



1. How do double logarithms emerge in the Lund plane?
2. How is the fractal dimension of the Lund diagram connected to QCD?  
Hint: Have a look at the figures on the next slides
3. Color factors


▶   $\propto \text{Tr} [T^a T^b T^a T^b] = -C_F \left( \frac{C_A}{2} - C_F \right)$


▶   $\propto F_{ab}^c \text{Tr} [T^a T^b T^c] = C_F \frac{C_A}{2}$

Where does the structure of the first term come from?

Hint: Find a connection to the second term.

4. Kinematical factors

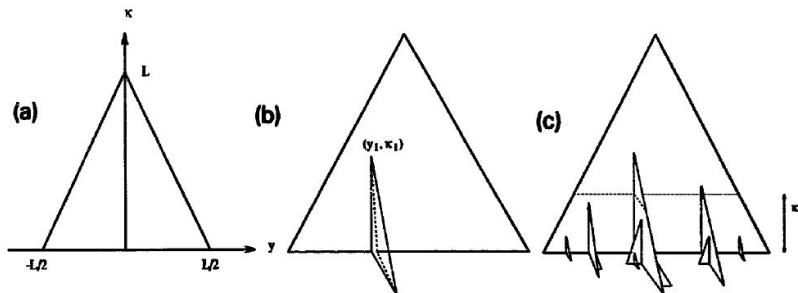
▶   $\propto \frac{p_i p_j}{(p_i p_1)(p_1 p_j)} \frac{p_i p_j}{(p_i p_2)(p_2 p_j)}$

▶   $\propto \frac{p_i p_j}{(p_i p_1)(p_1 p_j)} \frac{p_i p_1}{(p_i p_2)(p_2 p_1)}$

Can you derive these using the formulae given in the lecture?

5. Compute the quark collinear anomalous dimension, called  $B_q$  in the lecture

Figure reproduced from Nucl.Phys.B 355 (1991) 106-122  
[https://doi.org/10.1016/0550-3213\(91\)90304-G](https://doi.org/10.1016/0550-3213(91)90304-G)



**Fig. 1.** (a) The phase space available for a gluon emitted by a high energy  $q\bar{q}$  system is a triangular region in the  $(y, \kappa)$ -plane ( $\kappa = \ln k_{\perp}^2/\Lambda^2$ ;  $L = \ln s/\Lambda^2$ ). (b) If one gluon is emitted at  $(y_1, \kappa_1)$  the phase space for a second (softer) gluon is represented by the area of this folded surface. (c) Each emitted gluon increases the phase space for the softer gluons. The total gluonic phase space can be described by this multi-faceted surface. The length of the baseline corresponds to the quantity  $\lambda(L)$ , the length of the dashed line to  $\lambda(L, \kappa)$ .

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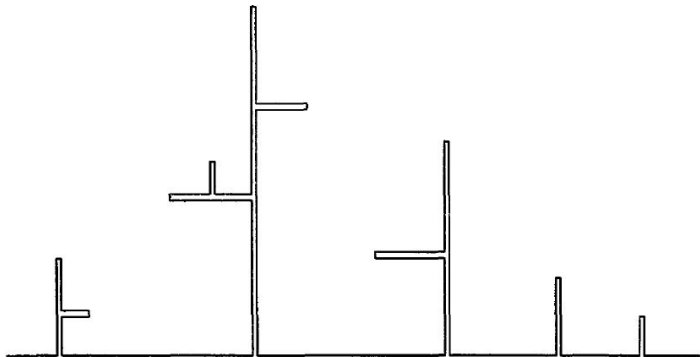


Fig. 3. The baseline of the surface in fig. 1c (more precisely the surface cut at the level  $\kappa = \ln m_0^2/\Lambda^2$ ).  
The length of this curve corresponds to the quantity  $\lambda$ .