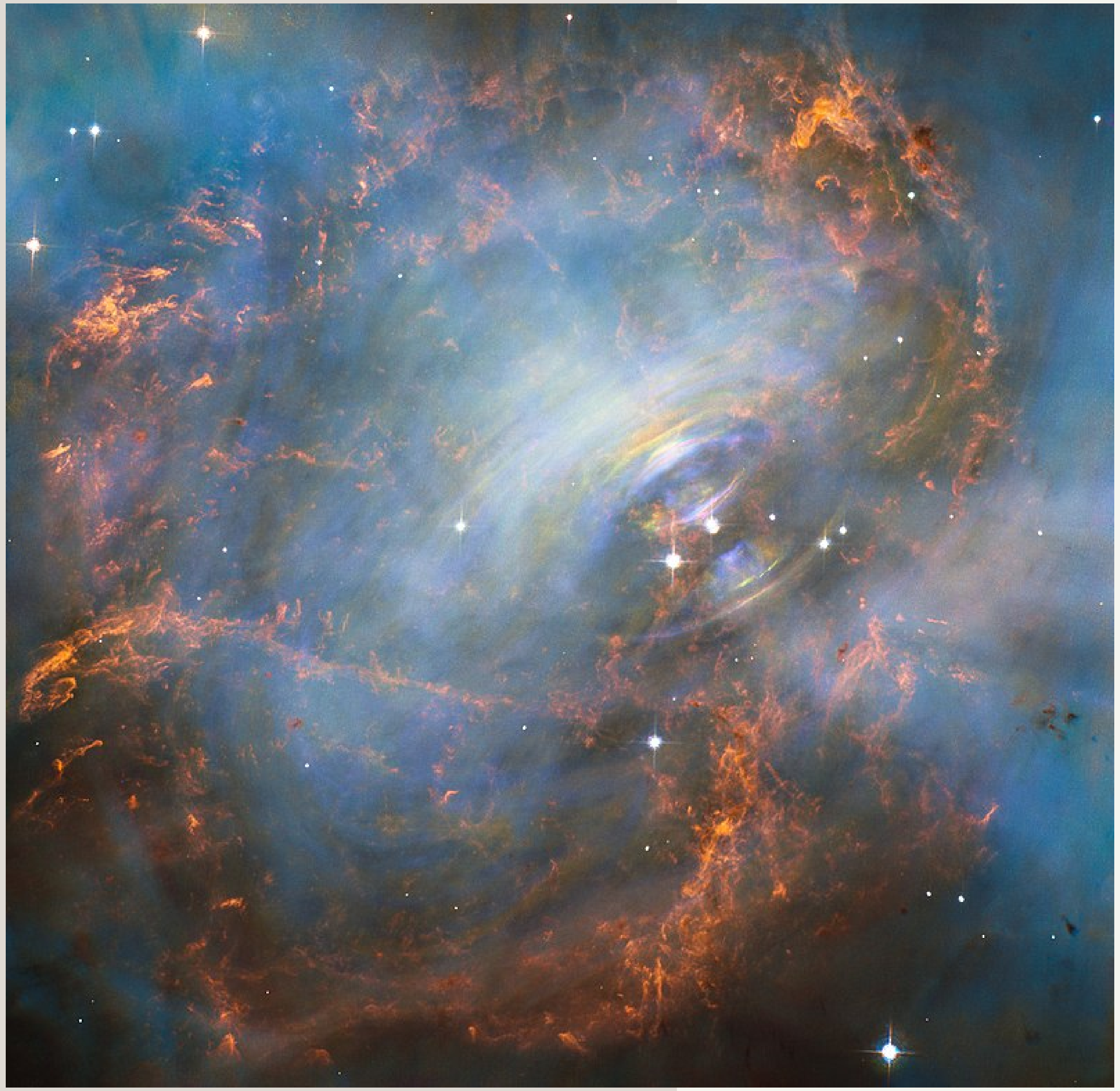

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Astrophysical solutions in the generalized SU(2) Proca theory

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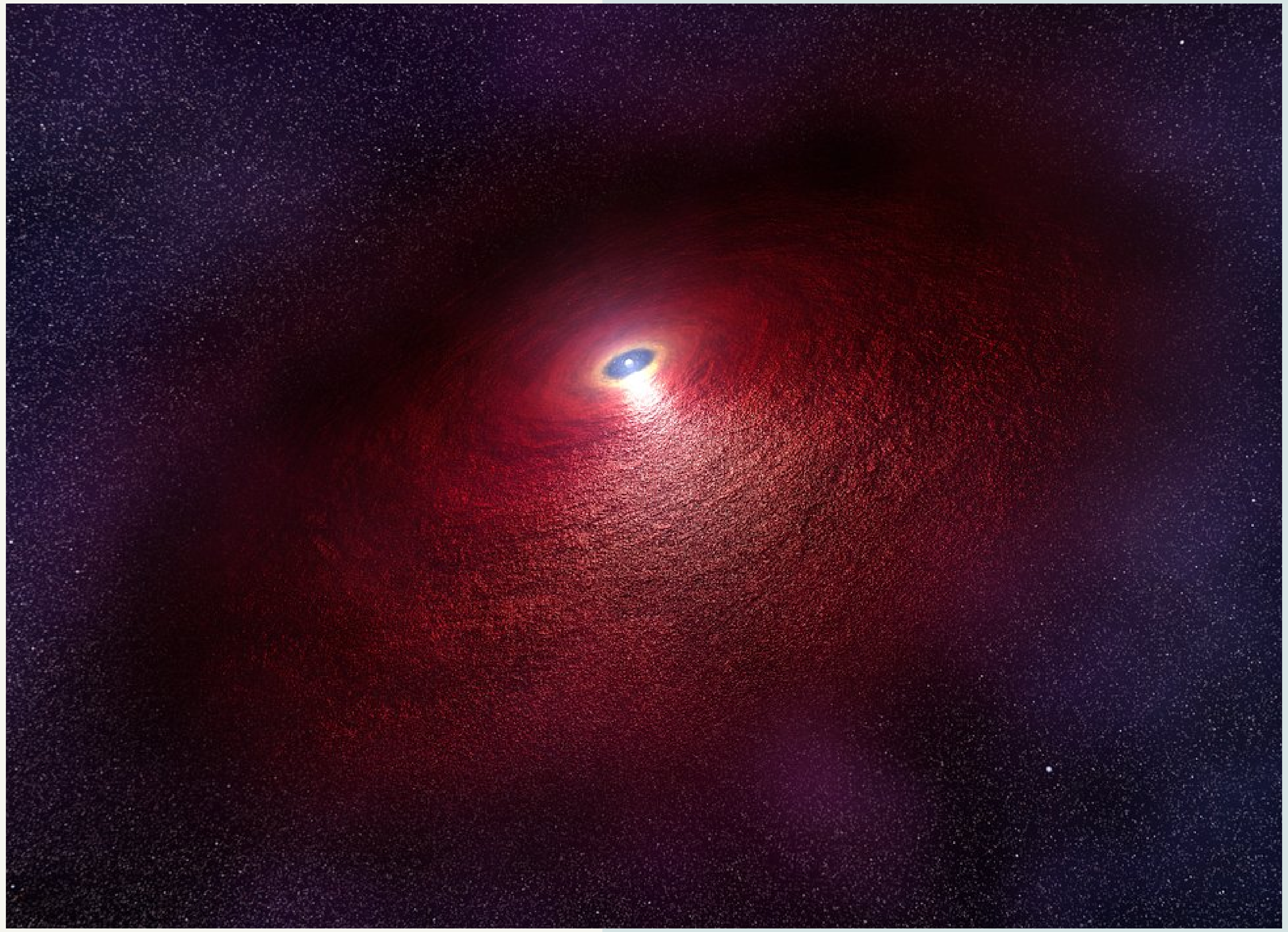
Contents



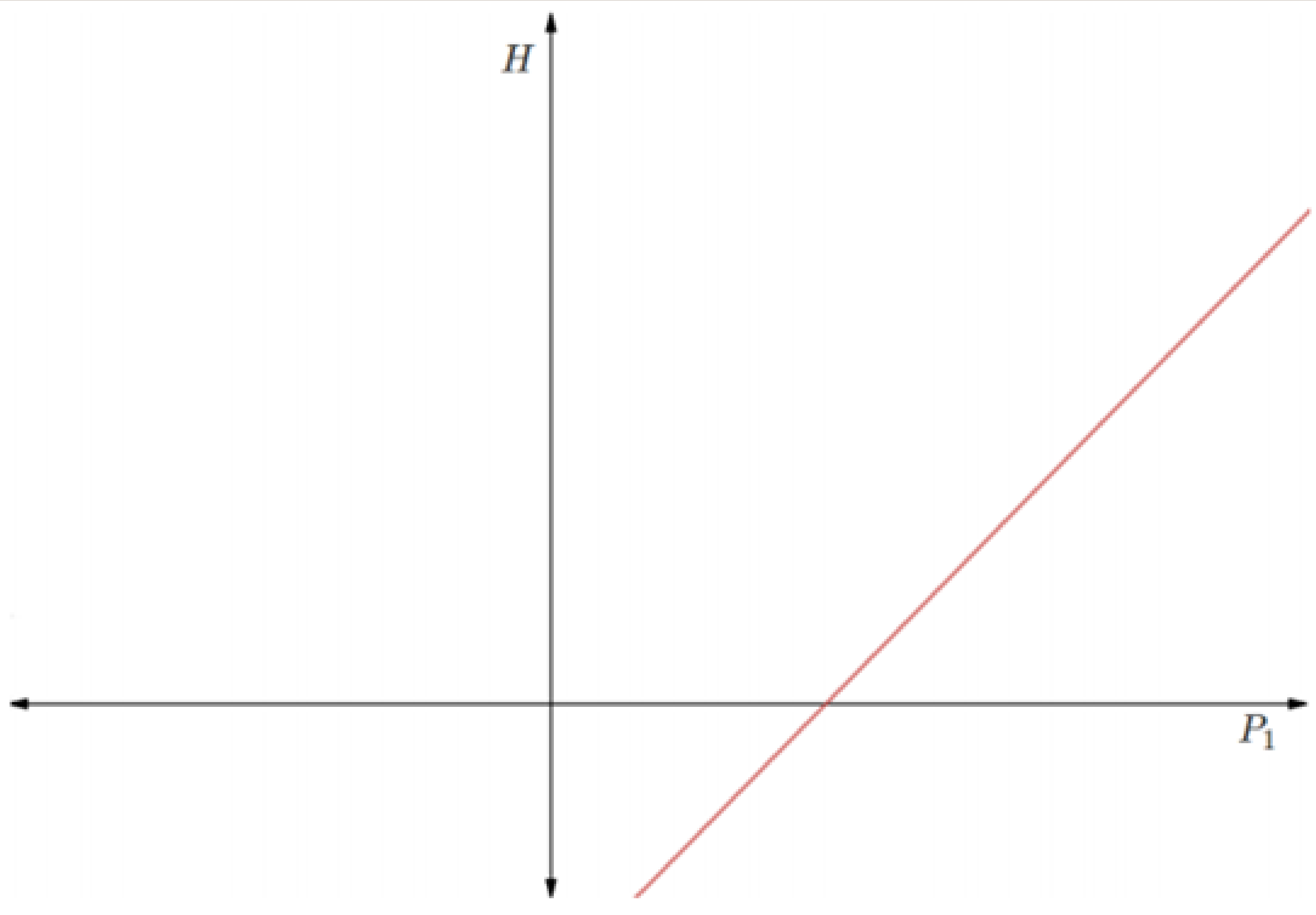
https://cutt.ly/UnxiMG9

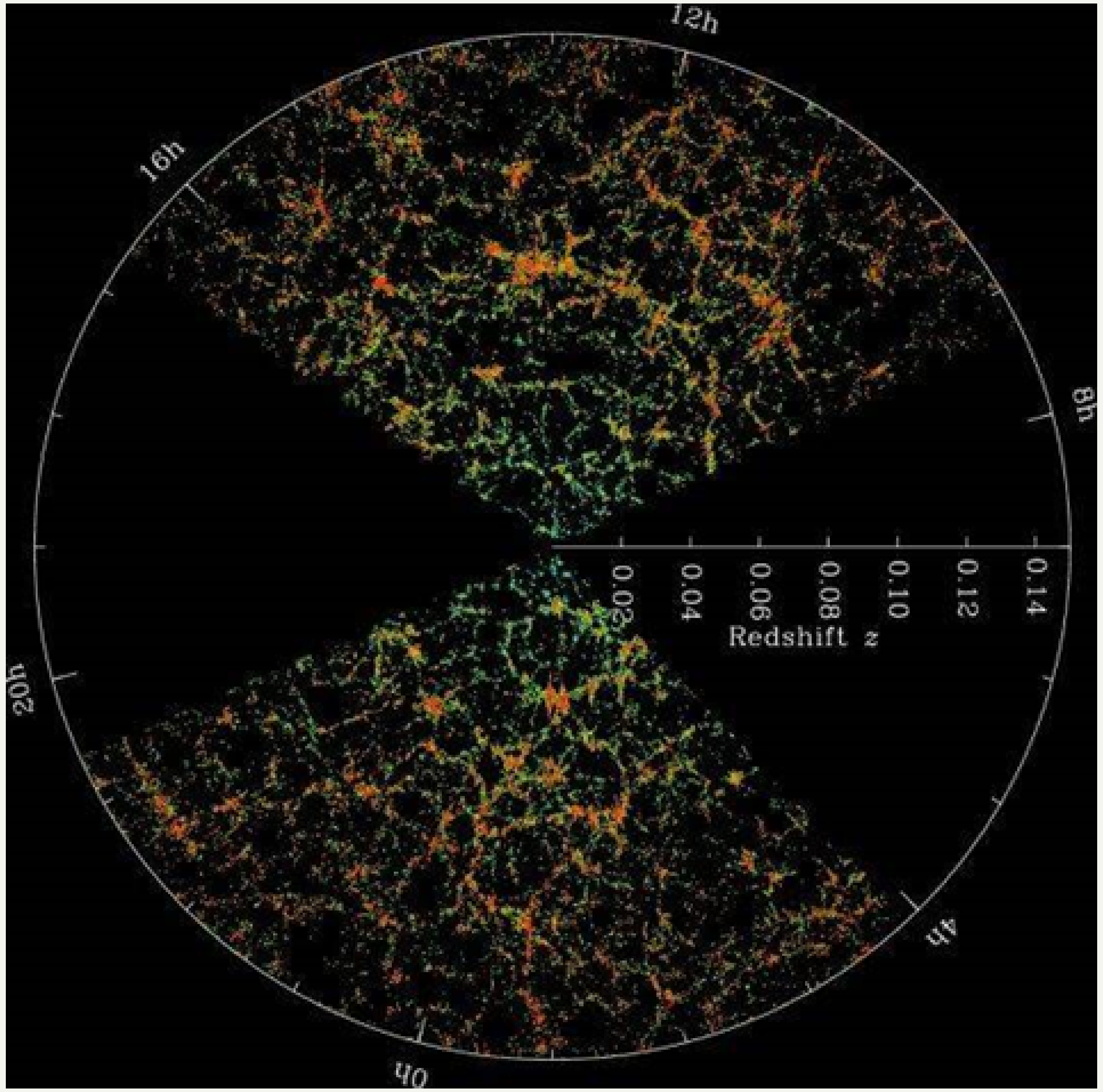
Introduction

- R. Bartnik and J. McKinnon, 1988.
- Generalized SU(2) Proca theory, 2020.
- Generalization of gauge bosons stars.
- Dark matter.

Ostrogradski's instability

M. Ostrogradski, Mem. Ac. St. Petersbourg, 1850.





Generalized SU(2) Proca theory

A. Gallego et. al., Phys. Rev. D.,2020.
Y. Rodríguez et al., Phys. Dark Univ.,2018.
https://bit.ly/2Z5m6XT.

Generalized SU(2) Proca theory

$$
\begin{aligned}
\mathcal{L} = \frac{1}{16\pi}\Big[& R - F^a_{\mu\nu} F_a^{\mu\nu} - 2\mu^2 A_a^\mu A_\mu^a \\
& + \alpha_1 \left(\mathcal{L}^1_{4,2} - 2\mathcal{L}^4_{4,2} - \frac{20}{3}\mathcal{L}^5_{4,2} + 5\mathcal{L}'_7 \right) \\
& + \alpha_3 \left(2\mathcal{L}^2_{4,2} + \mathcal{L}^3_{4,2} + \frac{7}{20}\mathcal{L}^4_{4,2} + \frac{14}{3}\mathcal{L}^5_{4,2} - 8\mathcal{L}^6_{4,2} + \mathcal{L}'_7 \right) \\
& + \chi_1 \mathcal{L}'_1 + \chi_2 \mathcal{L}'_2 \\
& + \chi_4 \left(\mathcal{L}'_4 - \frac{\mathcal{L}'_7}{2} \right) + \chi_5 \mathcal{L}'_5 + \chi_6 \left(\mathcal{L}'_6 - 3\mathcal{L}'_7 \right) \Big],
\end{aligned}
$$

Generalized SU(2) Proca theory

$$\mathcal{L}^1_{4,2} = \frac{1}{4}\left(A_b \cdot A^b\right)\left[S^{\mu a}_{\mu} S^{\nu}_{\nu a} - S^{\mu a}_{\nu} S^{\nu}_{\mu a}\right] + \frac{1}{2}\left(A_a \cdot A_b\right)\left[S^{\mu a}_{\mu} S^{\nu b}_{\nu} - S^{\mu a}_{\nu} S^{\nu b}_{\mu}\right],$$

$$\mathcal{L}^2_{4,2} = G^{a}_{\mu\nu} S^{\nu b}_{\sigma} A^{\nu}_{a} A^{\sigma}_{b} - G^{a}_{\mu\nu} S^{\mu b}_{\sigma} A^{\nu}_{b} A^{\sigma}_{a} + G^{a}_{\mu\nu} S^{\sigma b}_{\sigma} A^{\mu}_{a} A^{\nu}_{b},$$

$$\mathcal{L}^3_{4,2} = A^{\mu a} R^{\alpha}{}_{\sigma\rho\mu} A_{\alpha a} A^{\rho c} A^{\sigma}_{c} + \frac{3}{4}\left(A^{a}_{\mu} A^{\mu}_{a}\right)\left(A^{\nu}_{b} A^{b}_{\nu}\right) R,$$

$$\mathcal{L}^4_{4,2} = -\frac{3}{4}\left[\left(A^a \cdot A_a\right)\left(A^b \cdot A_b\right) + 2\left(A^a \cdot A^b\right)\left(A_a \cdot A_b\right)\right] R,$$

$$\mathcal{L}^5_{4,2} = G_{\mu\nu} A^{\mu a} A^{\nu}_{a}\left(A^b \cdot A_b\right), \quad \mathcal{L}^6_{4,2} = G_{\mu\nu} A^{\mu a} A^{\nu b}\left(A_a \cdot A_b\right),$$

Generalized SU(2) Proca theory

$$\mathcal{L}'_1 = A^a_\mu A^\mu_a A^b_\nu A^\nu_b,$$

$$\mathcal{L}'_2 = A^a_\mu A^\mu_b A^b_\nu A^\nu_a,$$

$$\mathcal{L}'_3 = A^b_\mu A_{\rho b} G^{\mu\nu a} G^\rho{}_{\nu a},$$

$$\mathcal{L}'_4 = A^b_\mu A_{\rho a} G^{\mu\nu a} G^\rho{}_{\nu b},$$

$$\mathcal{L}'_5 = A_{\mu a} A^b_\rho G^{\mu\nu a} G^\rho{}_{\nu b},$$

$$\mathcal{L}'_6 = A^b_\rho A^\rho_b G_{\mu\nu a} G^{\mu\nu a},$$

$$\mathcal{L}'_7 = A^b_\rho A^\rho_a G_{\mu\nu b} G^{\mu\nu a}.$$

The ansatz and the metric

$$ds^2 = -e^{2\Upsilon}dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

S. Weinberg, John Wiley & Sons,1972.

$$\mathcal{A} = \frac{1}{e}\Big[A_0\tau_r \mathrm{dt} + A_1\tau_r \mathrm{dr} + (\phi_1\tau_\theta - (\phi_2+1)\tau_\phi)\mathrm{d}\theta + ((1+\phi_2)\tau_\theta + \phi_1\tau_\phi)\sin\theta \mathrm{d}\phi\Big],$$

E. Witten, Phys. Rev. Lett., 1977.

$$\mathcal{A} = \frac{1}{e}\left[-(\phi_2+1)\tau_\phi \mathrm{d}\theta + (1+\phi_2)\tau_\theta \sin\theta \mathrm{d}\phi\right],$$

$$\phi_2 \equiv V.$$

Series solutions: around the origin

$$V = -1 + b_2 r^2 + b_4 r^4 + \mathcal{O}(r^6),$$

$$b_4 = \frac{\mu^2 b_2}{10} - \frac{3b_2^2}{10} + \frac{4b_2^3}{5e^2} + \frac{\alpha_1 b_2^3}{e^2} + \frac{7\alpha_3 b_2^3}{10e^2} + \frac{\chi_5 b_2^3}{5e^2} - \frac{\chi_6 b_2^3}{e^2}.$$

$$\rho_{efec}\Big|_{r=0} = \frac{6b_2^2}{e^2}.$$

Series solutions: around the origin

$$m = a_3 r^3 + a_5 r^5 + \mathcal{O}(r^7),$$

$$a_3 = \frac{2b_2^2}{e^2}, \quad a_5 = \frac{3\mu^2 b_2^2}{5e^2} - \frac{8b_2^3}{5e^2} + \frac{172\alpha_1 b_2^4}{3e^4} + \frac{7\alpha_3 b_2^4}{15e^4} - \frac{4\chi_6 b_2^4}{e^4},$$

Series solutions: around the origin

$$2\Upsilon = c_2 r^2 + c_4 r^4 + \mathcal{O}(r^6),$$

$$c_2 = \frac{4b_2^2}{e^2},$$

$$c_4 = \frac{2\mu^2 b_2^2}{5e^2} - \frac{8b_2^3}{5e^2} + \frac{24b_2^4}{5e^4} - \frac{16\alpha_1 b_2^4}{e^4} + \frac{9\alpha_3 b_2^4}{5e^4} - \frac{4\chi_5 b_2^4}{5e^4} - \frac{4\chi_6 b_2^4}{e^4},$$

Series solutions: spacial infinity

$$V = -1 + \frac{\tilde{b}_1}{r} + \frac{\tilde{b}_2}{r^2} + \frac{\tilde{b}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right),$$

$$\tilde{b}_2 = \frac{3(2M_\infty - \tilde{b}_1)\tilde{b}_1}{4},$$

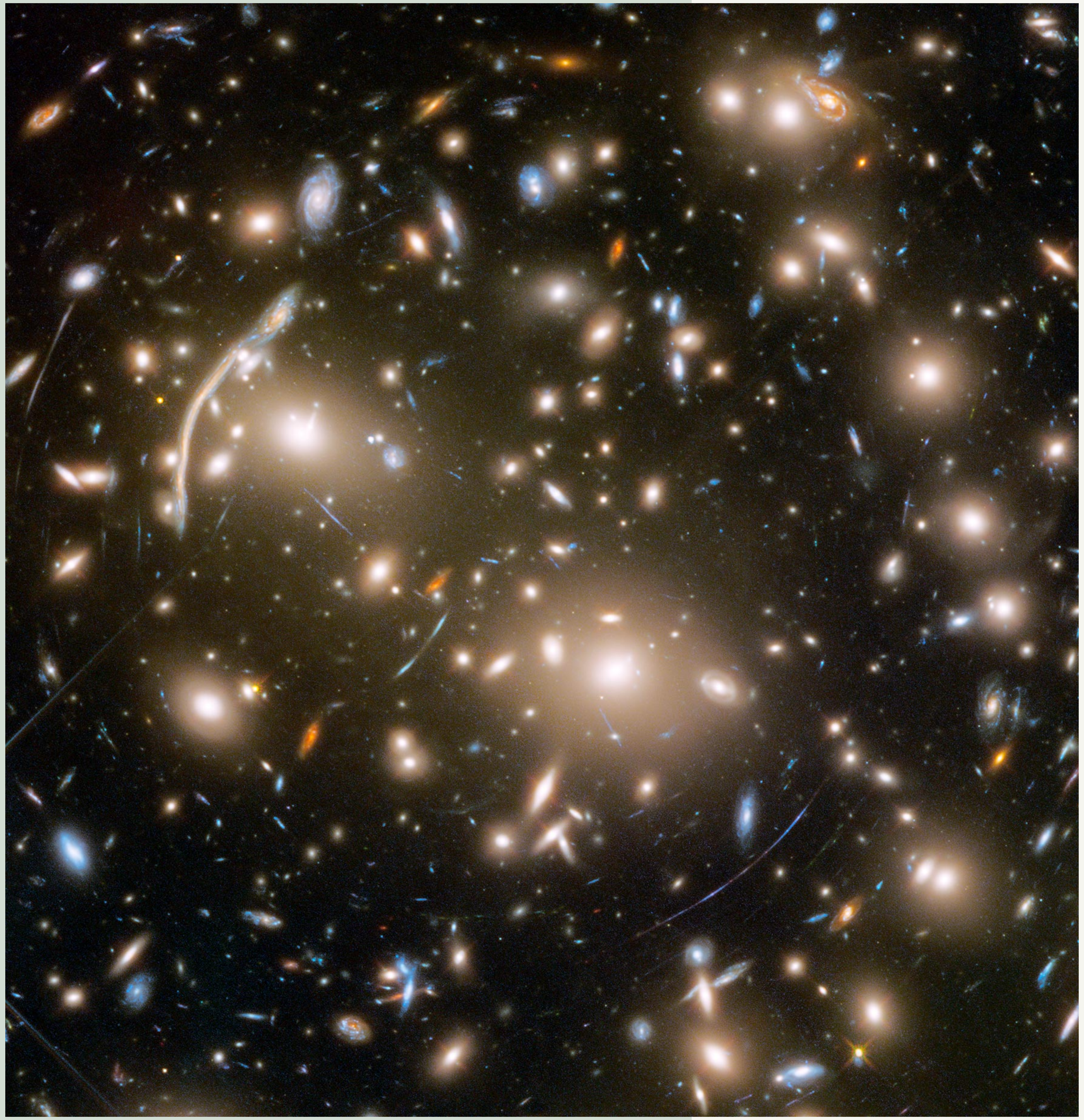
$$\tilde{b}_3 = \frac{\tilde{b}_1(48e^2M_\infty^2 - 42e^2M_\infty\tilde{b}_1 + (11e^2 - 2(2\chi_1 + \chi_2))\tilde{b}_1^2)}{20e^2},$$

Series solutions: spacial infinity

$$m = M_\infty + \frac{\tilde{a}_3}{r^3} + \frac{\tilde{a}_4}{r^4} + \frac{\tilde{a}_5}{r^5} + \mathcal{O}\left(\frac{1}{r^6}\right),$$

$$\tilde{a}_3 = -\frac{\tilde{b}_1^2}{e^2},\ \tilde{a}_4 = \frac{\tilde{b}_1^2(-5M_\infty + 4\tilde{b}_1)}{2e^2},$$

$$\tilde{a}_5 = -\frac{3(\tilde{b}_1^2(68e^2M_\infty^2 - 100e^2M_\infty\tilde{b}_1 + (37e^2 - 4(2\chi_1 + \chi_2))\tilde{b}_1^2))}{40e^4}.$$



Numerical solutions

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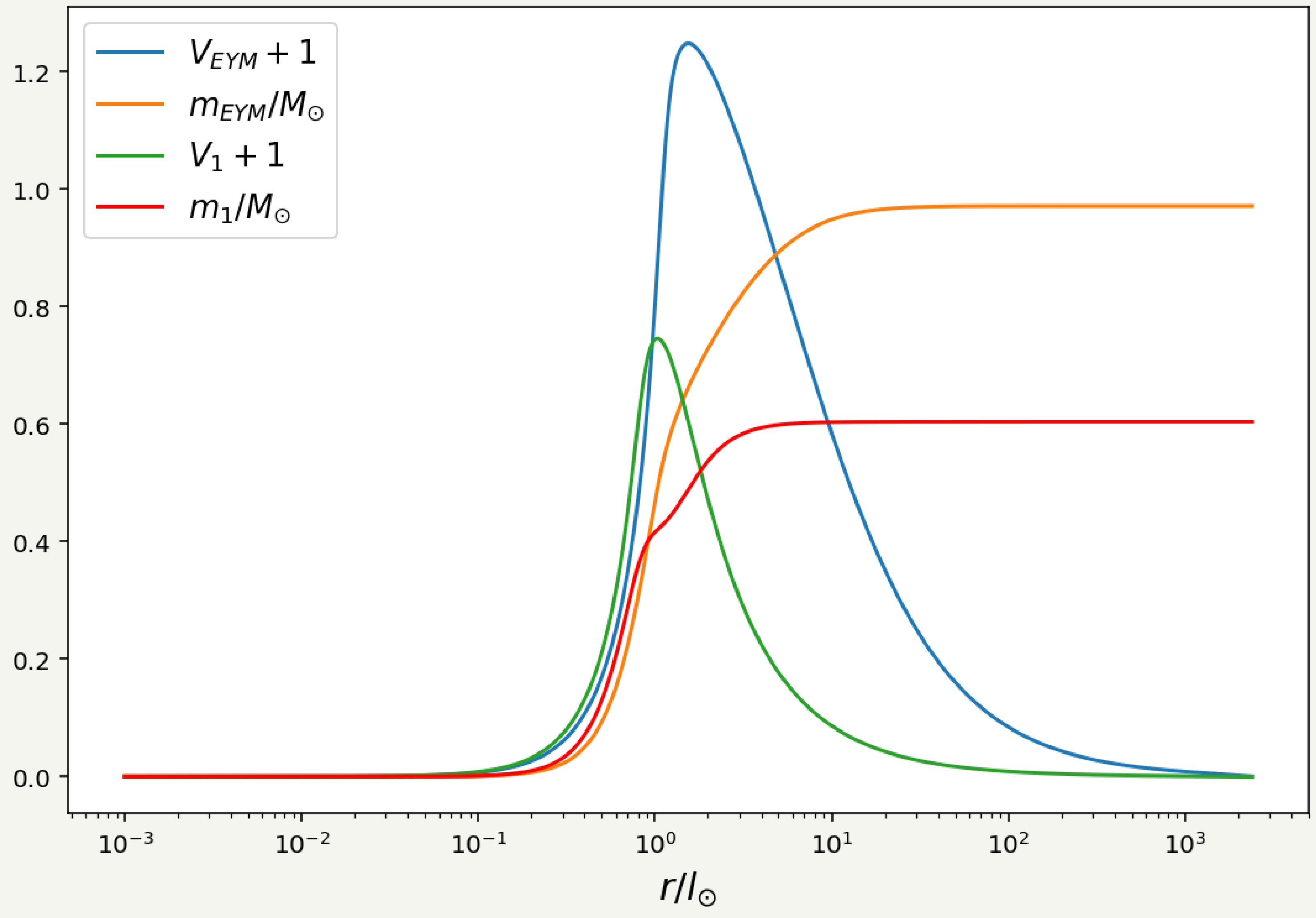
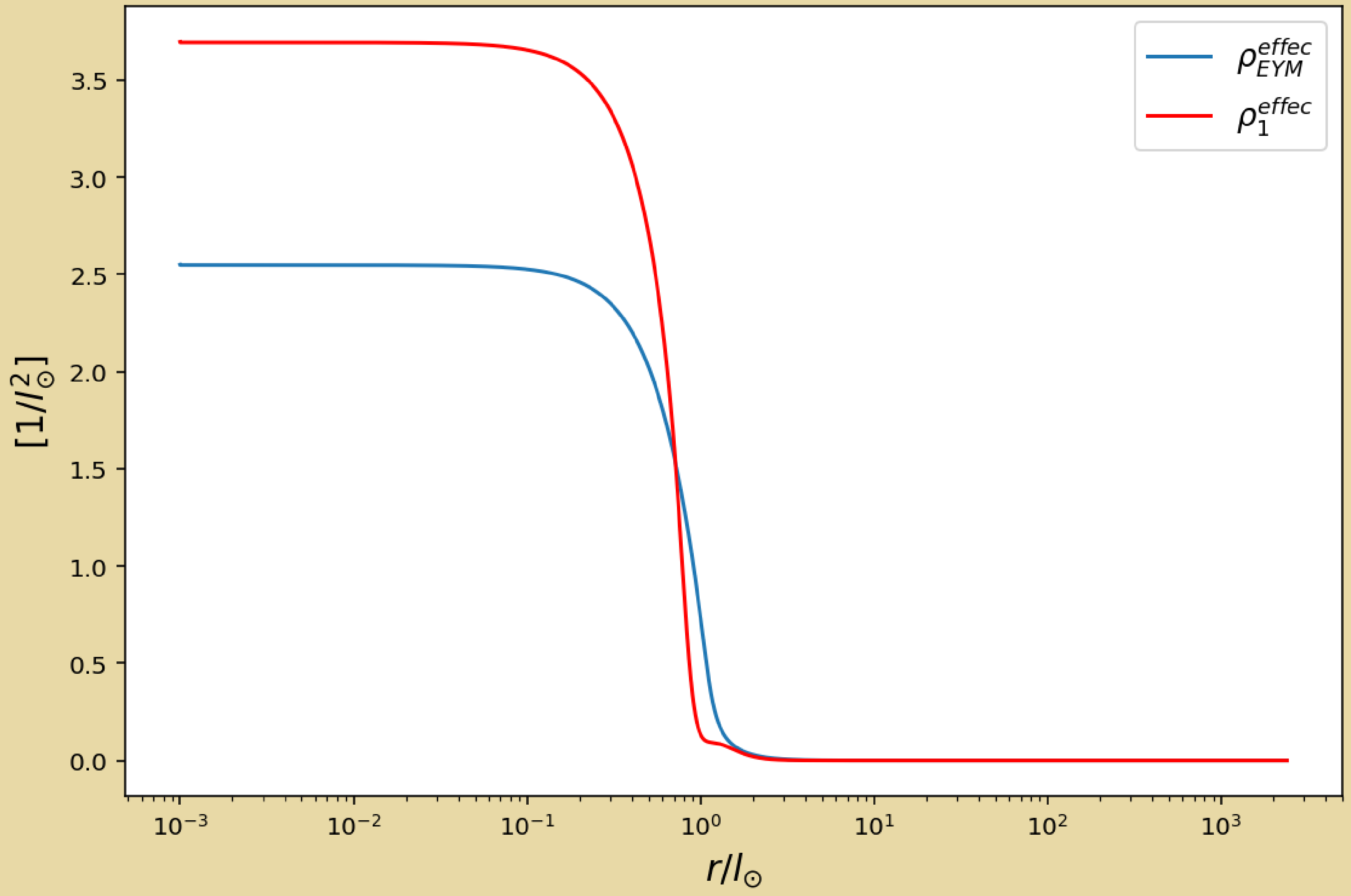
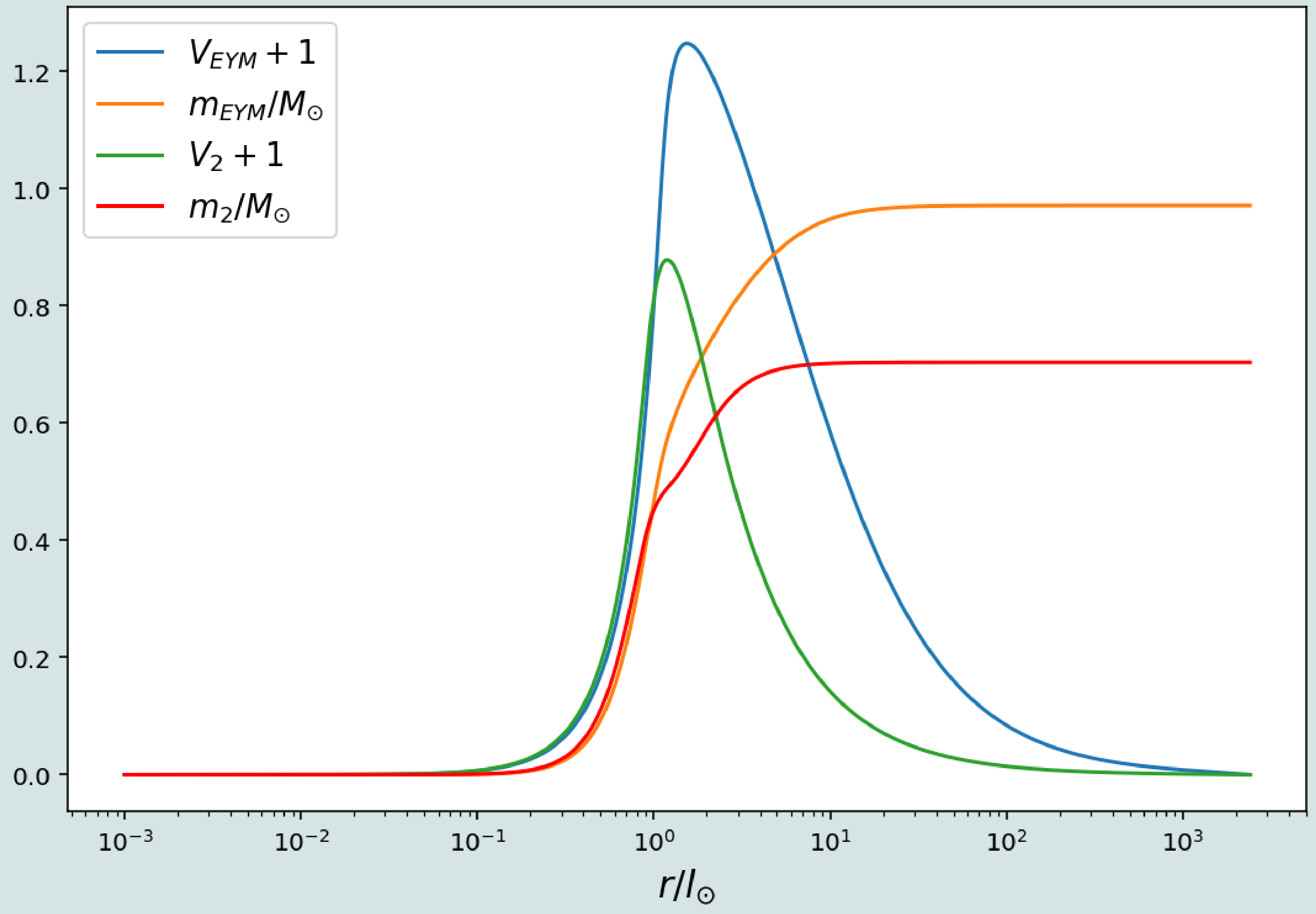
1.Einstein Yang-Mills, $\chi_1 = 1$

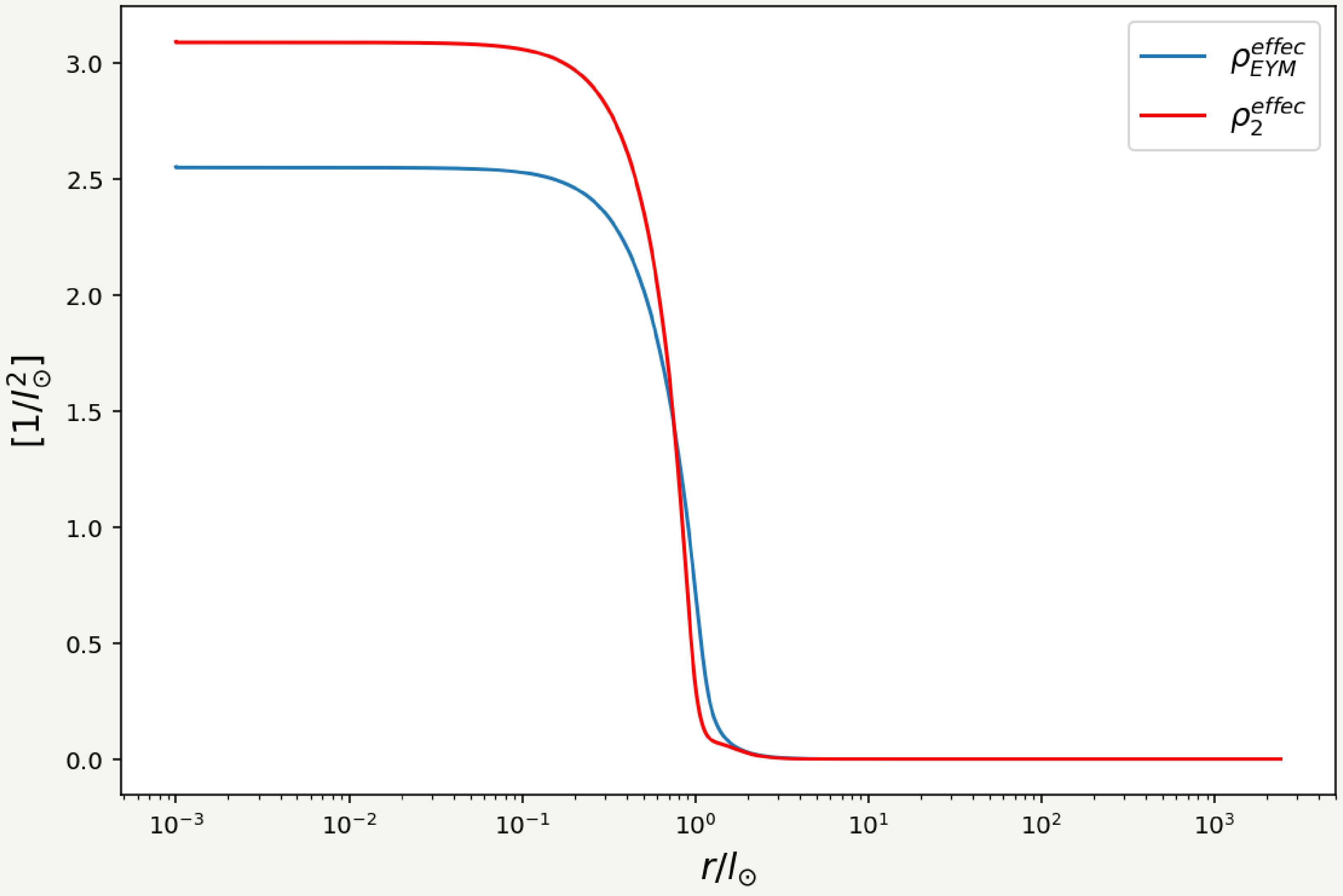
b_2 =0.7845233





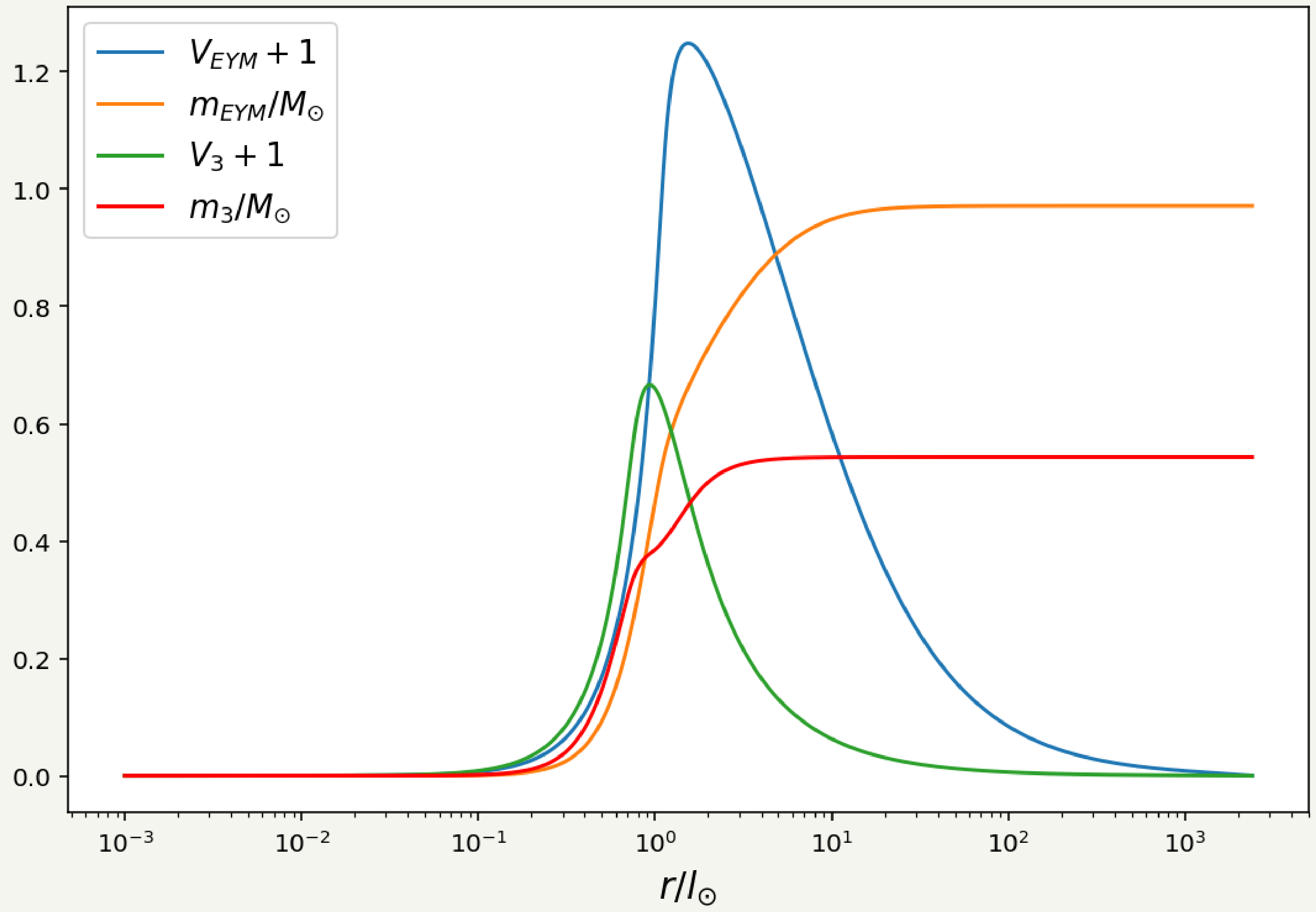
2.Einstein Yang-Mills, $\chi_2 = 1$
b_2=0.71725484

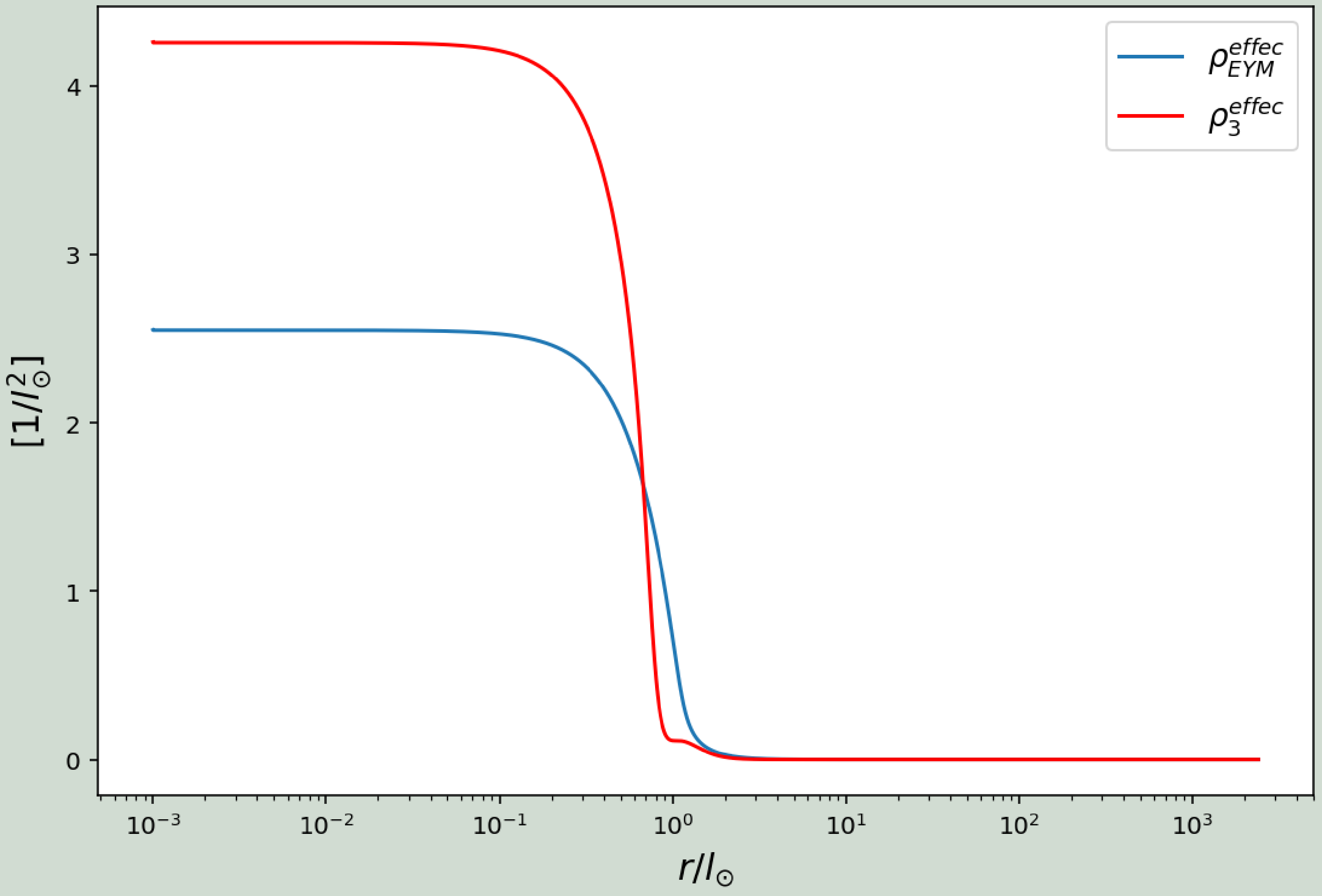




3.Einstein Yang-Mills, $\chi_1 = 1$ & $\chi_2 = 1$

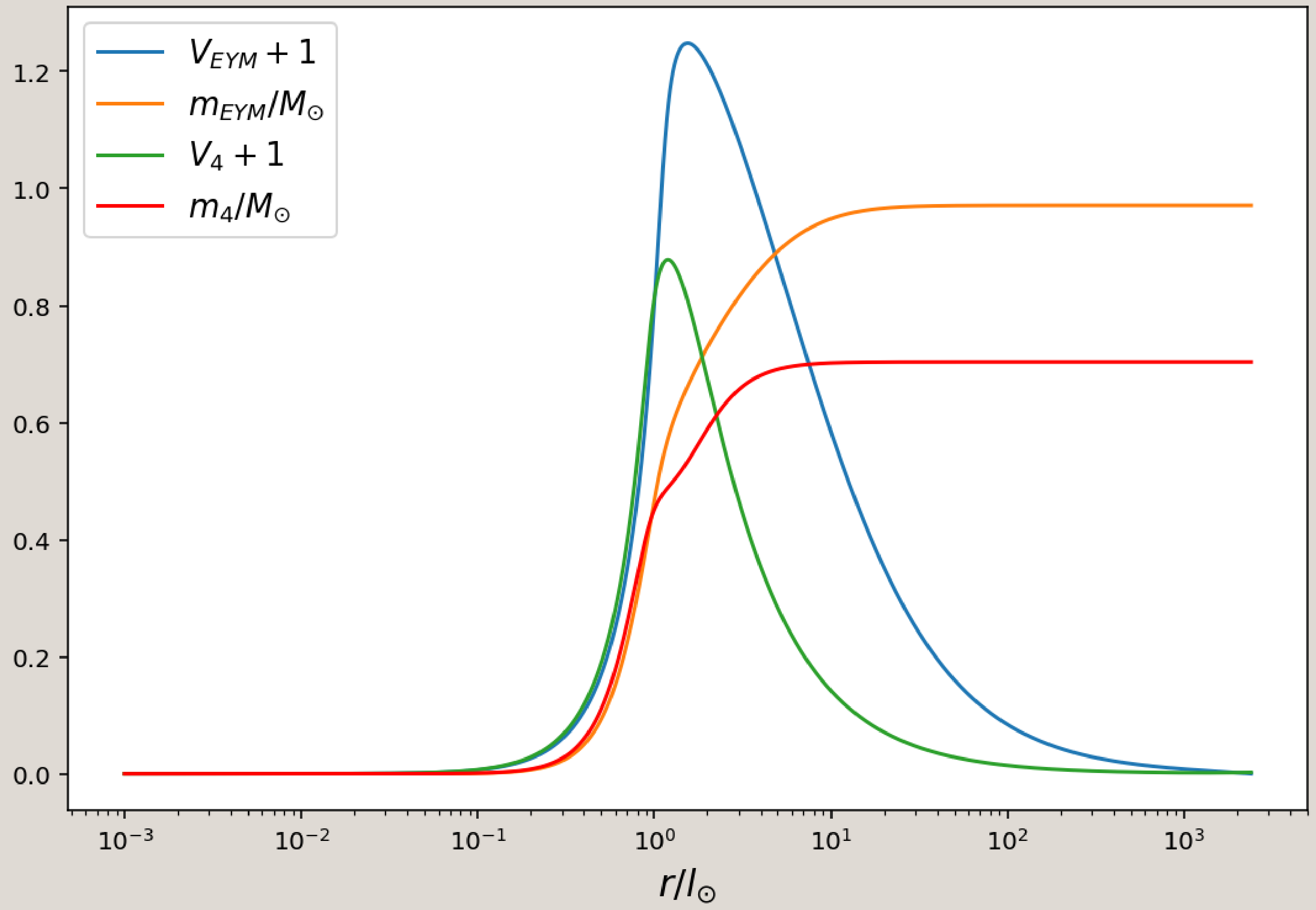
b_2=0.84223827

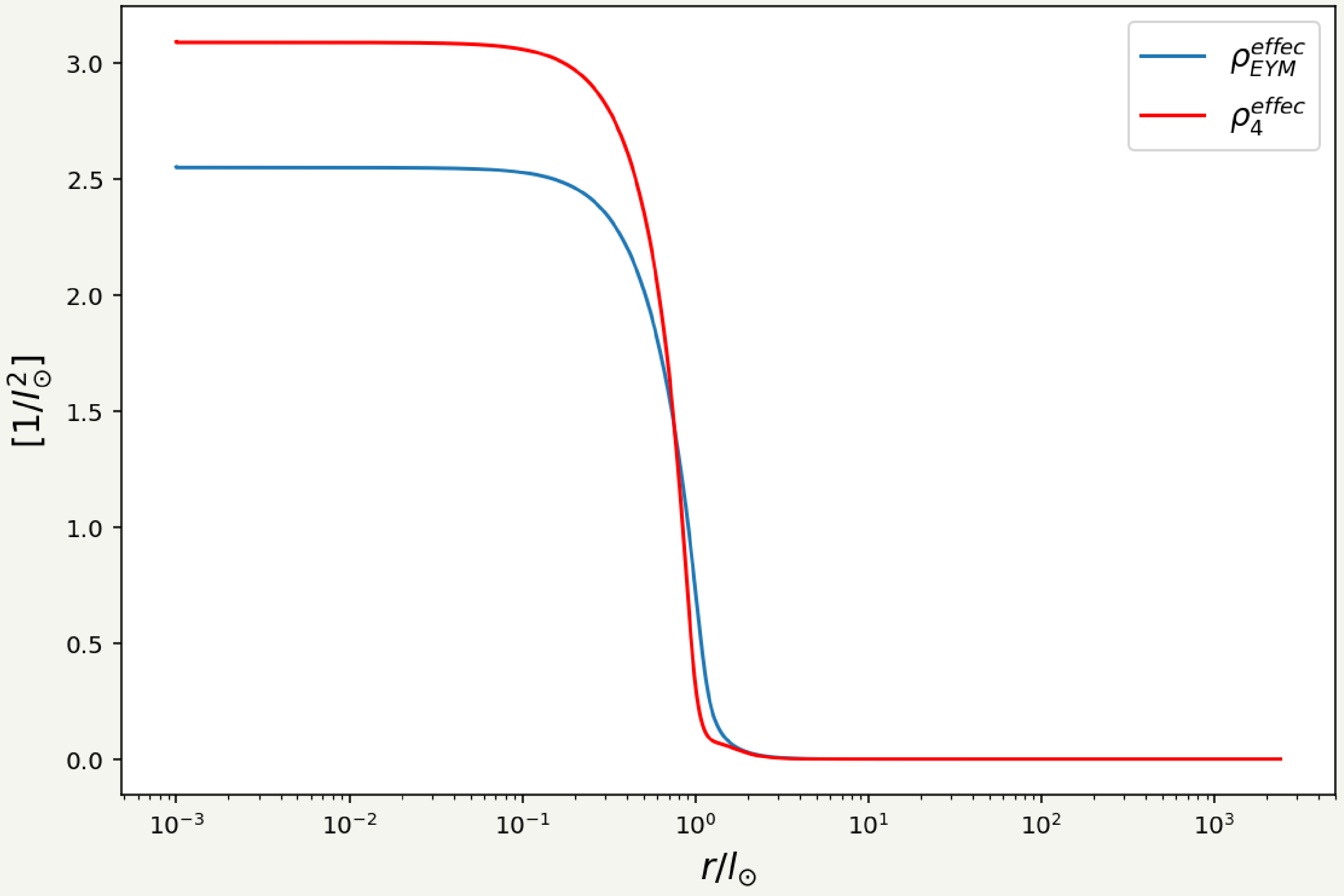




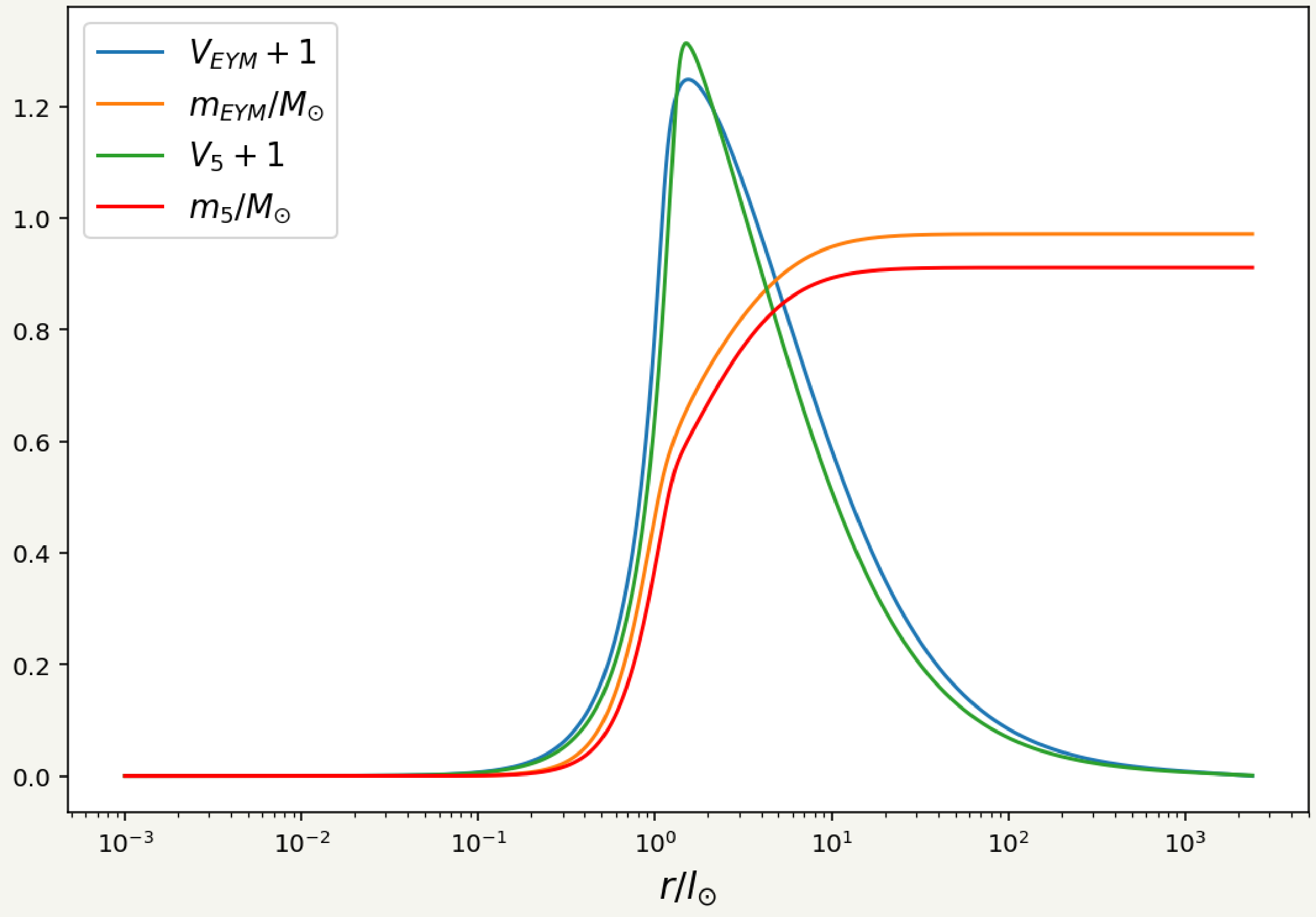
4.Einstein Yang-Mills, $\chi_1 = 1$ & $\chi_2 = -1$

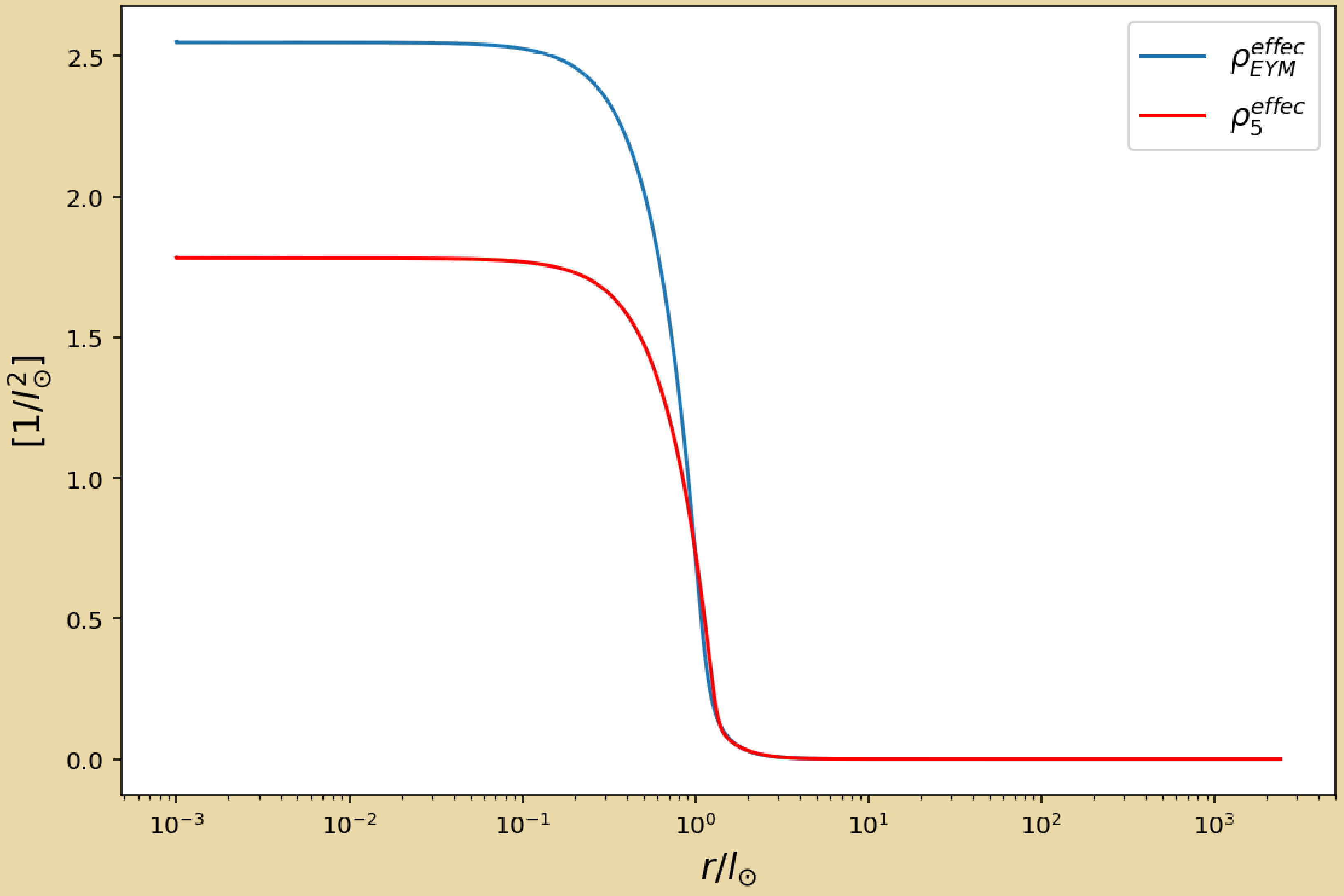
b_2=0.71725485





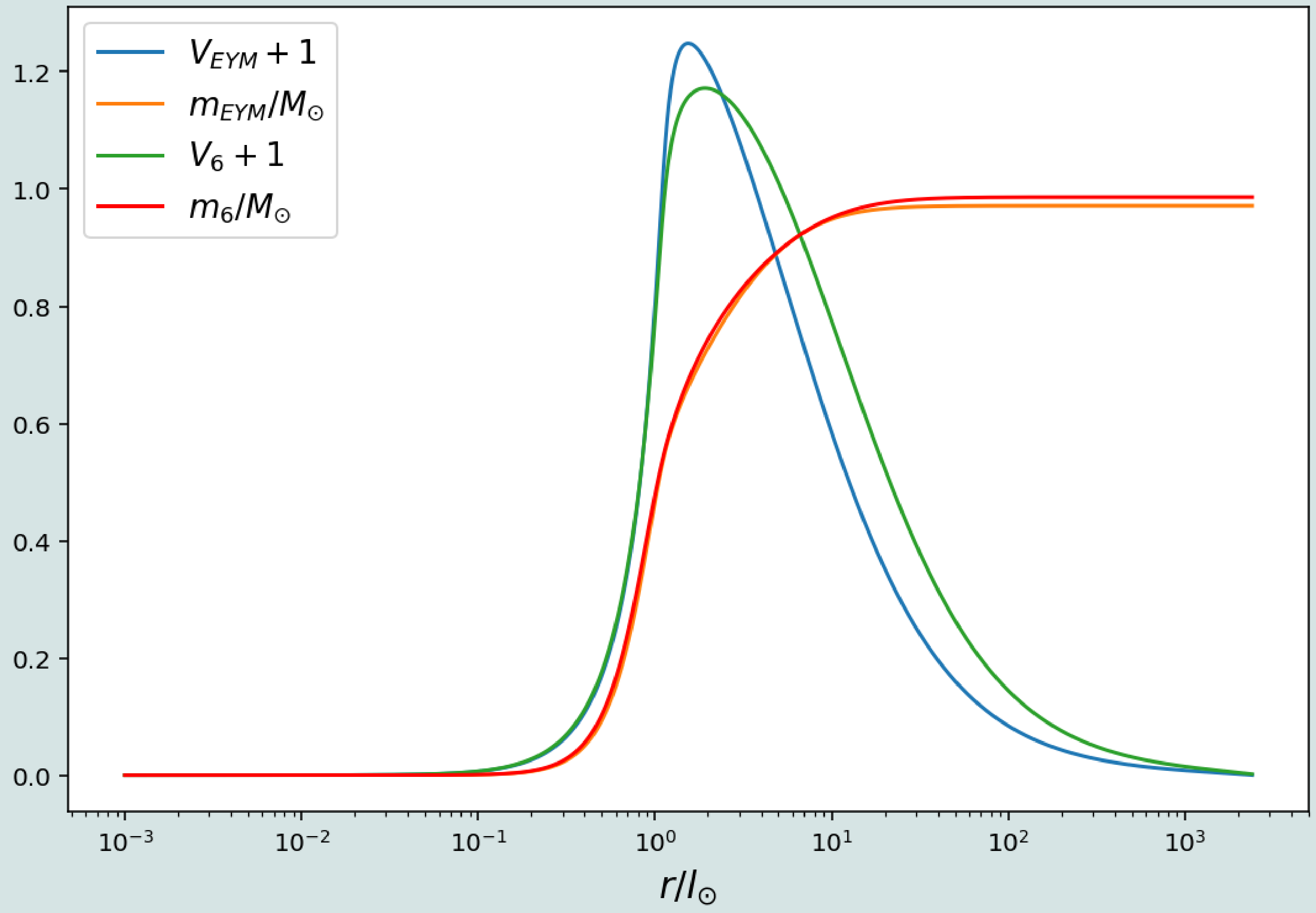
5.Einstein Yang-Mills, $\chi_5 = 1$
b_2=0.54489544

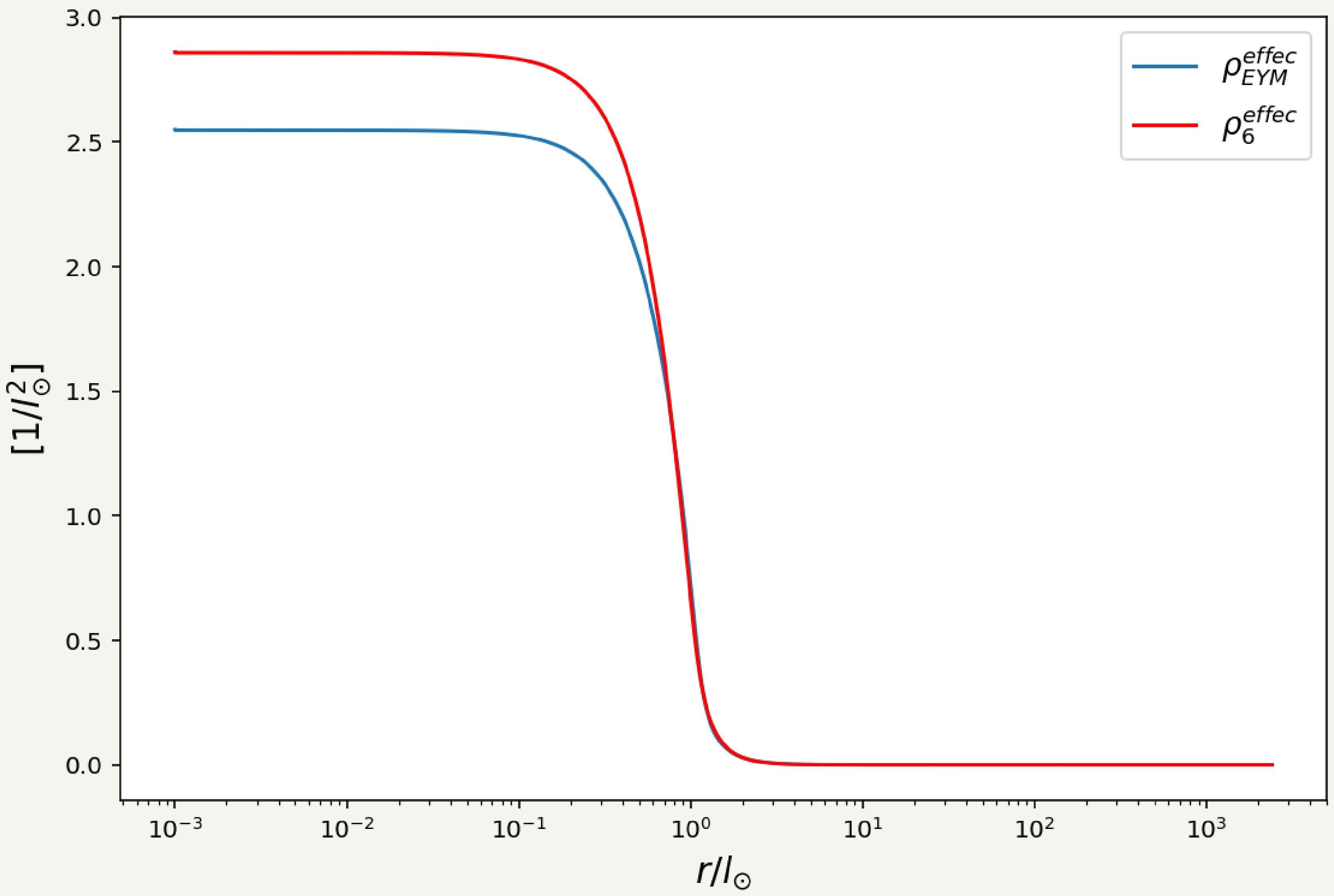




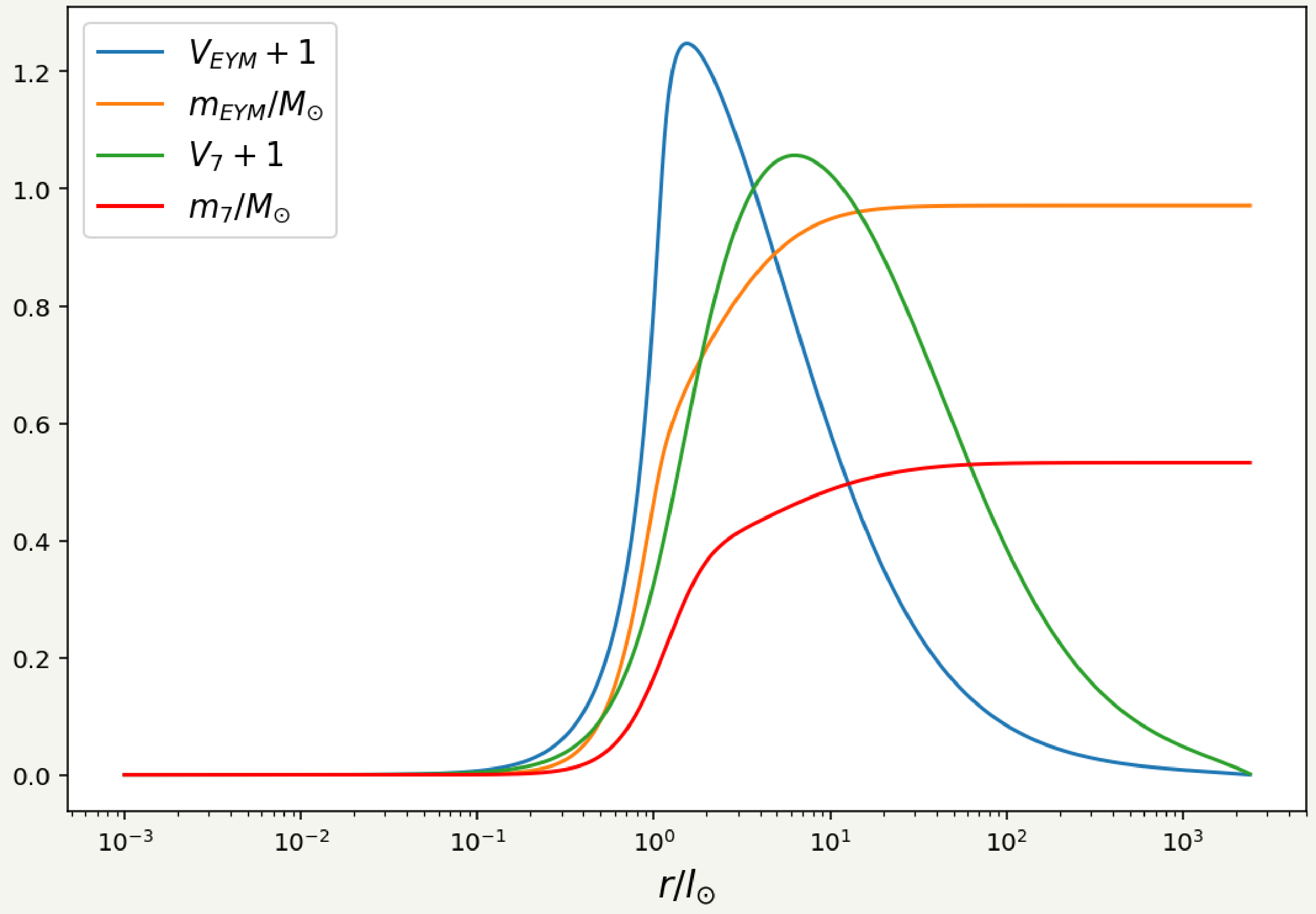
6.Einstein Yang-Mills, $\chi_5 = -1$

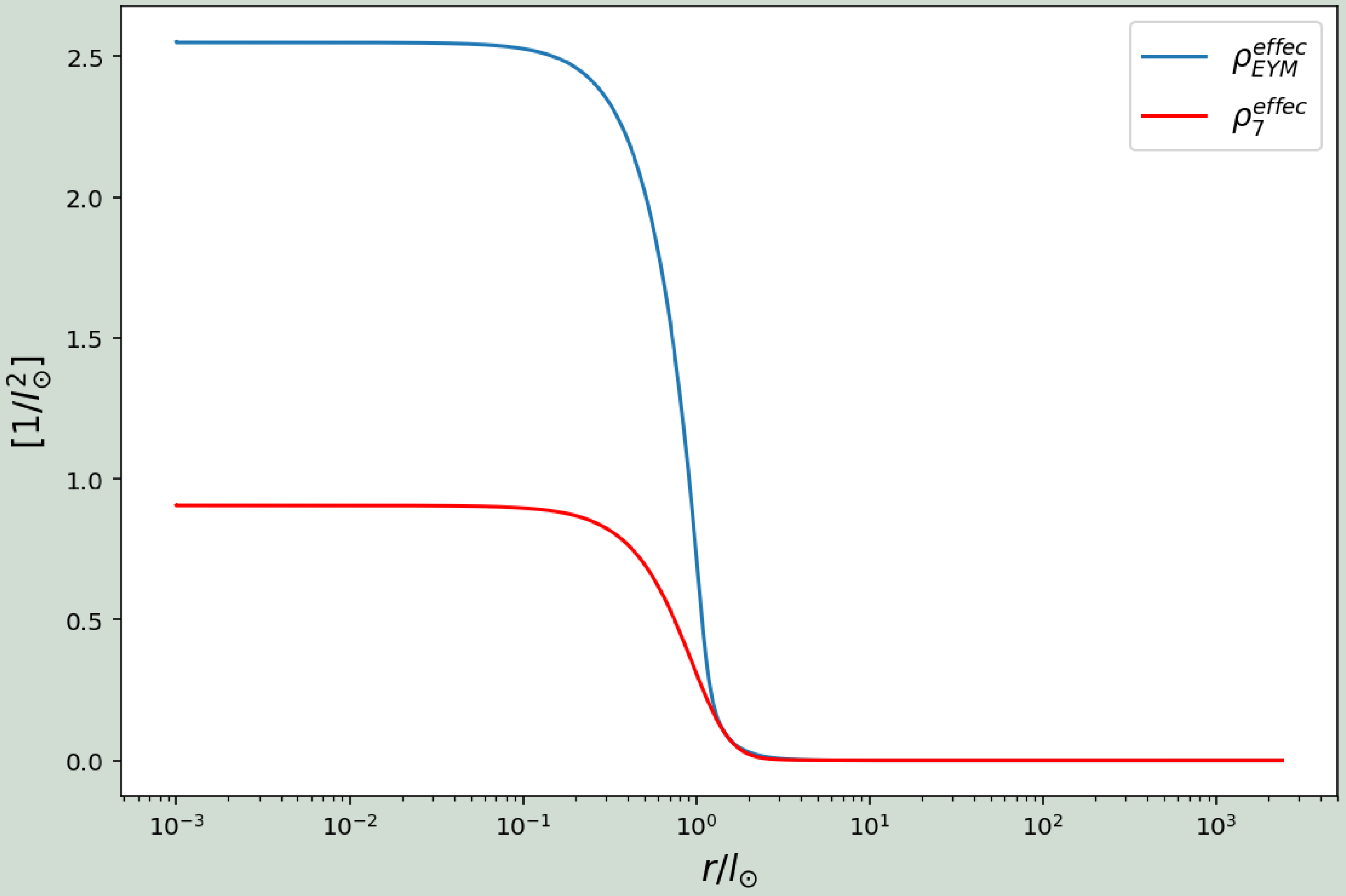
b_2 =0.6903282





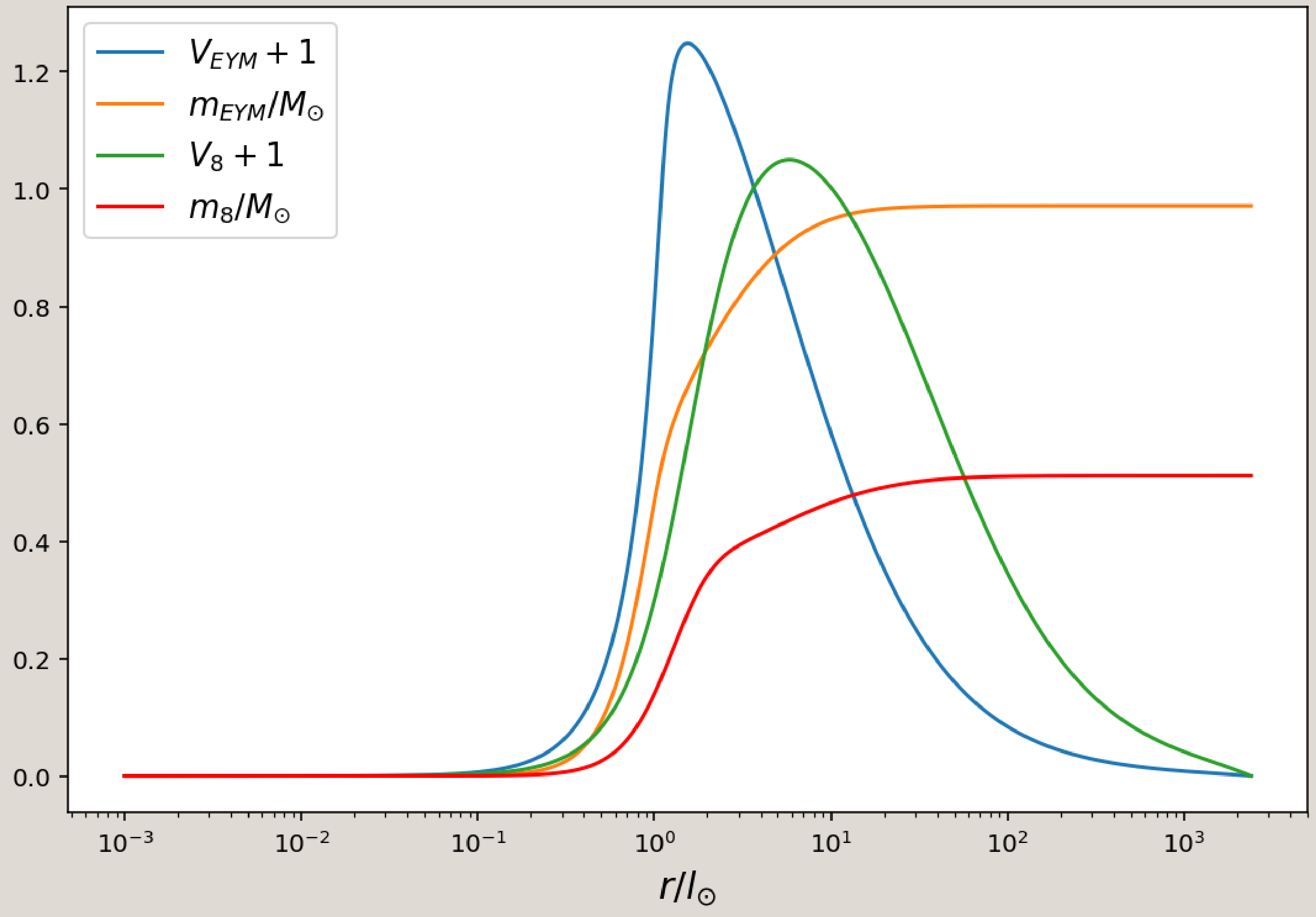
7.Einstein Yang-Mills, $\chi_6 = 1$
b_2=0.38846143

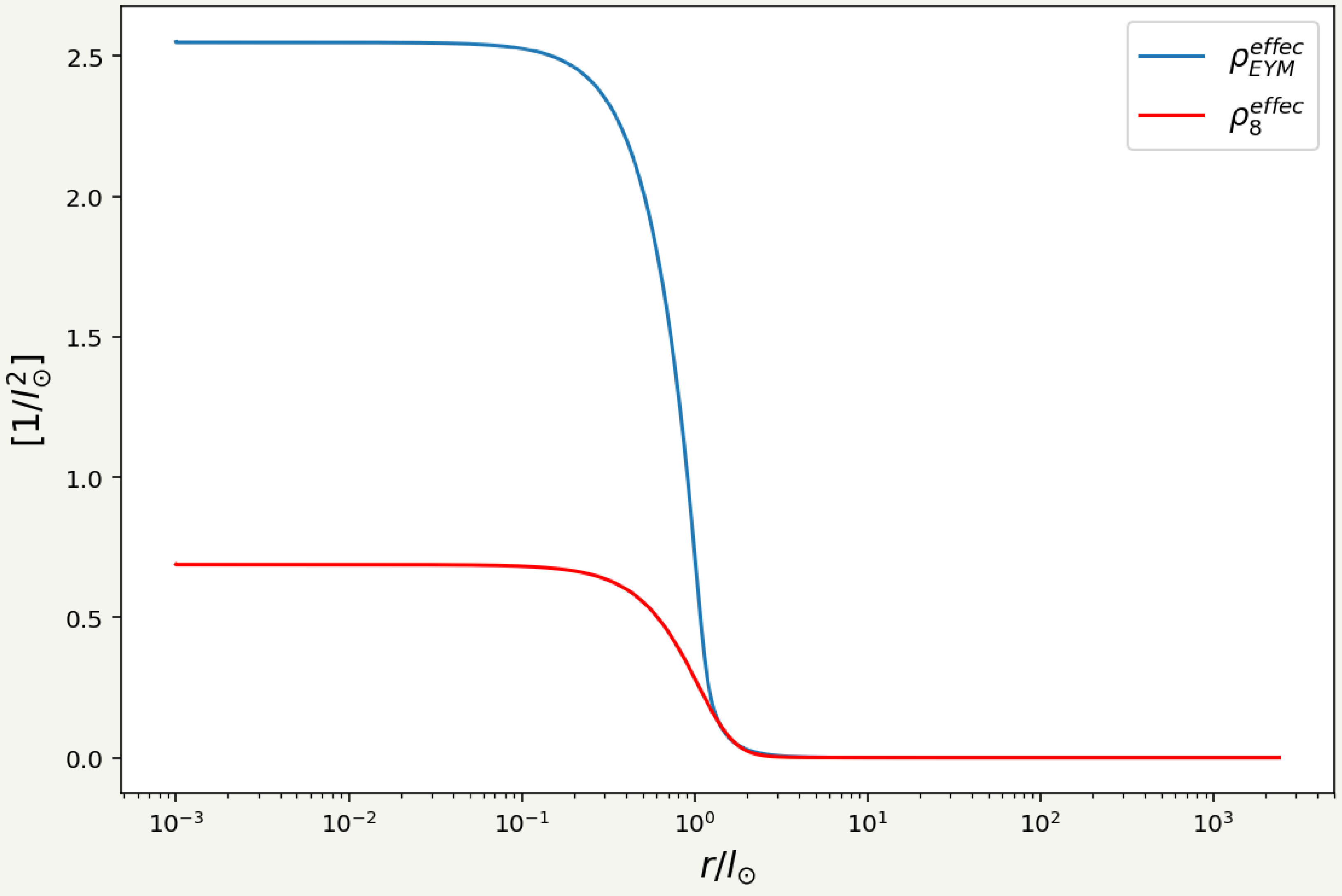




8.Einstein Yang-Mills, $\chi_5 = 1$ & $\chi_6 = 1$

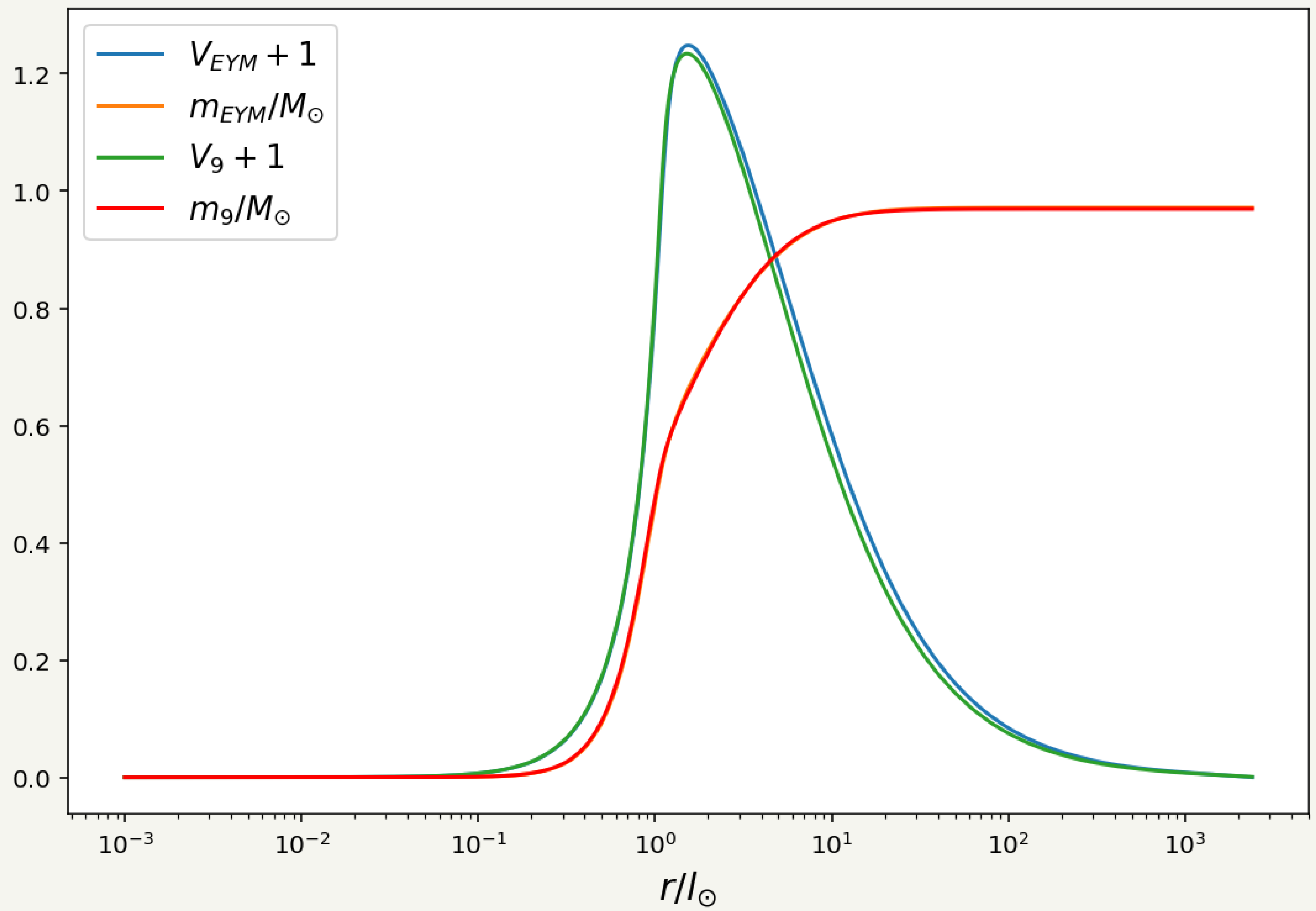
b_2=0.33862584

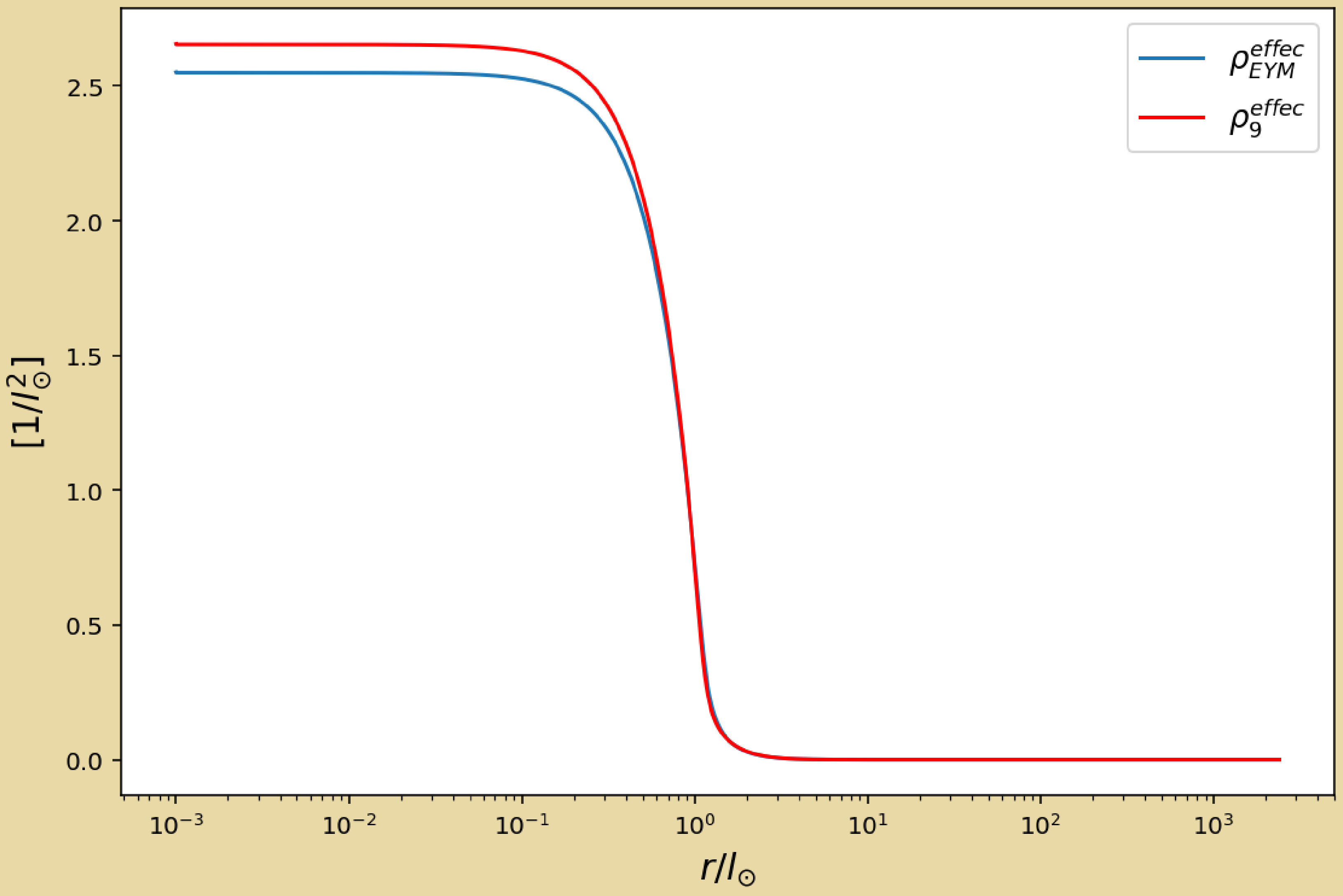




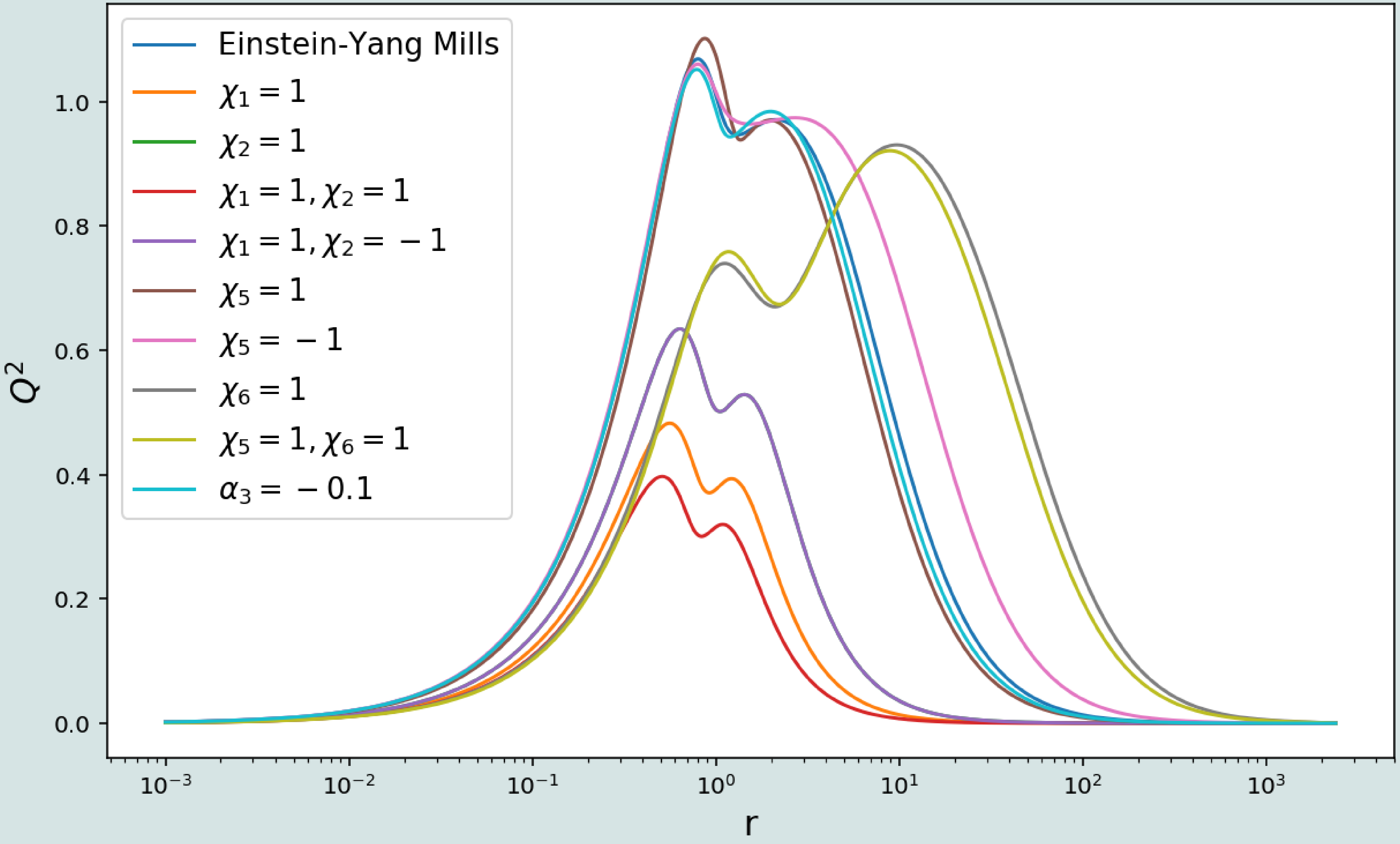
9.Einstein Yang-Mills, $\alpha_3 = -0.1$

b_2=0.66507009





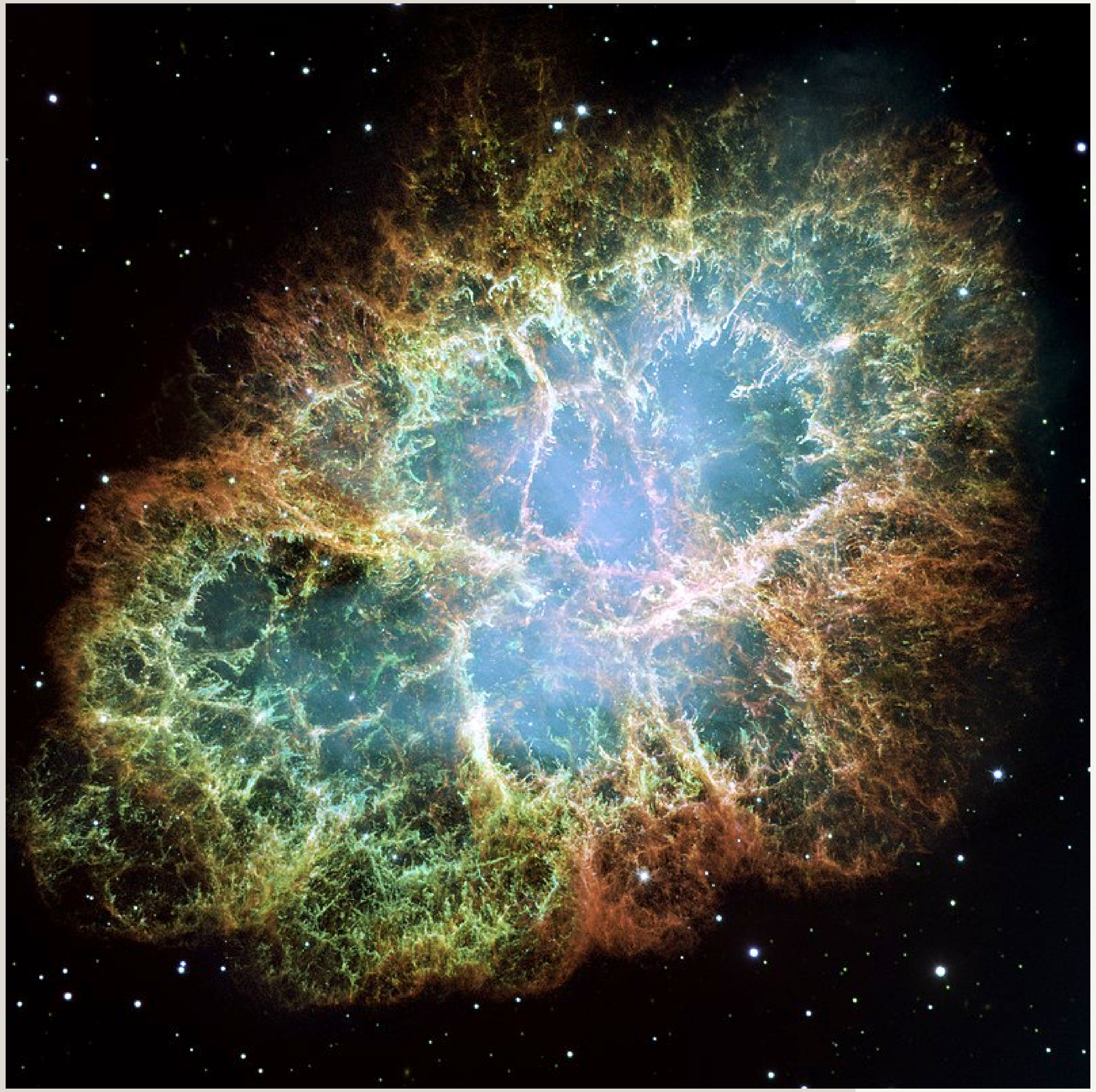
Charge



Einstein-Yang Mills
$\chi_1 = 1$
$\chi_2 = 1$
$\chi_1 = 1, \chi_2 = 1$
$\chi_1 = 1, \chi_2 = -1$
$\chi_5 = 1$
$\chi_5 = -1$
$\chi_6 = 1$
$\chi_5 = 1, \chi_6 = 1$
$\alpha_3 = -0.1$
Q^2
1.0
0.8
0.6
0.4
0.2
0.0
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
r

Prospects

- Exploring remaining free parameters of the theory.
- Study the stability of the solutions.
- Neutron stars (vectorization).
- Black holes.



¡Thanks a lot!

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