FAQ

1. Should it be Deep Inelastic or Deeply Inelastic Scattering?
2. After 50 years of DIS, is this still cool?
3. Why are there no neutrino-proton DIS data so far?
4. Solve the Homework problem on page 13.
5. Solve the Homework problem on page 25.

Homework Problems

1. Recap that the allowed kinematic region for $ep \rightarrow eX$ is $\mathbf{0 \leq x \leq 1}$ and $\mathbf{0 \leq y \leq 1}$. Construct the phase space in the (ν, Q^2)-plane yourself. [Ex. 8.11 in Halzen]

2. Show that $Q^2 = 2\, E\, E' \,(1 - \cos(\theta)) = 4\, E\, E' \sin^2(\theta/2)$ neglecting the lepton mass. Here, the z-axis coincides with the incoming lepton direction and θ is the polar angle of the outgoing lepton with respect to the z-axis

3. Show that in the target rest frame $x = [2\, E\, E' \sin^2(\theta/2)]/[M(E-E')]$ still neglecting the lepton mass and the energies **E**, **E'** are now in the target rest frame

Homework Problems

1. Show that the phase space for the outgoing lepton takes the following form in the variables **x** and **y** (without any approximation), where **F** is the flux and **S=2 p.l**:

$$\frac{d^3 l'}{(2\pi)^3 2E'} = \frac{2S^2 y}{(4\pi)^2 F} dx dy$$

2. Derive the following general expression for the doubly differential cross section:

$$\frac{d^2\sigma}{dx dy} = \frac{2S^2 y}{(4\pi)^2 F^2} \left[\frac{e^4}{Q^4} L_{\mu\nu} W^{\mu\nu} 4\pi \right] = \frac{4S^2}{F^2} \frac{2\pi\alpha^2}{Q^4} y \, L_{\mu\nu} W^{\mu\nu}$$

(Note that the factor **$4S^2/F^2 = 1 + O(m^2/S * M^2/S)$** and the mass term is negligibly small for incoming neutrinos, electrons, and muons even if the nucleon mass is taken into account.)

3. Show that the hadronic tensor in terms of the structure functions **F_1**, **F_2** is given by:

$$W^{\mu\nu} = -g_\perp^{\mu\nu} F_1(x, Q^2) + \frac{1}{p \cdot q} p_\perp^\mu p_\perp^\nu F_2(x, Q^2)$$

Homework Problems

Show that the hadronic tensor can be brought in the following forms:

$$4\pi W_{\mu\nu} = \sum_{\text{states } X} \int d\Phi_X (2\pi)^4 \delta^{(4)}(p+q-p_X) \Big\langle \langle N(p)|J_\nu^\dagger(0)|X\rangle\langle X|J_\mu(0)|N(p)\rangle \Big\rangle_{\text{spin}}$$

$$= \sum_{\text{states } X} \int d\Phi_X \int d^4y \; e^{iqy} \Big\langle \langle N(p)|J_\nu^\dagger(y)|X\rangle\langle X|J_\mu(0)|N(p)\rangle \Big\rangle_{\text{spin}}$$

$$= \int d^4y \; e^{iqy} \Big\langle \langle N(p)|J_\nu^\dagger(y) J_\mu(0)|N(p)\rangle \Big\rangle_{\text{spin}}$$

$$= \int d^4y \; e^{iqy} \Big\langle \langle N(p)|[J_\nu^\dagger(y), J_\mu(0)]|N(p)\rangle \Big\rangle_{\text{spin}}$$

- Use the integral representation for the delta-distribution:

$$(2\pi)^4 \delta^{(4)}(p+q-p_X) = \int dy \; e^{i(p+q-p_X)y} = \int dy \; e^{iqy} \; e^{i(p-p_X)y}$$

- The space-time translation of an operator in QM is generated by the 4-momentum operator:

$$\hat{O}(y) := e^{i\hat{P}\cdot y}\hat{O}(0)e^{-i\hat{P}\cdot y}$$

- Use the completeness relation:

$$\sum_{\text{states } X} \int d\Phi_X |X\rangle\langle X| = \mathbf{1}$$

- The second term in the commutator leads to $q+p_X-p=0$ violating mom. cons. $q+p-p_X=0$!