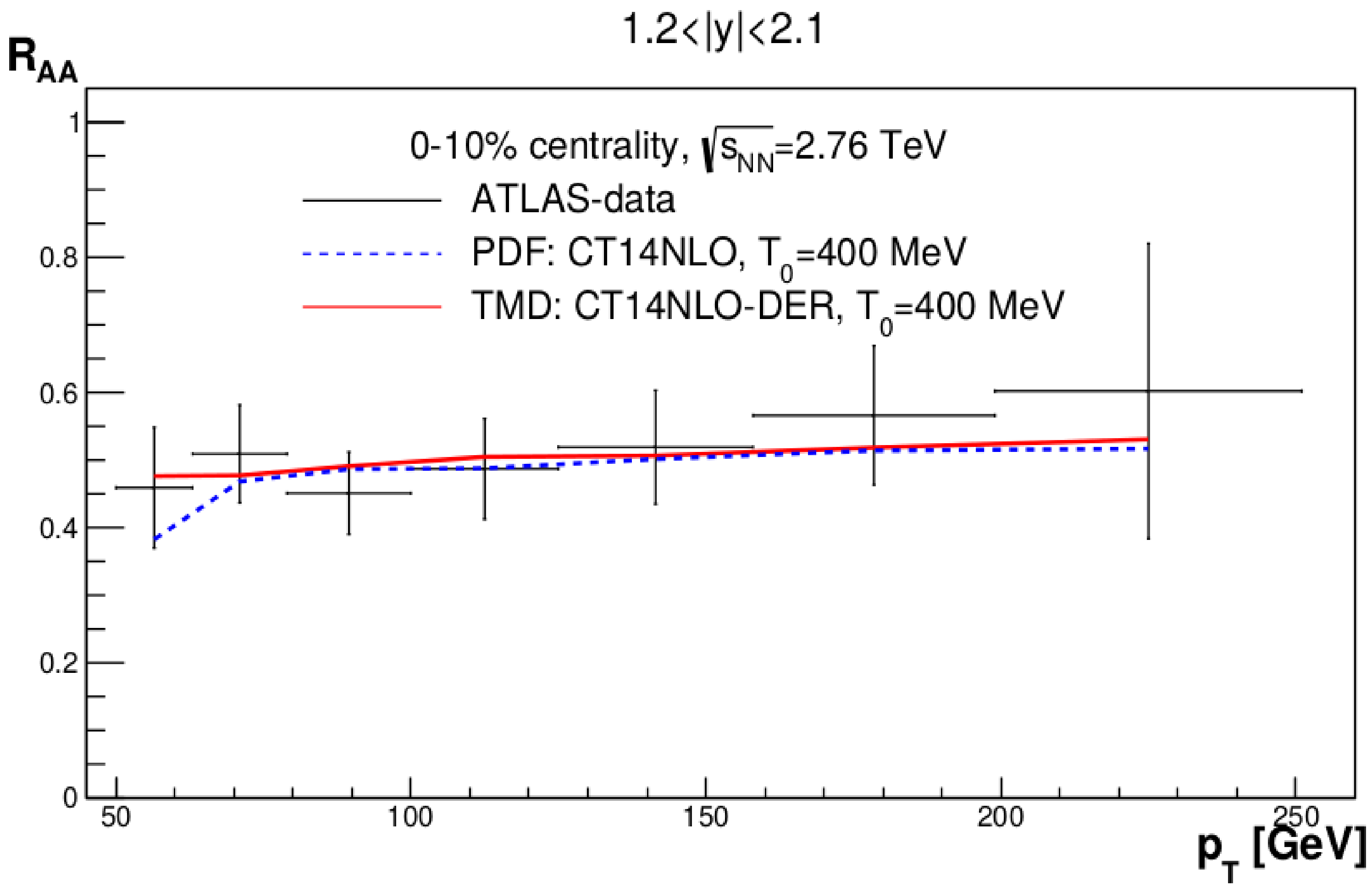
R_{AA} nuclear modificatio ratio



1911.05463
Van Hameren, Kutak,Placzek, Rohrmoser, Tywoniuk

Obtained using Monte Carlo
KaTie (hard cross-section) + MINCAS (jet quenching part)

Azimutal decorelations

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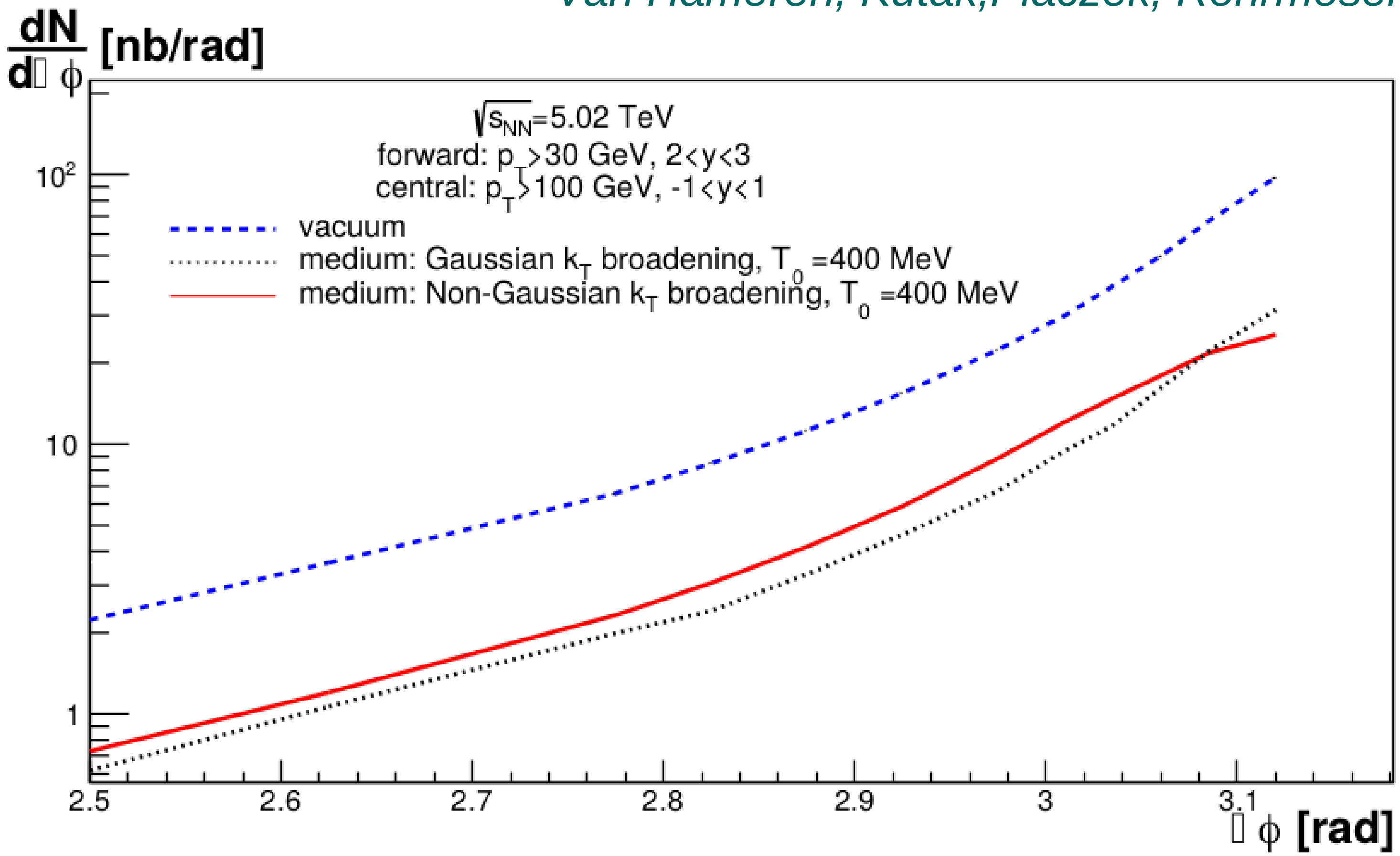
Van Hameren, Kutak,Placzek, Rohrmoser, Tywoniuk



Suppression at large angles
Enhancement at moderate angles

Obtained using Monte Carlo
KaTie (hard cross-section) + MINCAS (jet quenching part)

Other effects not discussed here

- *Vacum emissions, vacuum like emissions in medium: DGLAP. Medium like emissions: generalized BDMPS*
  Caucal, Iancu, Mueller, Soyez Phys. Rev. Lett. 120, 232001 (2018)
  Caucal, Iancu, Mueller, Soyez, JHEP 10 (2019) 273

- *Interferences of emissions in medium and outside of medium and expansion of medium - negative corrections to broadening*
  Zakharov Zh.Eksp.Teor.Fiz. 156 (2019) 615-637

- *Soft gluon approximation relaxed but limited to low x*
  Andres, Apolinario, Dominguez arxiv:2002.01517

- *Higher order corrections to jet quenching parameter*
  Mehtar-Tani, Tywoniuk arxiv 1910.02032

- *Rate equation for energy solved in expanding medium only energy distribution. No kt dependence*
  Adhya, Tywoniuk, Salgado, Spousta arxiv 1911.12193

Summary and outlook

- *we obtained solution of equation for gluon distribution in medium that depends on t, x,* k_T
- *We obtained system of equations for quarks and gluons and solved them*
- *combination of MINCAS with KaTiE: allows for calculation of jet-observables within* k_T *factorization approach*
- *results differ from pure Gaussian broadening. In back-to-back region cross section is suppressed. In moderate angles it is enhanced.*
- *Momentum transfer during branching is significant*

In the future we want to study more forward processes and in particular combine jet quenching and saturation, add vacuum like emissions,

Generating functional

$$D(x,\boldsymbol{k},t) = k^+ \frac{\mathrm{d}N}{\mathrm{d}k^+ \mathrm{d}^2\boldsymbol{k}} \equiv k^+ \left\langle \sum_{n=1}^{\infty} \sum_{j=1}^{n} \delta^{(3)}(\vec{k}_j - \vec{k}) \right\rangle$$

Inclusive gluon distribution as produced by hard jet

$$= k^+ \frac{\delta \mathcal{Z}_{p_0}[t, t_0|u]}{\delta u(\vec{k})} \bigg|_{u=1}$$

$$\mathcal{Z}_{p_0}[t, t_0|u] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \left(\prod_{i=1}^{n} \mathrm{d}\Omega_i \right) P_n(\vec{k}_1, \cdots, \vec{k}_n; t, t_0)\, u(\vec{k}_1) \cdots u(\vec{k}_n)$$

Blaizot, Dominguez, Iancu, Mehtar-Tani'13

t is lightcone time x+

P_n is a probability density to find exactly n gluons

Calculable in QCD

u(k) generic function