Polynomial Inflation and Dark Matter

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A simple model based on empirical facts

- Need cosmic inflation
- Need dark matter
- A minimal model:

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_{\rm EH} + \mathcal{L}_{\rm SM} + \mathcal{L}_\phi + \mathcal{L}_{H\phi} + \mathcal{L}_\chi \right)$$

1. Inflation:

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \left(b\,\phi^2 + c\,\phi^3 + d\,\phi^4 \right)$$

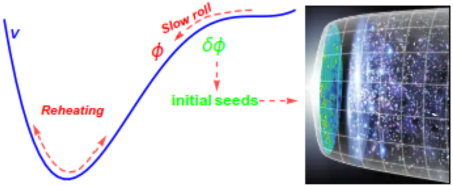
2. Reheating:

$$\mathcal{L}_{H\phi} = -\lambda_{12}\, \phi\, H^\dagger H - \frac{1}{2} \lambda_{22}\, \phi^2\, H^\dagger H$$

3. DM:

$$\mathcal{L}_\chi = i\, \bar{\chi}\, \gamma^\mu\, \partial_\mu \chi - m_\chi\, \bar{\chi}\, \chi - y_\chi\, \phi \bar{\chi}\, \chi$$

Inflation: a short review



- Monomial: $V(\phi) \sim \phi^{n}$, tensor-to-scalar ratio

$$r \propto \left(\frac{V'}{V}\right)^2 \sim \frac{4n}{N_{\rm CMB}}$$

- Planck 2018: $r < 0.061$; ruled out $n > 1$ $\Rightarrow$ less steep potentials are favored:
  - fraction power e.g. $V \sim \phi^{2/3}$ (not easy to realize in particle physics)
  - non-minimal coupling $V \sim \phi^n/(1+\xi\phi^2 R)^2$ (unitary problem with large ξ)
  - ...

Polynomial Inflation

- More natural: a polynomial

$$V(\phi) = \overset{\text{negligible}}{\cancel{Const.}} + d\,\phi^4 + c\,\phi^3 + b\,\phi^2 + \overset{\text{shifted away}}{\cancel{e\phi^1}}\,.$$

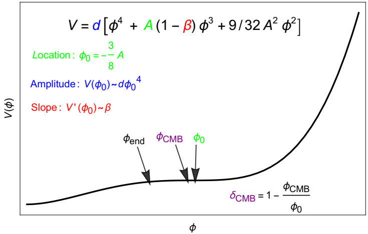
1. Large ϕ: $V \sim \phi^4 \Rightarrow$ Too steep ☹
2. Small ϕ: $V \sim \phi^2 \Rightarrow$ Too steep ☹
3. Intermediate regime: V flat due to negative ϕ^3 ☺

- If $b = \frac{9c^2}{32d} \Rightarrow$ inflection-point $\phi_0 = -\frac{3c}{8d}$
- Reparametrize:

$$V(\phi) = d\left[\phi^4 + \frac{c}{d}\,(1-\beta)\,\phi^3 + \frac{9}{32}\left(\frac{c}{d}\right)^2\phi^2\right] \equiv d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

1. $A \equiv -8/3\phi_0 \longleftrightarrow$ location ϕ_0
2. $\beta > 0$: $\longleftrightarrow$ Flatness
3. d: $\longleftrightarrow$ Amplitude

Slow-Roll Predictions



- SR parameters ($M_p \equiv 1$)

$$\epsilon_V = \frac{1}{2}\left(V'/V\right)^2;\ \eta_V = V''/V;\ \xi_V^2 = V'V'''/V^2$$

- Need $\phi_{\rm CMB}$ $\Rightarrow$ introduce δ:

$$\phi = \phi_0(1-\delta) \Rightarrow \delta_{\rm CMB} = 1 - \phi_{\rm CMB}/\phi_0$$

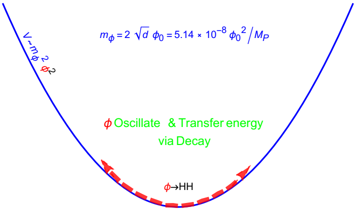
- Results:
  - $n_s \simeq 1 - 48\delta_{\rm CMB}/\phi_0^2$
  - $N_{\rm CMB} \propto \left(\frac{\pi}{2} - \arctan\left(\frac{\delta_{\rm CMB}}{\sqrt{2\beta}}\right)\right)$
  - $r \propto \left(2\beta + \delta^2\right)^2/\phi_0^2$
  - $\alpha \simeq -\frac{576(2\beta+\delta^2)}{\phi_0^4}$
  - $\mathcal{P}_\zeta \simeq \frac{d\phi_0^6}{5184\pi^2(\delta^2+2\beta)^2}$
- $n_s = 0.9649$, $N_{\rm CMB} = 65$, $\mathcal{P}_\zeta = 2.1 \cdot 10^{-9}$ $\Rightarrow$ fix parameters:

$$\delta_{\rm CMB} = 7.31 \times 10^{-4}\,\phi_0^2$$
$$\beta = 9.73 \times 10^{-7}\,\phi_0^4$$
$$d = 6.61 \times 10^{-16}\,\phi_0^2$$

- Predictions for r and α ($\phi_0 \lesssim 1$):

$$\boxed{r \sim 7.1 \times 10^{-9}\phi_0^6\,;\ \alpha \sim -1.4 \times 10^{-3}}$$

Reheating and Radiative Stability



- Decays to SM Higgs

$$\mathcal{L} \supset -\lambda_{12}\phi H^\dagger H$$

- Decay rate:

$$\Gamma_\phi \simeq \frac{\lambda_{12}^2}{8\pi m_\phi}$$

- Reheating Temperature:

$$T_{\rm rh} \simeq 1.41 g_\star^{-1/4} \Gamma_\phi^{1/2}$$

- BBN requires $T_{\rm rh} \gtrsim 4$ MeV ⇒ Lower bounds

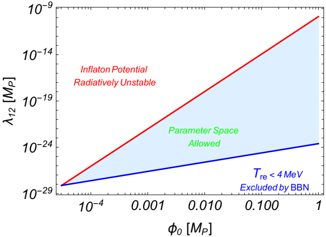
$$\frac{\lambda_{12}}{\phi_0} \gtrsim 2.4 \times 10^{-24}$$

- Radiative Stability require: Loop corrections not spoil flatness of inflaton potential ⇒ Upper bounds

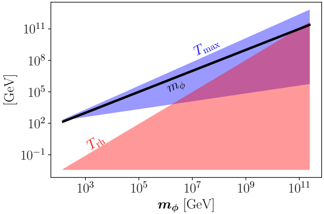
$$\left| \left(\frac{\lambda_{12}}{\phi_0}\right)^2 \ln\left(\frac{\lambda_{12}}{\phi_0}\right) - \left(\frac{\lambda_{12}}{\phi_0}\right)^2 \right| < 64\pi^2 d\,\beta$$

Reheating and Radiative Stability

- BBN+Radiative Stability $\Rightarrow$
    - Parameter space for couplings



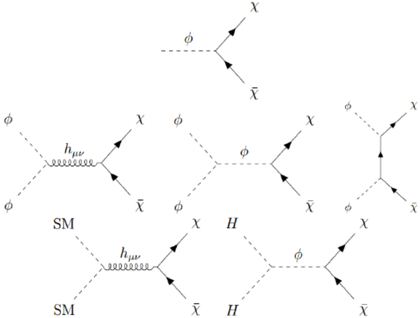
    - Parameter space for T



- $\phi_0 : 3\times10^{-5} M_p \lesssim \phi_0 \lesssim 1 M_P$; Inflaton mass: $100\,\text{GeV} \lesssim m_\phi \lesssim 10^{11}$ GeV
- $4\,\text{MeV} \lesssim T_{\rm rh} \lesssim 10^{11}$ GeV; $100\,\text{GeV} \lesssim T_{\rm max} \lesssim 10^{12}$ GeV
- Note $T_{\rm max} \sim \sqrt{T_{\rm rh}}\,(H_I M_P)^{1/4}$

DM production and relic density

- Six possible channels:

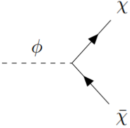


- Mainly focus on the first channel in this talk
- Boltzmann equation (BEQ) :

$$\frac{dn}{dt} + 3H\,n = \gamma$$

γ denotes interaction rate density

Inflaton direct decay



- Interaction rate density:

$$\gamma = 2\,\mathrm{Br}\,\Gamma\,\frac{\rho_\phi}{m_\phi}$$

- Branching ratio (Br $\ll 1$):

$$\mathrm{Br} \propto \frac{y_\chi^2 m_\phi^2}{\lambda_{12}^2} \propto \frac{y_\chi^2 m_\phi M_P}{T_{\rm rh}^2}$$

- During reheating ($T_{\rm rh} < T < T_{\rm max}$):

$$\rho_\phi(T) \propto \frac{T^8}{T_{\rm rh}^4}\,;\, H(T) \propto \frac{T^4}{M_P\,T_{\rm rh}^2}$$

- Convenient to use $N = n\,a^3$, rewrite BEQ:

$$\frac{dN}{dT} \sim -\frac{M_P\,T_{\rm rh}^{10}}{T^{13}}\,a^3(T_{\rm rh})\,\gamma$$

- DM yield $Y \equiv n/s$:

$$Y_0 = \frac{\overset{T_{\rm rh}^2}{\cancel{N}}(T_{\rm rh})}{s(T_{\rm rh})\,a^3(T_{\rm rh})} \propto \frac{M_P\,\overset{T_{\rm rh}^2/M_P}{\cancel{\Gamma}}}{m_\phi\,T_{\rm rh}}\mathrm{Br} \propto \frac{3}{2}\,\frac{T_{\rm rh}}{m_\phi}\mathrm{Br}$$

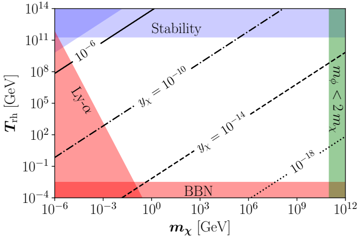
- To match the DM relics :

$$m_\chi\,Y_0 = \Omega_\chi h^2\,\frac{1}{s_0}\,\frac{\rho_c}{h^2} \simeq 4.3\times 10^{-10}\ \mathrm{GeV}$$

$$\boxed{y_\chi \simeq 1.2\times 10^{-13}\sqrt{\frac{T_{\rm rh}}{m_\chi}}}$$

Inflaton direct decay

- Parameter space (white region): $y_\chi \simeq 1.2 \times 10^{-13} \sqrt{\frac{T_{\rm rh}}{m_\chi}}$



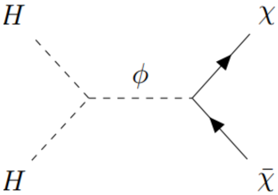
- Bounds:
  - Radiative stability: $T_{\rm rh} < 1.2 \times 10^{11}$ GeV (Higgs loop), $y_\chi < 10^{-5}$ (DM loop)
  - BBN: $T_{\rm rh} \gtrsim 4$ MeV
  - Lyα cold DM: $v_\chi = \frac{p_0}{m_\chi} \lesssim 10^{-8} c \Leftrightarrow \frac{m_\chi}{\rm keV} \gtrsim 2\, \frac{m_\phi}{T_{\rm rh}}$

$$p_0 = \frac{a_{\rm in}}{a_0}\, p_{\rm in} = \frac{a_{\rm in}}{a_{\rm eq}} \frac{\Omega_R}{\Omega_m} \frac{m_\phi}{2} \simeq 3 \times 10^{-14}\, \frac{m_\phi}{T_{\rm rh}}\ \text{GeV}\,,$$

- DM mass:

$$\boxed{\mathcal{O}(10^{-5})\ \text{GeV} \lesssim m_\chi \lesssim \mathcal{O}(10^{11})\ \text{GeV}}$$

Higgs scattering via UV freeze-in



- $m_\phi > T_{\rm rh} \Rightarrow$ UV freeze-in
- Interaction rate density:

$$\gamma \propto y_\chi^2\, \lambda_{12}^2\, \frac{T^6}{m_\phi^4}$$

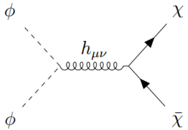
- DM yield:

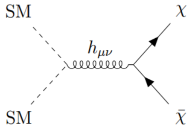
$$Y_0 \propto y_\chi^2\, \lambda_{12}^2\, \frac{M_P\, T_{\rm rh}}{m_\phi^4}$$

- DM yield :

$$Y_0 \ll Y_0^{\rm decay}$$

Gravitational channel





- DM yield:

$$Y_0 \propto \frac{T_{\rm rh}}{M_P^{5/2}}$$

- DM yield (with $m_\chi \ll T_{\rm rh}$ or $m_\chi \gg T_{\rm rh}$):

$$Y_0 \propto \left(\frac{T_{\rm rh}}{M_P}\right)^3 \text{ or } \frac{T_{\rm rh}^7}{M_P^3\, m_\chi^4}$$

- Radiative upper bounds: $T_{\rm rh} \lesssim 10^{11}$ GeV

$$\Rightarrow Y_0^{\rm gra} \ll Y_0^{\rm required}$$

Summary

- Propsoed a simple model to embed inflation and DM:

$$S = \int d^4x \sqrt{-g}\left(\mathcal{L}_{\rm EH} + \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi} + \mathcal{L}_{H\phi} + \mathcal{L}_{\chi}\right)$$

  - Inflation:

$$V = d\left[\phi^4 + A\,(1-\beta)\,\phi^3 + \frac{9}{32}A^2\phi^2\right]$$

with $A = -8/3\phi_0$; $\beta = 9.73 \times 10^{-7}\phi_0^4/M_p^4$; $d = 6.61 \times 10^{-16}\phi_0^2/M_p^2$.

    1. $r \simeq 7.1 \cdot 10^{-9}\phi_0^6/M_p^6$ ☹
    2. $\alpha \simeq -1.43 \cdot 10^{-3} \Rightarrow$ testable in future [S4 CMB] ☺
    3. $H_{\rm inf} \simeq 8.6 \cdot 10^{-9}\phi_0^3/M_p^2 \Rightarrow H_{\rm inf}$ as low as 1 MeV! ☺

  - DM: $\phi \to \bar{\chi}\chi$

$$\mathcal{O}(10^{-5})\ \text{GeV} \lesssim m_\chi \lesssim \mathcal{O}(10^{11})\ \text{GeV}$$

- Future prospects: Baryogenesis, Strong CP, ...

Thank you!