


$$g \log(2) = \lambda_g \log(2) + \nu_2(2i\pi)$$

Observation of Odderon Effects at LHC energies

A Real Extended Bialas-Bzdak Model Study

based on [T. Csörgő, I. Szanyi, *Eur. Phys. J. C* **81**, 611 \(2021\)](#) and other recent results

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Unitarity and the elastic amplitude

- unitarity of the scattering matrix S :

$$SS^\dagger = I$$

$$S = I + iT$$

$$T - T^\dagger = iTT^\dagger$$

- unitarity equation in impact parameter b representation:

$$2 \operatorname{Im} t_{el}(s, \vec{b}) = |t_{el}(s, \vec{b})|^2 + \tilde{\sigma}_{in}(s, \vec{b})$$

(s is the squared CM energy)

- the elastic amplitude can be written as a solution of the unitarity equation in terms of $\tilde{\sigma}_{in}$
- $0 \leq \tilde{\sigma}_{in}(s, \vec{b}) \leq 1$ and $\tilde{\sigma}_{in}$ can be calculated using the rules of the probability calculus based on Glauber's multiple scattering theory

Bialas-Bzdak p=(q,d) model

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2\vec{s}_q d^2\vec{s}'_q d^2\vec{s}_d d^2\vec{s}'_d D(\vec{s}_q, \vec{s}_d) D(\vec{s}'_q, \vec{s}'_d) \sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b})$$

- quark-diquark distribution inside the proton:

$$D(\vec{s}_q, \vec{s}_d) = \frac{1 + \lambda^2}{R_{qd}^2 \pi} e^{-\frac{s_q^2 + s_d^2}{R_{qd}^2}} \delta^2(\vec{s}_q + \lambda \vec{s}_d)$$

$$\lambda = \frac{m_q}{m_d}$$

$$\vec{s}_d = -\lambda \vec{s}_q$$

$$\vec{s}'_d = -\lambda \vec{s}'_q$$

[A. Bialas, A. Bzdak Acta Phys.Polon. B 38, 159-168 \(2007\)](#)

- interaction probability of the constituents:

$$\sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b}) = 1 - \prod_a \prod_b [1 - \sigma_{ab}(\vec{b} + \vec{s}'_a - \vec{s}_b)]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-|\vec{s}|^2/S_{ab}^2}$$

$$S_{ab}^2 = R_a^2 + R_b^2$$

$$a, b \in \{q, d\}$$

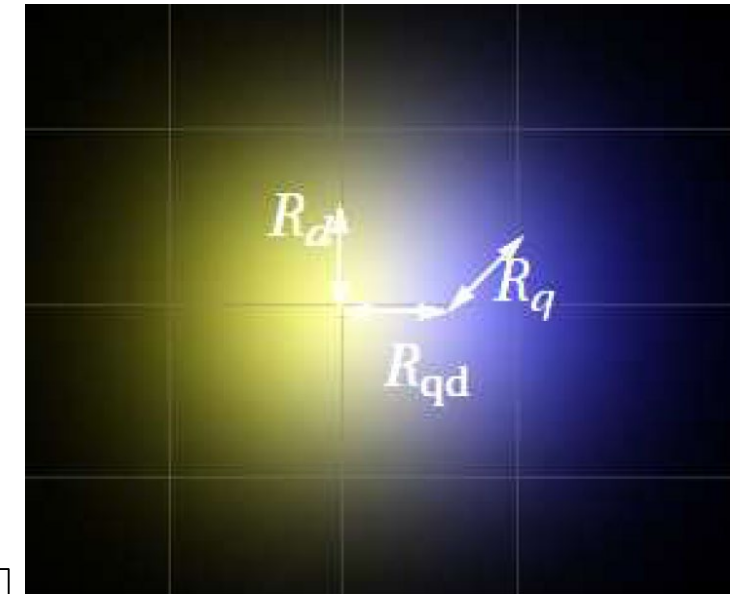
- inelastic cross-sections of quark, diquark scatterings :

$$\sigma_{ab,in} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{ab}(\vec{s}) d^2\vec{s}$$

$$\sigma_{qq,in} : \sigma_{qd,in} : \sigma_{dd,in} = 1 : 2 : 4$$

- free parameters:

$$A_{qq}, \lambda, R_q, R_d, R_{qd}, \quad (A_{qq} = 1 \text{ and } \lambda = 0.5 \text{ can be fixed})$$



Proton-(anti)proton scattering in the quark-diquark model.

Unitarily Real Extended Bialas-Bzdak (ReBB) model

- elastic scattering amplitude in the impact parameter space:

$$t_{el}(s, \vec{b}) = i \left[1 - e^{-\Omega(s, \vec{b})} \right]$$

arXiv:1505.01415

F. Nemes, T. Csörgő, M. Csanád, Int. J. Mod. Phys. A Vol. 30 (2015) 1550076

- the opacity function:

$$\Omega(s, \vec{b}) = \text{Re}\Omega(s, \vec{b}) + i \text{Im}\Omega(s, \vec{b})$$

$\text{Im}\Omega \neq 0$ as the real part of the amplitude is not negligibly small

$$\text{Re}\Omega(s, \vec{b}) = -\frac{1}{2} \ln[1 - \tilde{\sigma}_{in}(s, \vec{b})]$$

$$\text{Im}\Omega(s, \vec{b}) = -\alpha \tilde{\sigma}_{in}(s, \vec{b})$$

NEW FREE PARAMETER

- elastic scattering amplitude in momentum space:

$$T(s, t) = 2\pi \int_0^\infty t_{el}(s, |\vec{b}|) J_0(|\vec{\Delta}| |\vec{b}|) |\vec{b}| d|\vec{b}|$$

$$|\vec{\Delta}| \equiv \sqrt{-t} \text{ as } \sqrt{s} \rightarrow \infty$$

(t is the squared momentum transfer)

Measurable quantities

- differential cross section:

$$\frac{d\sigma}{dt}(s, t) = \frac{1}{4\pi} |T(s, t)|^2$$

- total, elastic and inelastic cross sections:

$$\sigma_{tot}(s) = 2\text{Im}T(s, t = 0)$$

$$\sigma_{el}(s) = \int_{-\infty}^0 \frac{d\sigma(s, t)}{dt} dt$$

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$$

- ratio ρ_0 :

$$\rho_0(s) = \lim_{t \rightarrow 0} \rho(s, t) \equiv \frac{\text{Re}T(s, t \rightarrow 0)}{\text{Im}T(s, t \rightarrow 0)}$$

- slope of $d\sigma/dt$:

$$B_0(s) = \lim_{t \rightarrow 0} B(s, t) \equiv \frac{d}{dt} \left(\ln \frac{d\sigma}{dt}(s, t) \right) \Big|_{t \rightarrow 0}$$

The pp and $p\bar{p}$ elastic scattering amplitude

- according to the Regge formalism the strong scattering amplitude for pp and $p\bar{p}$ scattering can be written in terms of $C = +1$ and $C = -1$ exchange components

$$T^{pp}(s, t) = T^+(s, t) - T^-(s, t)$$

$$T^{p\bar{p}}(s, t) = T^+(s, t) + T^-(s, t)$$

- for $\sqrt{s} \gtrsim 1$ TeV the mesonic reggeon exchanges are negligible only the gluonic Pomeron and Odderon exchanges are present implying that

$$T^+(s, t) \equiv T^P(s, t)$$

$$T^-(s, t) \equiv T^O(s, t)$$



$$T^P(s, t) = \frac{1}{2} (T^{pp}(s, t) + T^{p\bar{p}}(s, t))$$

$$T^O(s, t) = \frac{1}{2} (T^{p\bar{p}}(s, t) - T^{pp}(s, t))$$

- a simple and model independent consequence:

$$\text{if } \frac{d\sigma^{pp}}{dt}(s, t) \neq \frac{d\sigma^{p\bar{p}}}{dt}(s, t) \text{ for } \sqrt{s} \gtrsim 1 \text{ TeV then } T^O(s, t) \neq 0$$

Fit method

- least squares fitting with:

$$\chi^2 = \left(\sum_{j=1}^M \left(\sum_{i=1}^{n_j} \frac{(d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj}d_{ij}\sigma_{cj} - th_{ij})^2}{\tilde{\sigma}_{ij}^2} \right) + \epsilon_{bj}^2 + \epsilon_{cj}^2 \right) + \left(\frac{d_{\sigma_{tot}} - th_{\sigma_{tot}}}{\delta\sigma_{tot}} \right)^2 + \left(\frac{d_{\rho_0} - th_{\rho_0}}{\delta\rho_0} \right)^2$$

$$\tilde{\sigma}_{ij}^2 = \tilde{\sigma}_{aij} \left(\frac{d_{ij} + \epsilon_{bj}\tilde{\sigma}_{bij} + \epsilon_{cj}d_{ij}\sigma_{cj}}{d_{ij}} \right)$$

$$\tilde{\sigma}_{kij} = \sqrt{\sigma_{kij}^2 + (d'_{ij}\delta_k t_{ij})^2}, \quad k \in \{a, b\}$$

[A. Adare et al. \(PHENIX Collab.\)](#)

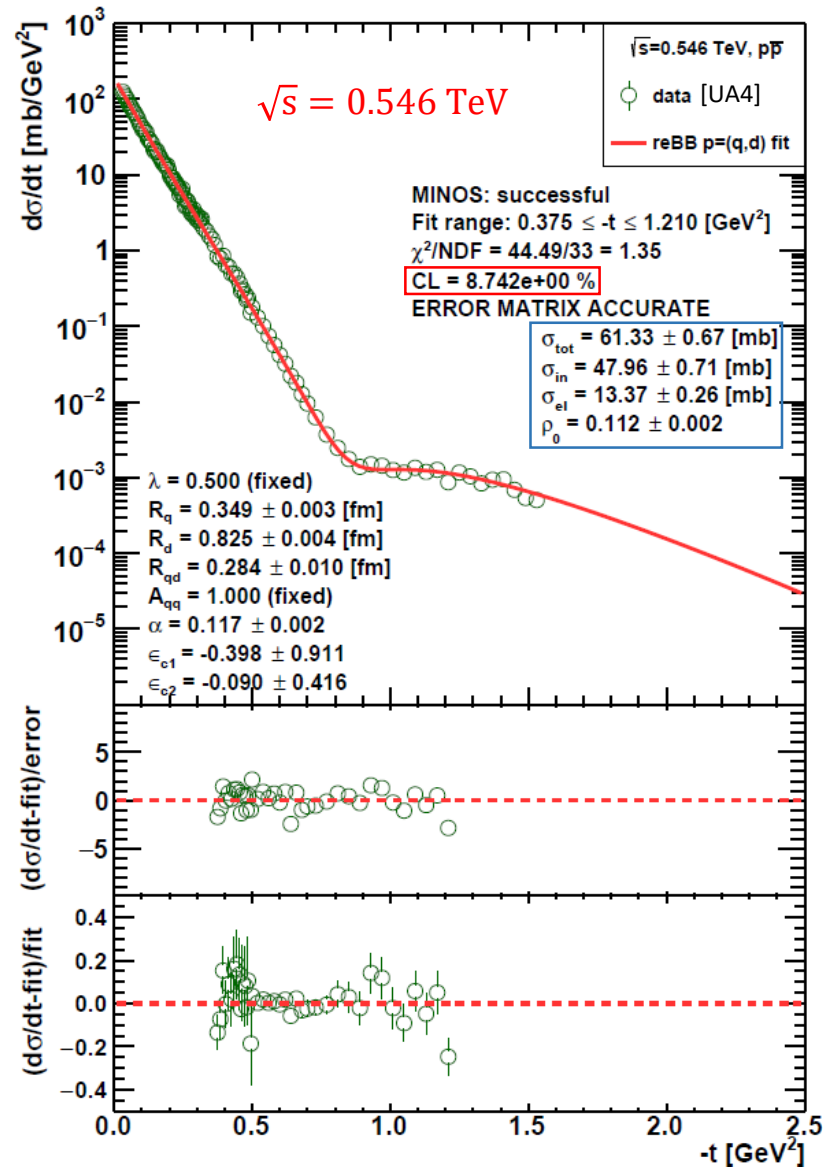
[Phys. Rev. C 77, 064907](#)

- it takes into account (in M separately measured t ranges):

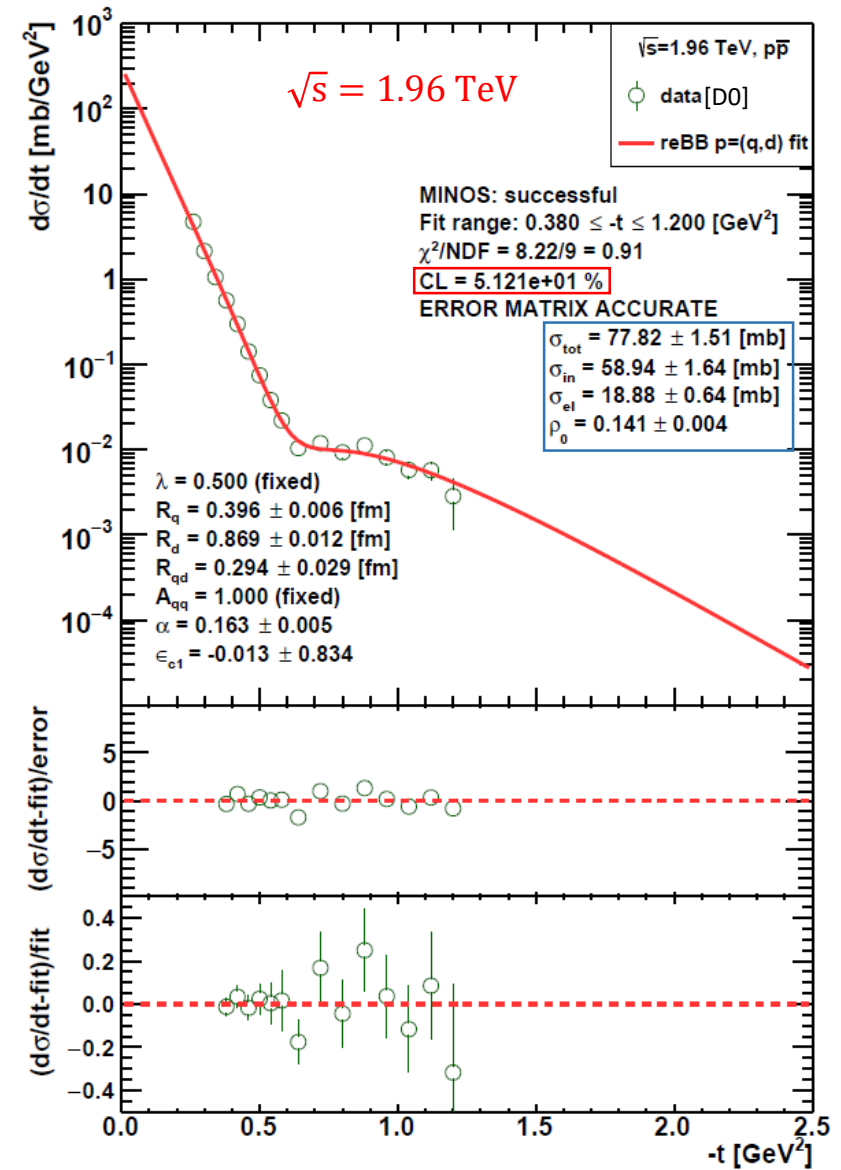
- the t -dependent statistical (a) and systematic (b) errors (both vertical σ_k and horizontal $\delta_k t$) $\rightarrow \epsilon_b$ parameters;
- the t -independent σ_c normalization uncertainties $\rightarrow \epsilon_c$ parameters;
- the measured total cross-section $d_{\sigma_{tot}}$ and ratio d_{ρ_0} and their total uncertainties $\delta\sigma_{tot}$ and $\delta\rho_0$.

- minimization with **CERN Root MINUIT**, parameter error estimation by **MINOS**.

Satisfactory ReBB model fits for $p\bar{p}$ $d\sigma/dt$ data

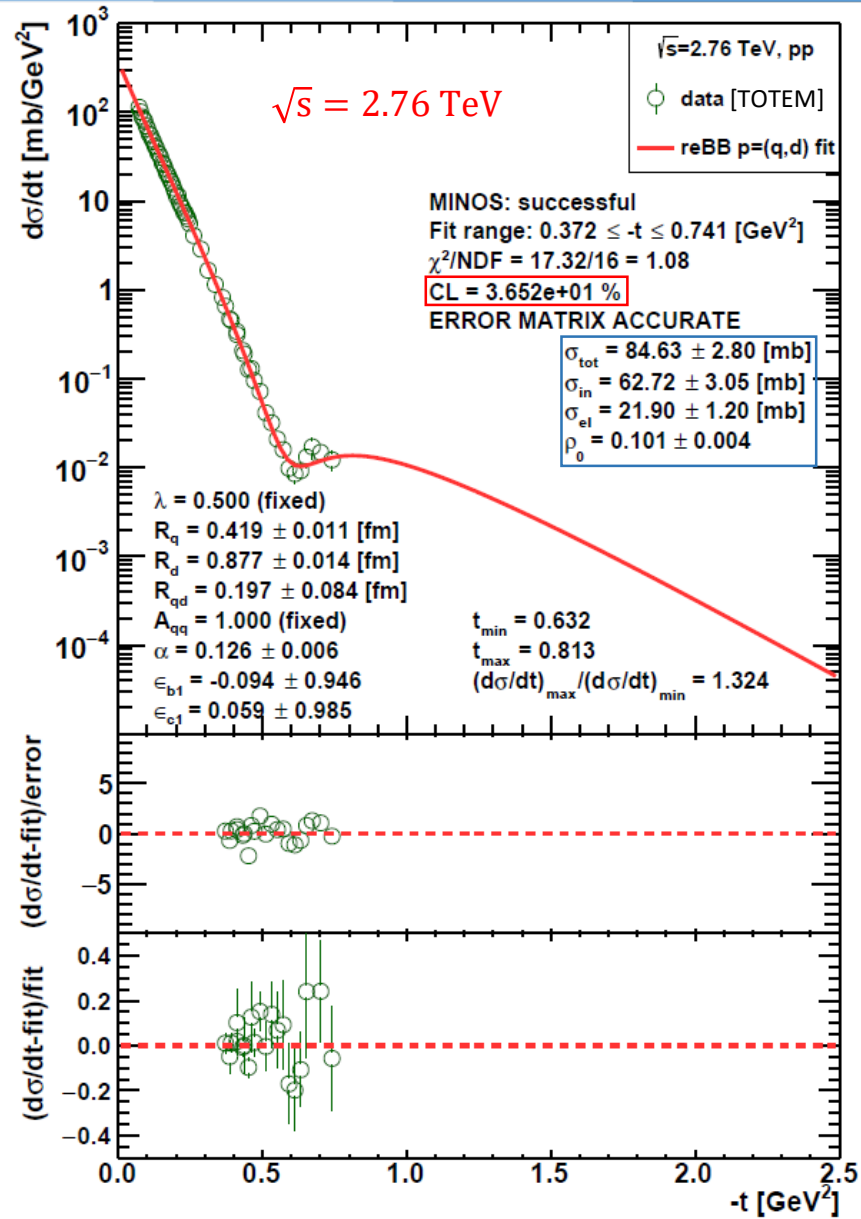


data: UA4 Collab. (SPS) Phys.Lett. 127B (1983)
UA4 Collab. (SPS) Phys.Lett. 155B (1985)

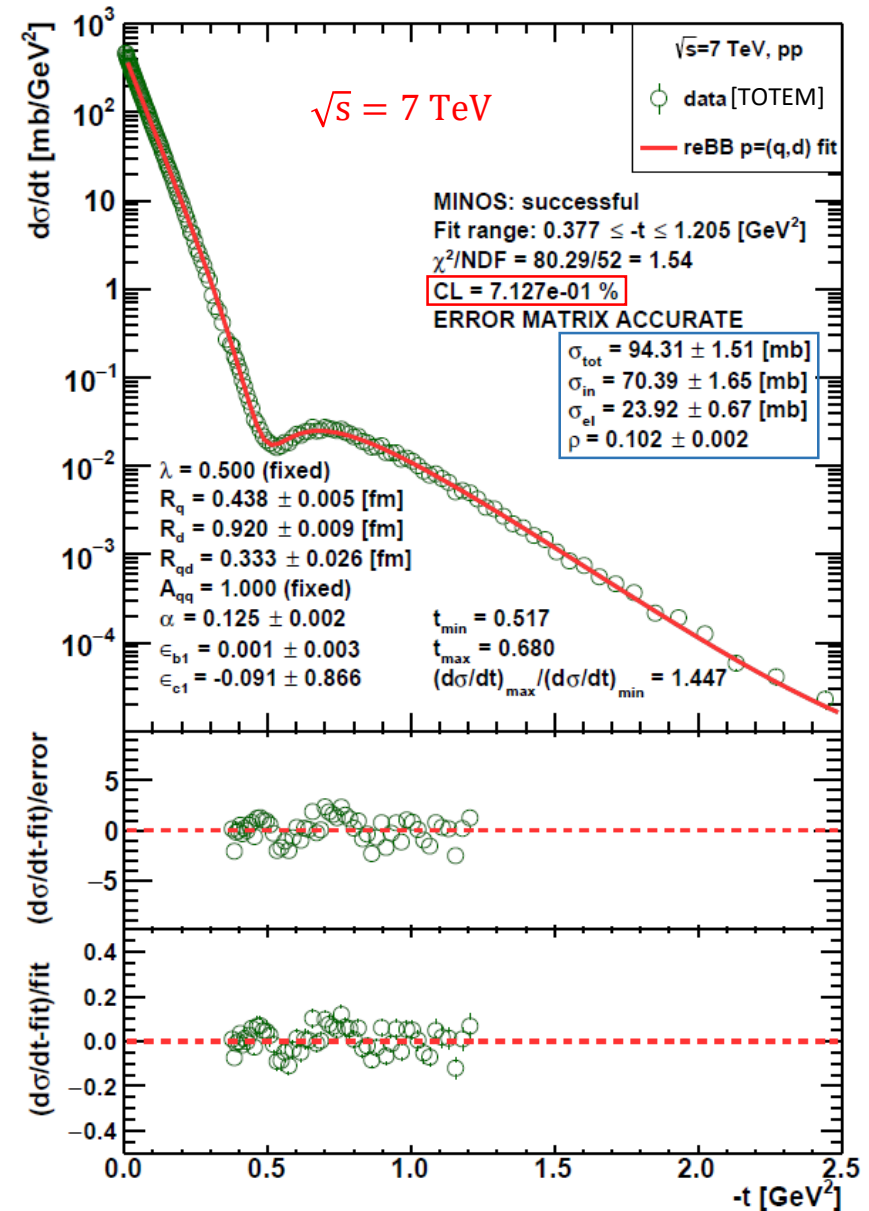


data: D0 Collab. (TEVATRON) Phys.Rev. D86 (2012)

Satisfactory ReBB model fits for pp dσ/dt data

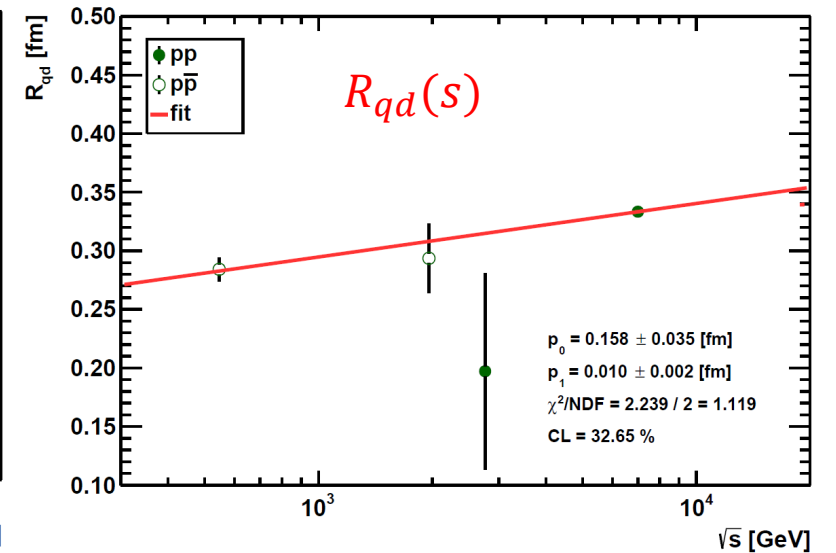
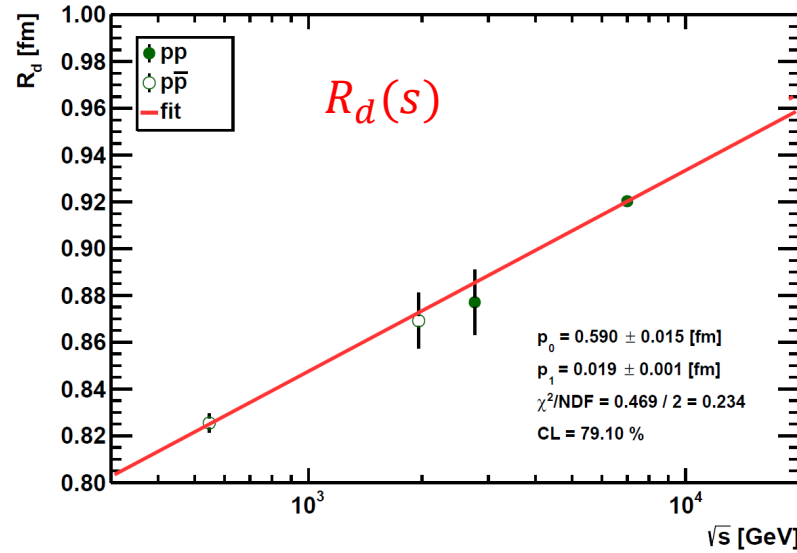
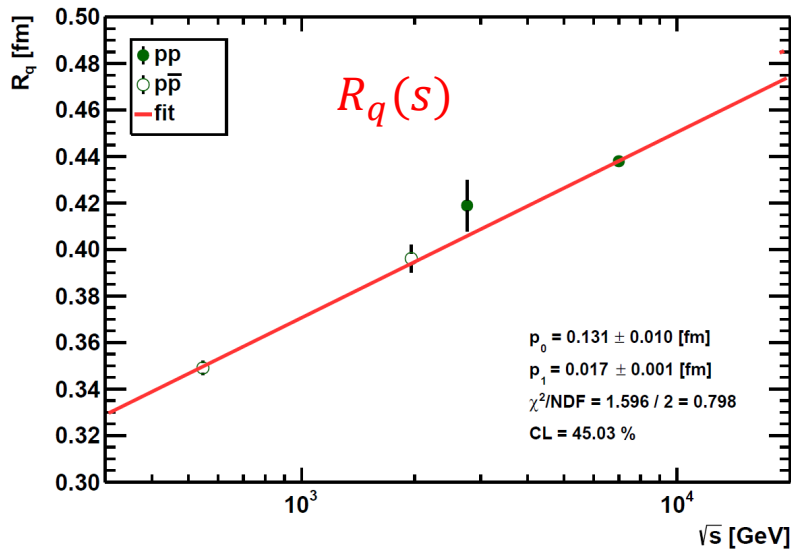


data: TOTEM Collab. (LHC) Eur. Phys. J. C 80 (2020) 91



data: TOTEM Collab. (LHC) Europhys.Lett. 98 (2012)

Energy dependences of the scale parameters



The energy dependences of the scale parameters, R_q , R_d and R_{qd} , are the same for pp and $p\bar{p}$ processes!

$$P(s) = p_0 + p_1 \ln(s/s_0)$$

$$P \in \{R_q, R_d, R_{qd}, \alpha\}$$

$$s_0 = 1 \text{ GeV}^2$$

Parameter	R_q [fm]	R_d [fm]	R_{qd} [fm]
χ^2/NDF	1.596/2	0.469/2	2.239/2
CL [%]	45.03	79.10	32.65
p_0	0.131 ± 0.010	0.590 ± 0.015	0.158 ± 0.035
p_1	0.017 ± 0.001	0.019 ± 0.001	0.010 ± 0.002

Parameters which define the energy dependence of the ReBB model scale parameters

New: proportionality between $\rho_0(s)$ and $\alpha(s)$

$$t_{el}(s, b) = i \left(1 - e^{i \alpha \tilde{\sigma}_{in}(s, b)} \sqrt{1 - \tilde{\sigma}_{in}(s, b)} \right)$$

$$\alpha \tilde{\sigma}_{in} \ll 1$$

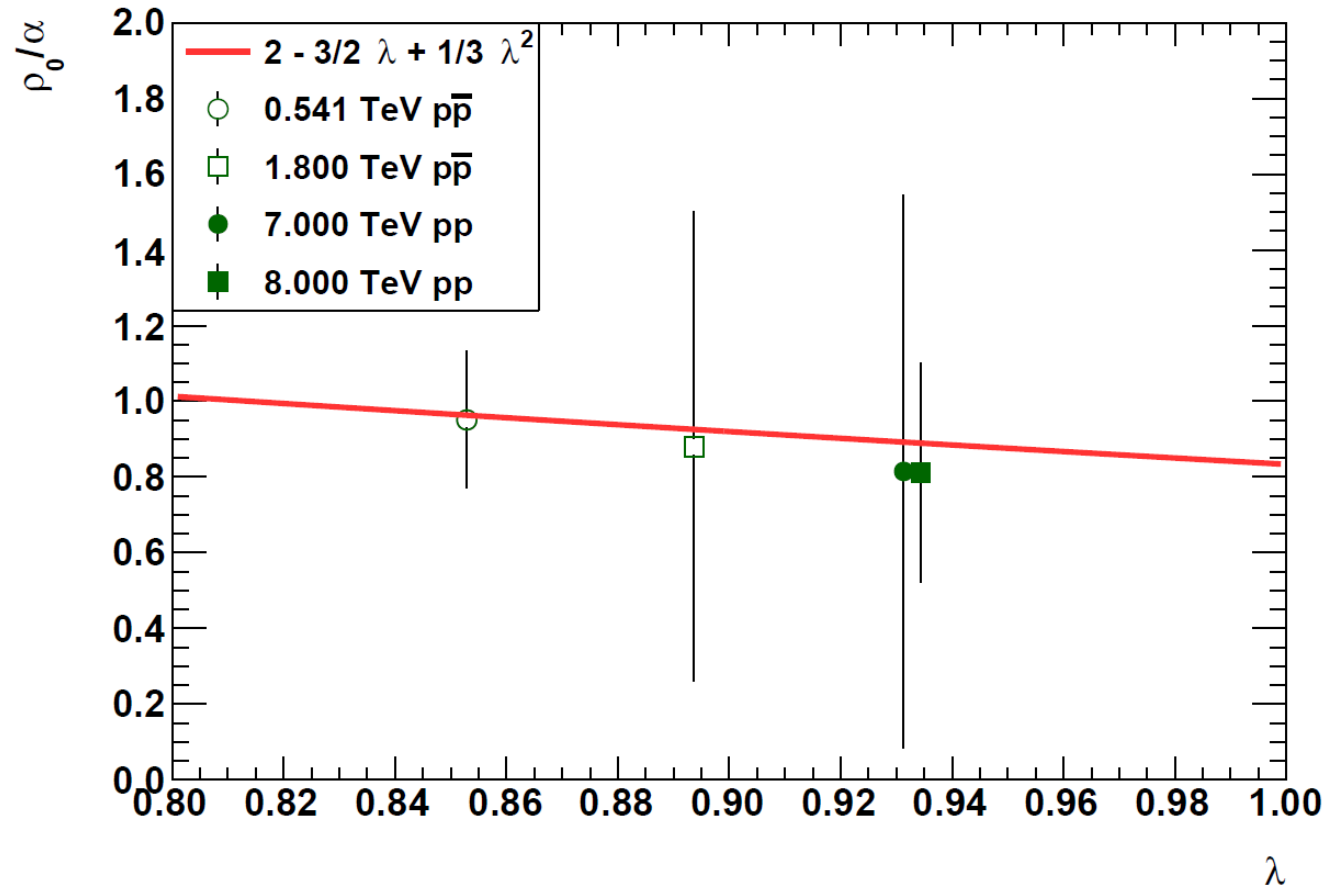
$$\text{Im } t_{el}(s, b) \simeq \lambda(s) \exp\left(-\frac{b^2}{2R^2(s)}\right)$$



$$\rho_0(s) = \alpha(s) \left(2 - \frac{3}{2} \lambda(s) + \frac{1}{3} \lambda^2(s) \right)$$

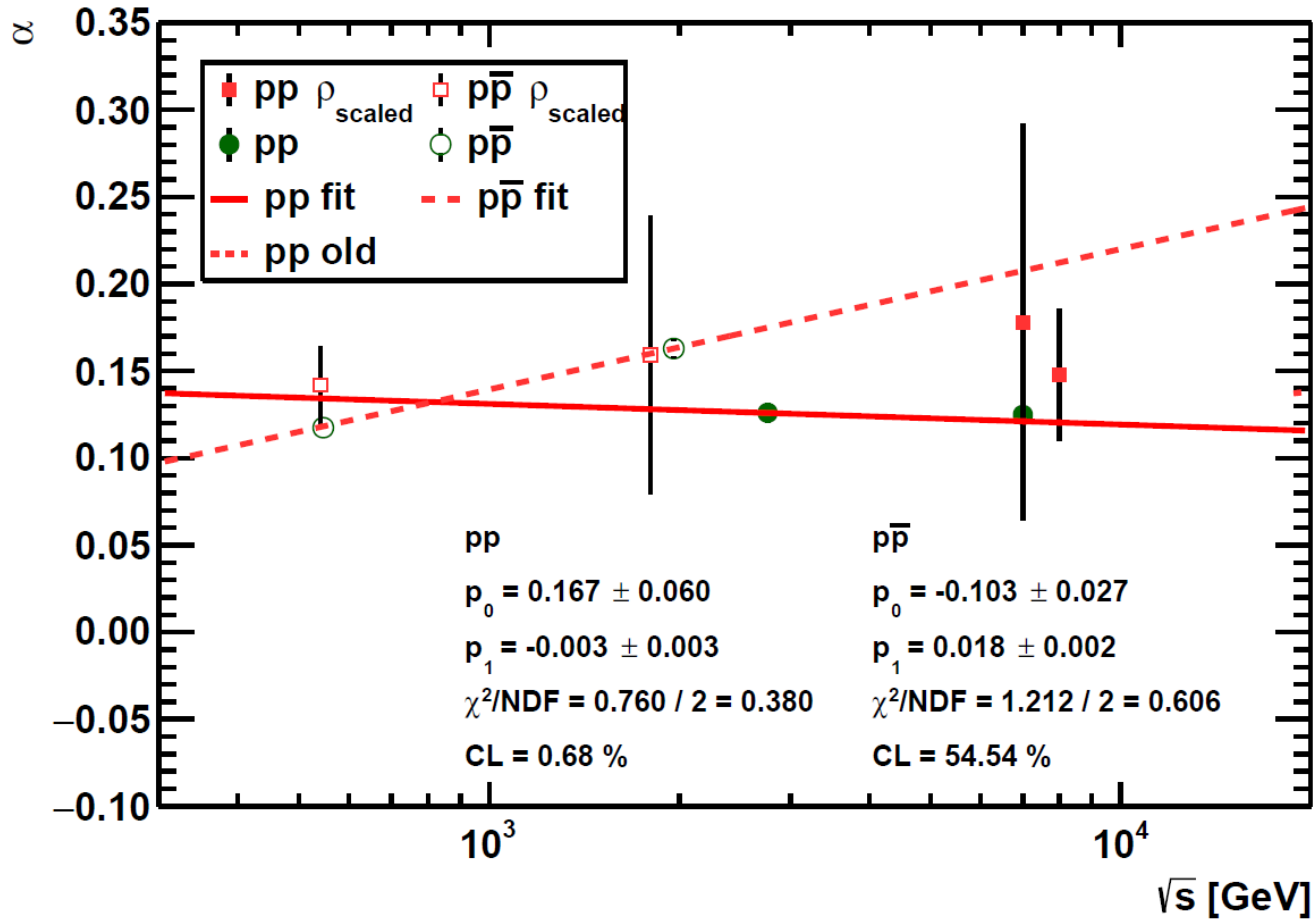
$$\lambda(s) = \text{Im } t_{el}(s, b = 0)$$

→ by rescaling one can get additional α parameter values at energies where ρ_0 is measured



The dependence of ρ_0/α on $\lambda = \text{Im } t_{el}(s, b = 0)$ in the TeV energy range. The data points are generated numerically by using the trends of the ReBB model scale parameters and the experimentally measured ρ -parameter values.

Energy dependence of the α parameter



$$P(s) = p_0 + p_1 \ln(s/s_0)$$

$$P \in \{R_q, R_d, R_{qd}, \alpha\}$$

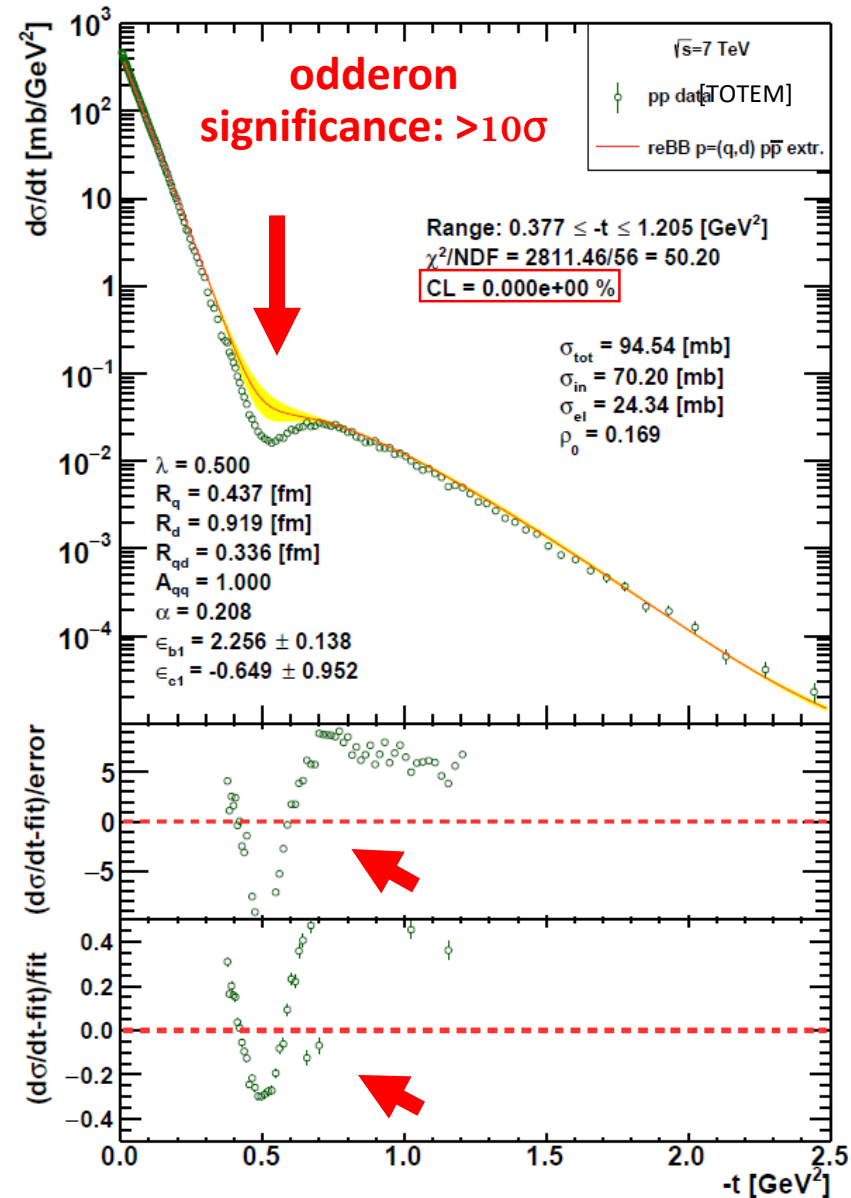
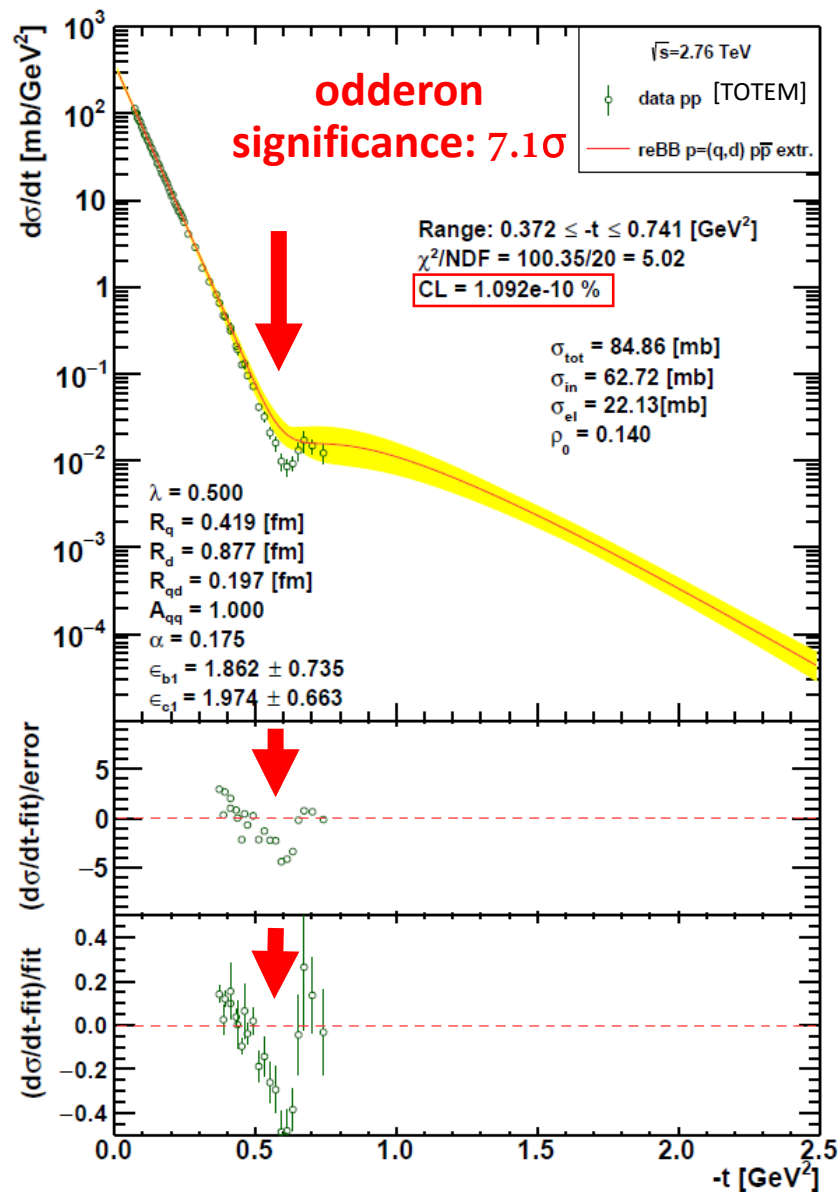
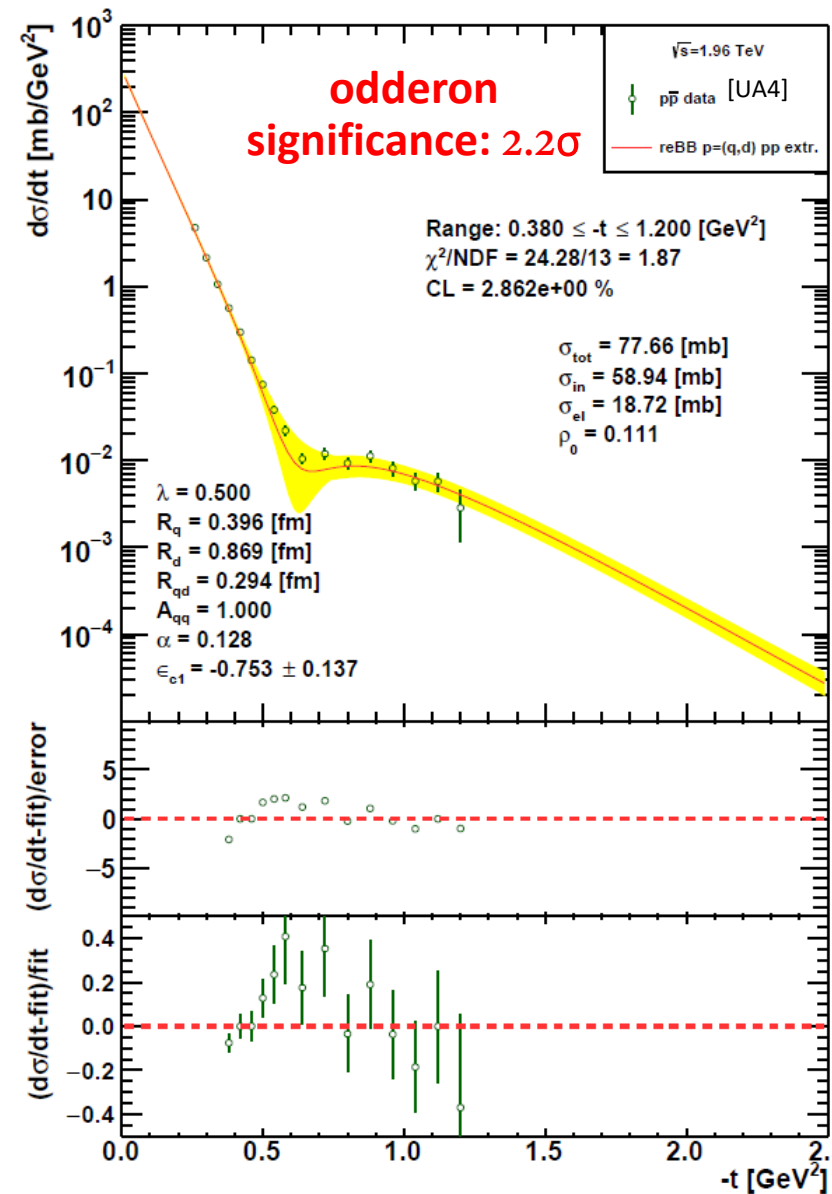
$$s_0 = 1 \text{ GeV}^2$$

Parameter	$\alpha (pp)$	$\alpha (p\bar{p})$
χ^2/NDF	0.760/2	1.212/2
CL [%]	0.68	54.54
p_0	0.167 ± 0.060	-0.103 ± 0.027
p_1	-0.003 ± 0.003	0.018 ± 0.002

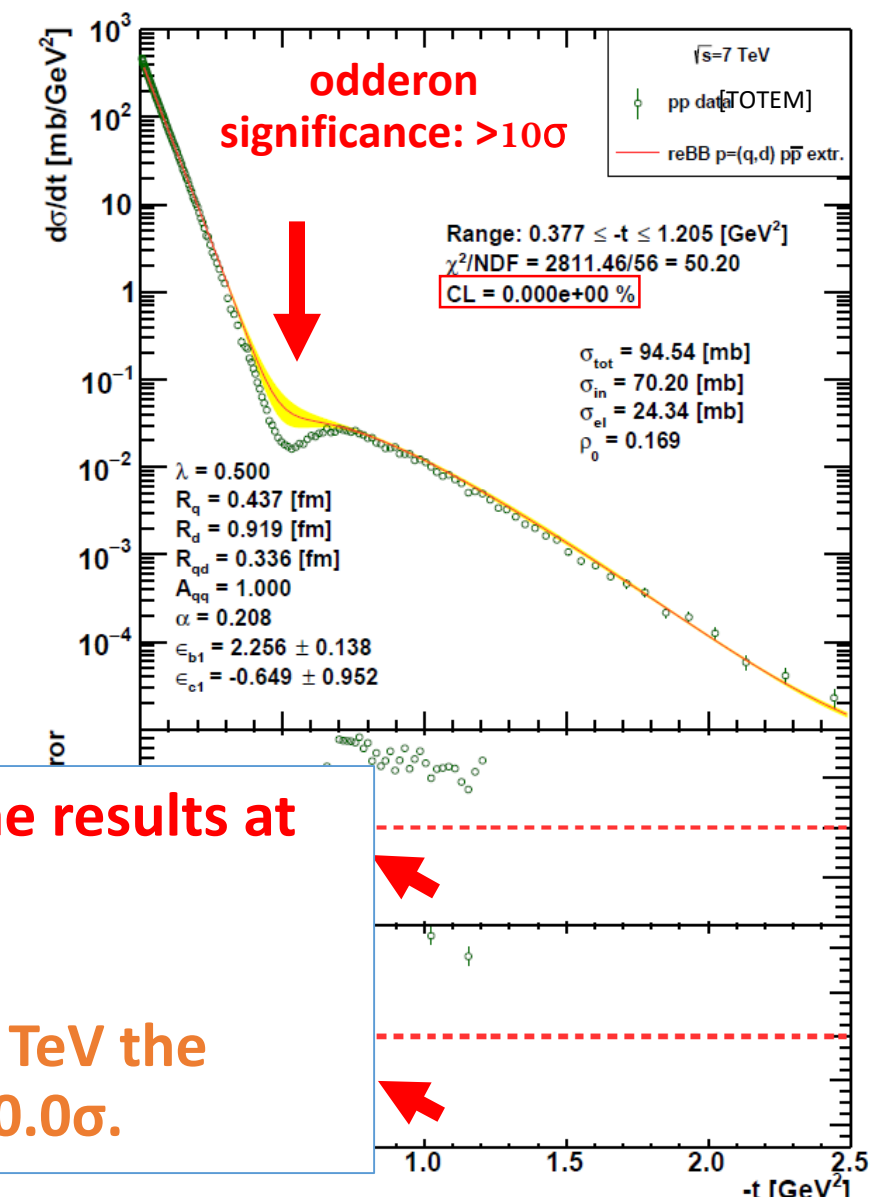
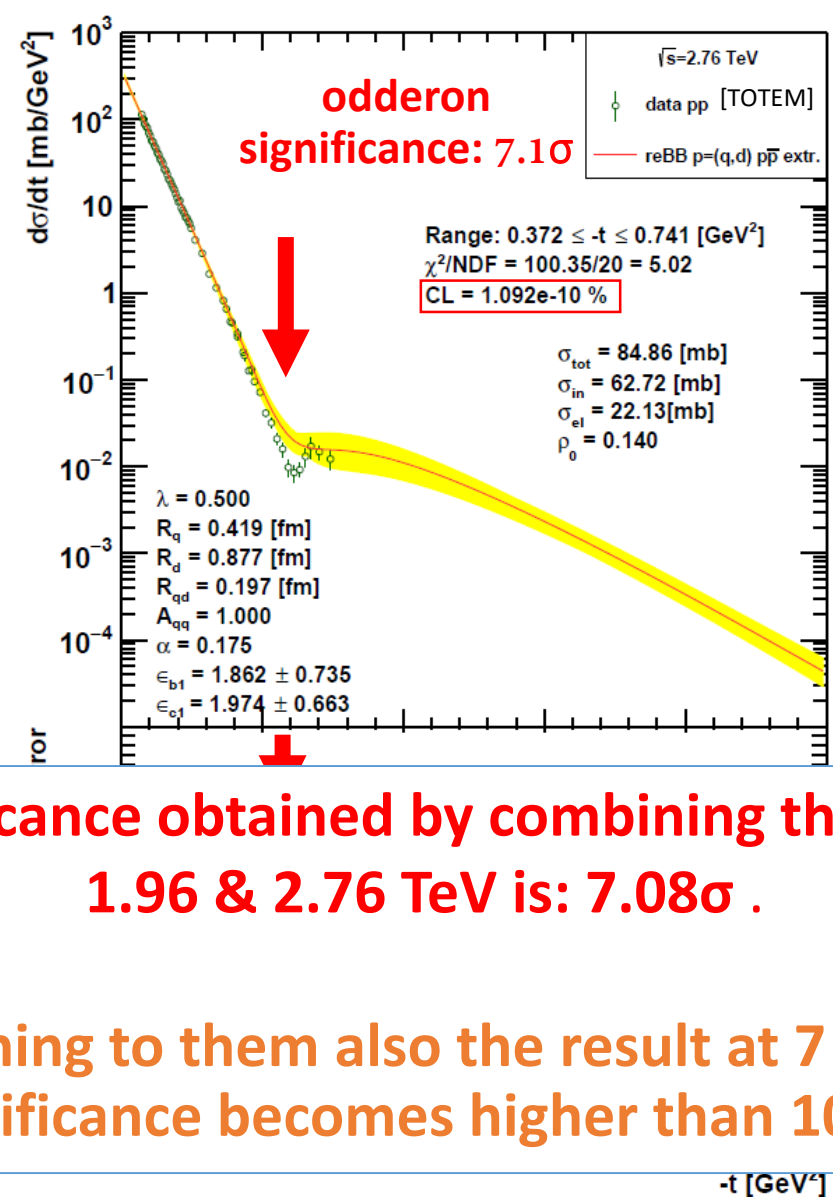
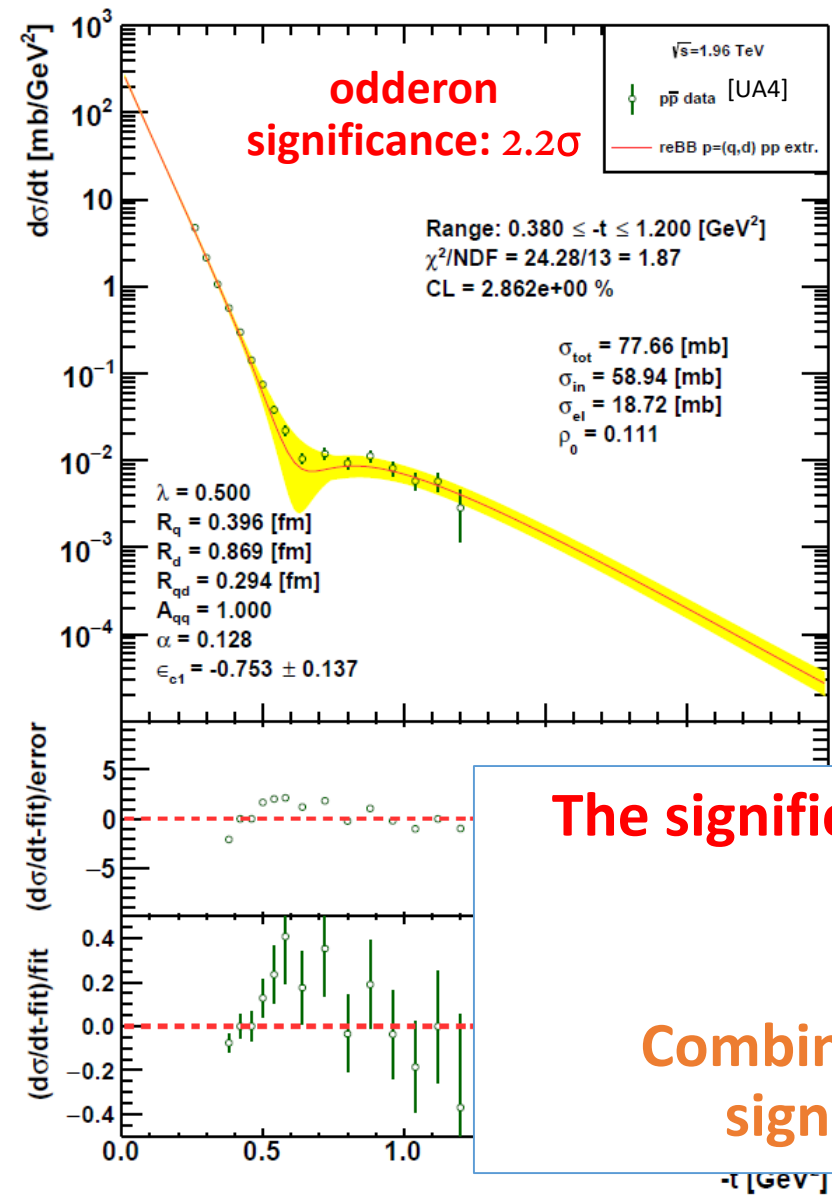
Parameters which define the energy dependence of the ReBB model α parameters for pp and $p\bar{p}$

The energy dependence of the α parameter is not the same for pp and $p\bar{p}$ processes
 → the Odderon is characterized by a single parameter, α !

Extrapolations → ODDERON



Extrapolations → ODDERON



The significance obtained by combining the results at 1.96 & 2.76 TeV is: 7.08σ .

Combining to them also the result at 7 TeV the significance becomes higher than 10.0σ .

Summary

- **ReBB model fits to pp and p \bar{p} d σ /dt data**

→ satisfactory description in the energy range of $0.546 \leq \sqrt{s} \leq 7$ TeV and squared momentum transfer range of $0.37 \leq -t \leq 1.2$ GeV²

- **determination of the energy dependence of the parameters**

→ $R_q(s)$, $R_d(s)$ and $R_{qd}(s)$ are the same for pp and p \bar{p} processes, $\alpha(s)$ is not

→ **Odderon effect**

- lack of pp and p \bar{p} d σ /dt measured data at the same energies at the TeV energy region

- **extrapolations to accomplish comparative study and obtain a significance for the Odderon effect**

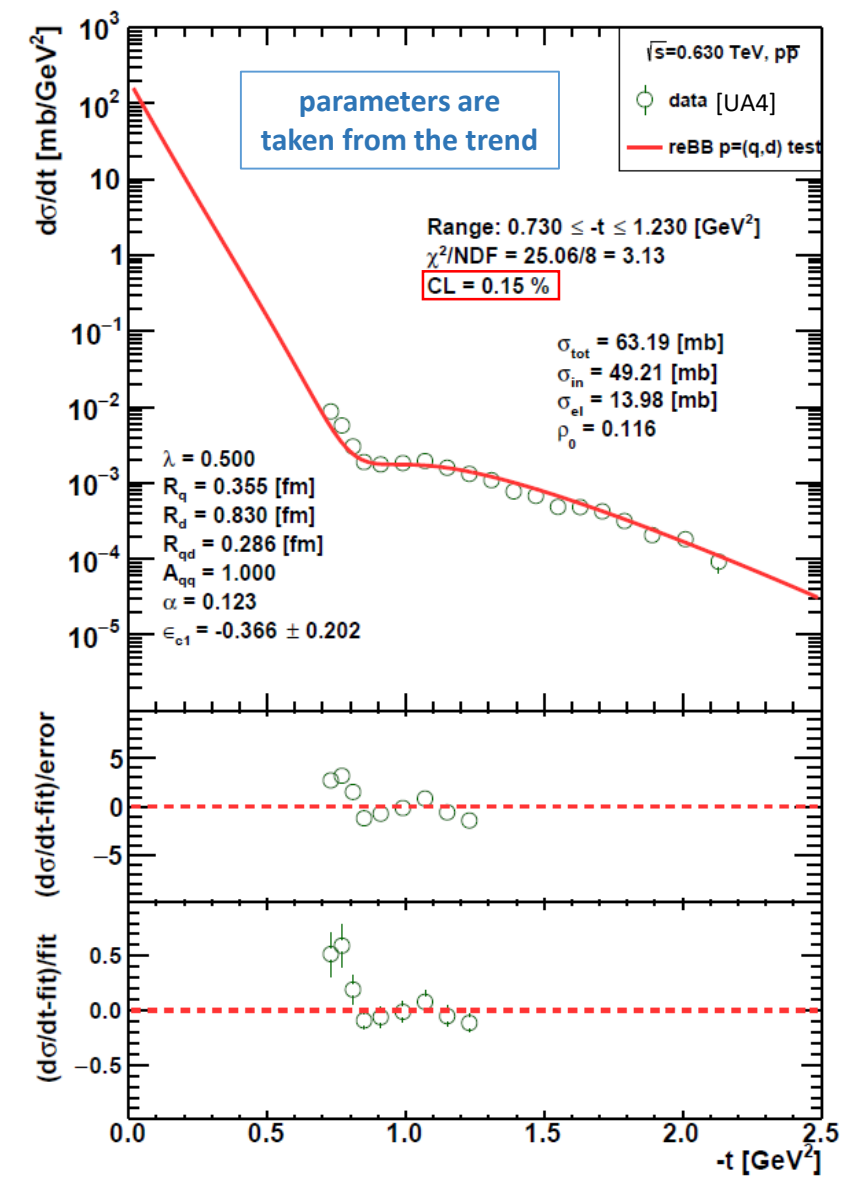
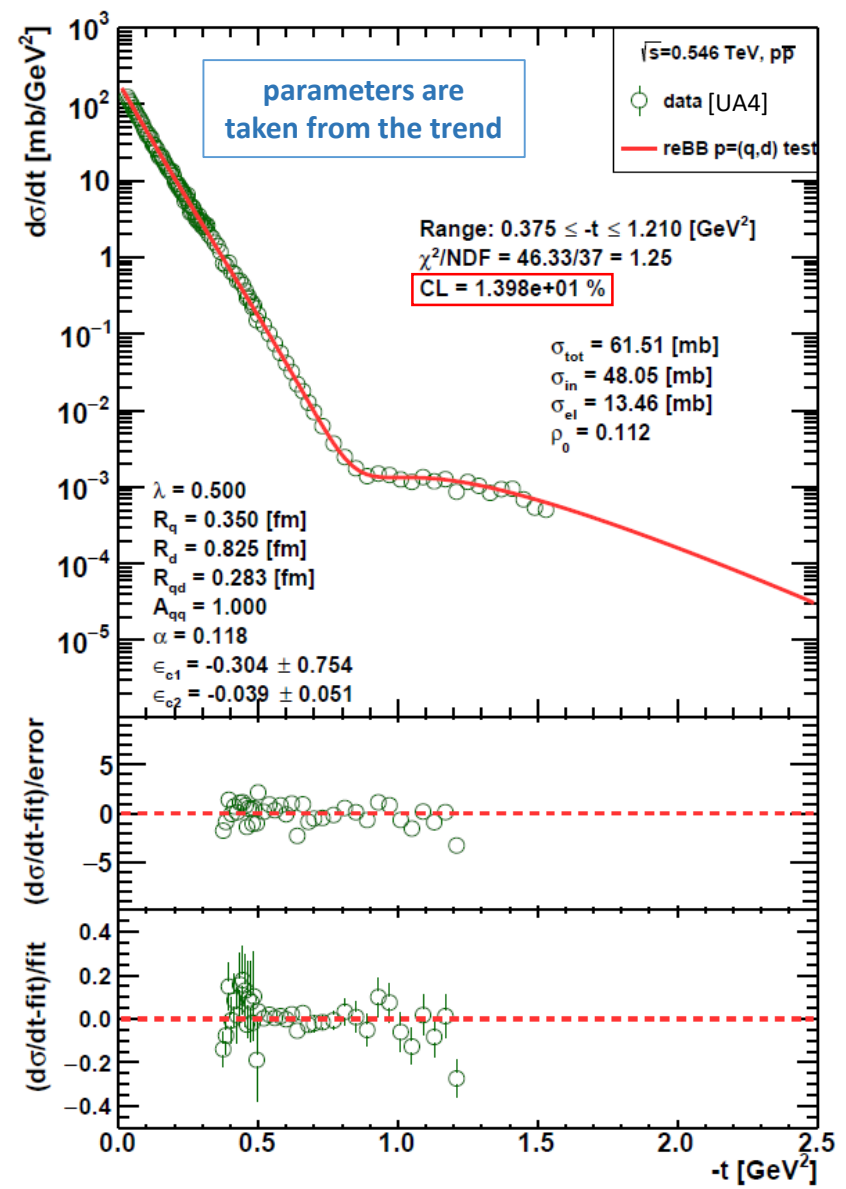
→ **difference between pp and p \bar{p} d σ /dt in the dip region**

→ **model-dependent evidence for Odderon exchange in t-channel at $\sqrt{s} \gtrsim 1$ TeV with a significance of at least 7.08σ**

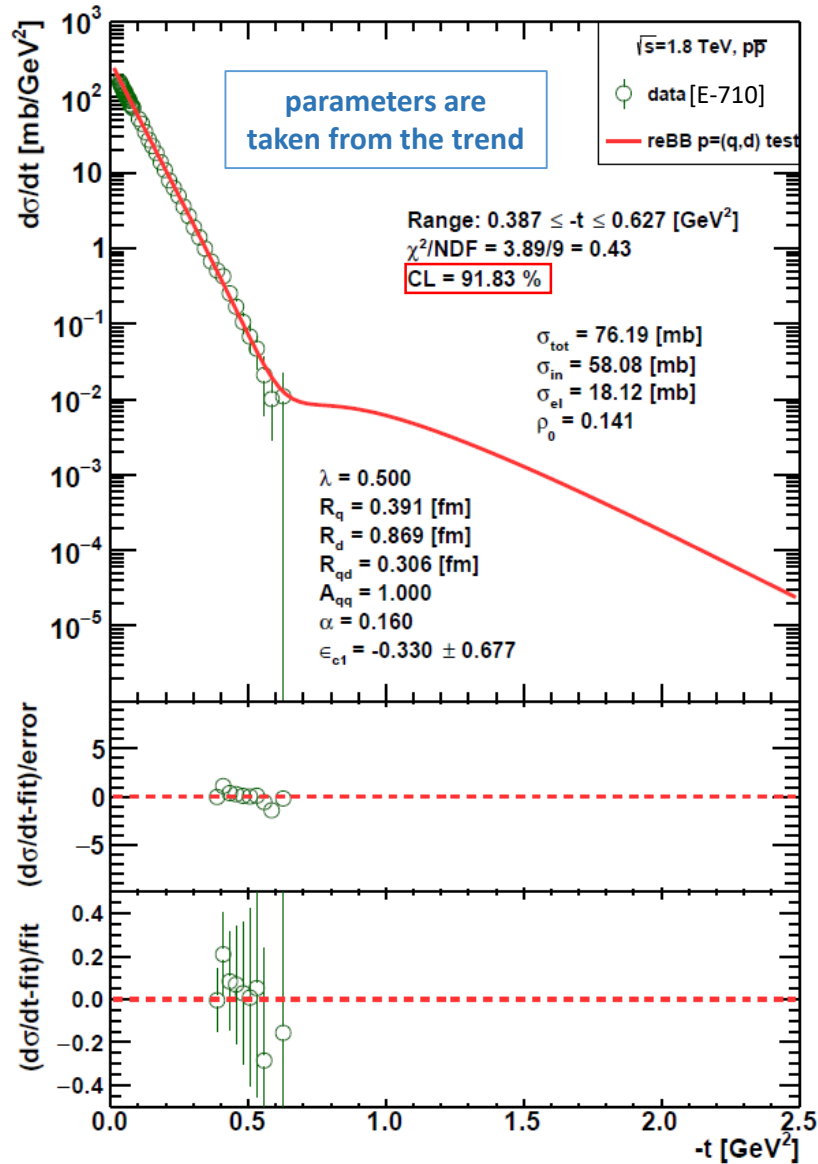
Thank you for your attention!

Backup slides

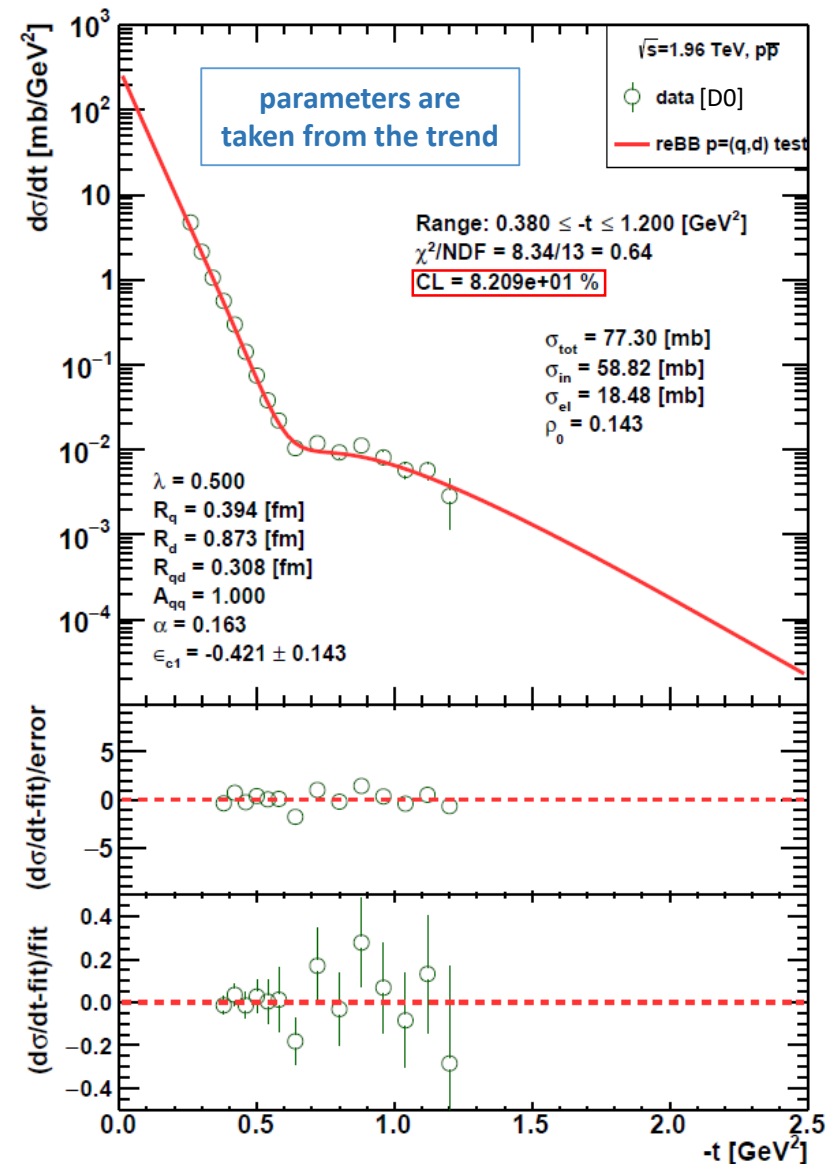
Tests @ 0.546 & 0.630 TeV ✓



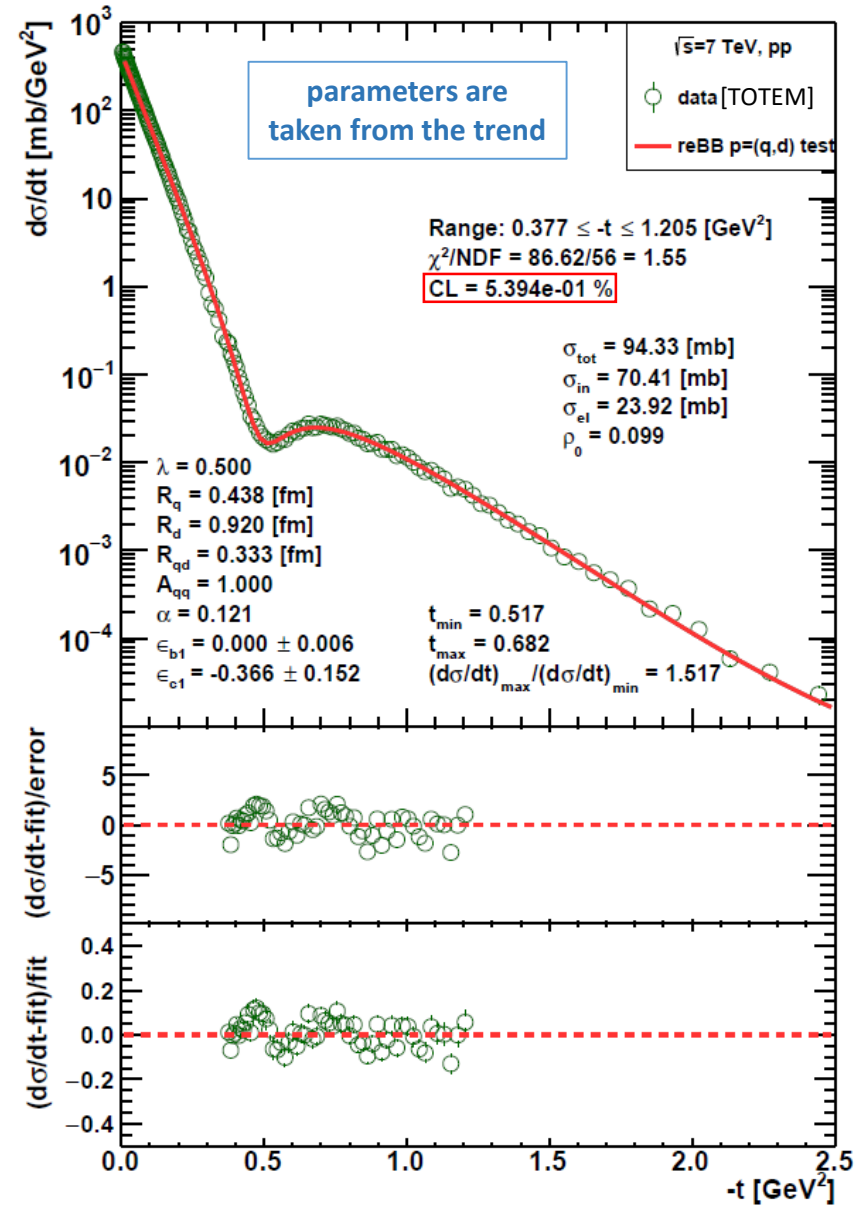
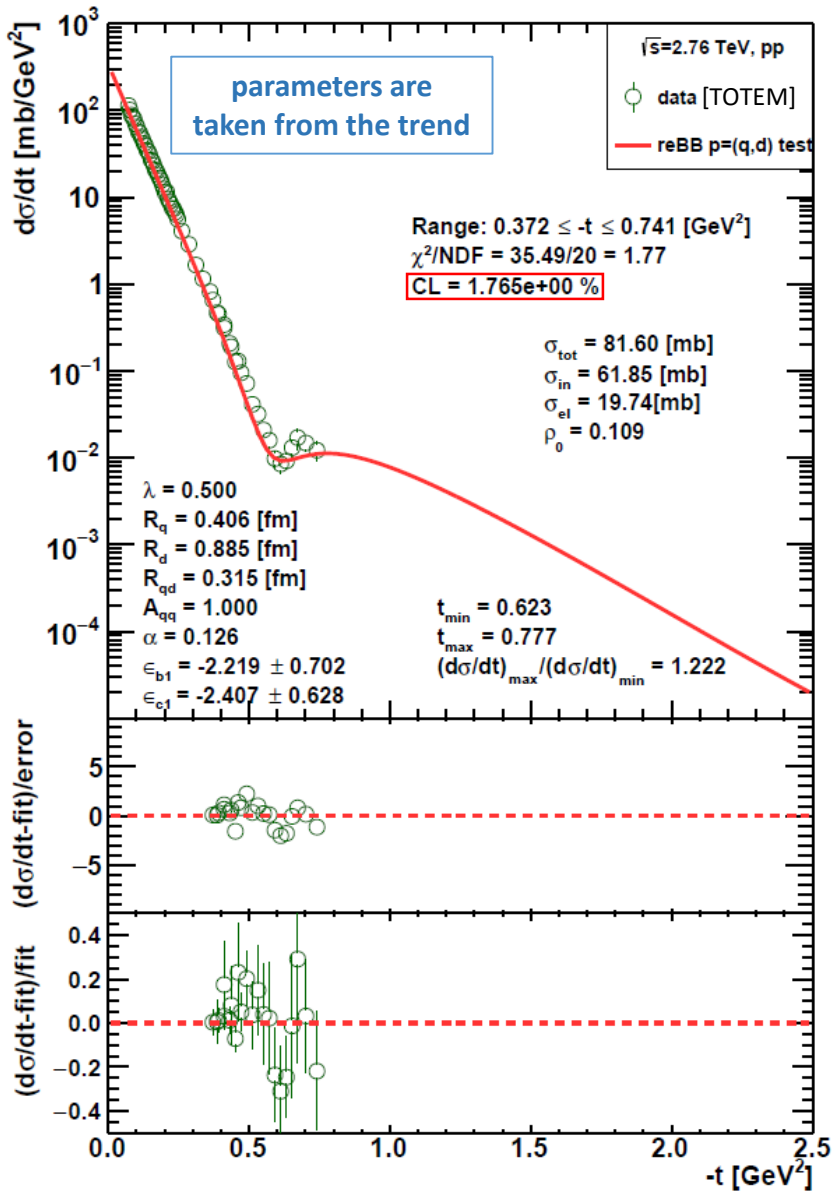
Tests @ 1.8 & 1.96 TeV



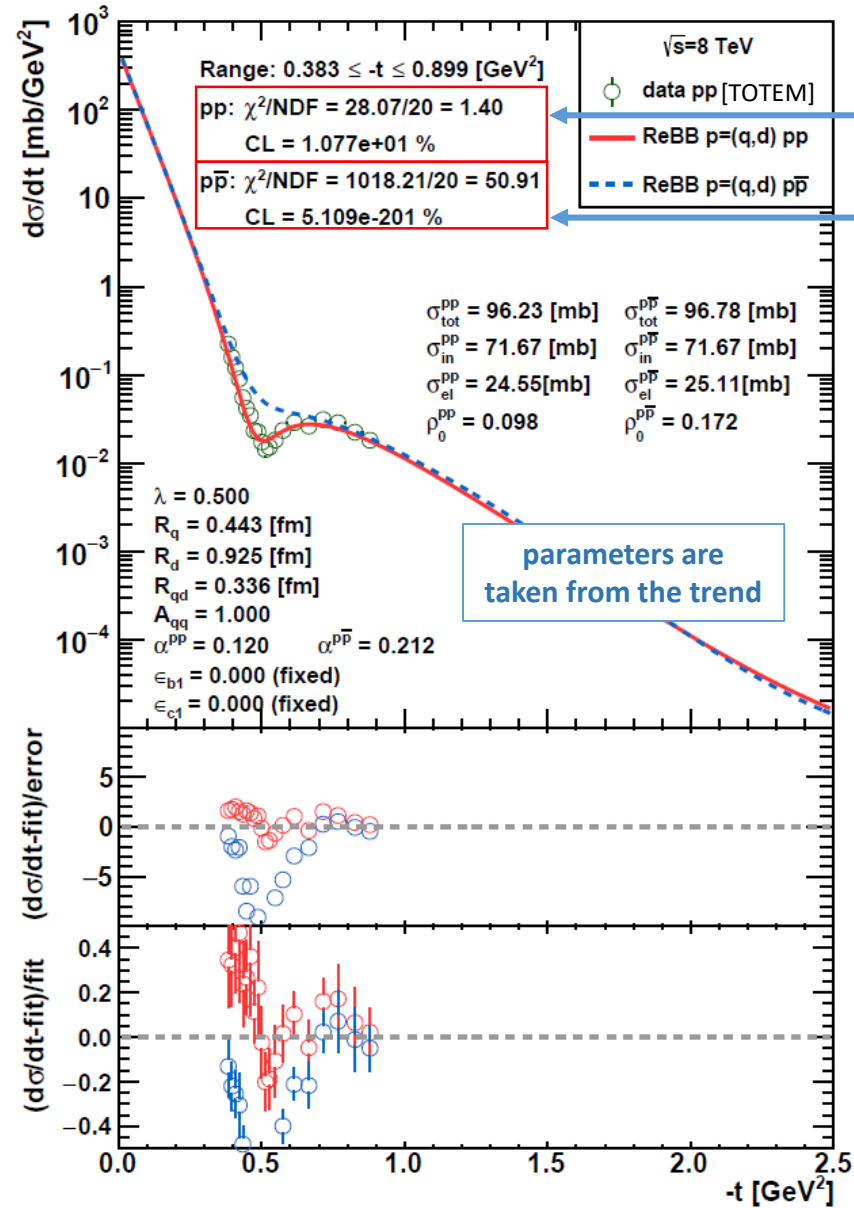
data: E-710 Collab. (TEVATRON) Phys.Lett. B247 (1990)



Tests @ 2.76 & 7.0 TeV ✓



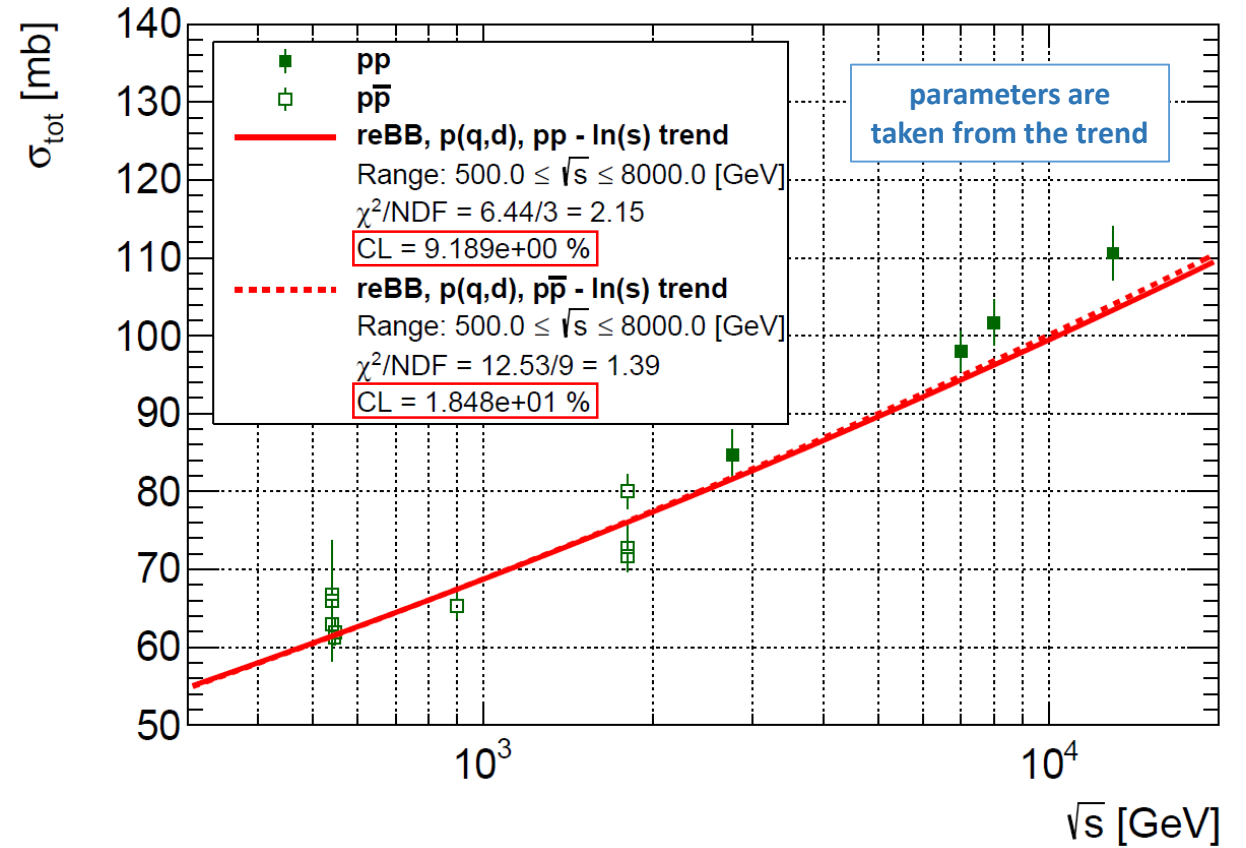
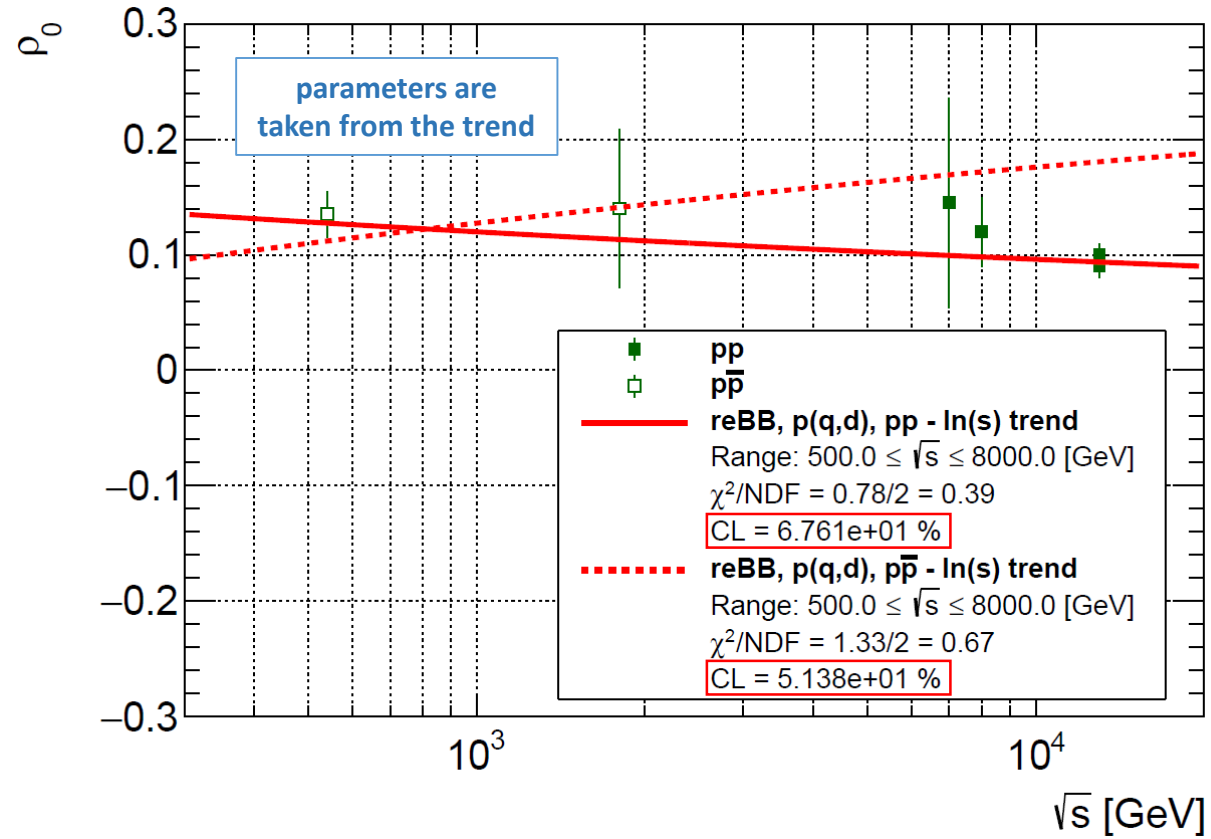
pp test and p \bar{p} extrapolation @ 8.0 TeV



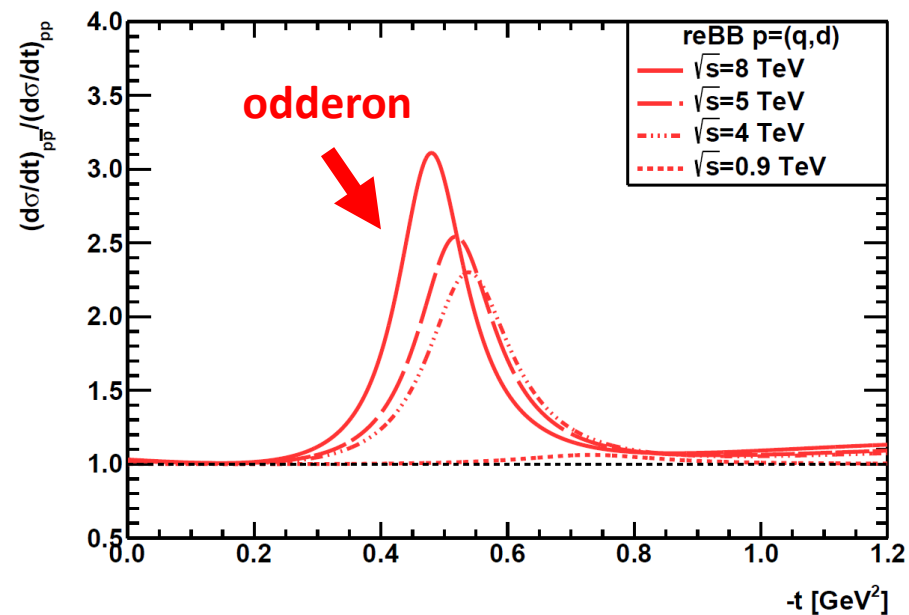
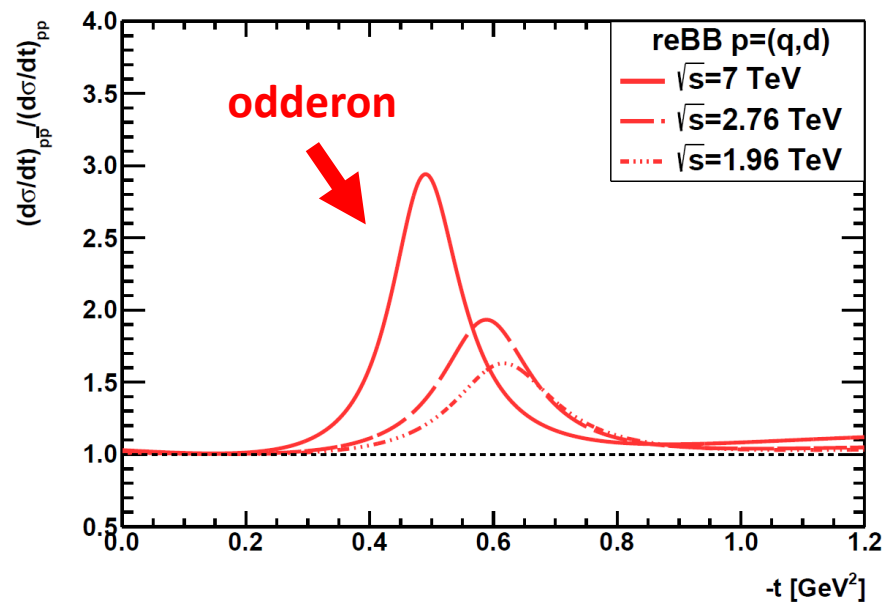
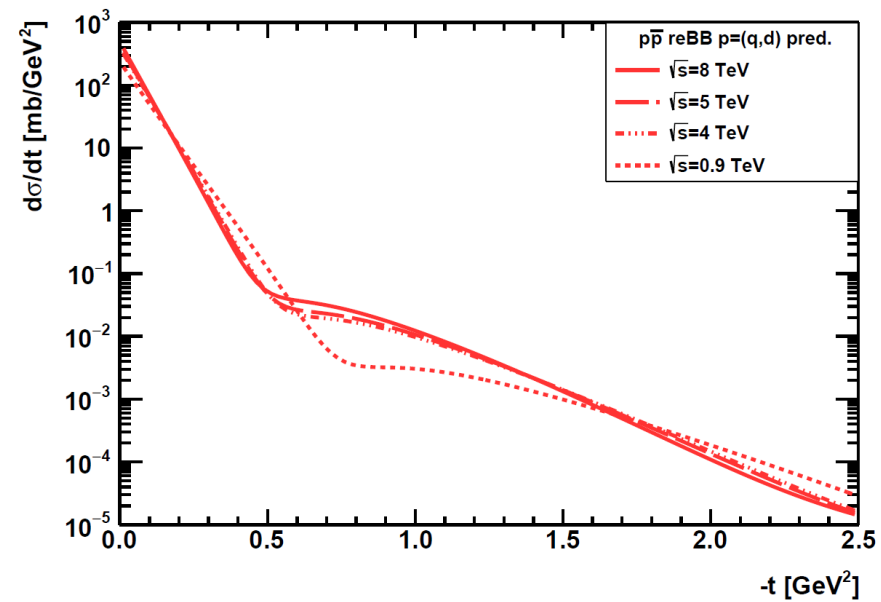
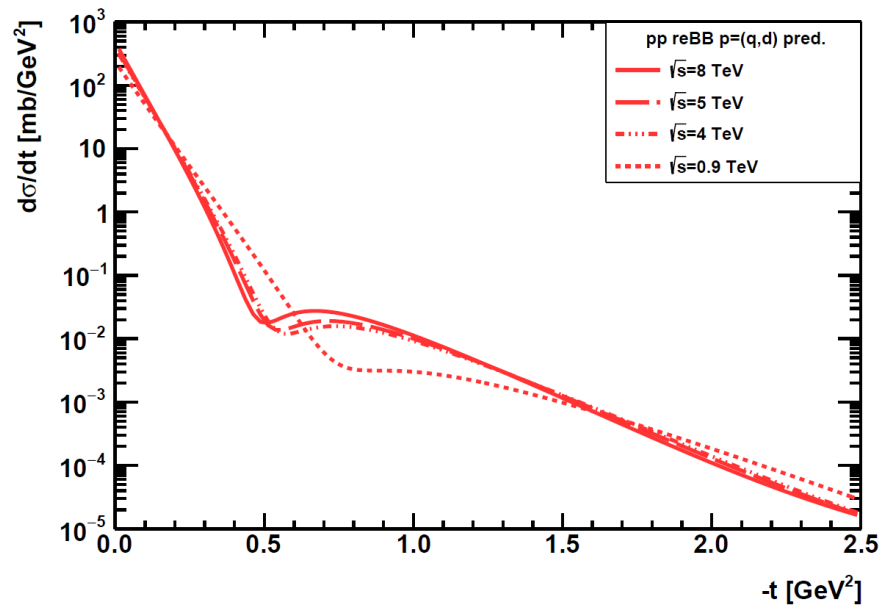
pp-pp agreement ✓

pp-p \bar{p} inagreement:
Odderon

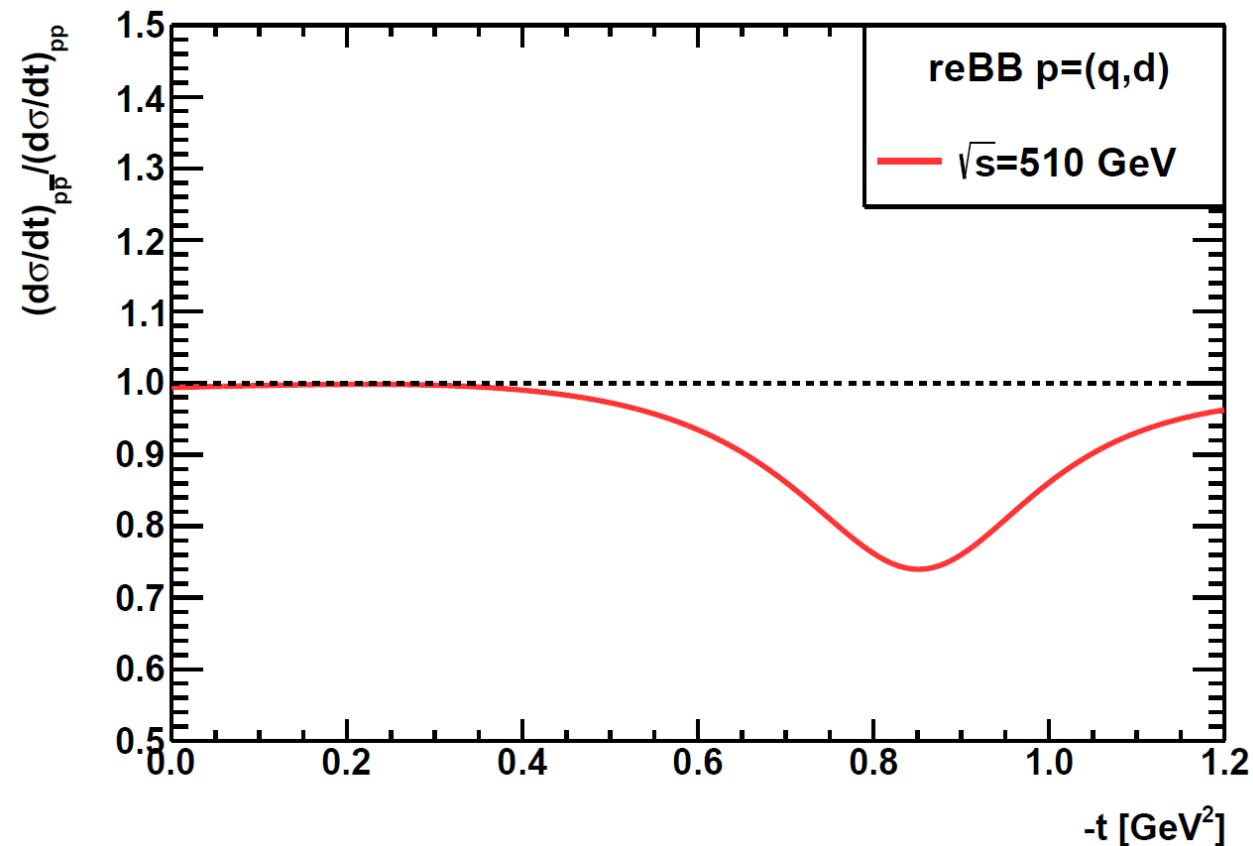
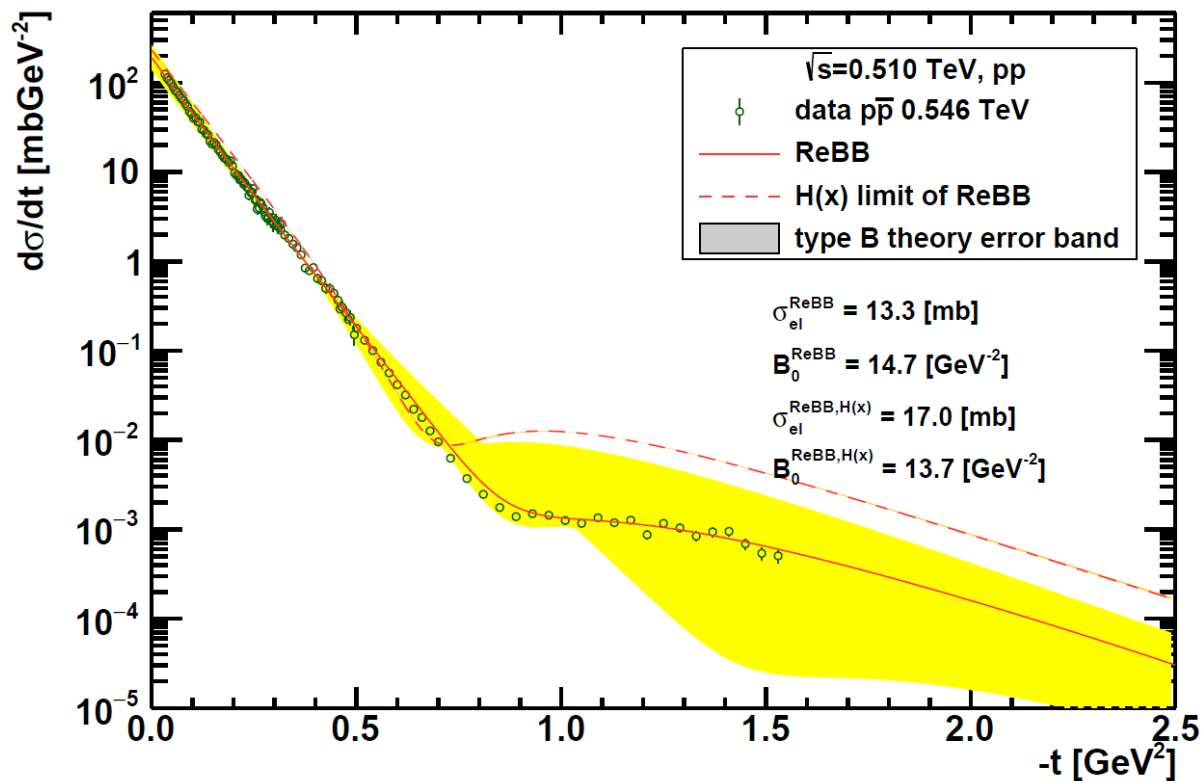
Tests for σ_{tot} and ρ_0 ✓



Predictions for pp and $p\bar{p}$ $d\sigma/dt$ and their ratios

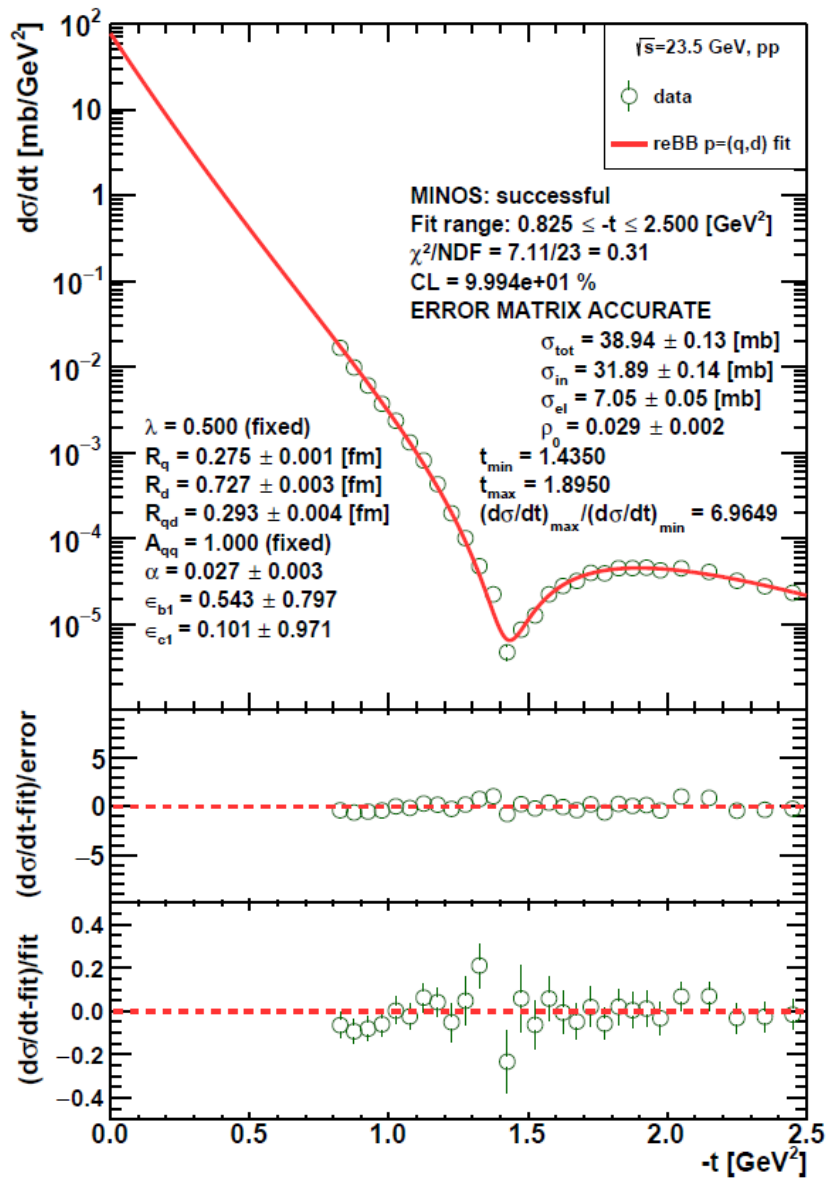


ReBB prediction for pp $d\sigma/dt$ @ 0.51 TeV

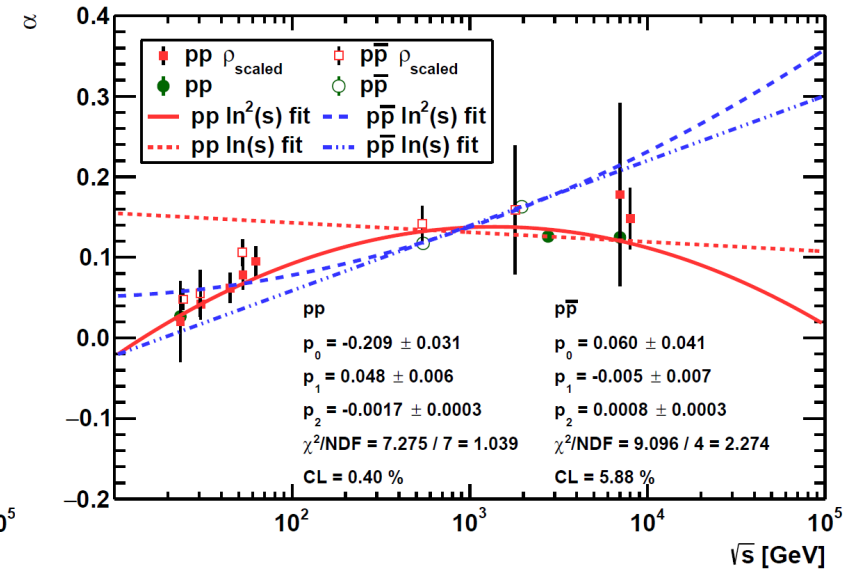
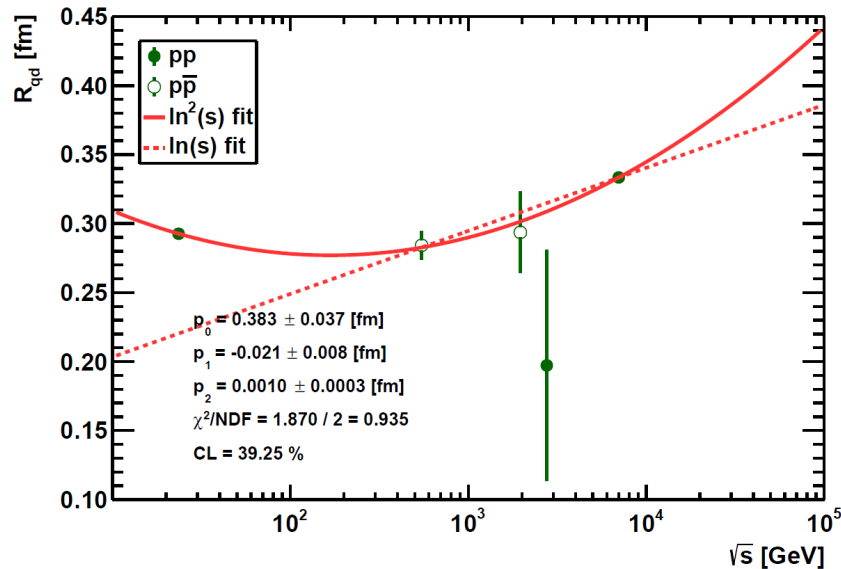
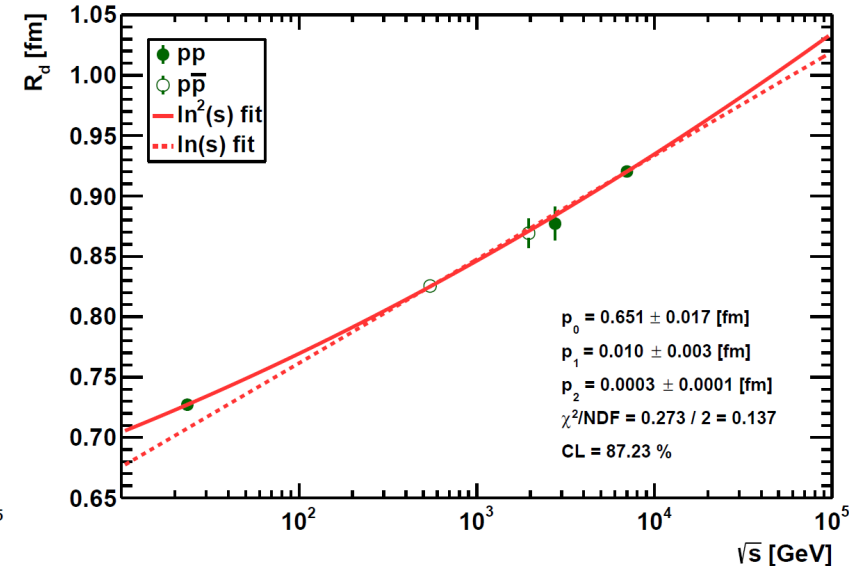
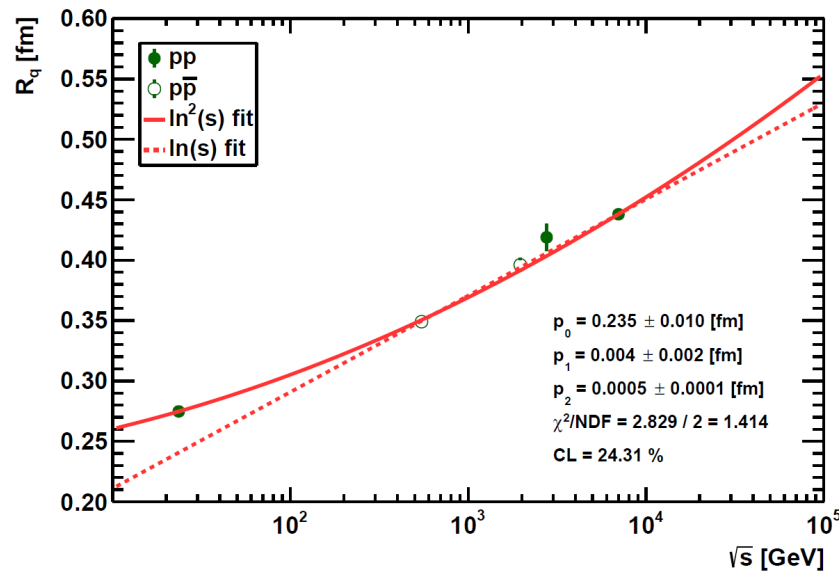


- pp data measured by the STAR Collab. is expected at 0.51 TeV

Fit at pp 23.5 GeV & $\ln^2(s)$ energy dependence



$$P(s) = p_0 + p_1 \ln(s/s_0) + p_2 \ln^2(s/s_0) \quad s_0 = 1 \text{ GeV}^2$$



Various significance combinations

