

# Improved low energy optics control at the CERN Proton Synchotron

Wietse Van Goethem, Alexander Huschauer

Acknowledgement:

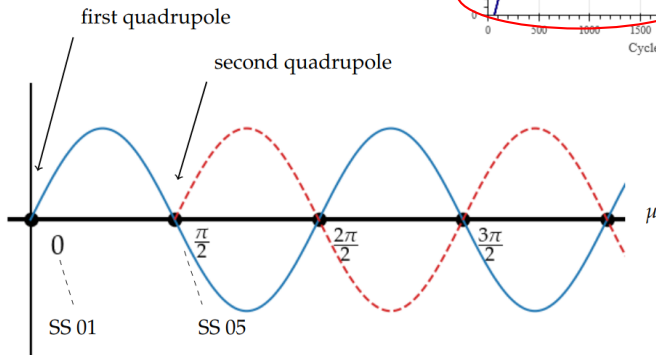
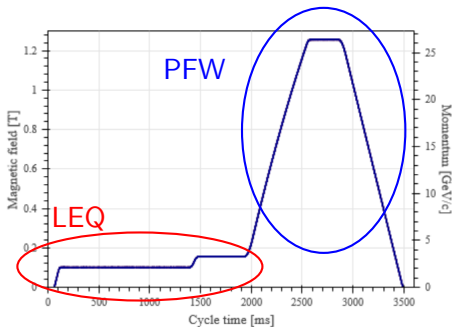
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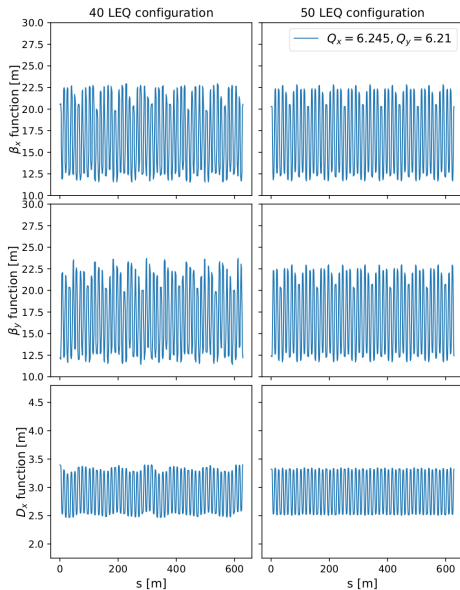
# Outline

- 1 Optics Beating Minimisation
- 2 Zero-Dispersion Optics
- 3 K-Modulation characteristics
- 4 Conclusion

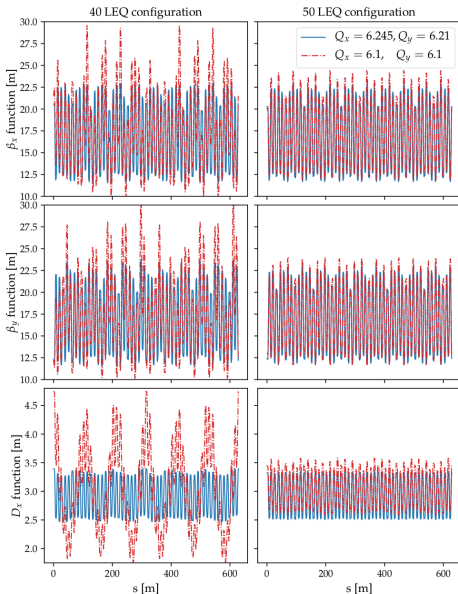
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  - consecutive optics beatings would annihilate at  $Q=6.25$



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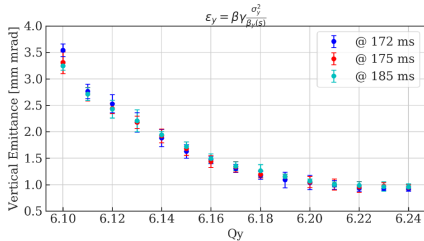
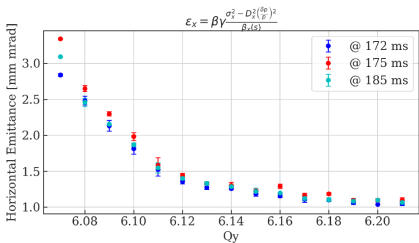


- **Initial design:** 50 symmetrically spread low energy quadrupoles (LEQs)
  - consecutive optics beatings would annihilate at  $Q=6.25$
- **Currently:** 40 quadrupoles sub-optimally spread across the lattice
- Large beta and dispersion beating near transverse integer tune values



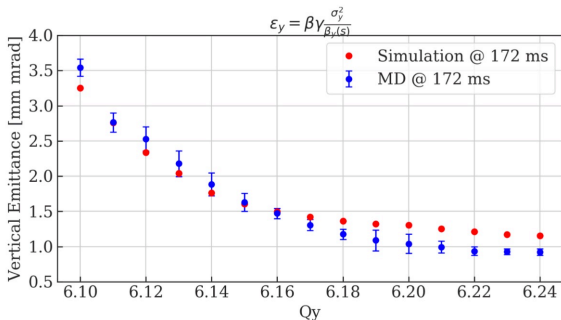
# Conclusions from Haroon's space charge study ([indico link](#))

- Measurements show clear beam blow-up as the beam is brought closer to the integer tune, where the quadrupole resonance sits.



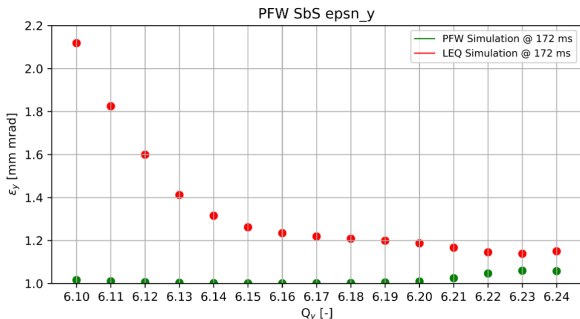
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- Measurements show clear beam blow-up as the beam is brought closer to the integer tune, where the quadrupole resonance sits.
- Model of PS benchmarked with space charge for the MD4224 case.
- Use of LEQs (rather than PFWs) to modify the tune increases the observed emittance growth.

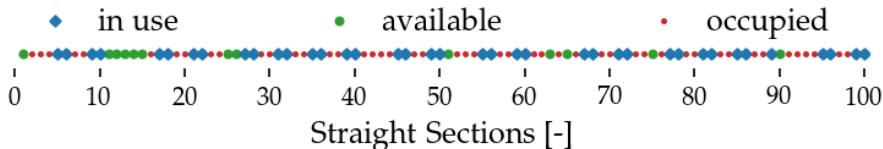




## Objective

Find a quadrupole configuration of 40 LEQs or less that minimizes the  $\beta_{x,y}$  and dispersion beating leading to reducing emittance blow-up

- The PS straight sections are able to hold LEQs if that section is not already occupied by another element (52 free sections)

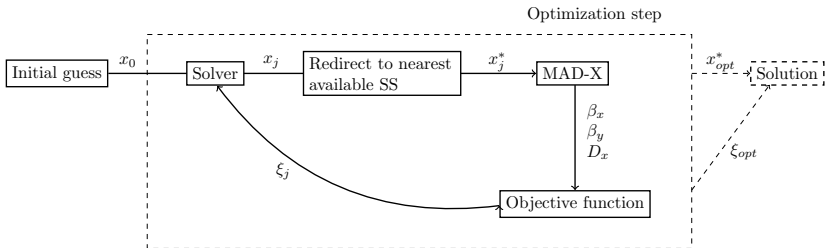


- Tests are done on a bare machine, proton injection energy lattice. Which replicate the 2018 reference measurements of tune and chromaticity with the LEQs turned off. Afterwards we use the LEQs to go to working point (6.1,6.1) to enhance beating

# Method 1: Optimisation algorithm

$$\text{minimise } \xi(x) \quad \text{subject to } g_i(x) \leq 0, \quad i = 1, 2, \dots, m$$
$$x \in \mathbb{R}^n$$

- Let  $x_j$  = position of quadrupole  $j$  with  $j = 1, \dots, 40$ 
  - optimisation step starts with a PS lattice without quadrupoles
  - Redirect to the closest available SS iteratively for every  $x_j$
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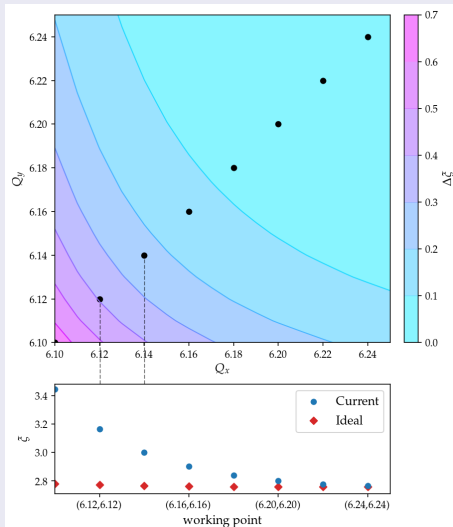
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  - $\xi^* = \frac{\xi}{\xi_0}$  with  $\xi_0$  the bare machine lattice where the LEQ strengths are set to zero
- solver needs to be suitable for non-differentiable and only testable search space
  - Zeroth-Order Optimization (ZOOpt)

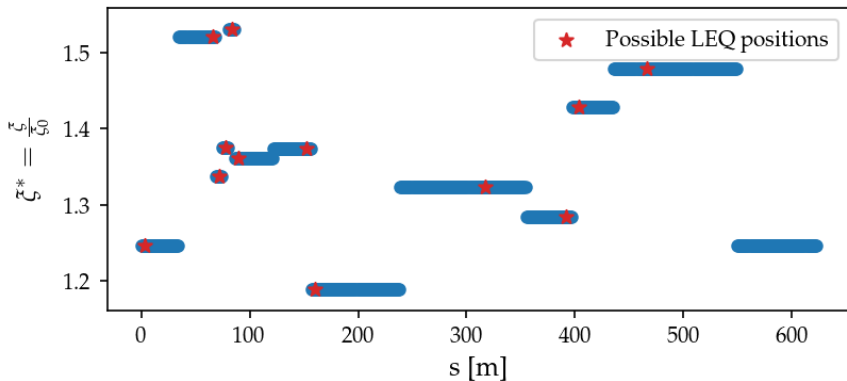
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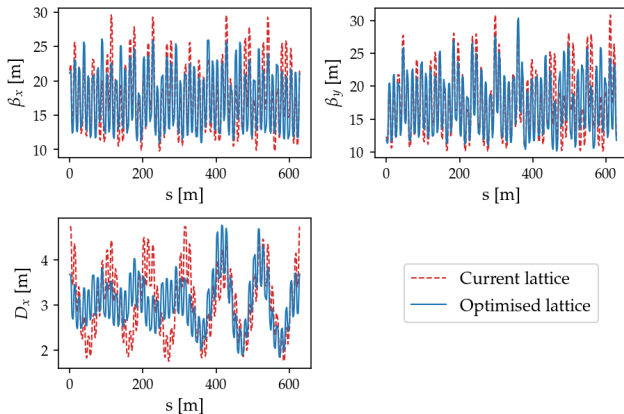
- $\xi$  increases when optics beating effects are more prevalent
  - Current lattice = 40-LEQ configuration
  - Ideal lattice = 50-LEQ configuration
- Different  $\xi$  definition can lead to different results



## Redirect to the closest available SS iteratively for every $x_j$

- Objective function  $\xi$  has discrete values
- The optimisation is ill-defined
- Large reliance on initial guess





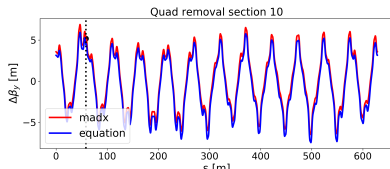
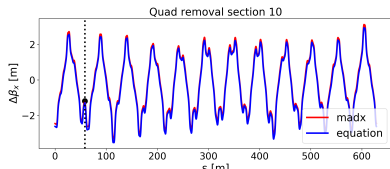
## Optimised lattice ( $\xi^* = 1.1037$ )

- Improvement of peak to peak values
- 10 changes to the current lattice → **difficult to test**
  - Remove from SS 55, 72, 95, 99 and 100
  - Add to SS 13, 14, 25, 26 and 63

## Method 2: Single Quadrupole Variation

- 1 Start with the current 40 LEQ configuration
- 2 Look at a specific straight section and use the following equation to calculate the effect of adding or removing a quadrupole on the beta functions at this location

$$\Delta\beta(s) = -\frac{\beta_0(s)}{2 \sin(2\pi Q)} \int_0^C \beta(s_1) \Delta k(s_1) \cos(|2\mu(s_1) - 2\mu(s)| - 2\pi Q) ds_1$$



- 3 Use this beta function to calculate the new phase advance  $\mu_x$  and tune  $Q_x$

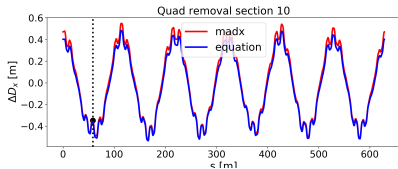


## Method 2: Single Quadrupole Variation

- 4 Find the new dispersion with  $\beta_x, \mu_x$  and  $Q_x$  by using the equation below

$$\Delta D_x(s) = \frac{\sqrt{\beta_x}}{2 \sin \pi(Q_x)} \int_0^C \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos [|\mu_x(\sigma) - \mu_x(s)| - \pi Q_x] - D_0(s)$$

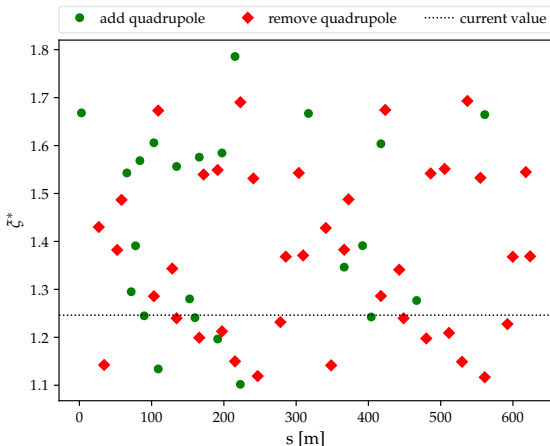
With  $\beta_x = \beta_{x;0} + \Delta\beta_x$ ,  
 $\mu_x = \mu_{x;0} + \Delta\mu_x$  and  
 $Q_x = Q_{x;0} + \Delta Q_x$



- 5 Calculate objective function  $f(x) = \frac{\sigma(\beta_x) + \sigma(\beta_y) + \sigma(D_x)}{3}$
- 6 iterate for every usable section

# Method 2: Single Quadrupole Variation

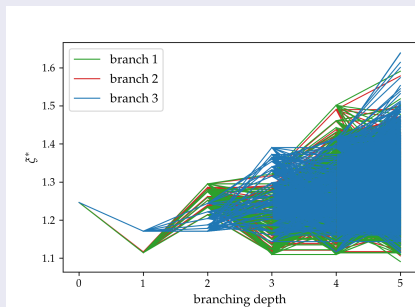
Predict the impact of **adding** or **removing** one LEQ.



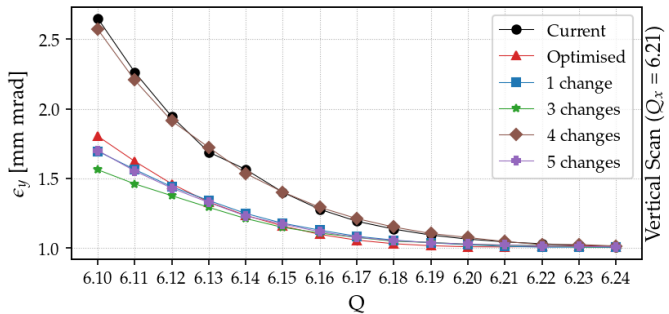
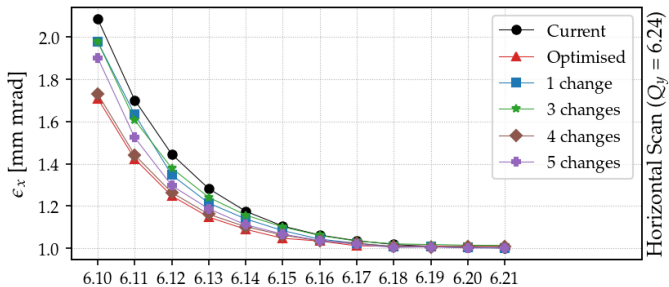
- 41-LEQ configuration
- 39-LEQ configuration
- apply iteratively to the configurations with the lowest  $\xi$ -value
  - branch-like structure

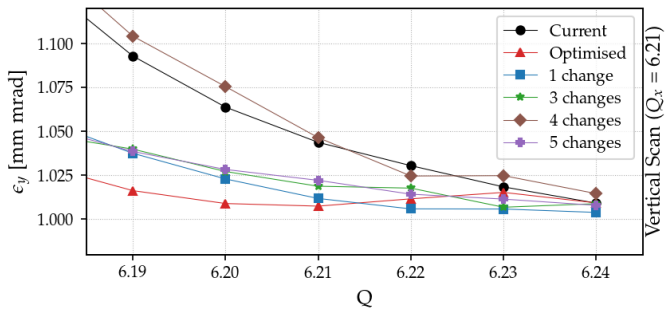
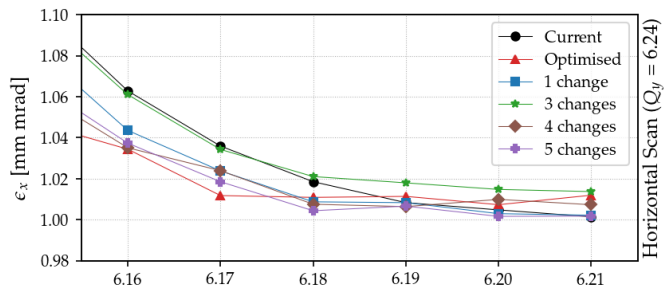
## branch-like structure

- Each point represents a quad-configuration
- Lowest  $\xi$  value at each depth is saved in table below
- one change already shows large  $\xi$  improvement



	remove LEQ in SS	add LEQ in SS	$\xi^*$
0 changes			1.2460
1 change	90		1.1141
2 changes	56	86	1.1707
3 changes	10, 90	26	1.1097
4 changes	10, 90	26, 36	1.1094
5 changes	21, 22, 90	13, 14	1.0908





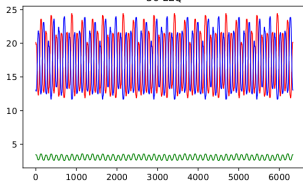
## conclusion

- Improvement found to the beta and dispersion beating with new LEQ-configurations
- Noteworthy improvement can be found with minimal and easy testable modification to the current lattice
- Increased flexibility for LHC type beams
- Test 1 change configuration in an experimental setup

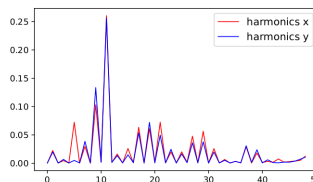
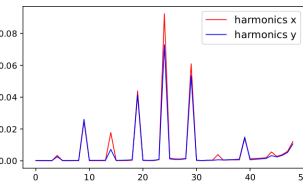
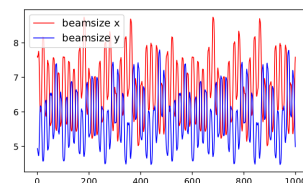
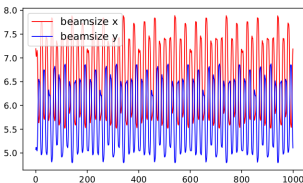
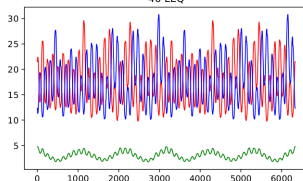
## Continuation studies

- Improve objective function  $\xi$  definition
  - Use periodicity as quality parameter
  - Study correlation between  $\xi$  and emittance blow-up
- Study quadrupole configurations where the LEQs are individually powered

50 LEQ



40 LEQ



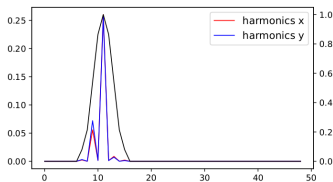
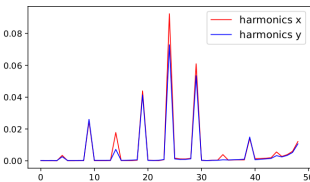
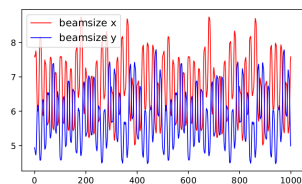
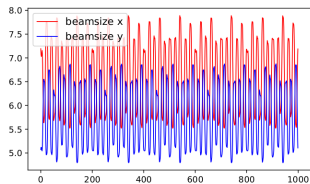
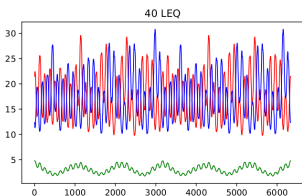
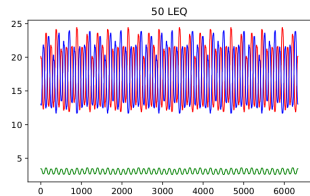
optics functions



beamsizes



harmonics



optics functions

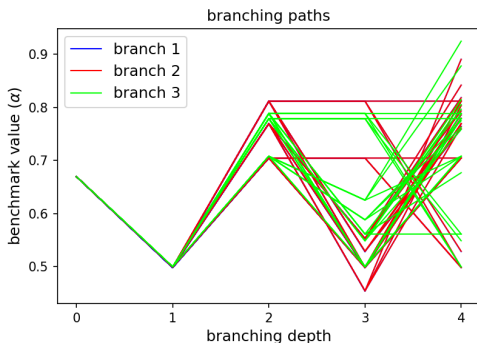


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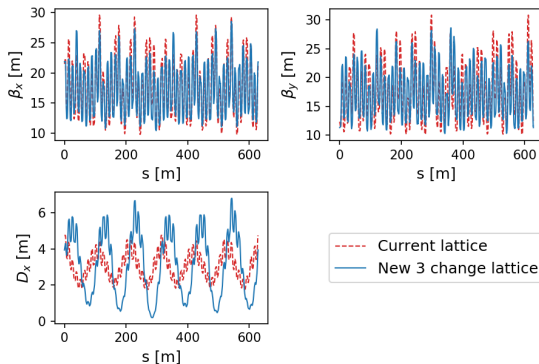
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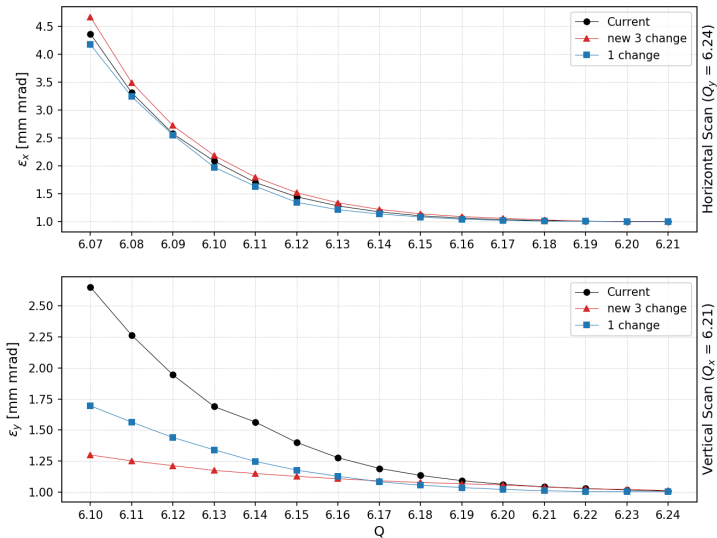
## branching study

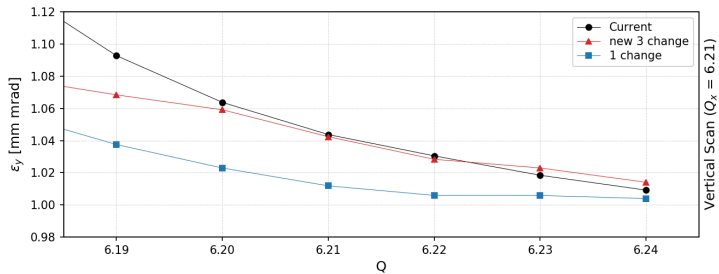
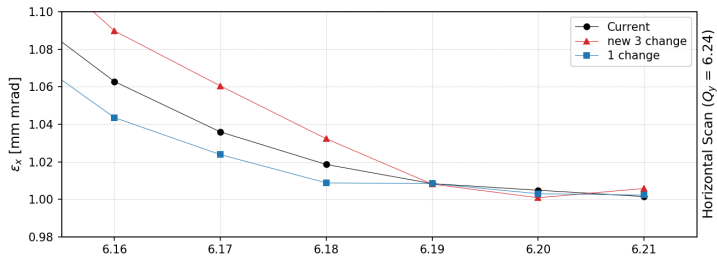
- 1 change is the same remove 90 configuration
  - Same result from different method
- 3 change configuration shows promise
  - Remove LEQ in SS 45,49,90
  - Also easily testable



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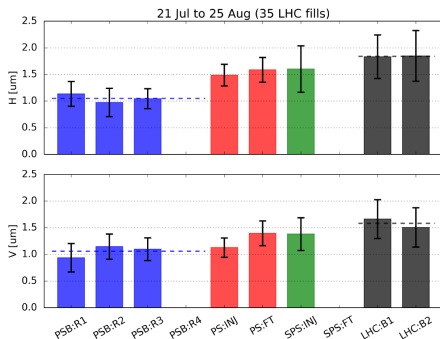




# Motivation

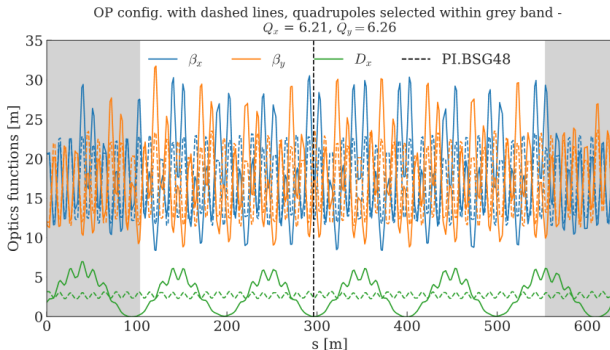
## Emittance growth between PSB and PS

- Multiple effects expected to contribute to discrepancy between PSB and PS horizontal emittance measurements
- Challenge of beam size measurements with resolution in the sub-mm range
- Important dispersive contribution to the beam size in both accelerators



# Zero-dispersion optics

- Would remove the necessity of deconvolving the horizontal and longitudinal distributions
- 10 Individually powered adjacent LEQs allows to achieve zero dispersion at any location around the ring (see presentation by Alex)
  - The LEQ-strength limit is reached
  - other optics functions are largely perturbed



# Zero-dispersion optics

## objective

Reach zero-dispersion optics while minding the LEQ strength limits and minimally perturbing the other optics

optics beatings is directly proportional to the quadrupole variation  $\Delta k$

$$\bullet \frac{\Delta\beta(s)}{\beta(s)} \propto \Delta k$$

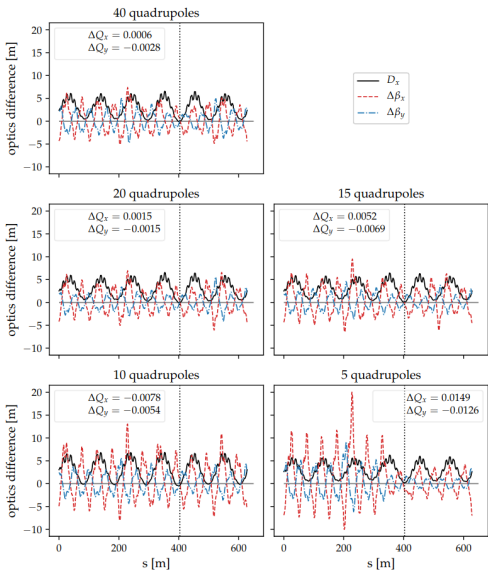
$$\frac{\Delta D(s)}{D(s)} \propto \Delta k$$

$$\Delta Q \propto \Delta k$$

## Quadratic optimisation algorithm

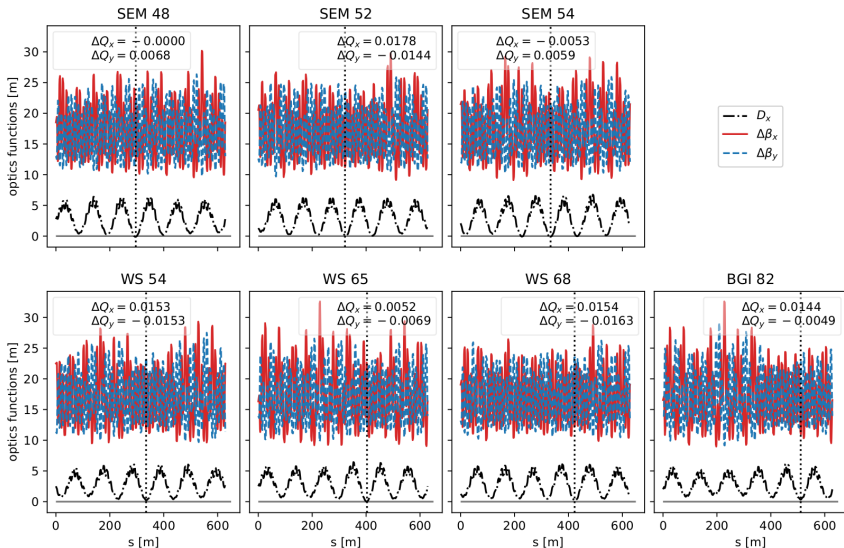
- The variables we want to optimise are the individual strengths  $\delta k_i$  of the LEQs
- constraints are the strength limits + a bound that forces the dispersion to zero:
  - $D^* = D_0 + \Delta D_{k_1} \times \delta k_1 + \Delta D_{k_2} \times \delta k_2 + \dots + \Delta D_{k_n} \times \delta k_n$
- minimize  $\delta k_1^2 + \delta k_2^2 + \dots + \delta k_n^2$  to minimally perturb other optics

- Iteratively remove the LEQ with the lowest weight during the optimisation
- Figure shows this process for WS 65
- If not enough quadrupoles are used, the approximations in the equations don't hold true
- 15 is arbitrarily chosen as the number of quadrupoles used in the following study



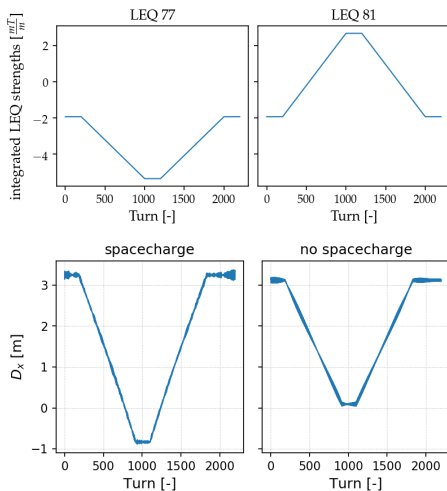


# Zero dispersion optics for beam size measurements using 15 LEQs



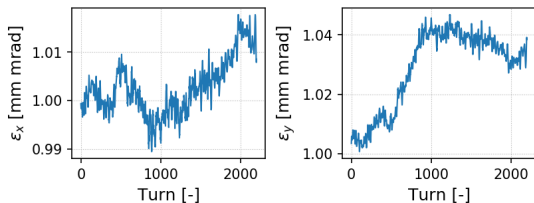
# Space charge study

- Replicate experimental conditions by ramping LEQs to and from zero dispersion optics
- only 2200 turns were used due to long simulation times
- Spacecharge forces causes dispersion to move to zero faster than expected
- Using knobs in an experimental setup allows to achieve zero-dispersion optics

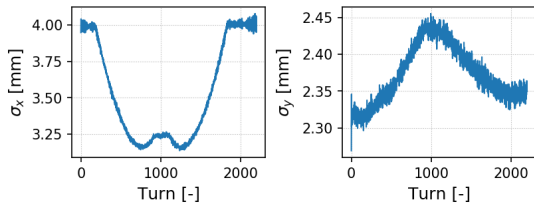


- Vertical emittance growth
- Vertical beam size should remain constant
  - suggests non-adiabatic ramping
  - can be resolved by simulating more turns
- Horizontal beam size doesn't stay at a minimum due to crossing through zero dispersion

## Emittance evolution



## Beam size evolution



## conclusion

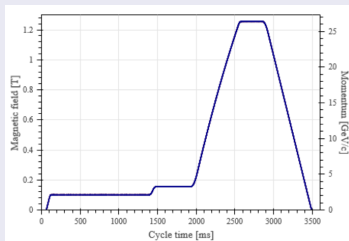
- Zero-dispersion optics are obtained with small perturbations to the working point
- Still some modifications present in the other optics
- Spacecharge effects causes the dispersion to go below zero

## Continuation studies

- Continue space charge simulation
  - more turns
  - other locations
- develop LEQ knobs for experimental testing

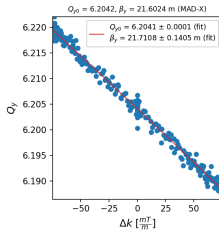
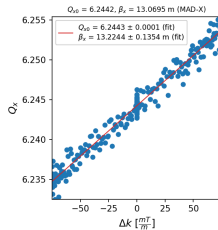
## Emittance measurement

- $\epsilon = \frac{\sigma^2(s) - D^2(s) \left(\frac{\Delta p}{p_0}\right)^2}{\beta}$
- The accuracy of  $\beta$  is unknown at WS
- $\beta$  is measured through K-Modulation at nearby LEQ



## K-modulation

- 1 induce sinusoidal  $\Delta k$
- 2 measure resulting  $\Delta Q$  with BBQ ( $\approx 5$ ms between measurements)
- 3 fit 
$$\bar{\beta} = \frac{4[\cos(2\pi Q_0) - \cos(2\pi(Q_0 + \Delta Q))]}{\Delta k L_{quad} \sin(2\pi Q_0)}$$



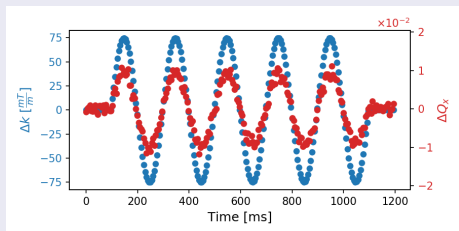
## error sources

### Transfer factor error

- Transfer factor is converts the applied electric current to quadrupole strength
- For the LEQs this comes from a single measurement outside the machine
- Iron yokes from MUs are near LEQs → can affect transfer factor

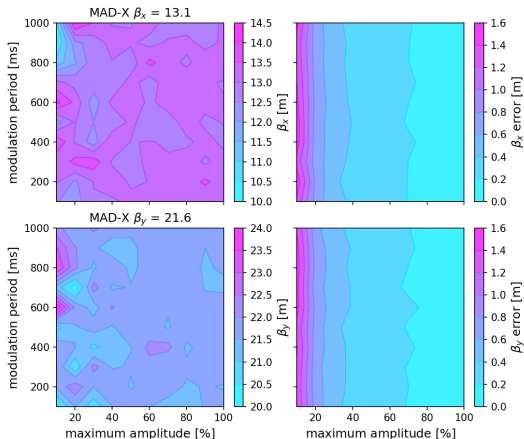
### Uncertainty on BBQ measurement

- replicate uncertainty by adding a gaussian value ( $\sigma = 1e-3$ )
- Modulation characteristics impact the  $\beta$  reconstruction
  - Amplitude
  - Period
  - Number of magnetic cycles



## K-modulation at LEQ 68

- large dependence on amplitude
- No dependence on modulation period
  - modulation period of 1000ms used for other tests
- increasing the number of cycles always results in better accuracy



# $\beta$ propagation to nearby WS

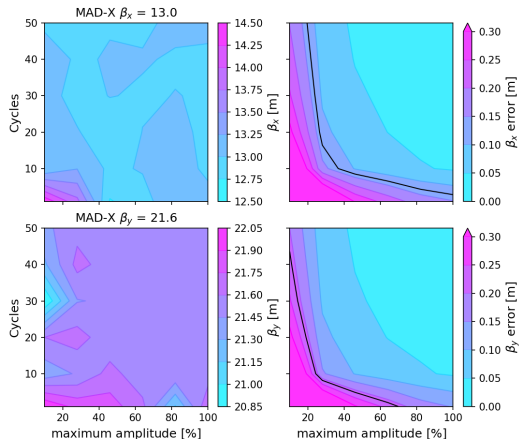
$$\begin{pmatrix} \beta_{WS} \\ \alpha_{WS} \\ \gamma_{WS} \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & m_{11}m_{22} + m_{12}m_{21} & -m_{22}m_{12} \\ m_{21}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_{LEQ} \\ \alpha_{LEQ} \\ \gamma_{LEQ} \end{pmatrix}$$

- Fit produces average  $\beta$  along the LEQ  $68 \approx \beta$  at  $\frac{L_{quad}}{2}$ 
  - Propagate through  $\frac{L_{quad}}{2}$
  - Propagate through drift section to WS 68
- Add gaussian uncertainty ( $\sigma = 1e-3$ ) on  $\alpha_{LEQ}$



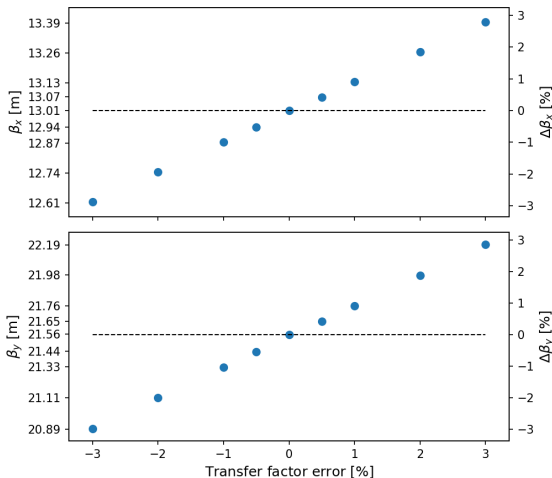
## Modulation characteristics at WS 68

- Black line represents 1% of the actual  $\beta$ -value
- Good accuracy possible for  $\approx 20$  magnetic cycles and  $\pm 50\%$  of the maximum amplitude



## transfer factor error at WS 68

- Unknown systematic error
- Effect on  $\beta$  directly proportional to transfer factor error



## conclusion

- General behaviour of K-modulation uncertainties in the PS is better understood
- Transfer factor needs a measurement in machine conditions

## Continuation studies

- propagate through machine using specific PS models

## Optics optimisation

- Reduction of emittance blow-up
- Increased flexibility for LHC-type beams
- 1 change configuration is easily testable
- Possible improvement for  $\xi$  definition

## Zero-dispersion optics

- Zero-dispersion optics are reached
- Small working point perturbation
- LEQ ramping requires more turns

## Quadrupole modulation

- Transfer factor needs better measurements
- modulation configuration for  $< 1\%$  accuracy