Exercises: Day #1: Stefan Prestel

The Monte-Carlo generator landscape is rich! Just to name a few:

Neutrino physics:
Genie, GiBUU, NuWro, NEUT...

Cosmic rays:
EPOS, QGSJET and SIBYLL

Heavy ions:
HIJING, AMPT, JEWEL...

LHC physics:
Herwig, Pythia, Sherpa
Madgraph, Whizard, Alpgen...

Exercise: Get together with friends and chat about an event generator in an unfamiliar field.

The same algorithm applies when picking a continuous "index" y, i.e. picking a random variable according to a distribution (e.g. a phase-space point)

The cumulative distribution becomes

$$C(y) = \int_{-\infty}^{y} dx p(x) \qquad \text{with} \int_{-\infty}^{\infty} dx p(x) = 1$$

which allows using $R \in [0, 1]$ and

$$C(y) = R \quad \Rightarrow \quad y = C^{-1}(R)$$

This is called inversion sampling.

Often, we're not so lucky that a uniquely invertible primitive function C^{-1} exists
...but we can often still use this method as part of a more flexible algorithm.

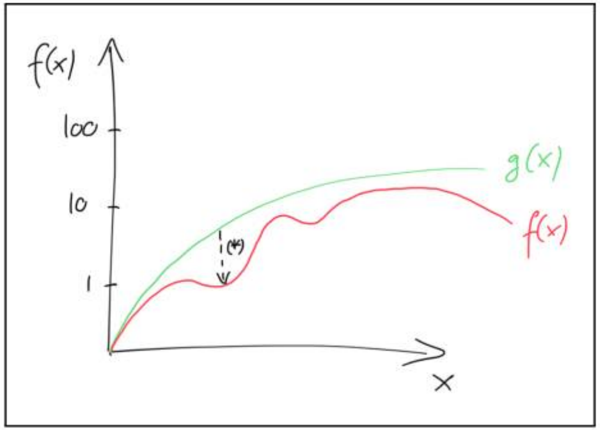
Exercise: Generate random variables $x > 0$ with distribution $f(x) = e^{-x}$

Rejection sampling will be much more efficient if combined with inversion sampling:

- Assume a simple distribution $g(x) > f(x)$, i.e.

$$f(x) = g(x) \underbrace{\frac{f(x)}{g(x)}}_{<1}$$

- Use inversion sampling to draw x from $g(x)$.
- Draw $R \in [0,1]$. Reject x if $\frac{f(x)}{g(x)} < R$



$\Rightarrow$ Accepted x now distributed according to $f(x)$. This algorithm is excessively used in Monte Carlo generators.

Comparison: Uniform sampling

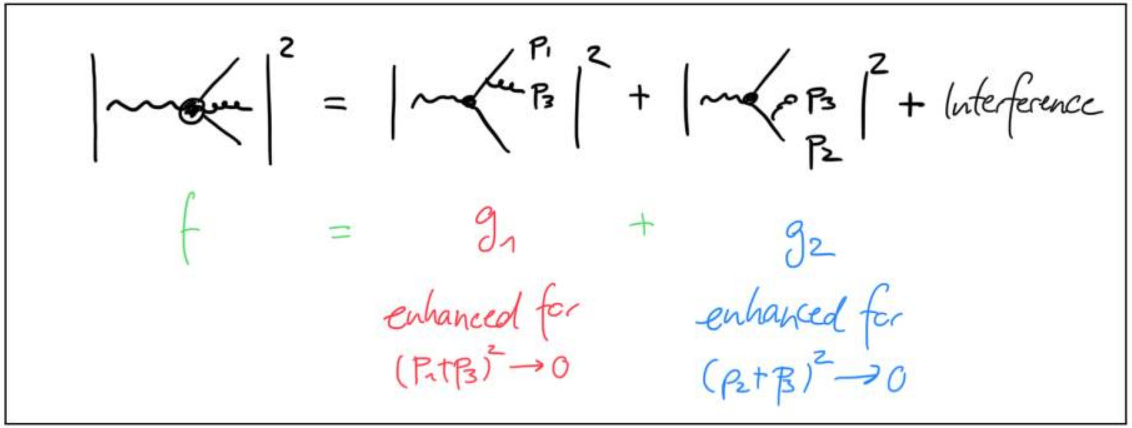
$$\mathrm{var}(f)_{MC} \approx \frac{\mathrm{var}(f)}{\sqrt{N}}$$

error worse in regions of large variance...

Importance sampling

$$\int dx g(x) \frac{f(x)}{g(x)} \approx \langle \frac{f}{g} \rangle \pm \sqrt{\frac{\langle f^2/g^2 \rangle - \langle f/g \rangle^2}{N}}$$

Exercise: Generate random variables $0 < z < 1 - \epsilon$ with distribution $P(z) = \frac{1+z^2}{1-z}$. Hint: Use a simpler denominator to get a simple $g(z)$...



Differential cross sections have a rich structure. In that case, importance sampling can be combined with the discrete transformation method into multichannel sampling:

- Use $f(x) \leq g_1(x) + g_2(x)$
- Choose index $i \in \{1,2\}$ [using $P_i = \int dx g_i(x)$]
- Draw x from $g_i(x)$. Overall, x is now distributed according to $g_1 + g_2$
- Draw $R \in [0,1]$, and accept if (i,x) pair if $\frac{f(x)}{g_1(x)+g_2(x)} < R$. Else reject & restart.

NB: also heavily used in parton showers.

Exercise: Draw x from the distribution $f(x) = \frac{1}{\sqrt{x(1-x)}}$ using two integration channels.

Exercises: Day #1: Dave Soper

1) The cross section for q qbar g production in electron-positron annihilation as a function of the final state energies and angles has singularities. Is the total cross section for q qbar or q qbar g production in electron-positron annihilation finite or infinite? Why?

2) If jets in nature are obvious, do you need to worry about the precise definition of a jet in order to measure a cross section for, say, $e^+ e^- \rightarrow$ 2 jets plus anything? Why?

3) Why is the thrust distribution infrared safe?

4) Can you verify the relations on slide 39 for the running coupling, given the result in the first line?

5) You may have heard talks about effective field theory.
What do people mean by effective field theory?

4) Can you verify the relations on slide 39 for the running coupling, given the result in the first line?

Result of the one loop renormalization group equation:

$$\begin{aligned}
\alpha_s(\mu) &\sim \alpha_s(M) - (\beta_0/4\pi)\log(\mu^2/M^2)\;\alpha_s^2(M) \\
&\quad +(\beta_0/4\pi)^2\log^2(\mu^2/M^2)\;\alpha_s^3(M) + \cdots \\
&= \frac{\alpha_s(M)}{1+(\beta_0/4\pi)\alpha_s(M)\log(\mu^2/M^2)} \\
&= \frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2)} \\
&= \frac{\alpha_s(M_Z)}{1+(\beta_0/4\pi)\alpha_s(M_Z)\log(\mu^2/M_Z^2)}
\end{aligned}$$