Graviballs and Dark Matter

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Benjamin GUIOT
Andrés Bórquez

Alexandre Deur

Klaus Werner

UNIVERSIDAD TECNICA FEDERICO SANTA MARIA

UNIVERSITY of VIRGINIA

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Introductory remarks

- Dark Matter indirectly observed thanks to its gravitational effects.
- The claim that relativistic particles cannot form sufficiently small clumps is not always true

Counterexamples:

  - Glueballs
  - Black holes

- What matters is the strength of the interaction.
- The gravitational interaction can be arbitrarily strong (just increase the energy or decrease the distance).

  The graviton looks like a natural candidate to the missing mass problem.

Graviballs

Gravitons could attract each other, forming a graviball (by analogy with glueballs).

Similar to black holes formation by ultra-Planckian $2 \to N$ scattering *(G. Dvali et al., Nucl.Phys. B893 (2015) 187-235; A. Addazi et al., JHEP 1702 (2017) 111)*.

Questions:

1) Is a graviball allowed by the theory? (our present focus).
2) Is this object a viable DM candidate? (at this point, speculative discussion).

I will present the case of a graviball made of two gravitons:

- The goal is to learn/develop the necessary tools for these calculations. We also want to motivate a more realistic/complete study.
- The case of two gravitons is simple, in particular with symmetric initial conditions.
- We will see that two gravitons can easily form a small bound system.

What we did

We used low-energy Quantum Gravity and semi-classical calculations, based on the potential extracted from the Feynman amplitudes:

1) Find the amplitude for $gg \to gg$. The result is already known at tree level, 1 loop, and since recently at 2 loops.
2) Extract the long-range potential, V(r), from these amplitudes.
3) Solve numerically the relativistic equation of motion (a system of non-linear differential equations).
4) Two videos are available in the arxiv ancillary files (arxiv 2006,02534).

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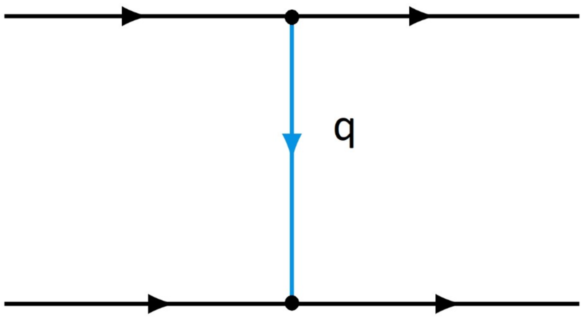
Quantum gravity as an EFT

- Quantum Gravity is a gauge theory. The requirement of general coordinate invariance, $x'_\mu = x'_\mu(x)$, leads to

$$S = \int d^4x \sqrt{g} \left[\Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + ... + \mathcal{L}_{\text{matter}} \right]$$

- Not renormalizable in the usual sense. But renormalizable in the EFT sense.

Energy expansion because each R includes two derivatives of the metric. In momentum space, these derivatives are associated with the 4-momentum transferred, q .



J. Donoghue, General relativity as an effective field theory: The leading quantum corrections, Phys.Rev. D50 (1994) 3874-3888, ***768 citations****.*

The potential (energy)

Obtained from non-analytic terms such as $\ln(q^2)$ or $(q^2)^{-1}$.

The long-distance potential is given by the Fourier transform of the non-analytic part of the amplitude:

$$V(r) = -\int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{r}.\vec{q}} \frac{\mathcal{M}(\vec{q})}{\sqrt{2E_1}\sqrt{2E_2}\sqrt{2E_3}\sqrt{2E_4}}$$

- We work in the non-relativistic limit $t = q^2 \simeq -\vec{q}^{\,2} \to 0$.
- In the C.O.M frame, the denominator is simply the Mandelstam variable s.

The potential (energy)

First example: The Coulomb potential $(e^-\mu^- \to e^-\mu^-)$

$$\mathcal{M}_{+,+;+,+} = \frac{e^2}{t}[4|\gamma^\mu|2\rangle\langle 1|\gamma_\mu|3] = \frac{e^2}{t}2[43]\langle 21\rangle = e^2\frac{s}{t} \simeq -e^2\frac{s}{\vec{q}^{\,2}}$$

$$\mathcal{M}_{+,-;+,-} = \frac{e^2}{t}[4|\gamma^\mu|2\rangle\langle 3|\gamma_\mu|1] = \frac{e^2}{t}2[41]\langle 23\rangle = -e^2\frac{u}{t} \simeq -e^2\frac{s}{\vec{q}^{\,2}}$$

$$V(r) = -\mathrm{FT}\left(\frac{\mathcal{M}}{s}\right) = \frac{e^2}{4\pi r} = \frac{\alpha}{r}$$

*We used to fact that $u \simeq -s$ when $t \to 0$.

The potential (energy)

Second example: Bending of light in Quantum Gravity, *N. E. J. Bjerrum-Bohr, John F. Donoghue, Barry R. Holstein, Ludovic Planté and Pierre Vanhove, Phys.Rev.Lett. 114 (2015) no.6, 061301*

- The authors considered the scattering of a massless particle (scalar or spin 1) from a heavy scalar object (the sun).
- They did the calculation up to one loop. Loops are complicated in quantum gravity, and you need to use different tricks or techniques (some groups work with the string based method).

$$V(r) = -\frac{2GM\omega}{r} - \frac{15(GM)^2\omega}{4r^2} + \text{ Quantum corrections}$$

- Gives the correct value, and includes the first post-Newtonian correction.
- Only the quantum correction part depends on the spin of the massless particle.
- Quantum corrections are negligible $\propto \hbar G^2 M\omega/r^3$

The graviton-graviton potential

Tree level:

- At tree level, the amplitude is known since several decades. It has been reported by several groups *(DeWitt; Delfino-Krasnov-Scarinci; Bern; Grisaru-van Nieuwenhuizen-Wu)*

$$\mathcal{M}_{++;++} = \left(\frac{\kappa}{2}\right)^2 \frac{s^3}{tu}; \qquad \kappa^2 = 32\pi G$$

By applying the usual procedure, $t \to 0$, $u \simeq -s$, $V(r) = -\mathrm{FT}\left(\mathcal{M}/s\right)$

$$V(r) = -\frac{2Gs}{c^4 r}$$

- The other amplitudes are zero (at tree level).

The graviton-graviton potential

One-loop amplitude:

- *D. C. Dunbar and P. S. Norridge, Nucl.Phys.B 433 (1995) 181-208.*
- Schematically, the amplitude reads:

$$A = F\left(\frac{1}{\epsilon}\frac{\ln s}{ut} + \frac{\ln t \ln s}{ut} + \frac{t^n \ln(t/s)^m}{s^p}\right) + \text{analytic terms}$$

Where $n, m \geq 0$ and $p > 5$.

- The IR divergence cancels at the cross section level with bremsstrahlung diagrams (just ignore it).
- One can also argue that the second term should be ignored.
- The third term gives negligible quantum corrections.

Surprisingly (?), in contrast to the case with at least one massive external particle, no classical corrections.

Simulation

- The 1-loop correction is negligible, so we work with the tree-level potential:

$$V(r) = -8G\frac{M_g^2}{r}, \quad M_g = \frac{\omega}{c^2}$$

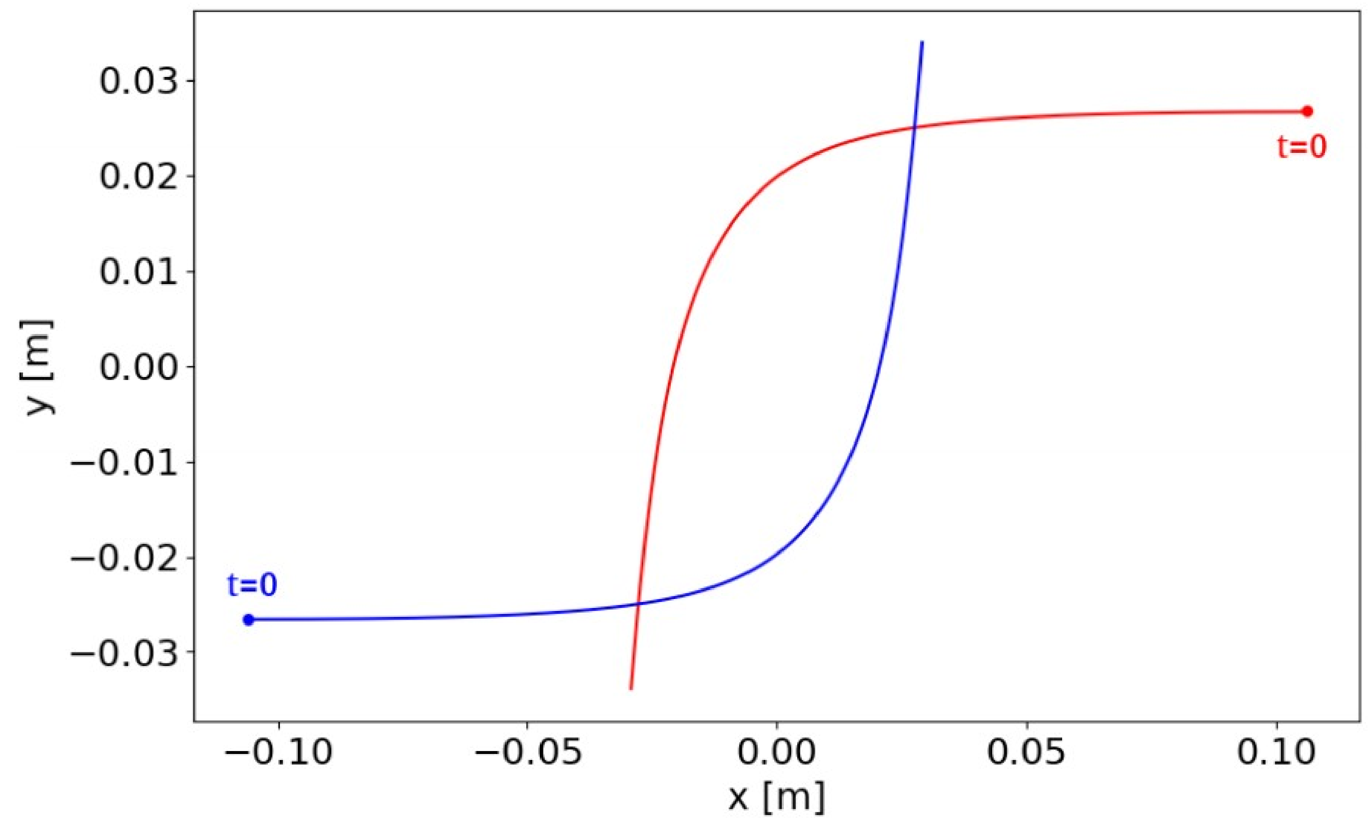
Initial momenta: $p_1 = (\omega, \omega, 0, 0)\,, \quad p_2 = (\omega, -\omega, 0, 0)$

- Use Runge-Kutta to solve the relativistic equations of motion:

$$\begin{aligned}
\frac{dx}{dt} &= v_x \\
\frac{dy}{dt} &= v_y \\
\frac{dv_x}{dt} &= \left(F_x - \frac{v_x}{c^2}(v_x F_x + v_y F_y)\right)/M_g \\
\frac{dv_y}{dt} &= \left(F_y - \frac{v_y}{c^2}(v_x F_x + v_y F_y)\right)/M_g
\end{aligned}$$

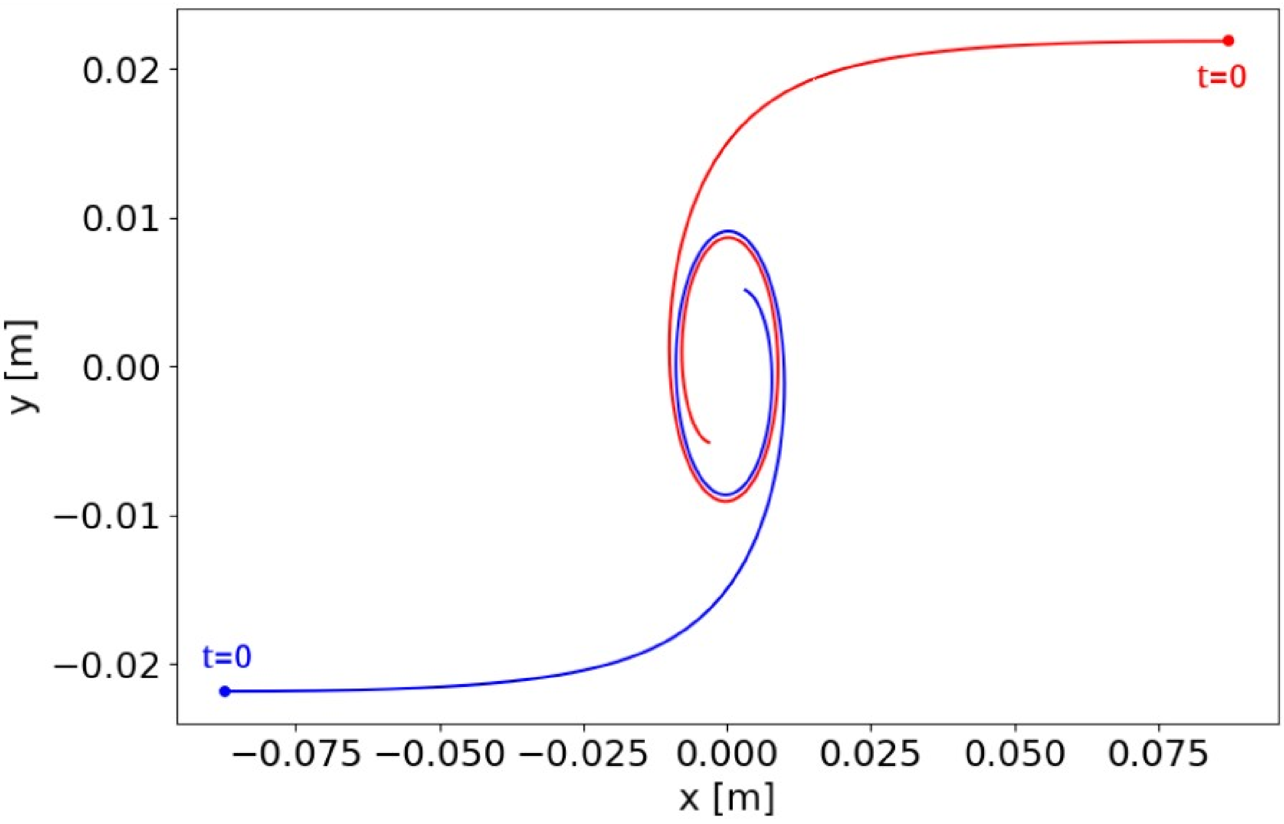
Results

- For large impact parameter b, bending of the gravitons trajectories.



Results

- For $b < b_{hor}$, formation of a graviball.
- $b_{hor} \simeq \frac{8GM_g}{c^2} \sim$ Few centimeters if $M_g = M_{\rm Earth} = 5.9\,10^{24}$ kg.
- Graviball at rest with a mass of $M_{\rm halo} = \sqrt{(p_1+p_2)^2} = 2\omega$.



Small distance behavior

- In the case of 2 massive particles, the corrections due to higher orders quadratic in the curvature is:

$$V(r) = -\frac{\kappa^2}{32\pi}\frac{m_1 m_2}{r}\left(1 + \frac{1}{3}e^{-M_1 r} - \frac{4}{3}e^{-M_2 r}\right)$$

With $M_1(c_1, c_2, G),\ M_2(c_1, c_2, G)$

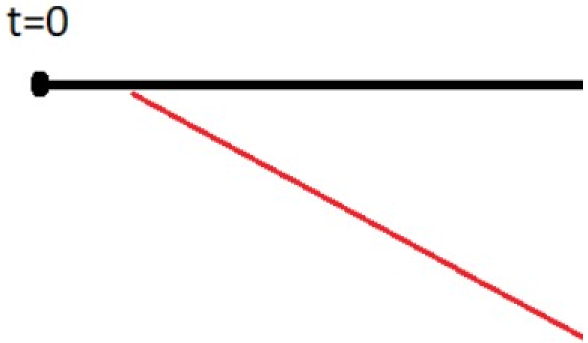
- For $c_1,\, c_2 \sim 1$, non-negligible corrections if $r < 10^{-35}$ m.

We do not expect the unknown small distance behavior to change our conclusion.

$$S = \int d^4x\sqrt{g}\left[\Lambda + \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R^{\mu\nu}R_{\mu\nu} + ... + \mathcal{L}_{\text{matter}}\right]$$

Why is the gravi-strahlung possible?

$$p_1 = \omega_1(1,1,0,0),\ p_e = \omega_e(1, \cos\theta, \sin\theta, 0)$$



$$V(r) = -\frac{4G\omega_1\omega_e(1-\cos\theta)}{r}$$

1) Modification of the potential by higher orders (small distance).

2) The initial distance between the two gravitons is larger than b_{hor}.

In the latter case, we see that only soft gravitons (small ω_e) can be emitted.

We find the result of general relativity back in the limit of large N:

$$V(r) \propto s = (p_1 + ... + p_N + p_e)^2 \simeq (p_1 + ... + p_N)^2$$

Dark Matter?

- Our results suggest two candidates for the missing mass problem:

1) Heavy graviballs, similar to the one just discussed.
2) Light graviballs.

Heavy graviballs:

- Similar to primordial black holes.
- Its mass is $M_{\rm halo} = 2M_g$ and the effect on the light of a distant galaxy is

$$\theta \simeq \frac{8GM_g}{bc^2} + \frac{15\pi G^2 M_g^2}{b^2 c^4}$$

For $b = 10^3$ m and $M_g = M_{\rm Earth}$, the first term gives 7.2 arcsec and the second 1.8 10^{-4} arcsec.

Eq. taken from Phys.Rev.Lett. 114 (2015) no.6, 061301 and I assumed that the Graviball is a scalar.

Dark Matter?

Light graviballs:

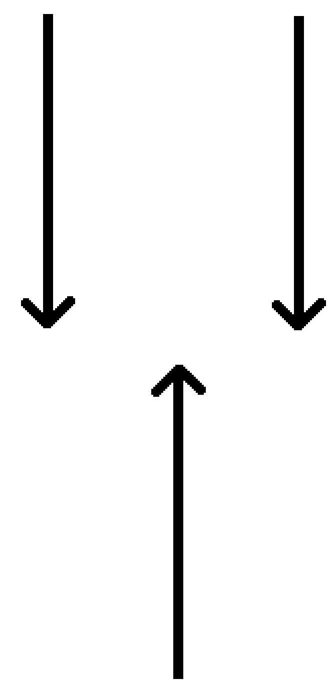
- $M_g \sim 10^{-6}\mathrm{kg} \rightarrow b_{hor} \sim 10^{-32}\mathrm{m}$
- Remember that for $c_1, c_2 \sim 1$ our calculations are valid up to $r \sim 10^{-35}$ m.
- Light enough to escape direct detection.
- 10^{50} of them would explain the missing mass (density about $10^{-15}\mathrm{m}^{-3}$).
- Could be confused with a new particle?

The 3-graviton graviball

- We modified the numerical code to make 4-momentum conservation explicit.
- A repulsive potential has been implemented at small distance $r \ll b_{hor}$.
- It allows for a first study of configurations leading to the graviball “evaporation”.

Remark: gravitons propagating in the same direction do not interact, $s = 0$.

A graviton could also escape if its energy is small enough. These considerations are important for the estimation of the **graviball lifetime (v.s. N)**.



Conclusion

- Graviballs seem to be allowed by low-energy quantum gravity.
- But we still need to study:
  1) the stability of this object.
  2) A realistic case with more than 2 gravitons, including gravi-strahlung, $2 \to N$ scatterings and creation of a pair of particle-antiparticle.
- Could be a viable solution to the missing mass, but a detailed phenomenological analysis should be performed.

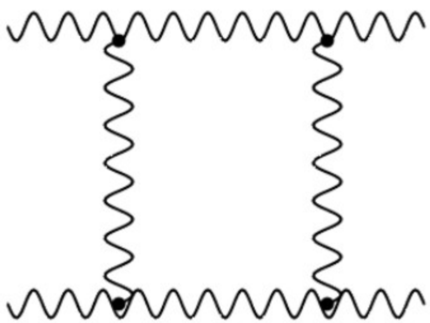
Thank you for your attention!

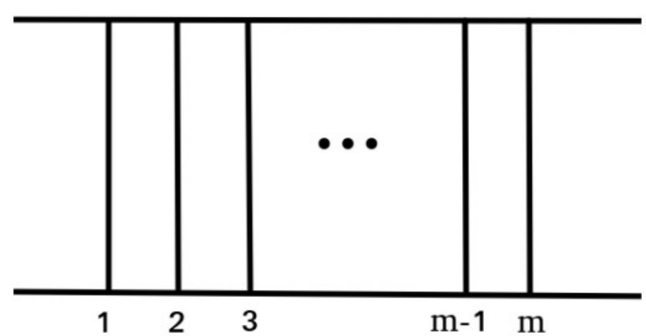
Back-up slide

The IR divergent terms of interest are:

$$\frac{2F}{\epsilon}\left(\frac{\ln(-\tilde{u})}{\tilde{s}\tilde{t}}+\frac{\ln(-\tilde{s})}{\tilde{u}\tilde{t}}\right)$$

Terms such $\frac{\ln t \ln s}{ut}$ give a contribution to the potential $\propto 1/\hbar$. Come usually from loop diagrams like the one shown to the left.





However, we don’t want to include such diagrams. the scattering of two ultra-Planckian gravitons is elastic and happens through the exchange of a large number of soft and nearly on-shell gravitons.