Axion quality from gauge flavour symmetries

Based in part on arXiv:2102.05055 (L. Darmé, EN) **and on work in progress** (Grillli Di Cortona, L. Darmé, C. Smarra)

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MOCa 2021 - Materia Oscura en Colombia - June 9th, 2021

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- If the axion exists, these problems must be solved ! It is conceivable that the solution could shed light on other unsolved issues of the Standard Model

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- A scalar potential invariant under a **global** U(1): $\Phi \to e^{i\xi}\,\Phi$, $\delta V(\Phi) = 0$
- U(1) **SSB**: $\Phi \to v_a\, e^{ia(x)/v_a}$. $a(x)$: $V(a) = 0 \to$ shift symmetry $a \to a + \xi v_a$
- Couplings between the scalars and some quarks $\bar{Q}_L\, \Phi\, q_R \to \bar{Q}_L\, v_a\, q_R\, e^{ia(x)/v_a}$
  U(1) is then enforced by <u>identifying</u> chiral PQ charges $X(Q) - X(q) = X(\Phi)$
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By redefining the quark fields in the basis of real masses $\bar{Q}_L\, v_a\, q_R$:

$$\Theta G\tilde{G} \quad \to \quad (a(x)/v_a + \Theta)\, G\tilde{G} \quad \to \quad (a(x)/v_a)\, G\tilde{G}$$

Instanton related non-perturbative QCD effects generate a potential
$V_{QCD}(a) = -(m_\pi f_\pi)^2 \cos(a/v_a)$ that drives $\langle a/v_a \rangle \to 0$ at the minimum

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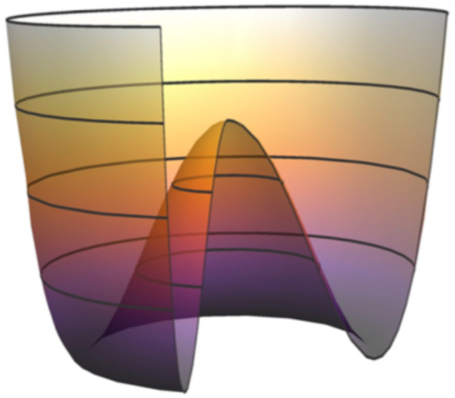
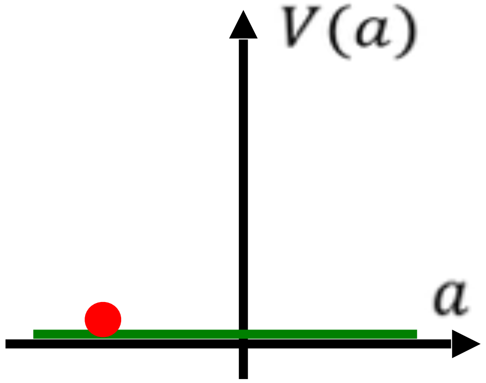
  $m_a = 0$, $\langle a_0 \rangle = \theta_0 f_a$, $\theta_0 \in [0, 2\pi]$





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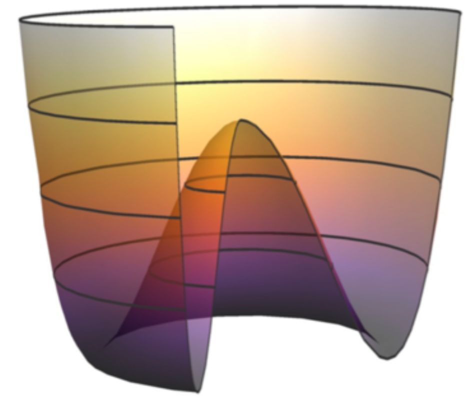

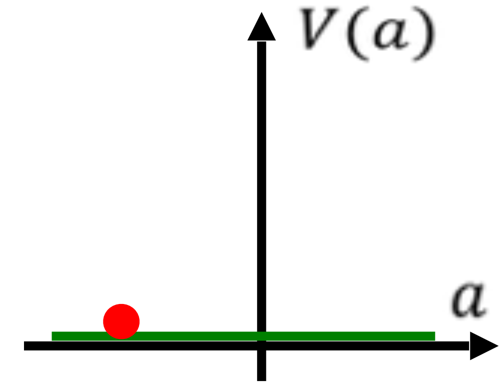

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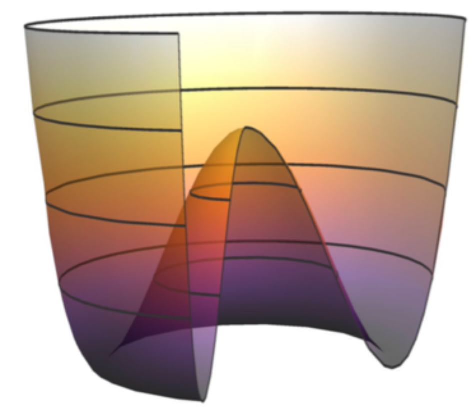

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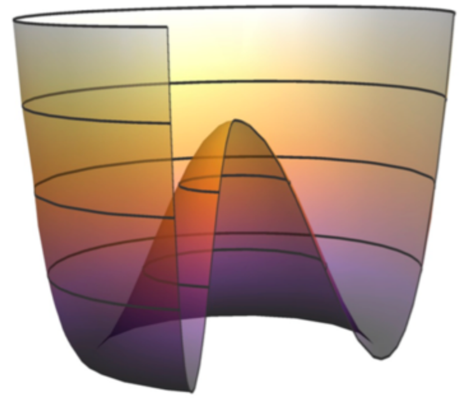

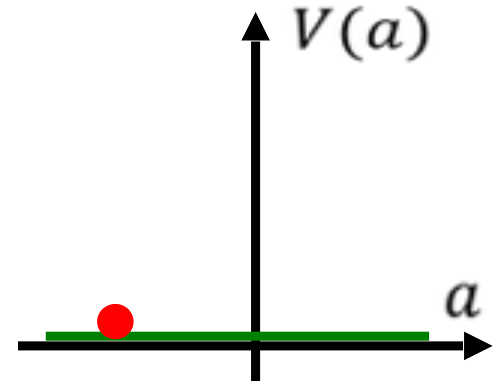

- As soon as $T \sim \Lambda_{QCD}$ ($\alpha_s \sim 1$):

  $U(1)_{PQ}$ explicit breaking ($e^{-2\pi/\alpha_s} \sim O(1)$)

  $m_a(T)$ turns on. When $m_a(T) > 3H \sim 10^{-9}$ eV,

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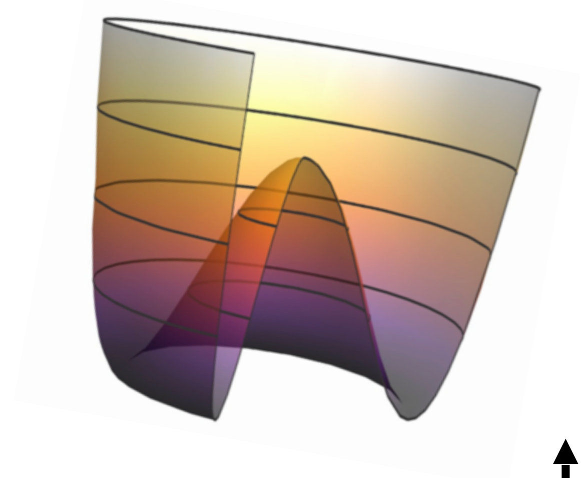

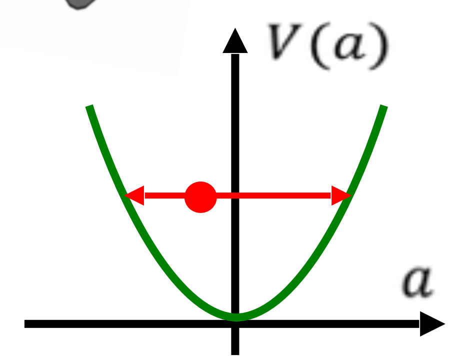

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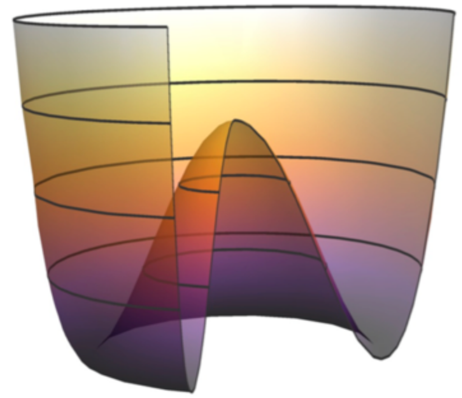

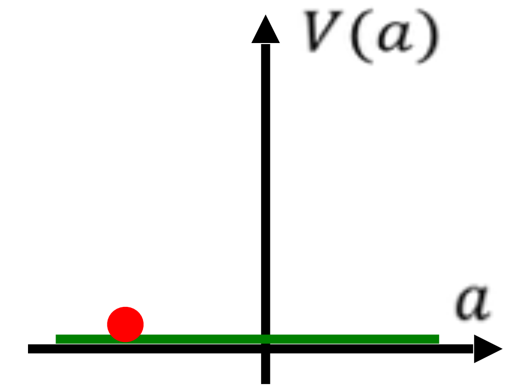

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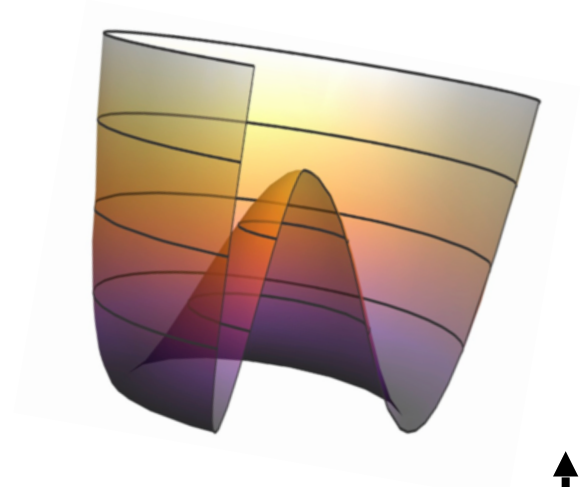

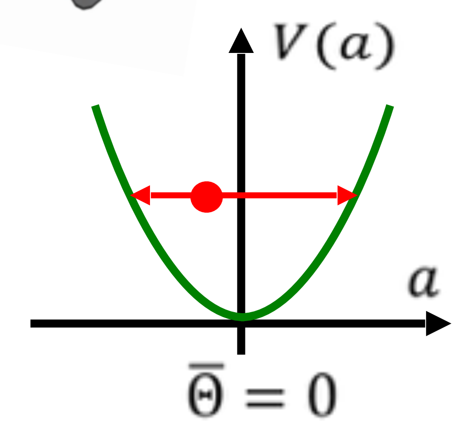

- Energy stored in oscillations behaves as CDM ($\rho_a \sim R^{-3}$)

[Preskill, Wise, Wilczek (1983), Abott, Sikivie (1983), Dine, Fischler (1983)]

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- In benchmark axion models, Φ is a complex scalar, and a gauge singlet. Renormalizable terms $\mu^3\Phi$, $\mu^2\Phi^2$, $\mu\Phi^3$, $\lambda\Phi^4$ do not break gauge or Lorentz and are not forbidden. However, they would destroy PQ invariance.

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- E.g. $g\sim 1$, $\Lambda \sim M_P$ and $v_a \sim 10^{10}$ GeV imply $d \gtrsim 10$ [with $g = g_{wh}$, $d \gtrsim 9$]
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- The axion scale: $v_a \gg 10^8$ GeV contributes to the EW stability problem (analogously to other SM completions involving a new large UV scale: seesaw, GUTs, etc.)

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- Non-Abelian $SU(n)_L \times SU(n)_R$, $a(x) \in Y_{n\times n}$. Svd: $Y = U\,\hat{Y}\,V^\dagger\,e^{ia/v_a}$

For $n > 4$ the ren. potential is very simple: $V(Y) = (T-\mu^2)^2 \pm A$

with $T = \mathrm{Tr}(Y^\dagger Y)$, $A = \mathrm{Tr}(\mathrm{mnr}[Y^\dagger Y,2]) = \frac{1}{2}[T^2 - \mathrm{Tr}(Y^\dagger Y Y^\dagger Y)]$

Automatic rephasing symm. $Y \to e^{i\xi}\,Y$. Anomaly from KSVZ quarks $\bar{Q}_L\,Y\,Q_R$

1^{st} ~~PQ~~ opt. $\Lambda^{4-n}\,\det Y$ $\dim = n$. This requires again $n \geq 10$

[Fong, EN '14 [in SU(3)xSU(3)], Di Luzio, Ubaldi, EN '17]

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Gauge invariants are constructed with Kronecker δ and Levi-Civita ε

δ-invariants can be red off the characteristic polynomial of $Y^\dagger Y$:

$$P(\xi) = \det(\xi I - Y^\dagger Y) = \Sigma_k \; (-1)^k \, C_k \; \xi^{n-k} \qquad C_k = Tr(mnr[Y^\dagger Y, k])$$

They are obviously all Hermitian $\Rightarrow$ accidental U(1): $Y \to e^{i\xi}\, Y$

ε-invariants (non-Hermitian): there is none $\varepsilon_{\alpha\beta\ldots\sigma}\, Y_{\alpha i}\, Y_{\beta j} \ldots Y_{\sigma r} = 0.$

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Note: for a $Y_{n\times n}$ square matrix $\varepsilon_{\alpha\beta\ldots\sigma} \; \varepsilon_{ij\ldots r} \, Y_{\alpha i} \, Y_{\beta j} \ldots Y_{\sigma r} \propto \det Y \neq 0$

Such automatic exact U(1) symmetries are peculiar of local `rectangular' symmetries

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$\mathrm{Rank}(Y_{3\times 2}) = 2$, one <u>massless</u> quark. Add $Z_a \sim (3,1)$: $M_q \subset \bar{Q}_L\, Y\, q_R + \bar{Q}_L\, Z\, t_R$

- Two mixed invariants $I_\varepsilon = \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ij}\, Y_{\alpha i}\, Y_{\beta j}\, Z_\gamma \neq 0$ $\quad U(1)_\varepsilon$: $\quad 2X_Y + X_Z = 0$

  $U(1)_Y \times U(1)_Z \to U(1)$ $\quad I_\delta = \varepsilon_{ij}\, (Z^\dagger Y)_i\, (Z^\dagger Y)_j$ $\quad U(1)_\delta$: $\quad X_Y - X_Z = 0$

Then $U(1)_Y \times U(1)_Z$ is completely broken, no residual U(1). No PQ symmetry ?

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$U(1)_Y \times U(1)_Z \to U(1)$ $\quad I_\delta = \varepsilon_{ij}\, (Z^\dagger Y)_i\, (Z^\dagger Y)_j$ $\quad U(1)_\delta$: $\quad X_Y - X_Z = 0$

Then $U(1)_Y \times U(1)_Z$ is completely broken, no residual U(1). No PQ symmetry ?

- Not so ! We need to consider the vacuum structure of Y and Z

$$Y = U_3\, \hat{Y}\, V_2^\dagger\, e^{i\phi_y} \quad \to \quad \langle Y \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \\ 0 & 0 \end{pmatrix} e^{i\frac{\varphi_y}{v_y}},$$

$$Z = U_3'\, \hat{Z}\, e^{i\phi_z} \quad \to \quad \langle Z \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} e^{i\frac{\varphi_z}{v_z}}, \qquad v_{y,z}^2 = T_{y,z}$$

Vacuum values of PQ breaking operators

• Exercise: take G_F = SU(3)$_L$ x SU(2)$_R$, take $Y_{ai} \sim (3,\bar{2})$ and the SU(3)$_c$ anomaly free set of quark multiplets $Q_L \sim (3,1)$; $q_R \sim (1,2)$; $t_R \sim (1,1)$

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<u>VEVs of non-Hermitian operators can only lower the potential so they are maximized</u>

$$V_{\rm nH} = \mu I_\epsilon + \lambda I_\delta + \text{h.c.} \quad \longrightarrow \quad -|\mu|\langle I_\epsilon\rangle - |\lambda|\langle I_\delta\rangle$$

$$\begin{cases} \max\langle I_\epsilon\rangle & \langle Z\rangle \sim (0,0,z_3)^T, \quad \langle I_\delta\rangle = 0 \\ \max\langle I_\delta\rangle & \langle Z\rangle \sim (z_1,z_2,0)^T, \quad \langle I_\epsilon\rangle = 0 \end{cases}$$

$$\cos[\varphi_\mu + \varphi_\epsilon(x)]\, |\mu I_\epsilon| \to -|\mu|\langle I_\epsilon\rangle$$

Vacuum values of PQ breaking operators

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Scalar bosons associated with these symmetries remain massless [Georgi & Pais '75]

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- We can easily identify the NGB that remains (perturbatively) massless and that enjoy the required shift symmetry.
  In the vacuum determined by I_ε, charges are related by $X_Z = -2\,X_y$

$$a(x) = \frac{v_y}{v_a}\varphi_y - 2\frac{v_z}{v_a}\varphi_z$$

$$a(x) = \frac{v_y}{v_a}\varphi_y - 2\frac{v_z}{v_a}\varphi_z, \quad v_a^2 = v_y^2 + 4v_z^2 \qquad \text{s.t. for} \qquad \xi \in [0, 2\pi) \quad \begin{cases} \varphi_y & \to \varphi_y + \xi\, v_y \\ \varphi_z & \to \varphi_z - 2\,\xi\, v_z \\ a(x) & \to a(x) + \xi\, v_a \end{cases}$$

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The non-Abelian local G_F thus is a flavour symmetry !

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Any non-Abelian gauge symmetry generating a $U(1)_{PQ}$ is a flavour symmetry

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- We are led to consider models of flavour with a generic structure

$$\mathcal{L} \sim \overline{Q}\, Z\, q + \frac{1}{\Lambda}\left(\kappa_d \overline{Q}\, Y\, d\, H + \kappa_u \overline{Q}\, X\, u\, \tilde{H} + \kappa_3 \overline{Q}\, Z\, u_3\, \tilde{H}\right) + \frac{1}{\Lambda^2}\left(\kappa_q \overline{Q}\, W\, q + \dots\right) + \dots$$

with Z, X, Y scalar multiplets of some G_F. Possibly involving also combinations of scalar fields W = W[Z,X,Y]. It can contain EW vectorlike quarks (e.g. $q_R \in SU(2)_W$). SM quarks masses and mixings generated dynamically by specific ⟨Z⟩, ⟨X⟩, ⟨Y⟩ configurations, with hierarchical singular values [for a proof of principle of the viability, Fong & EN '13]

The guiding principle is that a PQ symmetry of the required high quality must arise automatically from G_F and the field content.

A glimpse on the generation of Yukawa hierarchies

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Consider $G_F = SU(4)_L \times [SU(3)_d \times SU(2)_u]_R$ and the quark/scalar multiplets:

$$Q_L \sim (4,1,1), \quad q_R \sim (1,1,1), \quad d_R \sim (1,\bar{3},1), \quad u_R \sim (1,1,\bar{2}), \quad t_R \sim (1,1,1)$$

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"Flavour relevant" renormalizable invariants and their action in the approx. in which the svd L-matrices $U_X, U_Z \to I_4$ (neglecting mixings, only hierarchies)

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"Flavour relevant" renormalizable invariants and their action in the approx. in which the svd L-matrices $U_X, U_Z \to I_4$ (neglecting mixings, only hierarchies)

$$\begin{array}{llllll}
A_Y & = \mathrm{Tr}\,[\mathrm{Mnr}(Y^\dagger Y, 2)] & = & y_1^2\, y_2^2 + y_1^2\, y_3^2 + y_2^2\, y_3^2, & (\eta_A > 0) & \hat{Y} \to (y, 0, 0) \\
D_X & = \mathrm{Det}[X^\dagger X] & = & x_1^2\, x_2^2, & (\eta_D < 0) & \hat{X} \to (x, x) \\
\mathcal{E}_{YZ} & = \epsilon_3 \epsilon_4\, YYYZ & \to & -2\, y_1\, y_2\, y_3\, z_4 & & \hat{Y} \to (y, \epsilon_y, \epsilon_y) \\
T_{XY} & = \mathrm{Tr}\,[XX^\dagger YY^\dagger] & \to & x_1^2\, y_1^2 + x_2^2\, y_2^2, & (\eta_T < 0) & \hat{Y} \to (y, \epsilon_y, \epsilon'_y),\ \hat{X} \to (x, \epsilon_x) \\
T_{ZX} & = \mathrm{Tr}\,[ZZ^\dagger XX^\dagger], & T_{ZY} = \mathrm{Tr}\,[ZZ^\dagger YY^\dagger] \to 0 & & &
\end{array}$$

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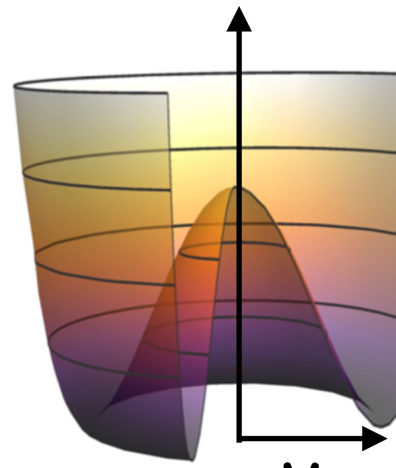
We are currently studying flavour groups that we would never have considered had it not been for the axion !

The axion scale problem: scale vs. compact space radius

Consider the usual Mexican hat potential for a complex Φ hosting the axion



- Scale of PQ symm. breaking: $\langle\Phi\rangle = v_a$
  (phase transition, primordial GW,…)
- Axion compact field space radius $a \in [0, 2\pi\, f_a)$
  (suppression of axion couplings: $a(x)/f_a$)

Here $v_a = f_a$, but conceptually they are different quantities.

When the axion is hosted in more than one scalar multiplet: $\Phi_i \sim v_i\, e^{a_i/v_i}$

$a = \Sigma_i\, (v_i/f_a)\, a_i$ with $f_a^2 = \Sigma_i\, X_i^2\, v_i^2$ enhancement by large charge values

[Clockwork mechanism: Choi & Im '16, Kaplan & Rattazzi '16, Giudice & McCullough '17 …]

Consider a gauge group $[SU(3)\times SU(2)]^{n+1}$ and $Y \sim (1_{n-1}, 2_n, 3_n)$, $\Sigma\sim(3_{n-1},\bar{2}_n, \bar{3}_n)$

The potential $V = \Sigma_n\, \varepsilon_3\, \varepsilon_2\, Y_n\, Y_n\, \Sigma_{n+1}\, Y_{n+1}$ has automatic symm. $X_{n+1} = 2\, X_n$ $(X_\Sigma = 0)$

Then $f_a^2 = \Sigma_n\, X_n^2\, v_n^2 \approx (1/3)\, v^2\, 4^{n+1}$ (after taking all $v_n \approx v$)

If quarks couple to Y_1 : $\bar{Q}\, Y_1\, q$ so that X_q are small, all axion interactions are suppressed as $1/f_a$. For $n \sim 20$, $v \sim 100$ GeV, $v/M_P \sim 10^{-17}$

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- Recall $\langle Y \rangle \sim (y_1, y_2, 0)^T$. To ensure a mass for t_R, we need to choose $\langle Z \rangle \sim (0,0,z_3)^T$ that is $\langle Y \rangle$ and $\langle Z \rangle$ must be "misaligned".

The vacuum is defined by $\langle I_\delta \rangle = 0$ and $\langle I_\varepsilon \rangle \neq 0$ $\quad X(I_\varepsilon) = 2X_Y + X_Z = 0$

- Let us now compute the anomaly $\quad A_{PQ} = \Sigma_{q_L} X_L \; - \; \Sigma_{q_R} X_R$

$$3\,X_Q - 2\,X_q - X_t \;=\; 2(X_Q - X_q) + (X_Q - X_t) \;=\; 2X_Y + X_Z \;=\; X(I_\varepsilon) = 0$$

Thus $\langle I_\varepsilon \rangle$ breaks $U(1)_Y \times U(1)_Z \to U(1)_\varepsilon$ which is non-anomalous !
Then $U(1)_\varepsilon$ is not a PQ symmetry, and its (exactly massless) NGB does not solve the strong CP problem.

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Is this just an unlucky accident occurring with the flavour $SU(3)_L \times SU(2)_R$ gauge symmetry ?

An upper limit on the quality of the PQ symmetry

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Consider a gauge symmetry $G_F = [\Pi_\ell\ SU(m_\ell)]_L \times [\Pi_r\ SU(n_r)]_R$ acting on a certain set of scalar multiplets in bi-fundamentals $Y^{\ell r} \in SU(m_\ell) \times SU(n_r)$ of the m_ℓ, n_r gauge factors, and on $N = \Sigma_\ell\, \lambda_\ell\, m_\ell = \Sigma_r\, \lambda_r\, n_r$ LH and RH quarks also in fundamentals ($\lambda_{\ell,r}$: isospin multiplicity). We can write a certain number of gauge invariant quark-scalar couplings: $\Sigma\ \eta^{\ell r} \bar{Q}_\ell\, Y^{\ell r} q_r$ ($\eta^{\ell r}$: O(1) constants; $^{\ell r}$ \`names' not indices; H/Λ when needed)

Assuming that all the quarks acquire masses ($\det M_q \neq 0$), it can be shown that:

1. for any global U(1) there exists at least one scalar operator $O(Y)$ with a non-vanishing VEV and charge equal to the U(1)-SU(3)$_c$ anomaly: $X_{O(Y)} = A_C \neq 0$
2. modulo the coupling constants $\eta^{\ell r}$ we have: $\langle X_{O(Y)} \rangle \simeq \Lambda^{4-N} \det \mathcal{Y}_q^{\mathrm{eff}}$

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1. implies that any anomalous U(1) suffers explicit breaking at least at $d = N$. This provides an upper limit on the quality of G_F-protected PQ symmetries.
2. implies that this source of breaking is removed as $\det \mathcal{Y}_q^{eff} \to 0$. Providing an unexpected connection between PQ quality and Yukawa hierarchies !