Deep Inelastic Scattering (DIS)

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MCnet-CTEQ Summer School 2021

Virtual
5-16 September 2021

Homework Problem

Leading order parton model expressions for structure functions in **e+N→e+X** (γ-exchange) and **ν_μ+N→μ^-+X** DIS, assuming a diagonal **CKM**-matrix, **m_c=0**

$$
\begin{aligned}
F_2^{ep} &= \tfrac{4}{9}x\left[u+\bar{u}+c+\bar{c}\right] \\
&+ \tfrac{1}{9}x\left[d+\bar{d}+s+\bar{s}\right] \\
F_2^{en} &= \tfrac{4}{9}x\left[d+\bar{d}+c+\bar{c}\right] \\
&+ \tfrac{1}{9}x\left[u+\bar{u}+s+\bar{s}\right] \\
F_2^{\nu p} &= 2x\left[d+s+\bar{u}+\bar{c}\right] \\
F_2^{\nu n} &= 2x\left[u+s+\bar{d}+\bar{c}\right] \\
F_2^{\bar{\nu} p} &= 2x\left[u+c+\bar{d}+\bar{s}\right] \\
F_2^{\bar{\nu} n} &= 2x\left[d+c+\bar{u}+\bar{s}\right] \\
F_3^{\nu p} &= 2\left[d+s-\bar{u}-\bar{c}\right] \\
F_3^{\nu n} &= 2\left[u+s-\bar{d}-\bar{c}\right] \\
F_3^{\bar{\nu} p} &= 2\left[u+c-\bar{d}-\bar{s}\right] \\
F_3^{\bar{\nu} n} &= 2\left[d+c-\bar{u}-\bar{s}\right]
\end{aligned}
$$

- Verify this list!
  Check for isospin symmetry,
  How are the structure functions F_3 in neutrino and anti-neutrino DIS related?

- These different observables are used to dis-entangle the flavor structure of the PDFs

Homework Problem

Verify the following sum rules in the parton model:

Adler (1966)

$$\int_0^1 \frac{dx}{2x}\left[F_2^{\nu n} - F_2^{\nu p}\right] = 1$$

Bjorken (1967)

$$\int_0^1 \frac{dx}{2x}\left[F_2^{\bar{\nu} p} - F_2^{\nu p}\right] = 1$$

Gross Llewellyn-Smith (1969)

$$\int_0^1 dx\left[F_3^{\nu p} + F_3^{\bar{\nu} p}\right] = 6$$

Gottfried (1967) if $\bar{u} = \bar{d}$

$$\int_0^1 \frac{dx}{x}\left[F_2^{ep} - F_2^{en}\right] = \frac{1}{3}$$

Experimentally: @Q²=4 GeV²
I_G=0.235 +/- 0.026
Conclusion? [cf. hep-ph/0311091]

Homework (19??)

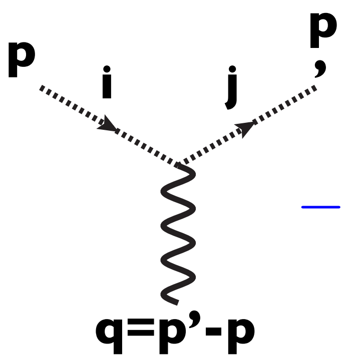
$$\frac{5}{18} F_2^{\nu N} - F_2^{eN} = ?$$

$$F_2^{lN} := (F_2^{lp} + F_2^{ln})/2 \quad (l = \nu, e)$$

Homework Problem

- Calculate the structure functions **$F_1(x)$** and **$F_2(x)$** for a massless spin-0 parton 'i' of electric charge **e_i** and parton distribution **$q_i(x)$**.

The scalar-scalar-photon vertex reads:



$$-iee_q(p' + p)^\mu \delta_{ij}$$

The result for the partonic tensor is:

$$\hat{w}_i^{\mu\nu} = \frac{2}{Q^2} e_i^2 x^3 p_\perp^\mu p_\perp^\nu \delta(\xi - x)$$

How do the results change if the incoming parton remains massless but the outgoing parton has a mass m?

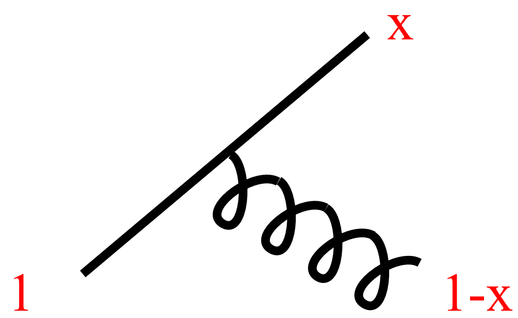
The leading order Splitting Functions

Group theory SU(3): $T_F = 1/2$, $C_F = 4/3$, $C_A = 3$

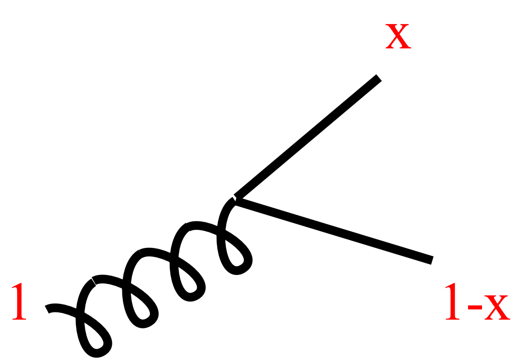
$$P_{ij} = P_{ij}^{(1)} + \frac{\alpha_s}{2\pi} P_{ij}^{(2)} + \dots$$

Read backwards

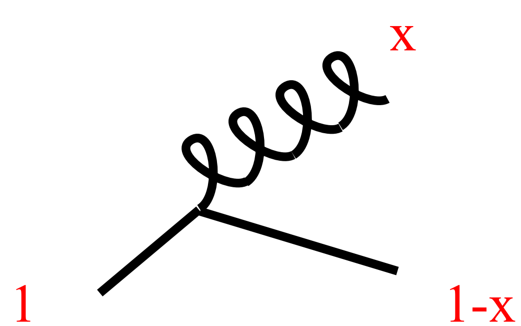
Note singularities



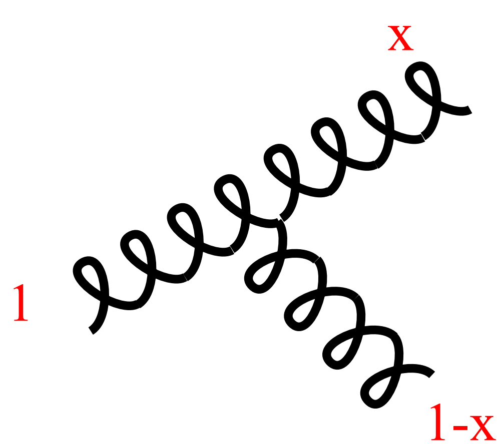
$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$



$$P_{qg}^{(1)}(x) = T_F \left[(1-x)^2 + x^2\right]$$



$$P_{gq}^{(1)}(x) = C_F \left[\frac{(1-x)^2 + 1}{x}\right]$$



$$P_{gg}^{(1)}(x) = 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F\right] \delta(1-x)$$

Homework

Definition of the Plus prescription:

Definition using a test function f(x)

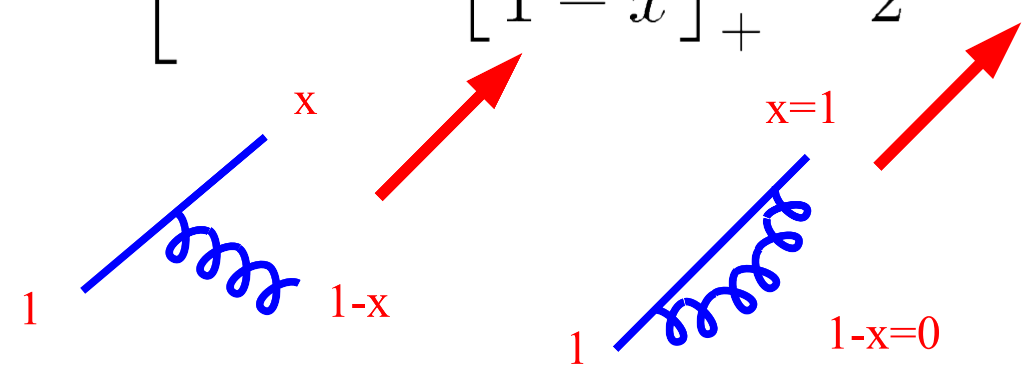
$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} = \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

Compute:

$$\int_a^1 dx \, \frac{f(x)}{(1-x)_+} = ???$$

Verify:

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$



Homework

Appendix E in R. D. Field, Applications of Perturbative QCD

Definition as a limiting procedure:

Hard → $\Theta(1-x-\beta)$; Virtual: → $\delta(1-x-\beta)$

$$[F(x)]_+ := \lim_{\beta to 0} \left(F(x)\Theta(1-x-\beta) - \delta(1-x-\beta) \int_0^{1-\beta} F(y)dy \right)$$

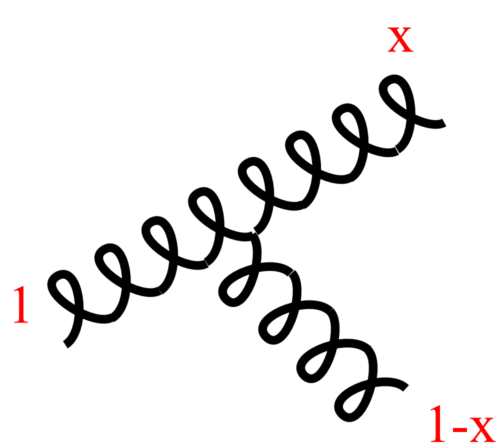
Verify:

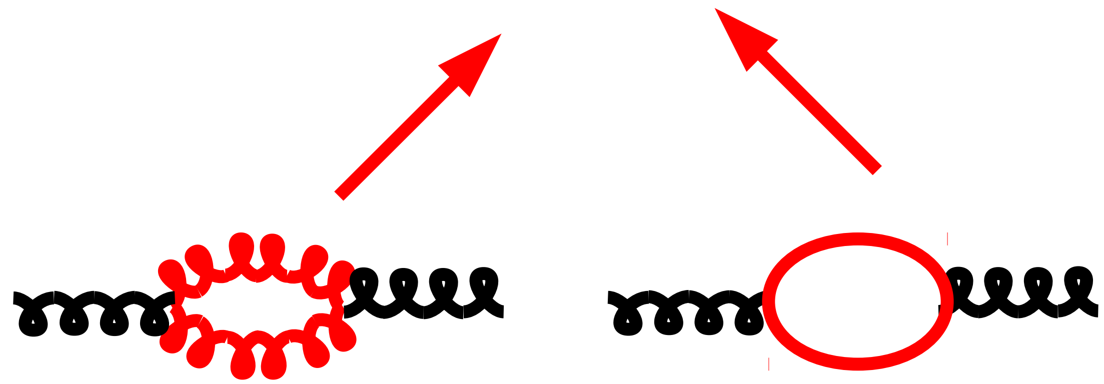
$$\frac{1}{(1-x)_+} = \lim_{\beta \to 0} \left(\frac{1}{1-x}\Theta(1-x-\beta) + \ln(\beta)\delta(1-x-\beta) \right)$$

Observe

$$P_{gg}^{(1)}(x) = 2C_F\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+\left[\frac{11}{6}C_A - \frac{2}{3}T_F\,N_F\right]\;\delta(1-x)$$



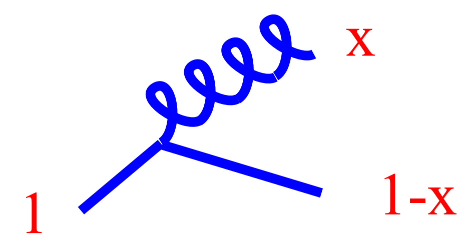


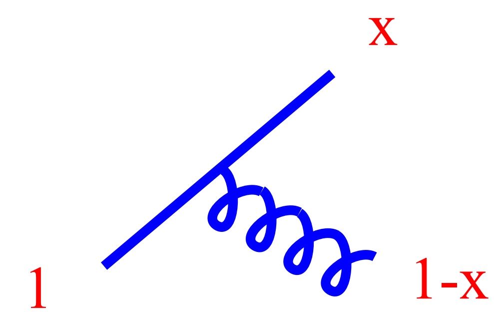
Homework: Symmetries and Limits

Verify the following relation among the regular parts (from the real graphs)

For the regular part show:

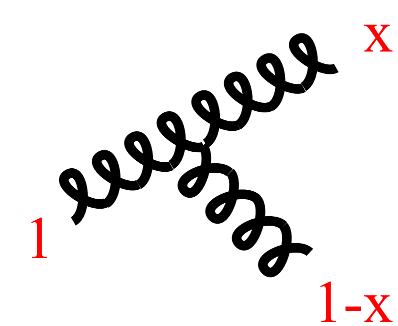
$$P_{gq}^{(1)}(x) = P_{qq}^{(1)}(1-x)$$





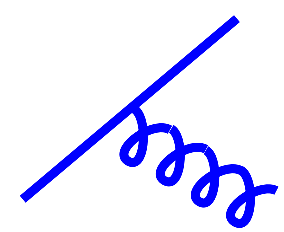
For the regular part show:

$$P_{gg}^{(1)}(x) = P_{gg}^{(1)}(1-x)$$

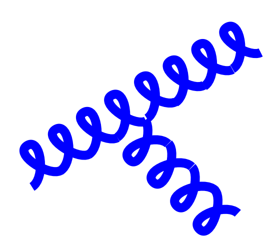


Verify, in the soft limit:

$$P_{qq}^{(1)}(x) \underset{x\to 1}{\longrightarrow} 2C_F \frac{1}{(1-x)_+}$$



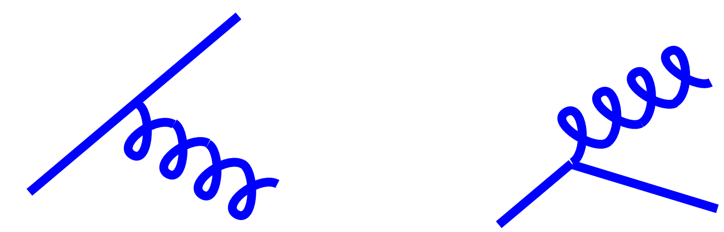
$$P_{gg}^{(1)}(x) \underset{x\to 1}{\longrightarrow} 2C_F \frac{1}{(1-x)_+}$$



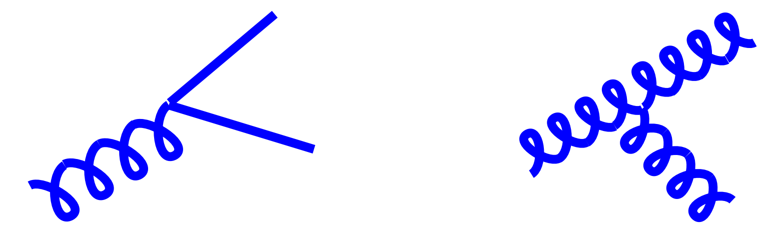
Homework: Conservation rules

Verify conservation of momentum fraction

$$\int_0^1 dx\, x\, \left[P_{qq}(x) + P_{gq}(x)\right] = 0$$



$$\int_0^1 dx\, x\, \left[P_{qg}(x) + P_{gg}(x)\right] = 0$$



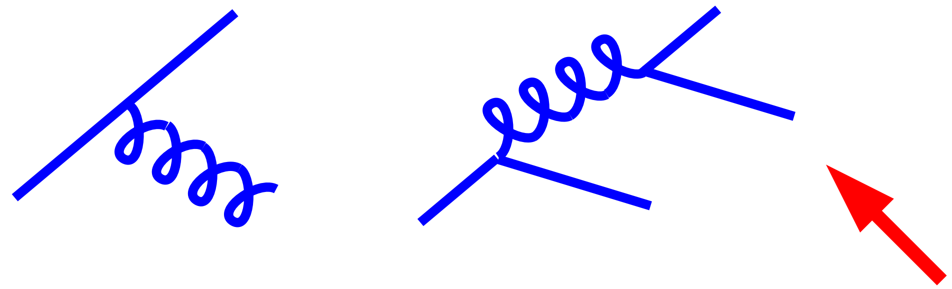
Verify conservation of fermion number

$$\int_0^1 dx \quad \left[P_{qq}(x) - P_{q\bar{q}}(x)\right] = 0$$

Homework: Using the real to guess the virtual

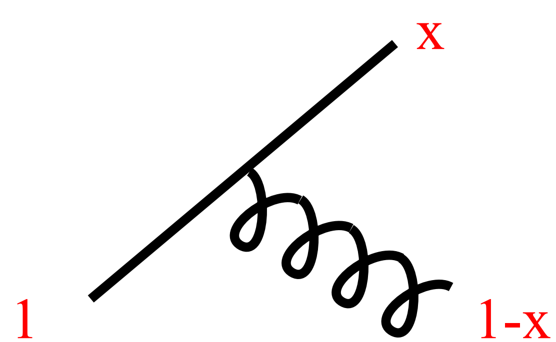
Use conservation of fermion number to compute
the delta function term in P(q←q)

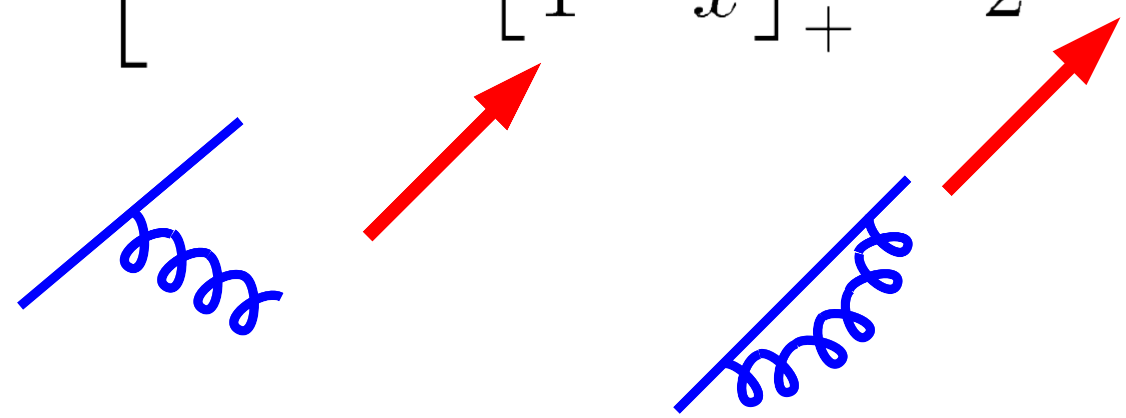
$$\int_0^1 dx \quad [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$



This term only starts at NNLO

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$





Powerful tool: Since we know real and virtual must balance, we can use to our advantage!!!