September 13, 2021, CTEQ School

Jets

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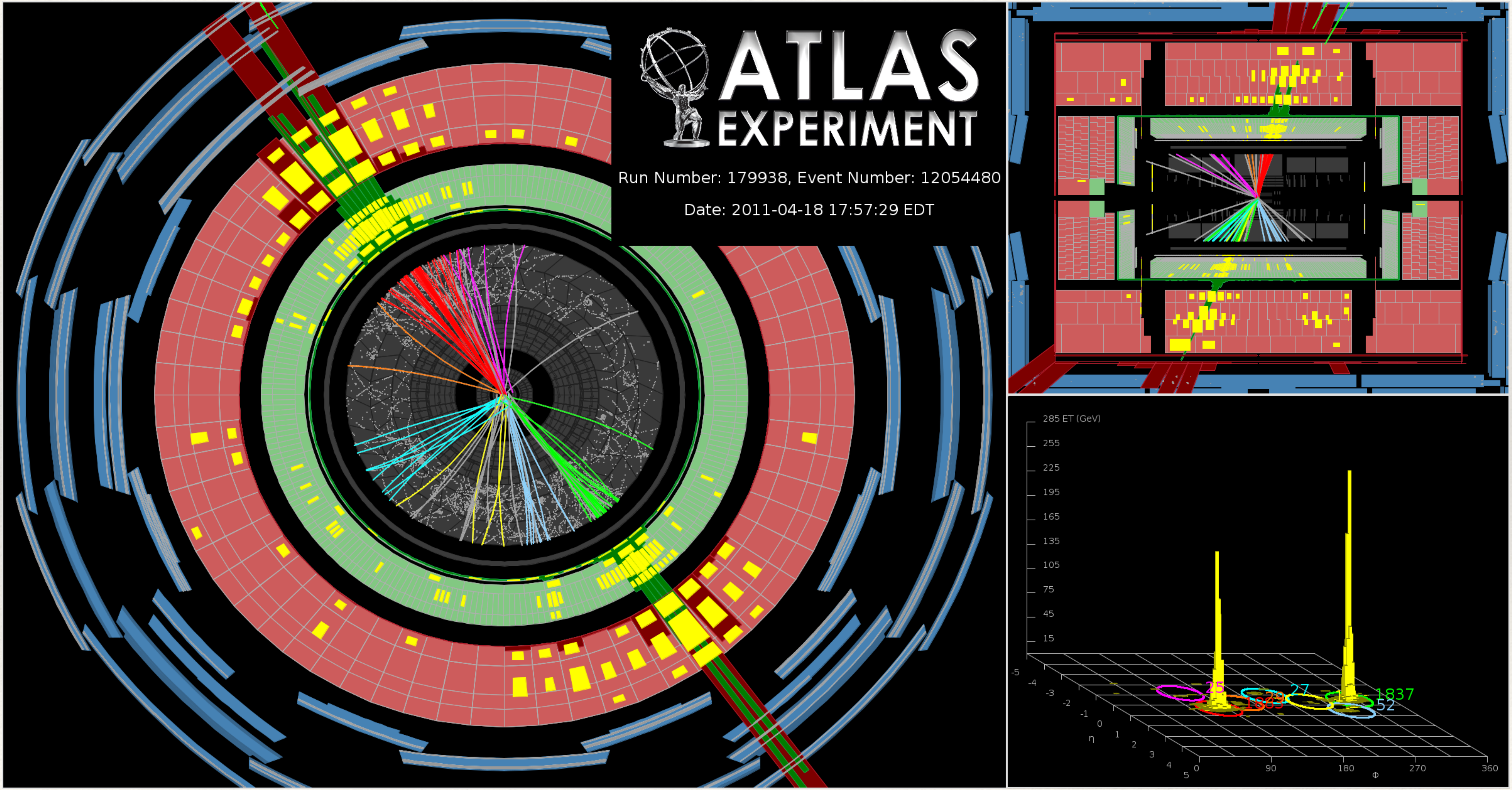
DESY

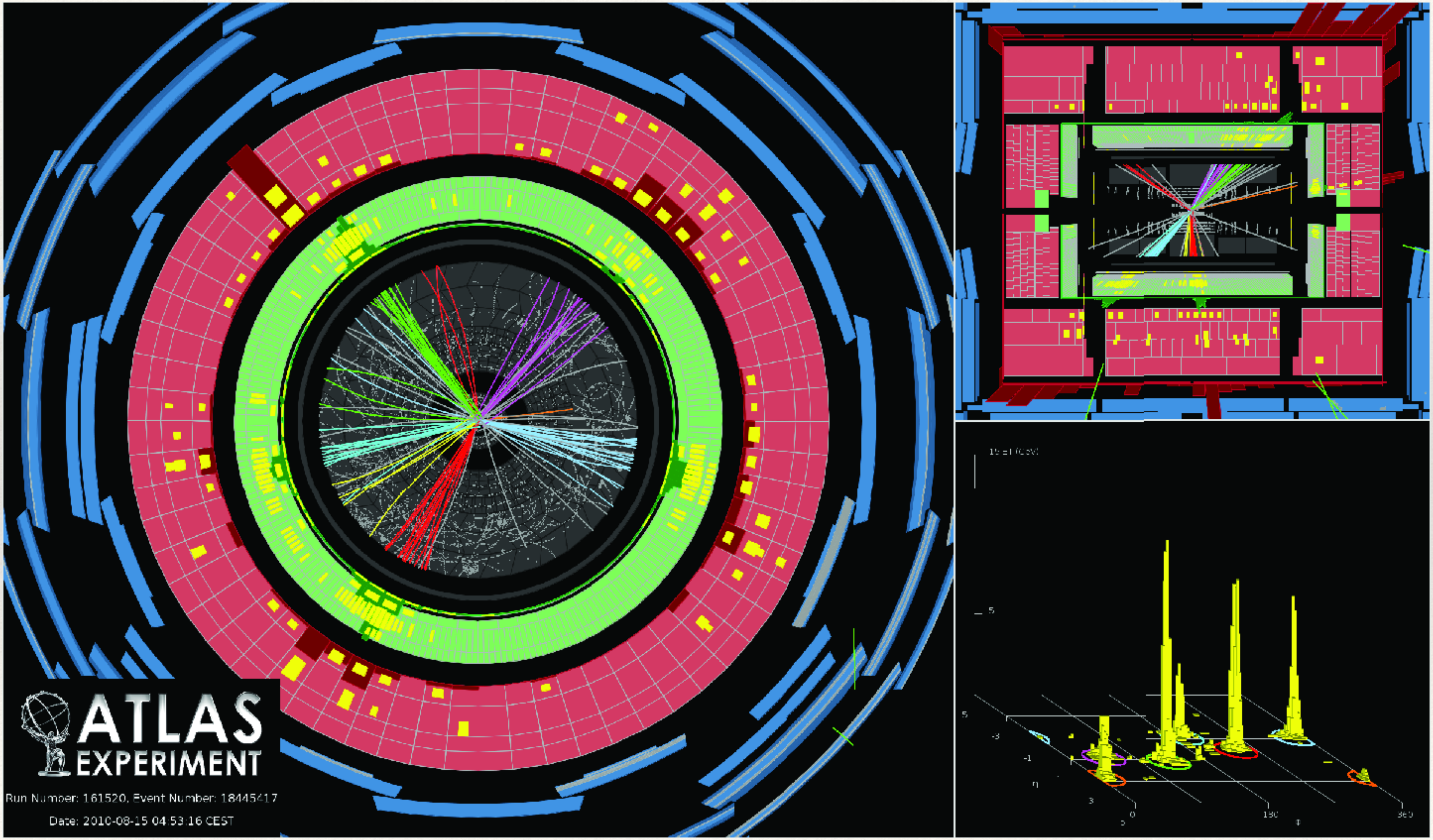
What are Jets?

A di-jet ATLAS event



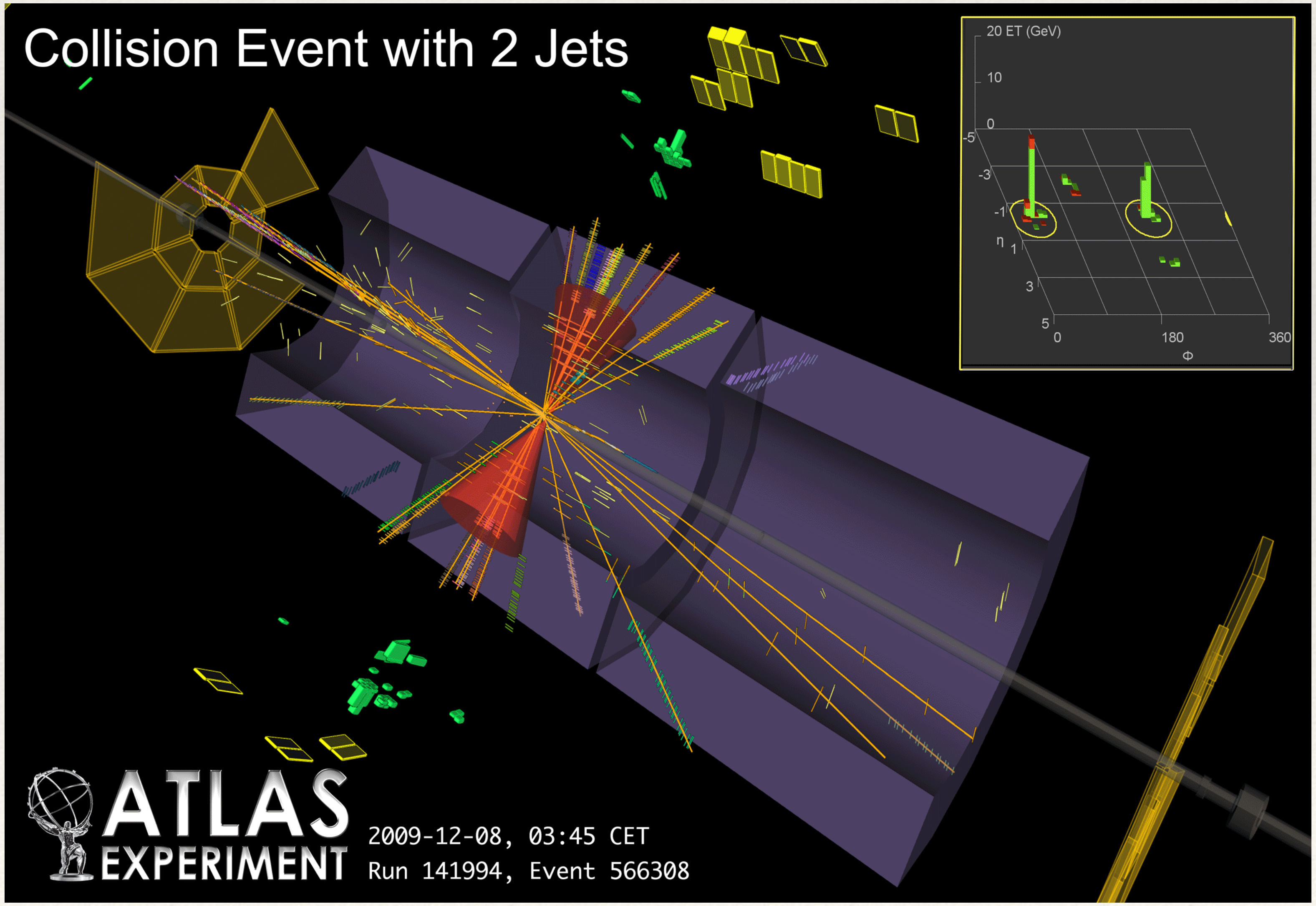
What are Jets?

A multi-jet (6-jet) event



What are Jets?

- The pT is concentrated in a few narrow sprays of particles
- These sprays are called jets.
- Events with big total pT are rather rare…
- … but when they happen, the pT is always in jets

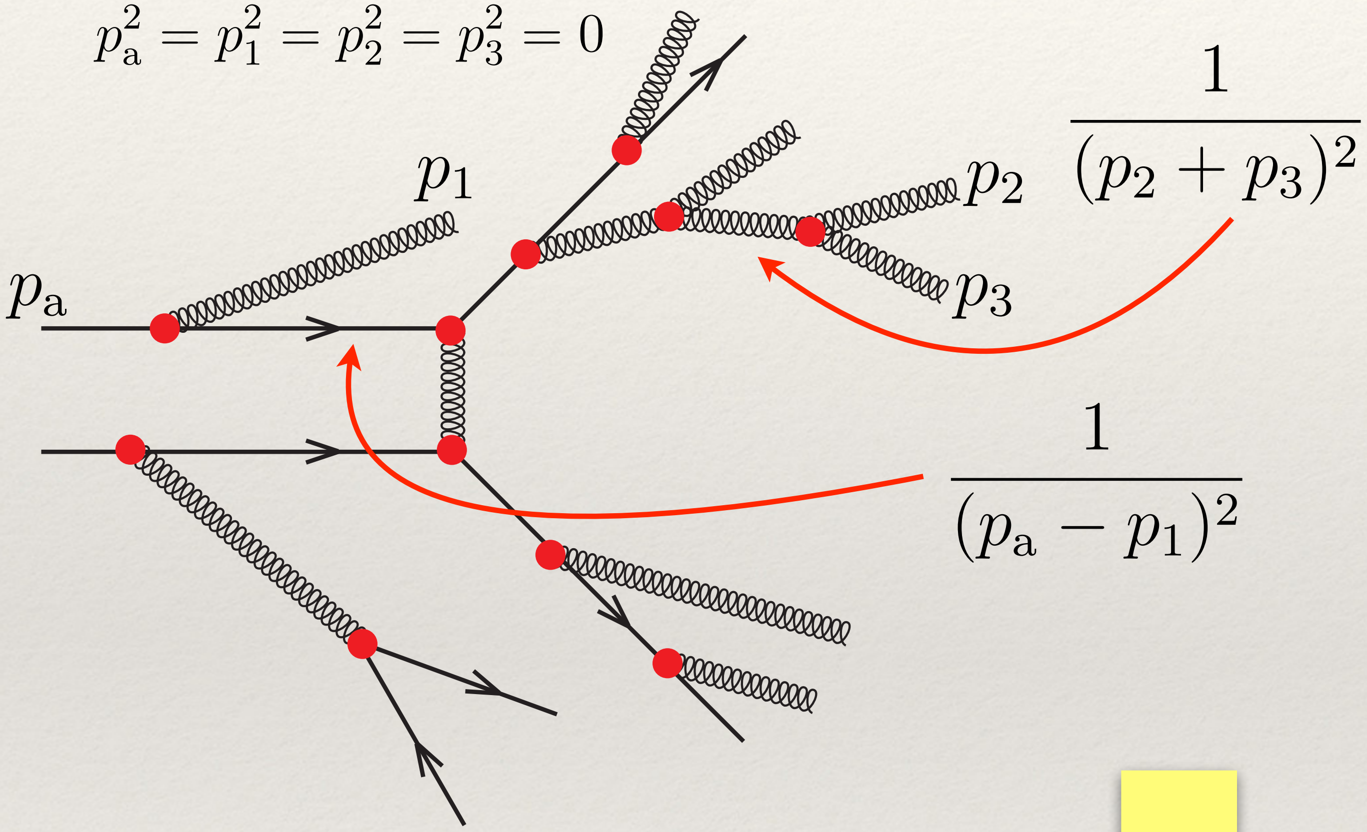


Collision Event with 2 Jets

2009-12-08, 03:45 CET
Run 141994, Event 566308

Why are the Jets there?

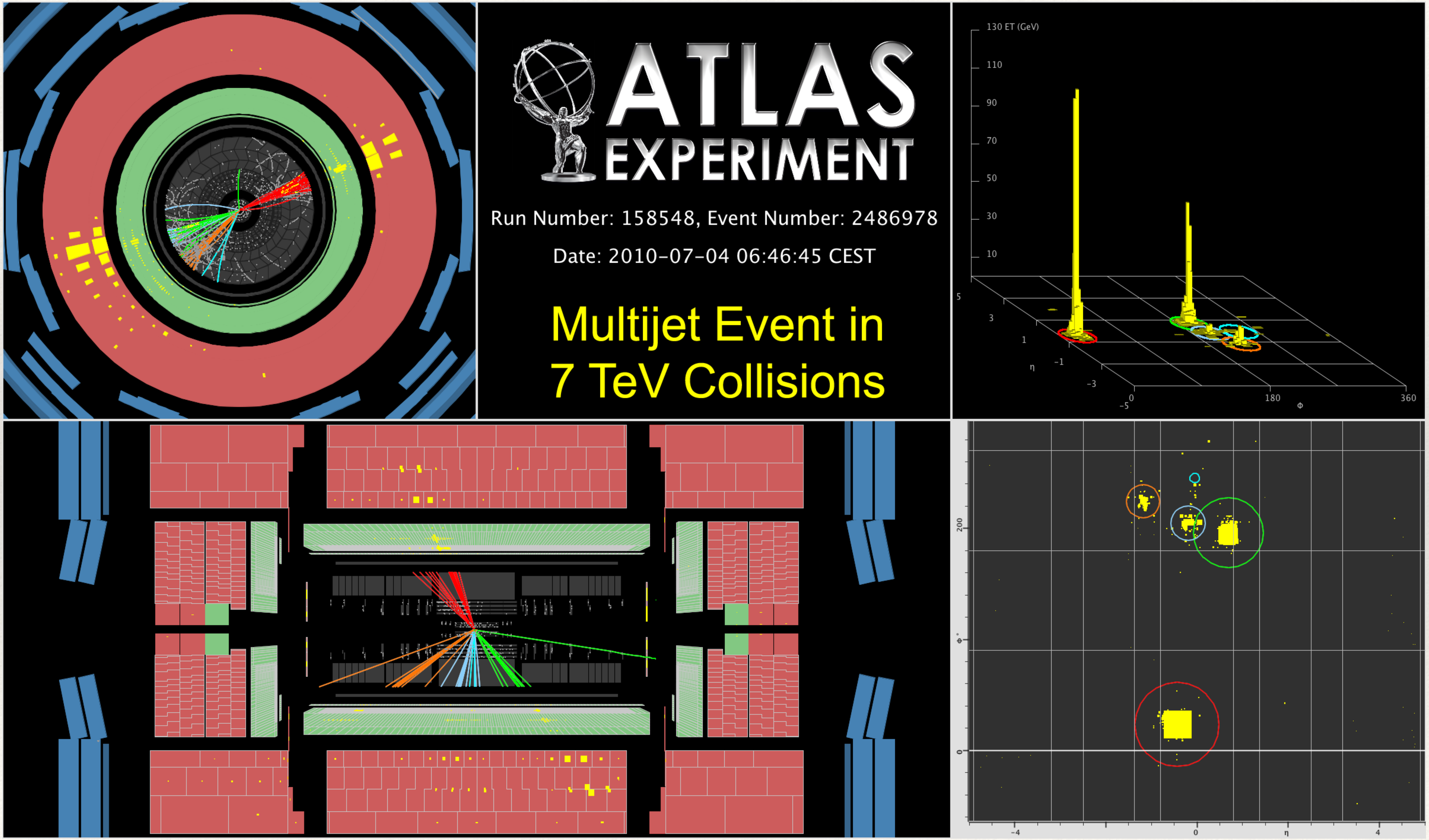
Here is a Feynman graph for quad-quark scattering with additional radiation that can contribute to the jet events.



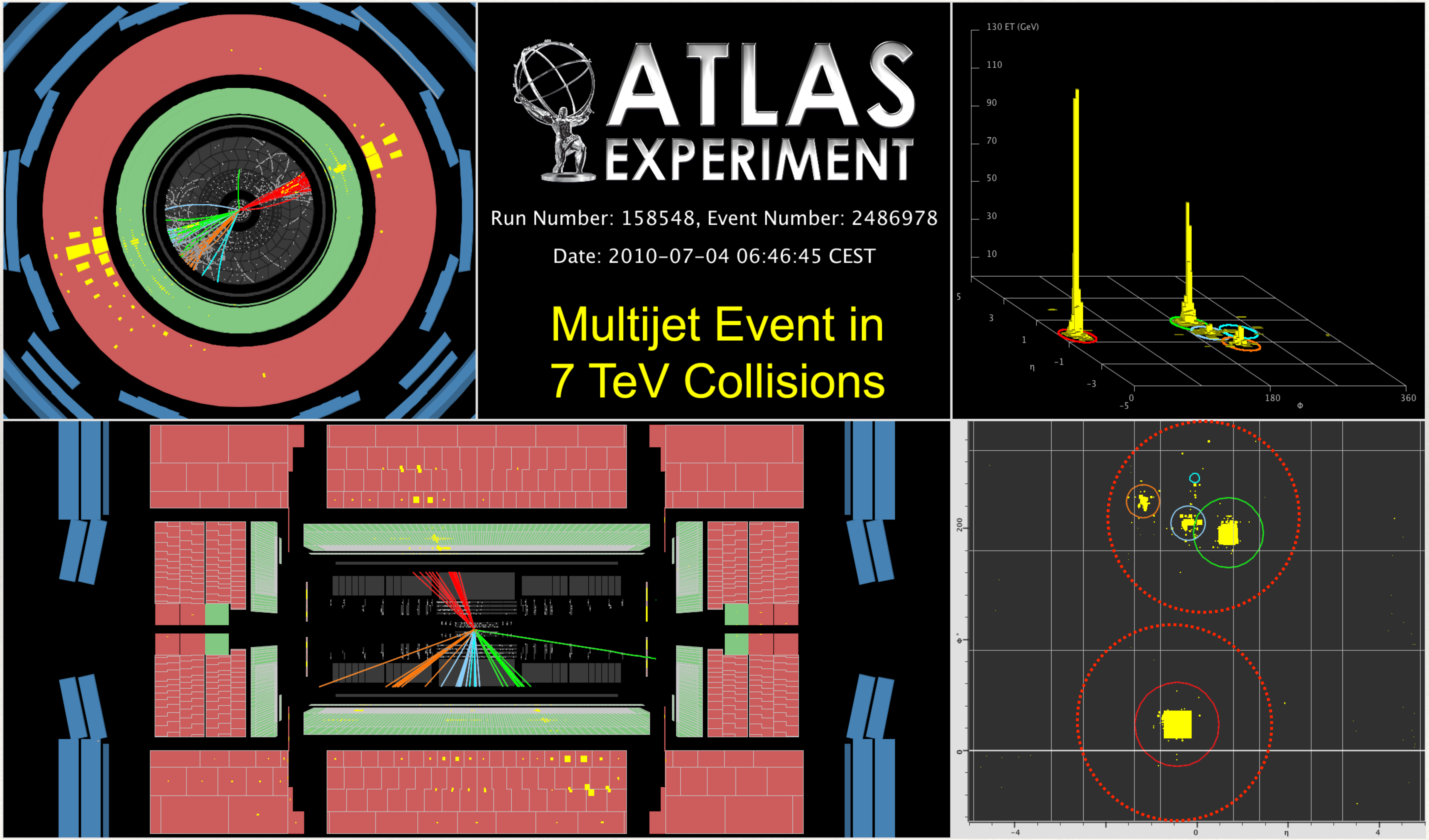
- Initial state
  - If $p_1 \to 0$, then $1/(p_a - p_1)^2 \to \infty$
  - If $p_1 \to \lambda p_a$, then $1/(p_a - p_1)^2 \to \infty$
- Final state
  - If $p_2 \to 0$, then $1/(p_2 + p_3)^2 \to \infty$
  - If $p_3 \to 0$, then $1/(p_2 + p_3)^2 \to \infty$
  - If $p_3 \to \lambda p_2$, then $1/(p_2 + p_3)^2 \to \infty$

The probability is big to get a spray of collimated particles plus some low momentum particles with wide angle.

How many Jets are there?

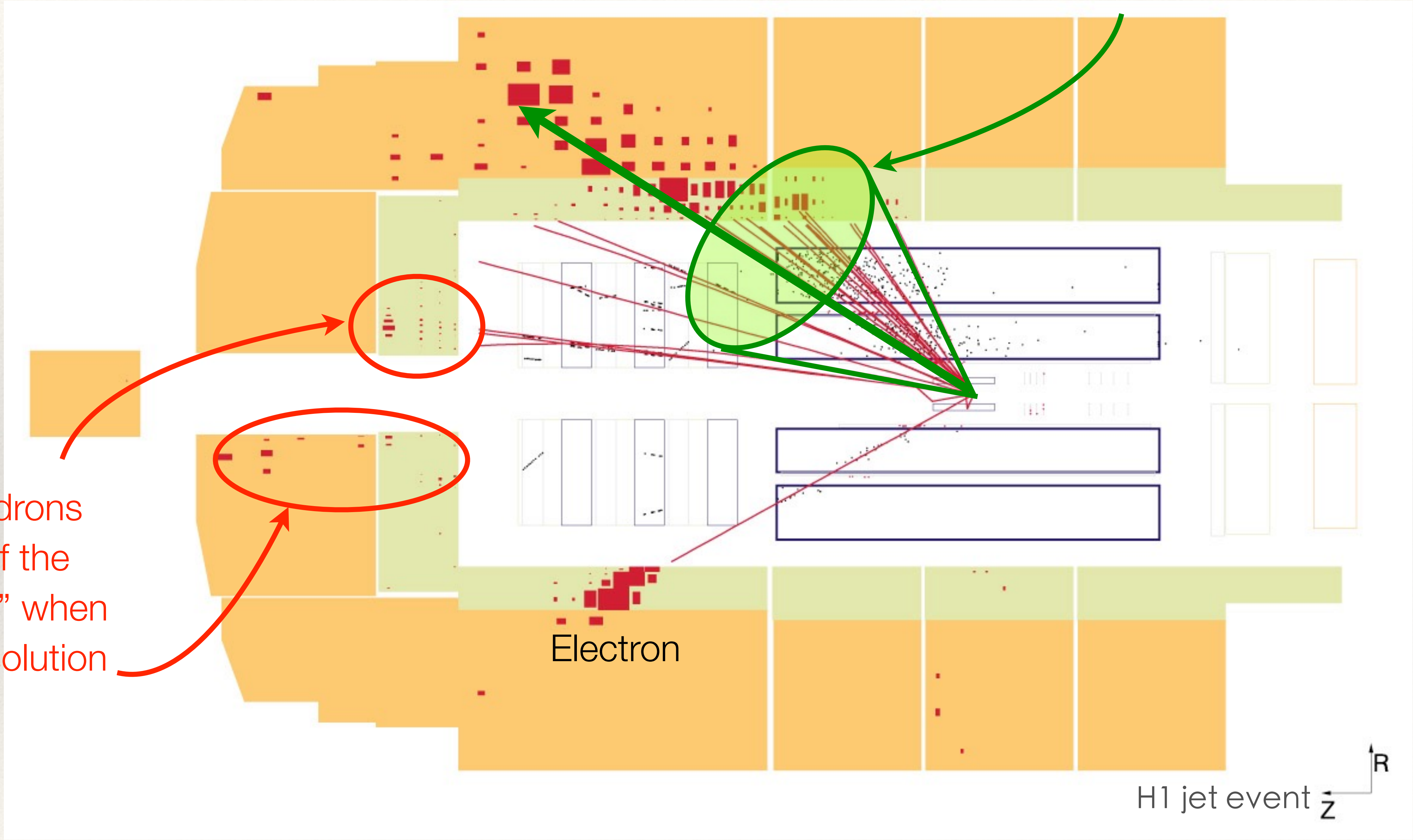


How many Jets are there?



How many Jets are there?

Jet structure at large resolution scale:

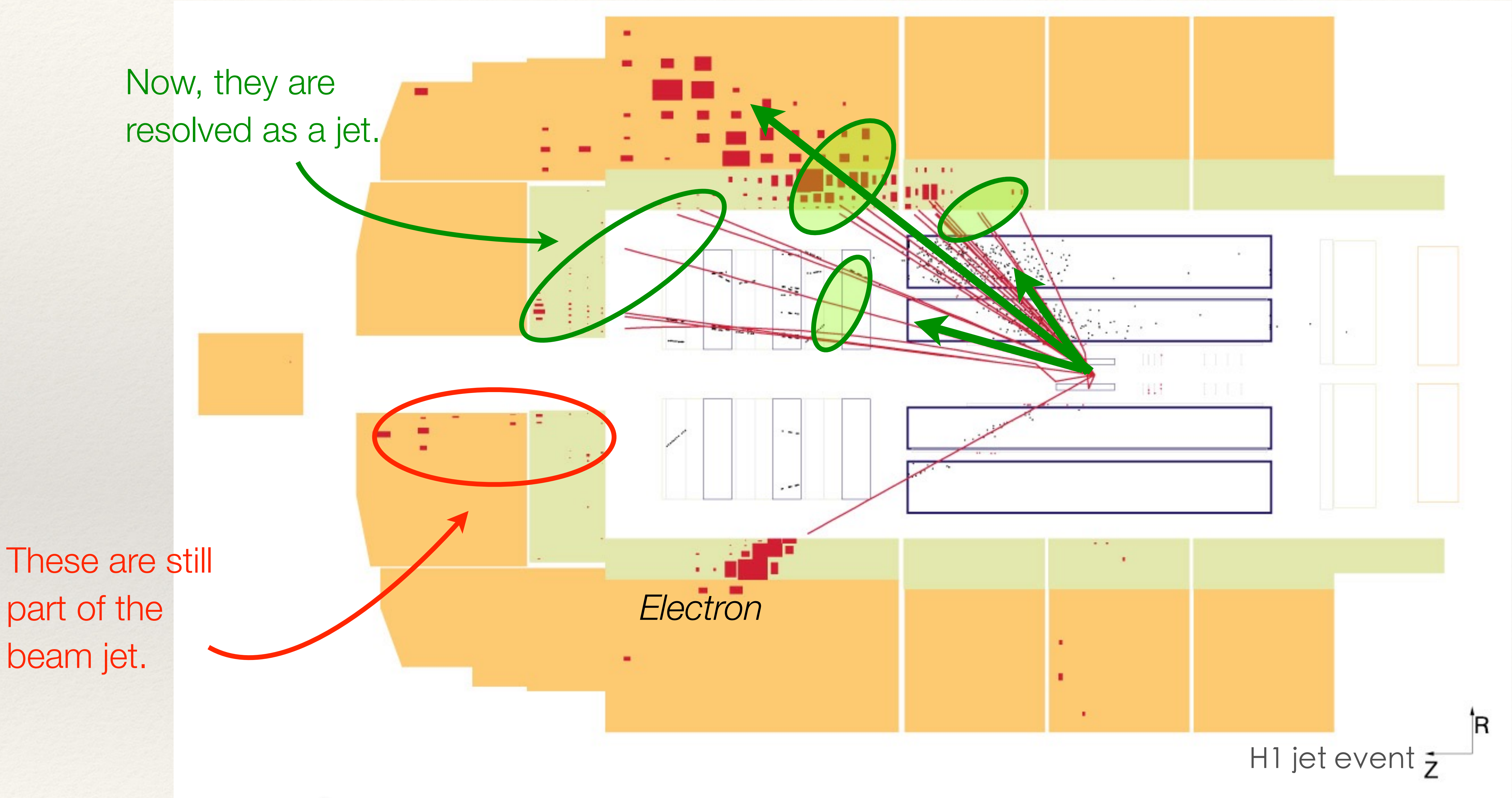


The jet algorithm finds one fat jet.

These hadrons are part of the "beam jet" when the jet resolution is crude.

How many Jets are there?

Jet structure at small resolution scale:



Now, they are resolved as a jet.

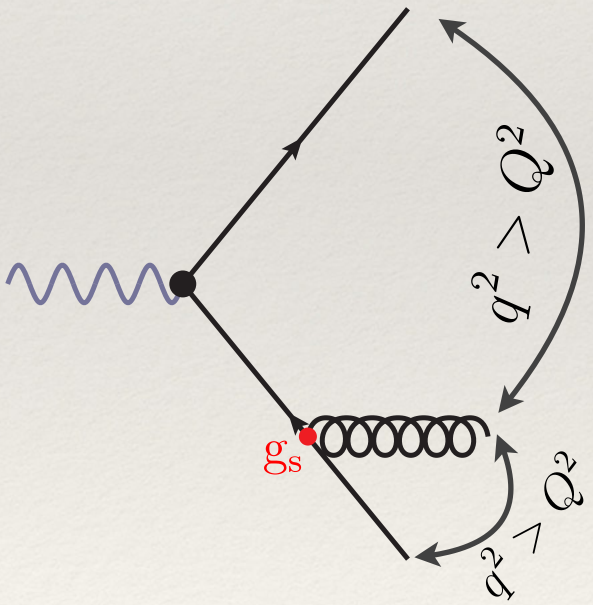
These are still part of the beam jet.

The number of the jets depend on the typical resolution scale (theory), detector sensitivity and angular resolution (experiment) .

Jets in pQCD

- Let us consider a 3-jet event in e+e- annihilation with the typical resolution scale Q.
- At each vertex in a diagram, there is a factor of the strong coupling, $\mathrm{g}_\mathrm{s}^2/(4\pi) = \alpha_\mathrm{s}$
- The simplest graph that contributes to this process is the tree level graph

Tree level graph

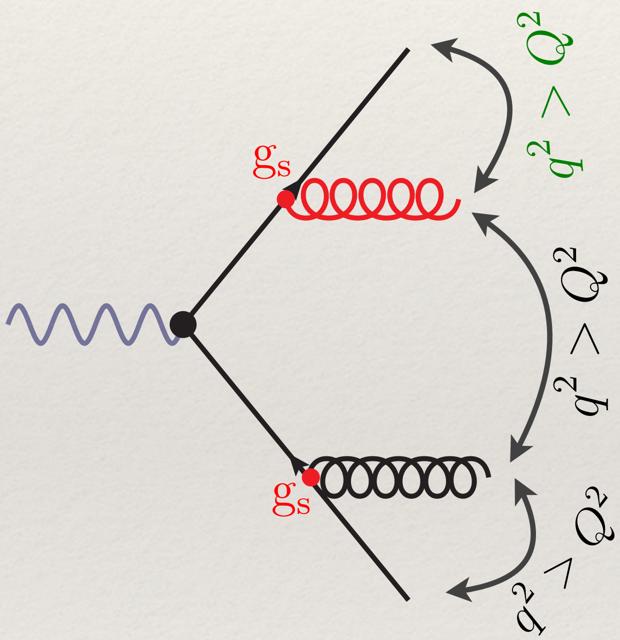


All the three patrons are well separated from each others and the "distance" is measured by some hardness variable like transverse momentum or virtuality.

Jets in pQCD

- In the perturbation theory should consider radiative correction.
- We can consider one more gluon in the final state… .

Resolvable real radiation



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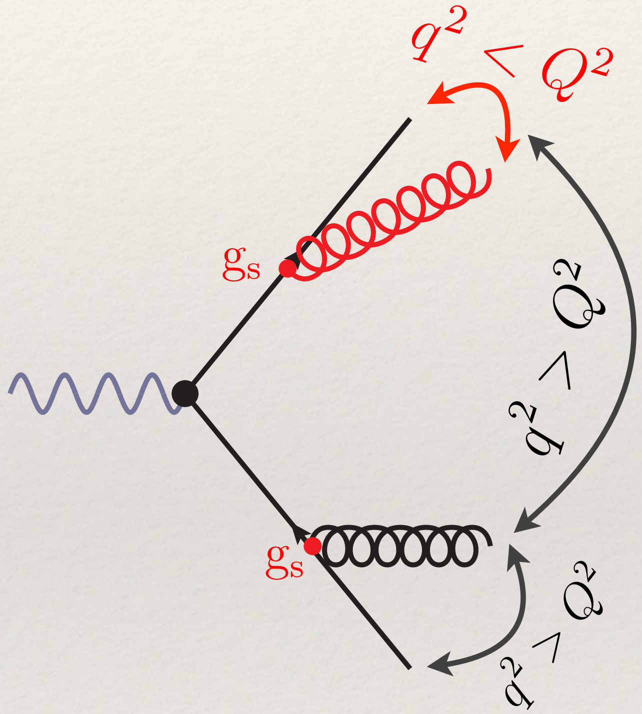
Unresolvable real radiation



4-jet configuration

all the four patrons are well separated

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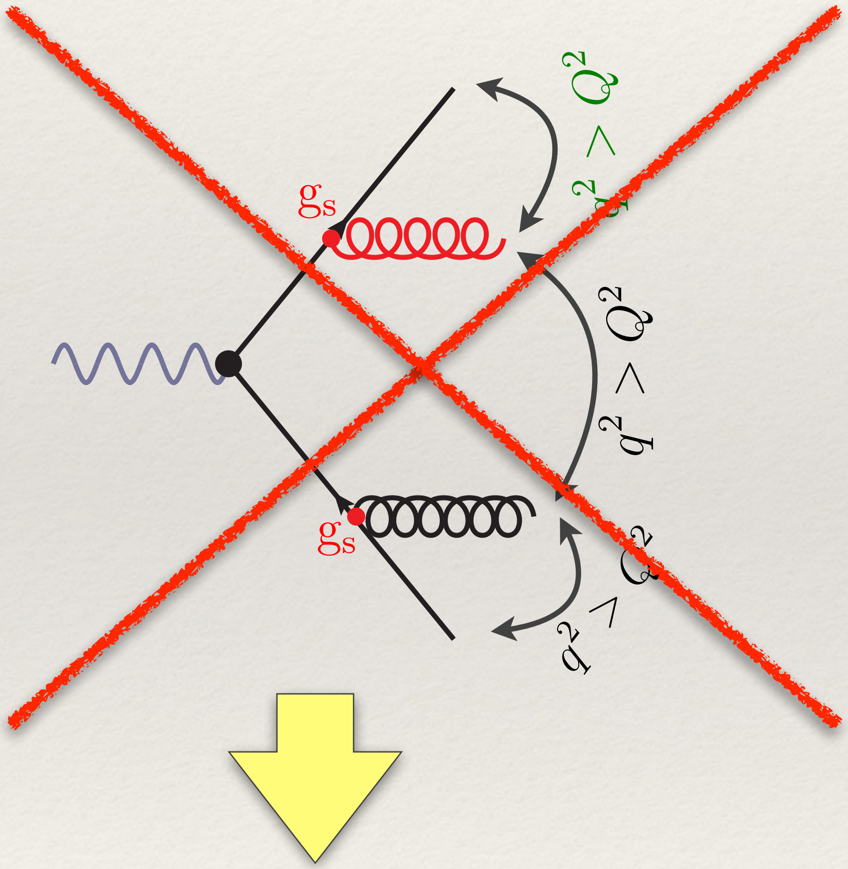
Resolvable real radiation



4-jet configuration
all the four patrons are well separated

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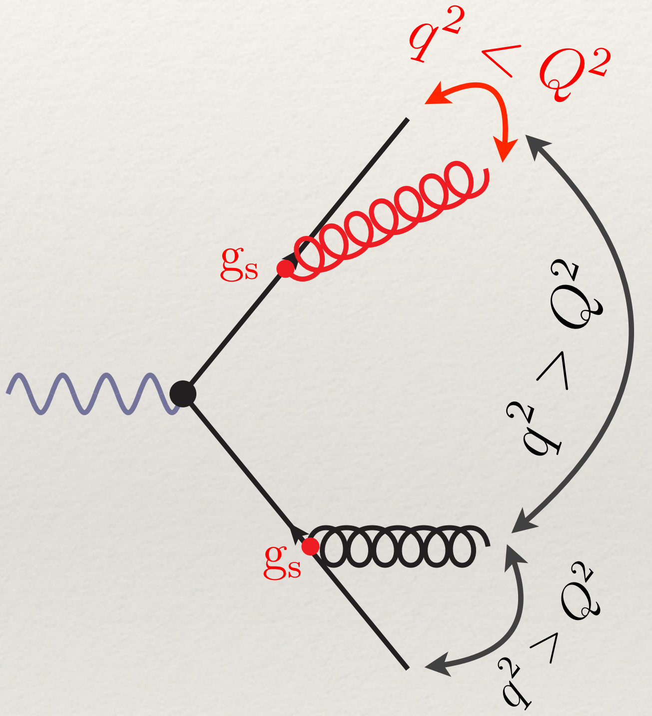
Unresolvable real radiation



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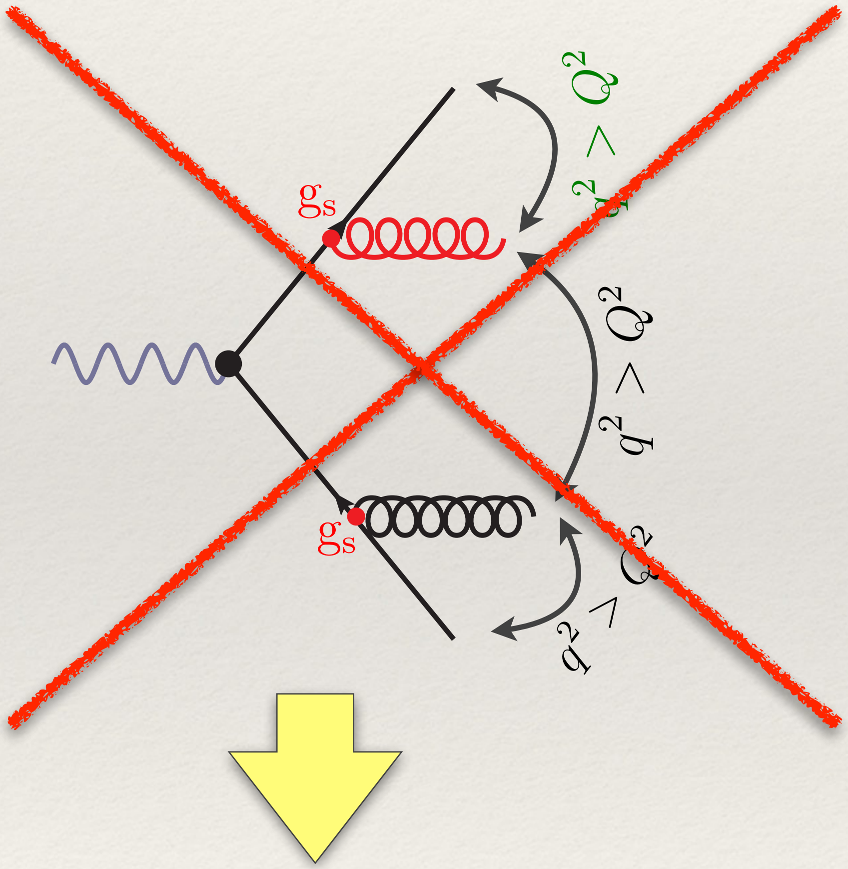

4-jet configuration
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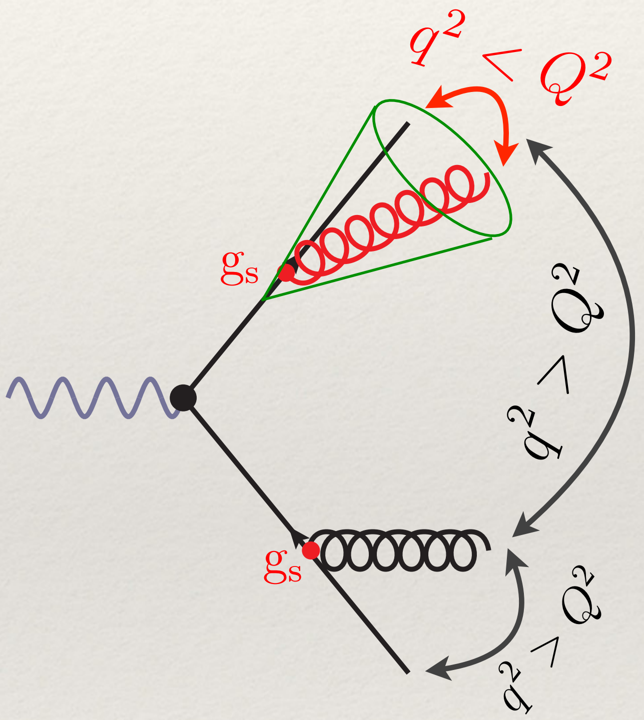
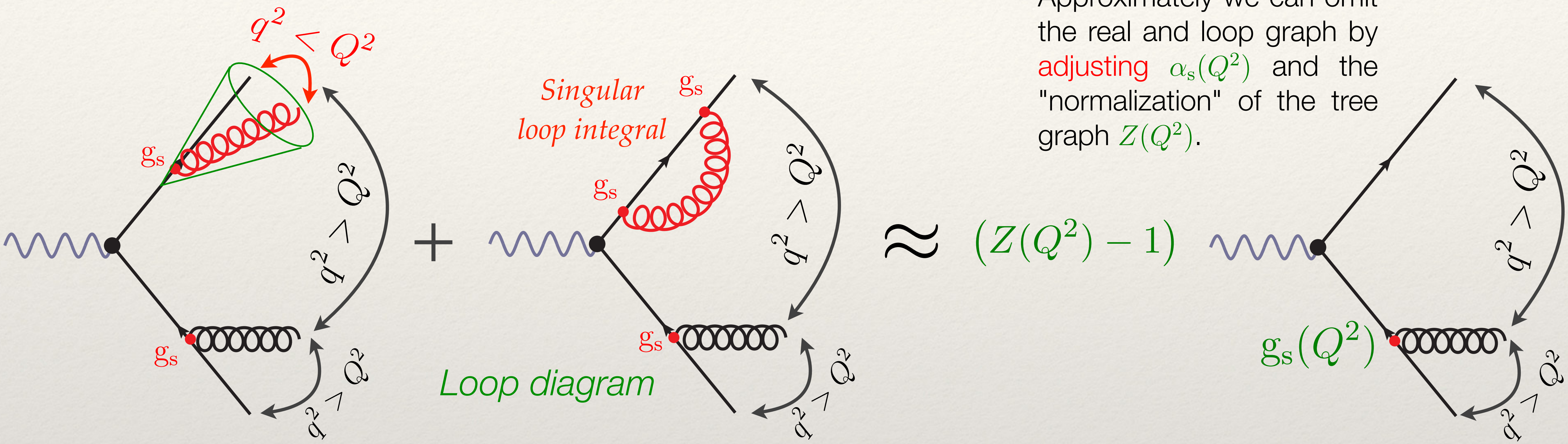
Unresolvable real radiation



- Everything inside the green cone is unresolvable and integrated out.
- It is a singular integral.
- This singularity has to be cancelled. Otherwise we cannot make pQCD predictions for jet production.

Jet in pQCD

We have to also consider the virtual corrections, thus we have graphs like…



Approximately we can omit the real and loop graph by adjusting $\alpha_s(Q^2)$ and the "normalization" of the tree graph $Z(Q^2)$.

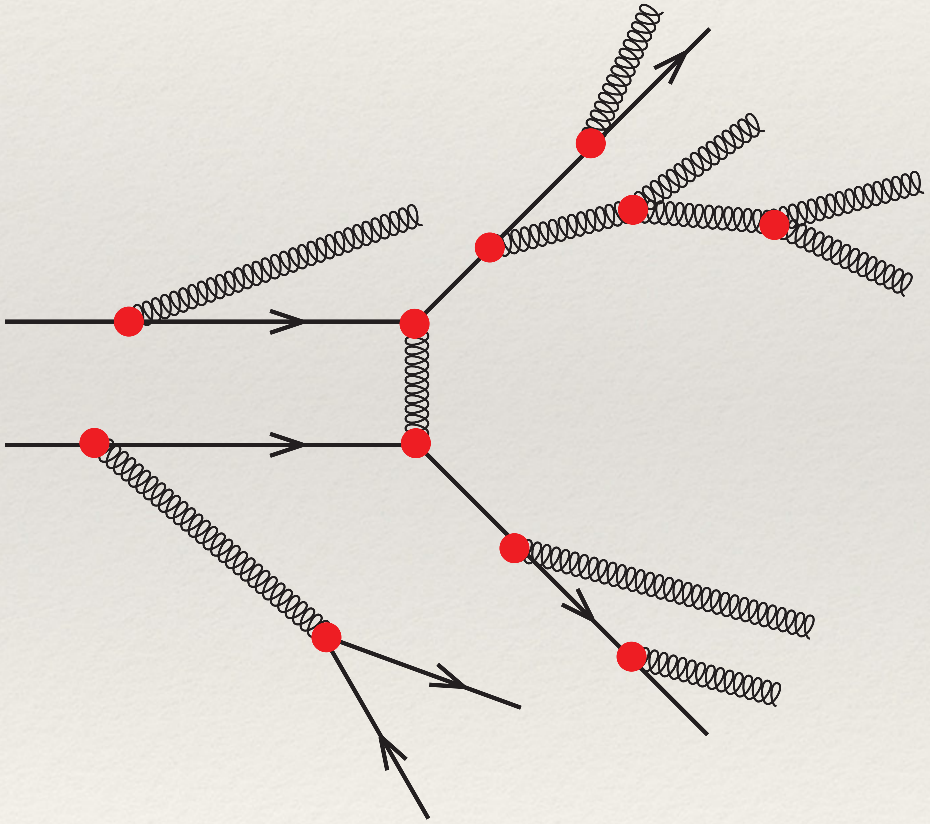
- *Singularities has to be cancelled between the two graphs!!!*
- *This cancelation has to be ensured by the jet definition!!!*

INFRARED SAFETY

Infrared Safety

The jet algorithm has to be infrared safe. This means it has to be *insensitive for any small scale physics* (soft or collinear radiation).

- We construct jets from particle momenta $\{p_1, p_2, \ldots, p_m\}$.
- We get *N* jets with momenta $\{P_1, P_2, \ldots, P_N\}$.



- If any p_i becomes very small, we should get the same jets by leaving particle i out.
- If any two momenta p_i and p_j become collinear, we should get the same jets by replacing the particles by one with momentum $p_i + p_j$.

Jet Cross Sections

In the general case the cross section is given by

$$\sigma[F] = \sum_m \frac{1}{m!} \int d\{p, f\}_m \, |M(\{p, f\}_m)|^2 \, \underbrace{F(\{p\}_m)}_{\text{Jet measurement function}}$$

$$F(\{p\}_m) \equiv F(p_1, p_2, \ldots, p_m)$$

INFRARED SAFETY (formal definition):

$$F(p_1, p_2, \ldots, p_m, p_{m+1}) \xrightarrow{p_{m+1} \to 0} F(p_1, p_2, \ldots, p_m)$$

$$F(p_1, p_2, \ldots, p_m, p_{m+1}) \xrightarrow{p_m || p_{m+1}} F(p_1, p_2, \ldots, p_m + p_{m+1})$$

The measurement is insensitive to soft and collinear radiation.

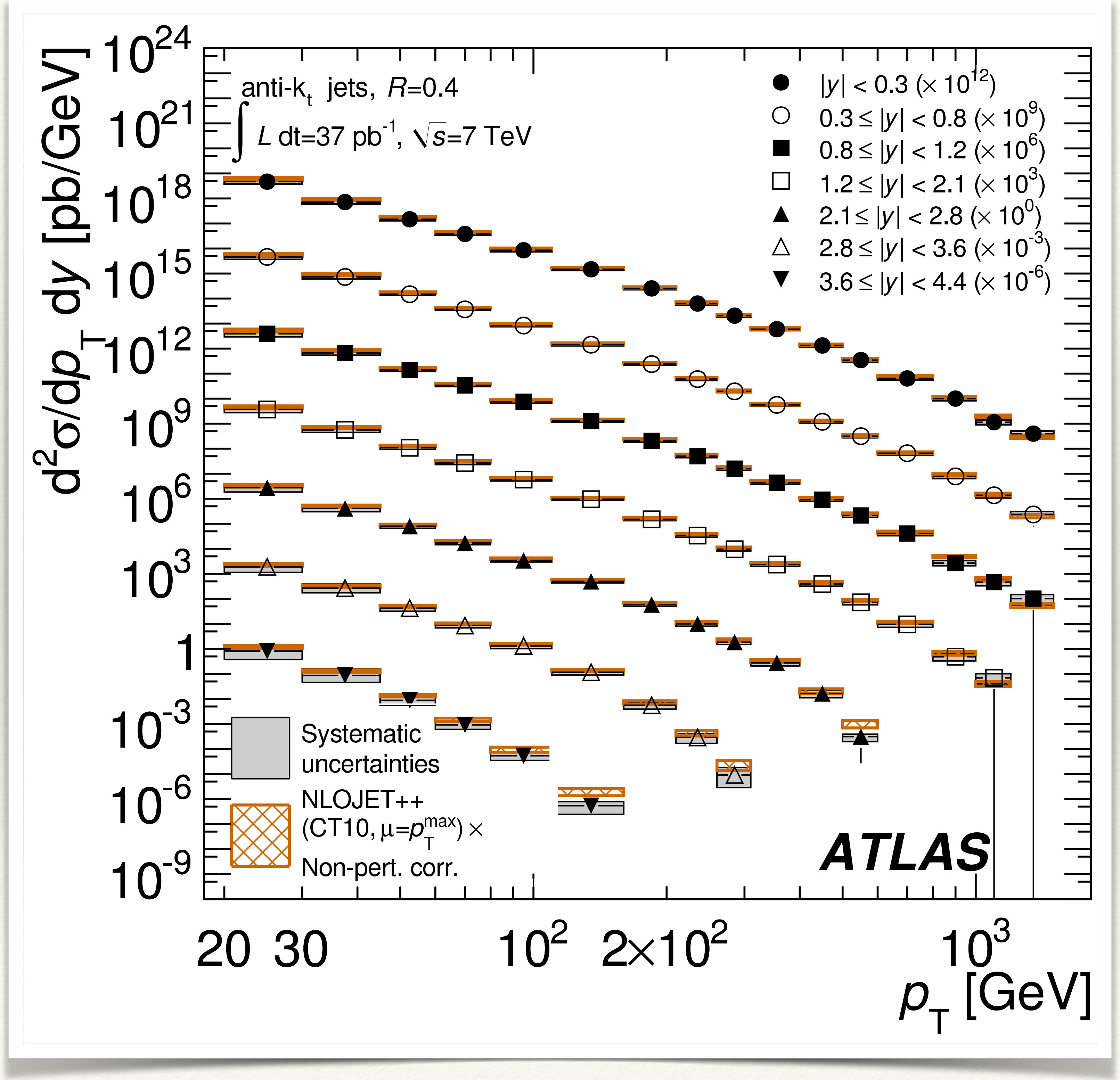
One can consider for example the inclusive one jet cross section

$$\sigma[F] \Longrightarrow \frac{d\sigma}{dp_T \, dy}$$

$$F(\{p\}_m) \Longrightarrow \delta\big(p_T - \underbrace{P_T(\{p\}_m)}_{\substack{\text{Transverse momentum} \\ \text{of the observed jet}}}\big)\,\delta\big(y - \overbrace{Y(\{p\}_m)}^{\text{Rapidity of the observed jet}}\big)$$

One Jet Inclusive Cross Section

A result from ATLAS



- Note nine order of magnitude variation in cross section at one $|y|$.
- Compared to NLO pQCD prediction and the agreement is very good! (No sign of new physics…)

Now, it is high time to give a definition of the jet algorithm.

Jet Algorithms

- There are two kind of algorithms for defining jets:
  - cone algorithms
  - successive combination algorithms
- Both can be infrared safe.
- I will discuss just the successive combination algorithms.
- This traces back to the JADE collaboration at DESY.

THE KT JET ALGORITHM

- Choose an angular resolution parameter *R*
- Start with the list of protojets, specified by their momenta $\{p_1, p_2, \ldots, p_m\}$.
- Start with an empty list of finished jets, {}.
- The result is a list of finished jets with their momenta, $\{P_1, P_2, \ldots, P_N\}$.
- Many are low pT debris, just ignore them.

kT Jet Algorithm

1. For each pair of protojets define

   $$d_{ij} = \min\left\{p_{T,i}^2, p_{T,j}^2\right\}\left[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2\right]/R^2$$

   and for each protojet define

   $$d_i = p_{T,i}^2$$

2. Find the smallest of the d_{ij} and the d_i

   $$d_{\text{min}} = \min_{i,j}\{d_i, d_{ij}\}$$

3. If d_{min} is a d_{ij}, merge protojets i and j into a new protojets k with momentum

   $$p_k = p_i + p_j$$

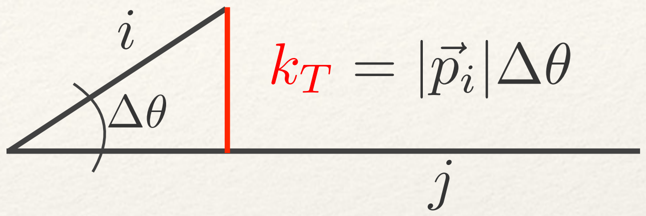
4. If d_{min} is a d_i, then protojet i is ``not mergable". Remove it from the list of protojets and add it to the list of finished jets.

5. If protojets remain, go to step 1.

kT Jet Algorithm

Why the name?

$$d_{ij} = \min\left\{p_{T,i}^2, p_{T,j}^2\right\}\left[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2\right]/R^2 \approx k_\perp^2/R^2$$



Infrared safety of this:

- Suppose $p_j \to 0$
  - Then when it merges with other protojet,

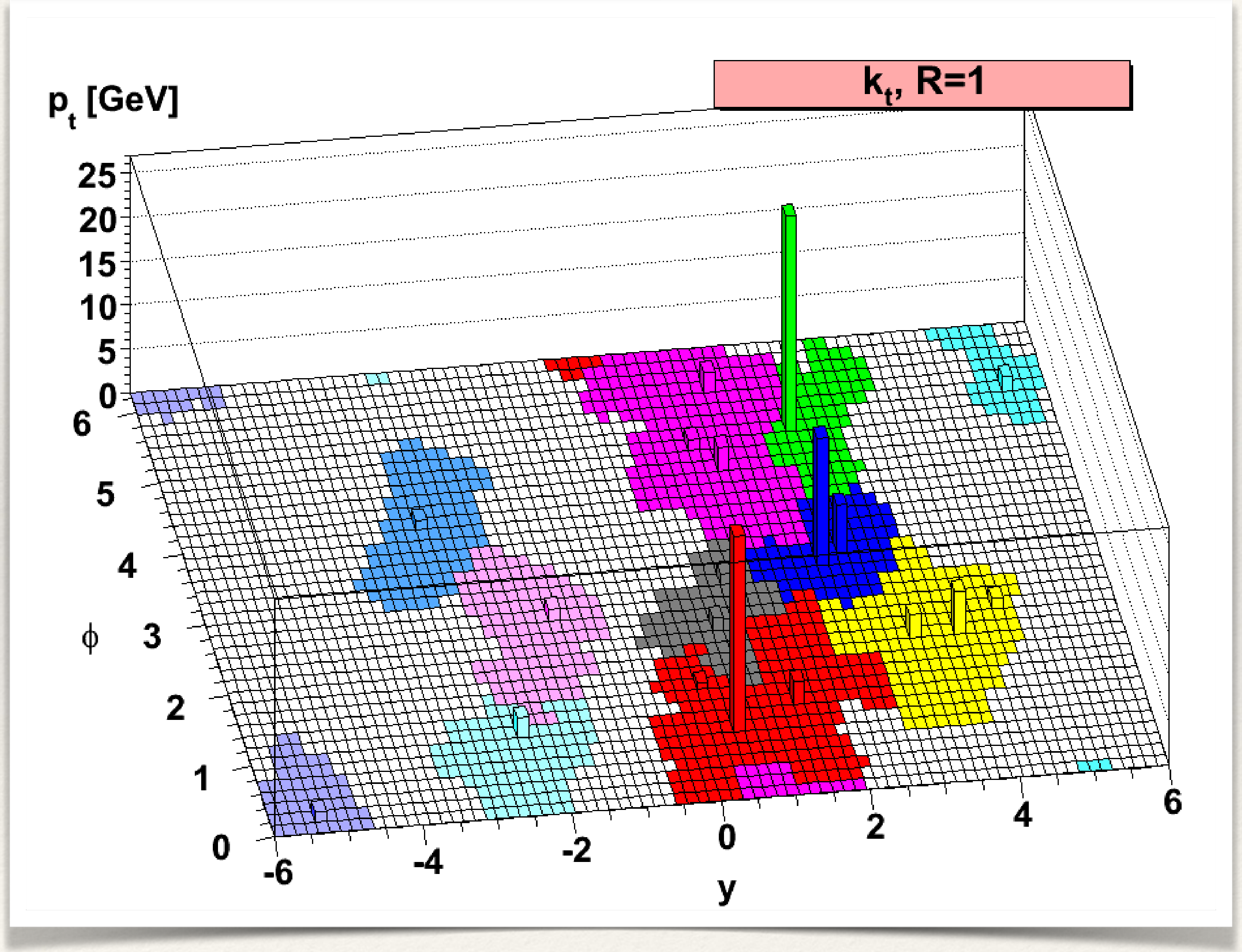
$$p_k = p_i + p_j \to p_i$$

  - If it never mergers with other protojets , then it just remains as a low p_T jets a the end.
- Suppose $p_i = \lambda p_j$
  - Then protojets i and j are always merged at the beginning to

$$p_k = p_i + p_j$$

Example with kT Algorithm

Here is an event from Cacciari, Salam and Soyes (2008). An event was generated by HERWIG++ along with (lots of) random soft particles.



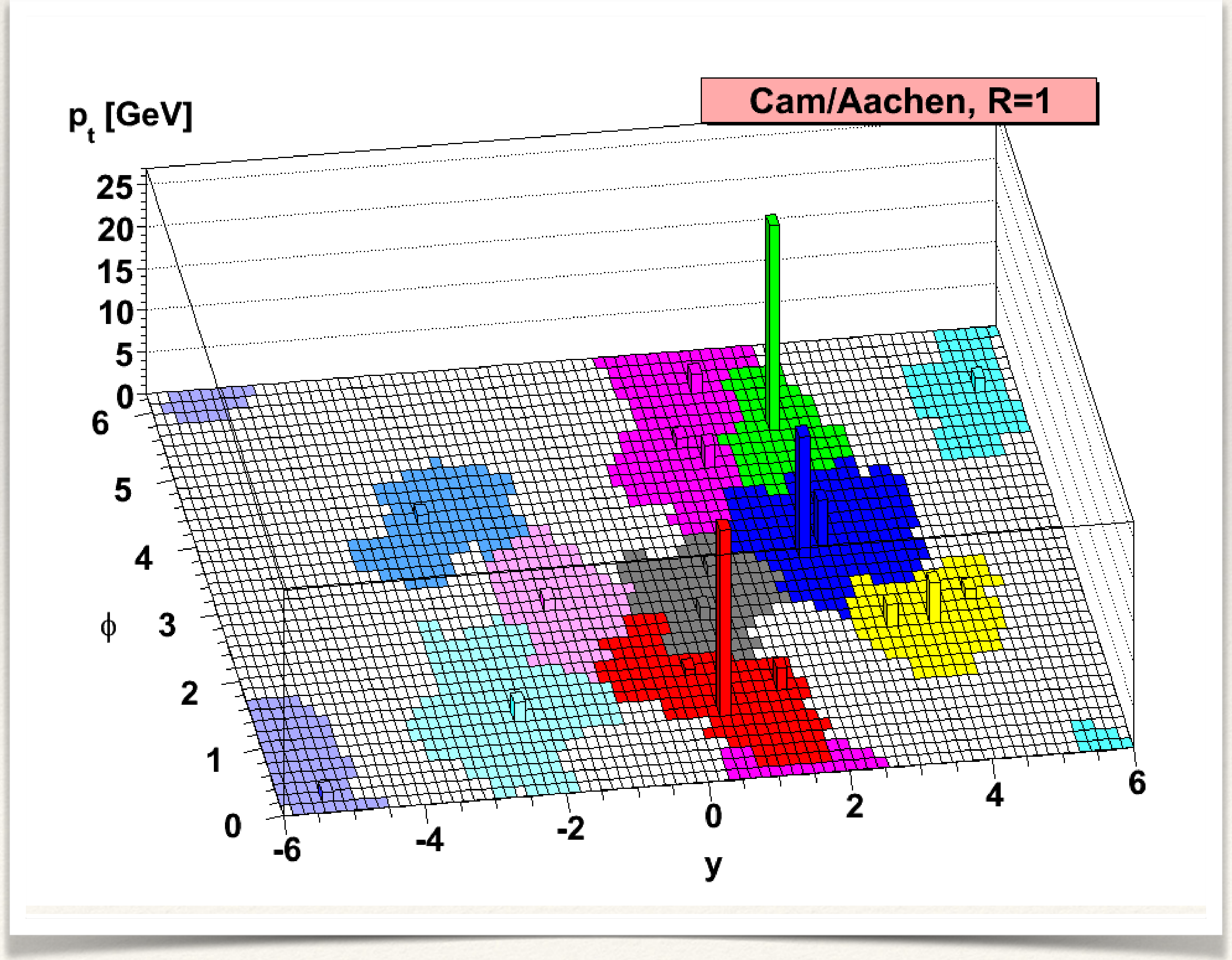
- The detector area that goes into each jet is irregular.
- The kT algorithm has the tendency to "*suck*" in low pT radiation and contaminate the jets with underlaying event.

Cambridge-Aachen Algorithm

This is a variation on the general successive combination algorithm. The only difference is in the "distance" measure.

Only the angles count!

$$d_{ij} = \left[(y_i - y_j)^2 + (\phi_i - \phi_j)^2\right]/R^2$$
$$d_i = 1$$



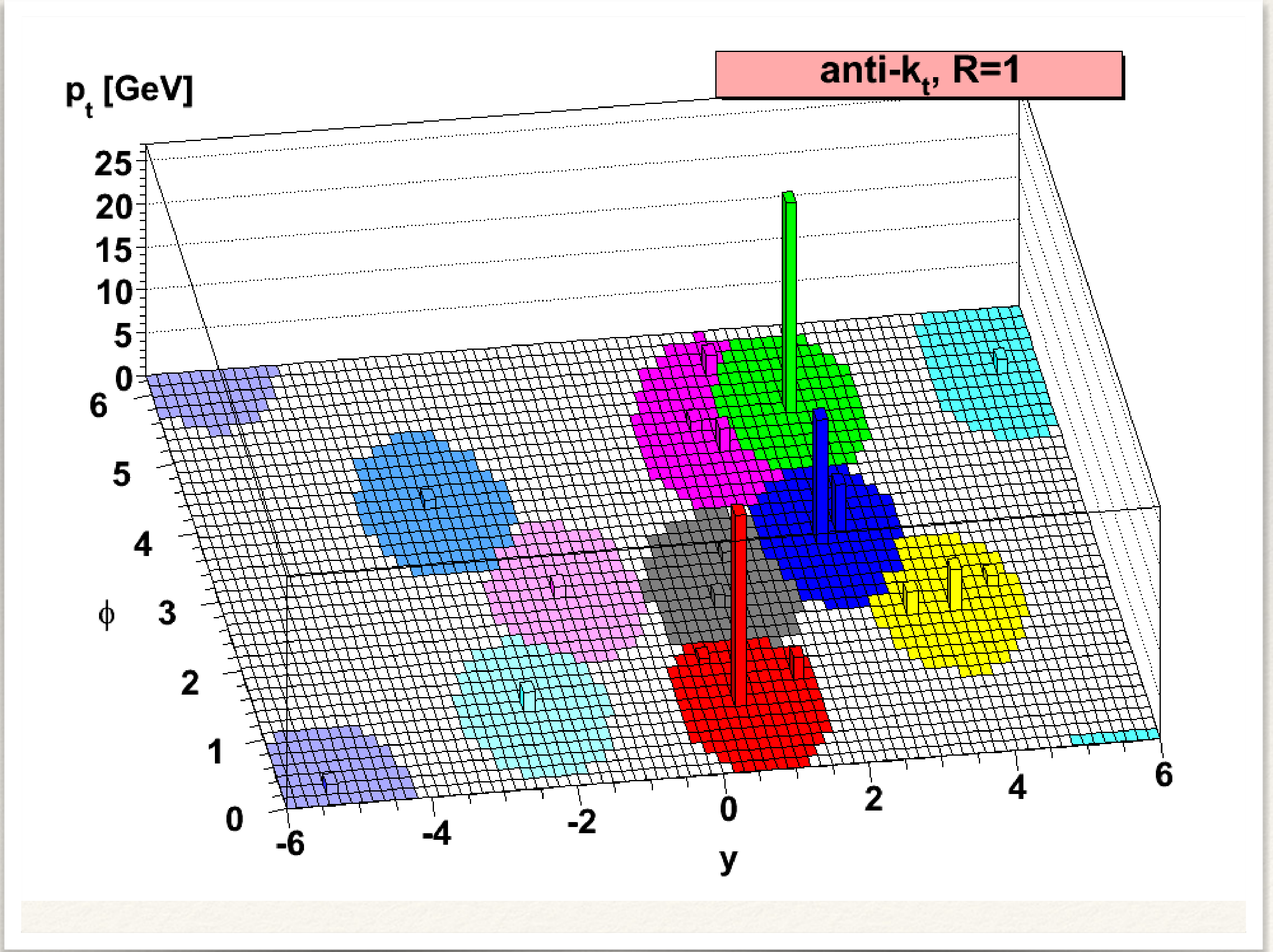
With this algorithm the jets still have irregular shape.

Anti-kT Algorithm

This is another variation on the general successive combination algorithm. The only difference is in the “distance” measure.

$$d_{ij} = \min\left\{\frac{1}{p_{T,i}^2}, \frac{1}{p_{T,j}^2}\right\}\left[(y_i - y_j)^2 + (\phi_i - \phi_j)^2\right]/R^2$$

$$d_i = \frac{1}{p_{T,i}^2}$$

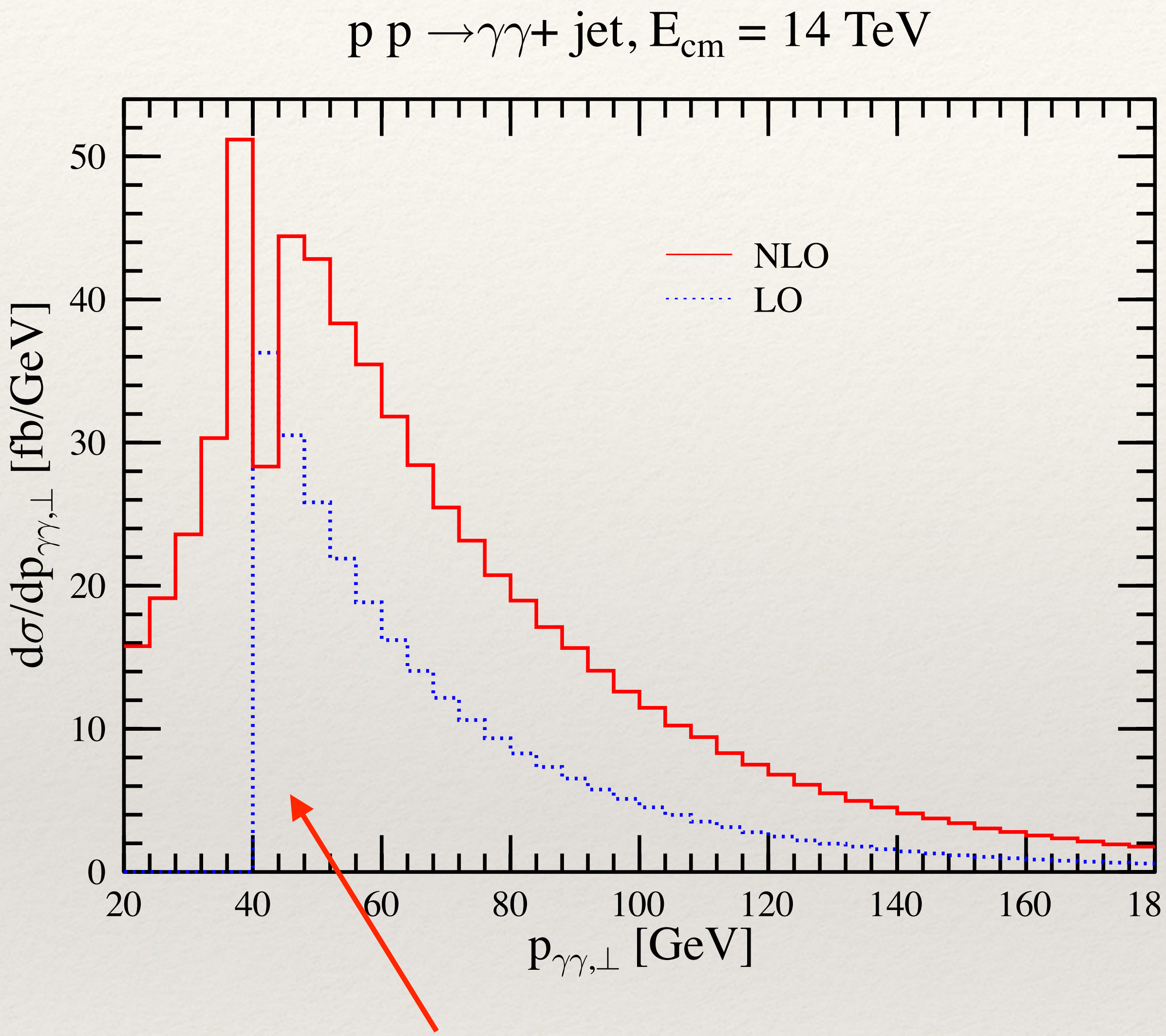


The highest pT protojet has the priority to absorb nearby softer protojets.

The high pT jets are round.

When Fixed Order Breaks Down

Let us consider 2photon + 1jet inclusive production and plot the di-photon pT distribution



$p_{T,\mathrm{jet}} > 40\,\mathrm{GeV}$

- For this distribution the characteristic scale is

$$\mu_J^2 = (p_{\gamma\gamma,\perp} - 40\,\mathrm{GeV})^2$$

- The NLO distribution has discontinuity at 40GeV. It is -∞ from the right and +∞ from the left.
- The singularities are logarithms (it appears finite because of the bin smearing effect).
- The effective expansion variable is

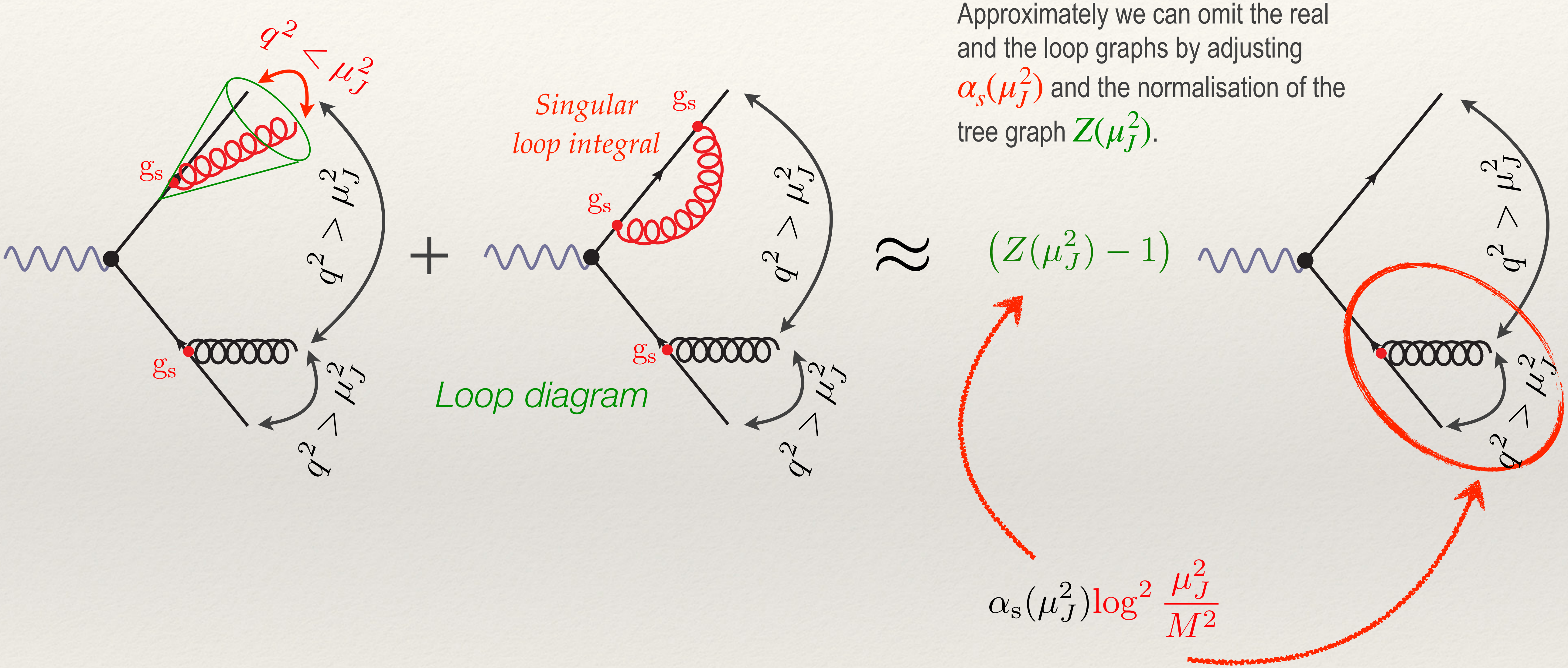
$$\alpha_s(Q^2)\,\log^2\frac{Q^2}{(40\,\mathrm{GeV})^2}$$

- This effect has to be summed up all order. NLO calculation is not enough.

$$\alpha_s(\mu_J^2)\,\log^2\frac{\mu_J^2}{(40\,\mathrm{GeV})^2}$$

Jet in pQCD

We have to also consider the virtual corrections, thus we have graphs like…

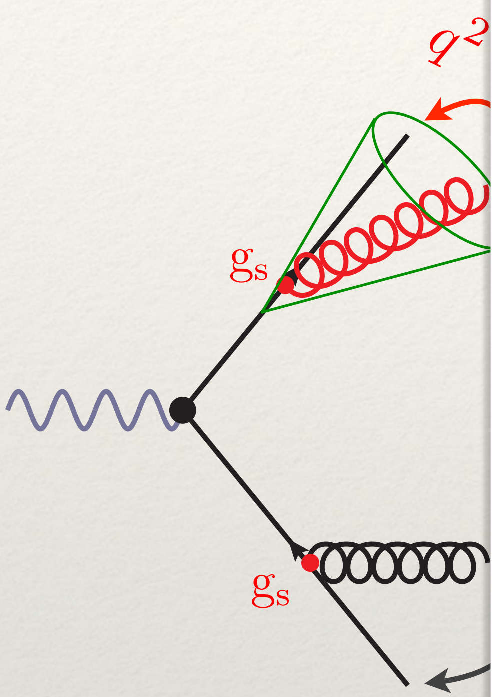


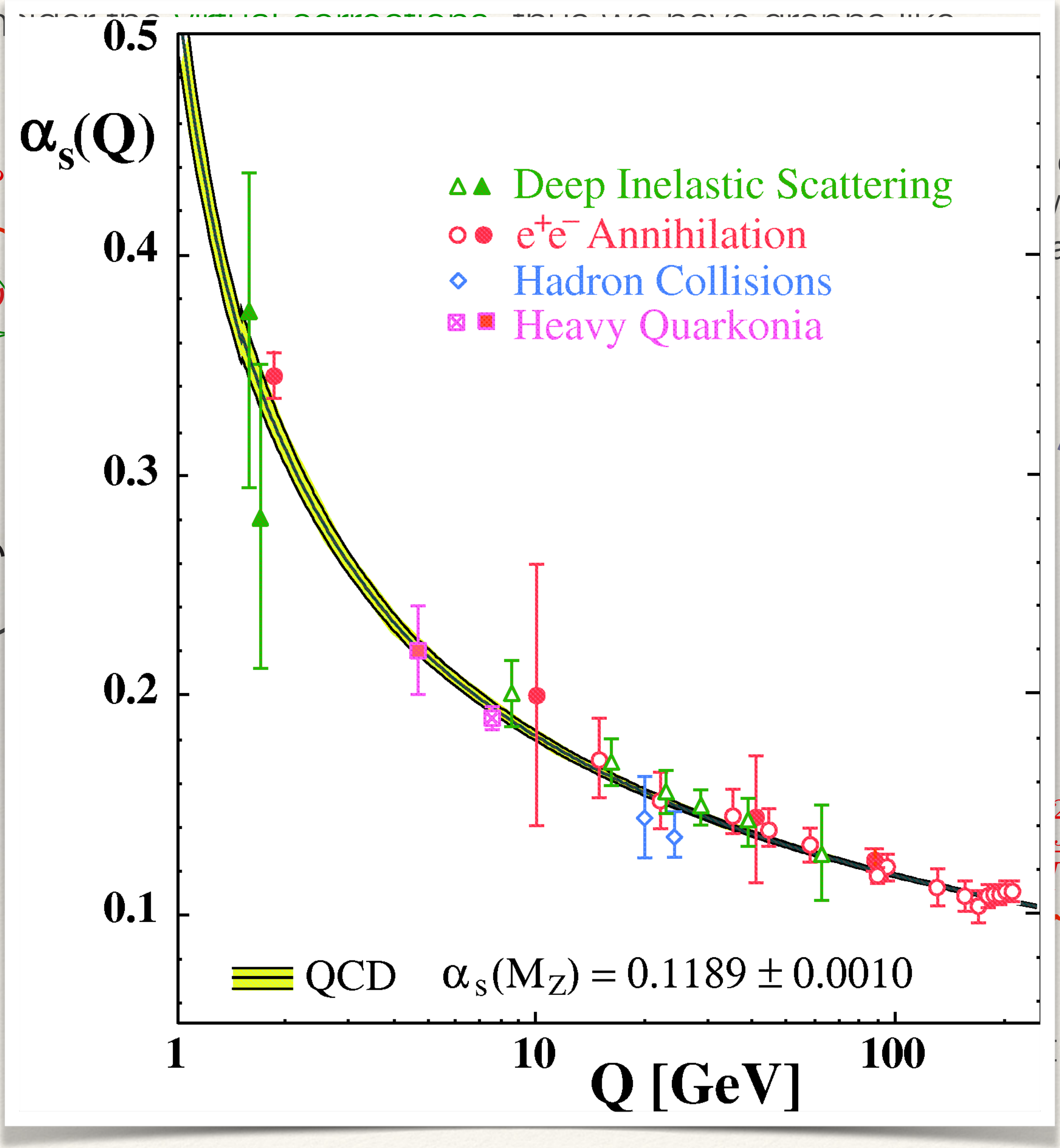
Approximately we can omit the real and the loop graphs by adjusting $\alpha_s(\mu_J^2)$ and the normalisation of the tree graph $Z(\mu_J^2)$.

When μ_J^2 gets small the coupling and the logarithm blow up.

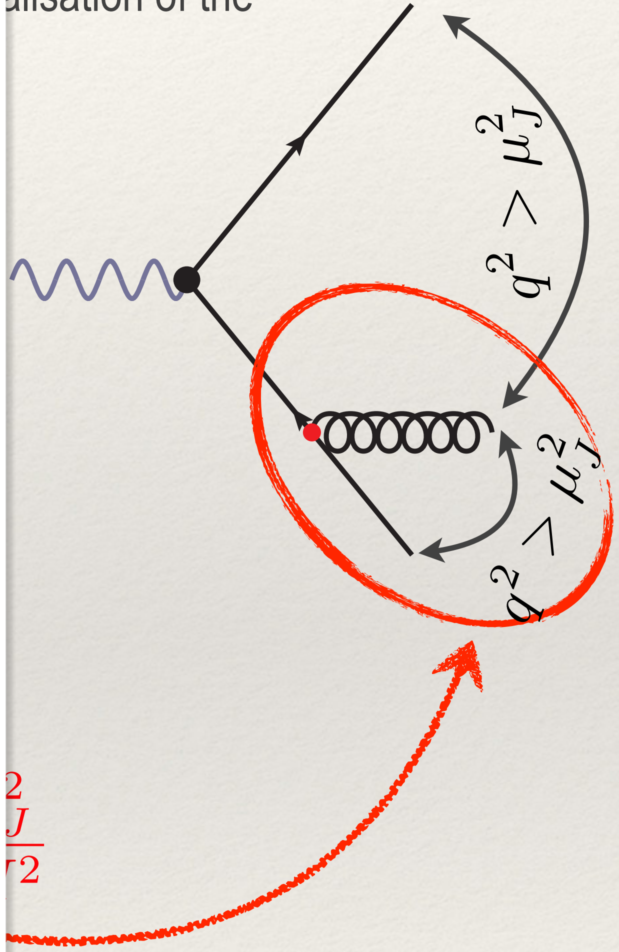
Jet in pQCD

We have to also con





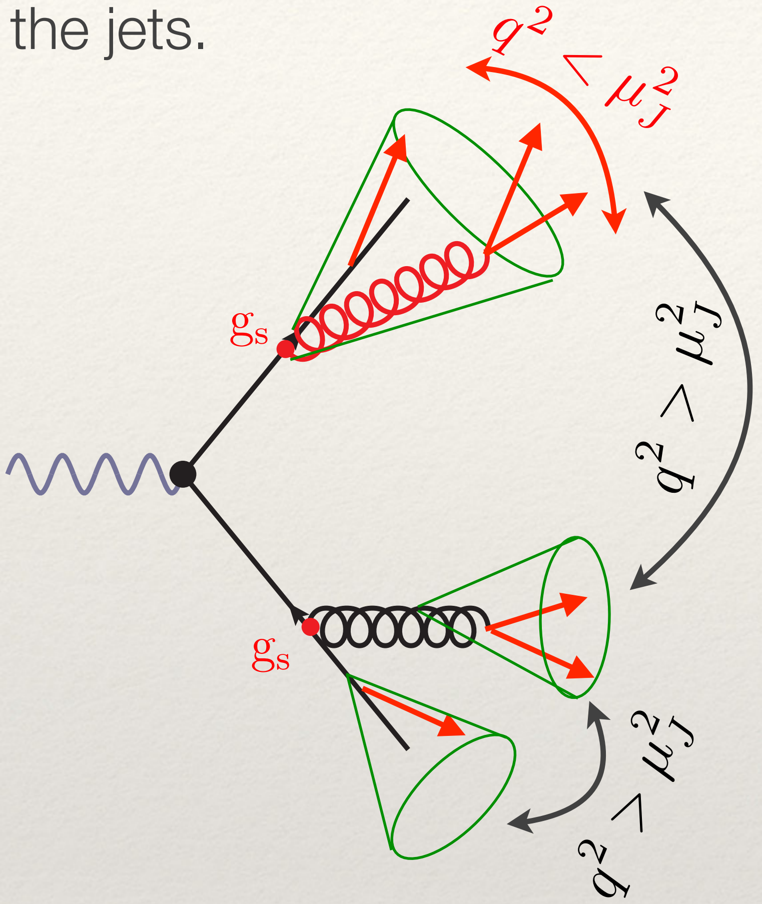
omit the real
adjusting
alisation of the



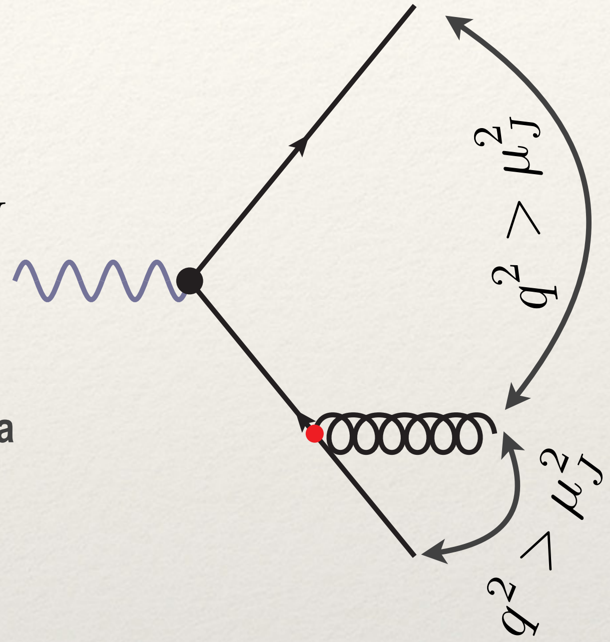
he logarithm blow up.

Jets in pQCD

At higher order level the picture is more complicated and we have many unresolvable radiation in the jets.



$$\sim \left[\alpha_\mathrm{s}(\mu_J^2)\log^2\frac{\mu_J^2}{M^2}\right]^N$$



This large logarithms are always a problem in the fixed ordered calculations.

Let us try to understand the origin of these large logarithms!

Usual cross section formulae:

$$\sigma[O_J] = \sum_m \frac{1}{m!}\int d\{p,f\}_m O_J(\{p\}_m, \mu_J^2)|M(\{p,f\}_m)|^2$$

Same thing but with abstract linear algebra:

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J(\mu_J^2)\big|\rho(\mu^2)\big)$$

Fixed Order Calculations (NLO, NNLO,…)

You might have seen this NLO formulae a hundred times in the last week:

$$\sigma[O_J] = \int_m d\sigma_m^B[O_J] + \int_{m+1} \underbrace{\left[d\sigma^R_{m+1}[O_J] - d\sigma^B[O_J] \otimes dV(\mu^2)\right]} + \int_m \underbrace{\left[d\sigma^V_m[O_J] + d\sigma^B[O_J] \otimes \int_1 dV(\mu^2)\right]} + \cdots$$

Subtracted real contribution
Every radiation under the $q^2 < \mu^2$ is considered as unresolvable and subtracted.

Virtual and integrated unresolvable real contributions

Writing this in a more general and more abstract way

$$\sigma[O_J] = \underbrace{\left(1\middle|\mathcal{O}_J(\mu_J^2)\right)\left(1 + \frac{\alpha_s(\mu^2)}{2\pi}\mathcal{D}^{(1)}(\mu^2) + \cdots\right)} \left(\big|\rho^{(0)}(\mu^2)\big) + \frac{\alpha_s(\mu^2)}{2\pi}\underbrace{\left[\big|\rho^{(1)}(\mu^2)\big) - \mathcal{D}^{(1)}(\mu^2)\big|\rho^{(0)}(\mu^2)\big)\right]} + \cdots\right)$$

It is finite because the **observable is IR safe** and the singularities cancel between the real and virtual subtractions.

It is finite since the real and virtual singularities are **canceled** by the corresponding counterterms.

Fixed Order Calculations (NLO, NNLO,…)

The subtraction schemes are based on the factorisation properties of the QCD amplitudes and at all order level (formally) we have:

Universal singular operator based on
the factorisation of the QCD amplitudes.

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J(\mu_J^2)\,\overbrace{\mathcal{D}(\mu^2)}\,\underbrace{\mathcal{D}^{-1}(\mu^2)\big|\rho(\mu^2)\big)} + \mathcal{O}(\alpha_\mathrm{s}^{k+1}L^{2k+2})$$

The **inverse of the $\mathscr{D}(\mu^2)$ operator**
provides the subtractions for the QCD
matrix elements.

- The $\mathscr{D}(\mu^2)$ operator acts on a partonic state and can create emissions **those are not resolvable** above the μ^2 scale. This is good approximation as long as the $\mathscr{D}$ operator doesn't create resolvable emissions. Otherwise we have some large logarithm problem.
- To maintain the accuracy we should choose the renormalisation scale to be small, much smaller than the characteristic scale of the measurement operator,

$$\mu^2 \ll \mu_J^2$$

- With this choice, we have

$$\mathcal{O}_J(\mu_J^2)\mathcal{D}(\mu^2) \approx \mathcal{D}(\mu^2)\mathcal{O}_J(\mu_J^2)$$

**Cross section doesn't depend on the
soft and collinear radiation.**

Fixed Order Calculations (NLO, NNLO,…)

How about the hard part of the cross section?

Universal singular operator based on the factorisation of the QCD amplitudes.

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J(\mu_J^2)\,\overbrace{\mathcal{D}(\mu^2)}\,\underbrace{\mathcal{D}^{-1}(\mu^2)\big|\rho(\mu^2)\big)} + \mathcal{O}(\alpha_{\mathrm{s}}^{k+1}L^{2k+2})$$

The hard part of the cross section

- The hard part of the cross section is process dependent and calculated from exact tree and loop matrix elements. These calculations are very complicated and we want to keep this part perturbative. Usually calculated at NLO or may be NNLO level.
- To do this we have to keep the renormalisation scale to be big something like the typical scale of the hard process,

$$\mu^2 \approx Q^2$$

- This is in conflict with the *soft* part of the cross section, which prefers small renormalisation scale.
- To solve this conflict we have to sum up the large logarithms at all order level. This is the job of the parton showers and analytical summation.

$$\sigma[O_J] = \big(1\big|\mathcal{O}_J(\mu_J^2)\mathcal{D}(\mu_{\mathrm{f}}^2)\underbrace{\mathcal{U}(\mu_{\mathrm{f}}^2,\mu_{\mathrm{H}}^2)}\,\mathcal{D}^{-1}(\mu_{\mathrm{H}}^2)\big|\rho(\mu_{\mathrm{H}}^2)\big)$$

Represents the evolution between the hard and the soft scale. Parton shower or analytic summation.

$$\mu_{\mathrm{f}}^2 \approx 1\mathrm{GeV}^2$$

$$\mu_{\mathrm{H}}^2 \approx Q^2$$

Conclusions

- QCD gives us jets.
- Jets are real and seen in experiments.
- To measure jet cross sections, you need a careful definition of jets.
- At the LHC we use successive combination algorithms, such as kT, Cambridge-Aache or anti-kT algorithm.
- The definition needs to be infrared safe.
- Infrared safety allow us to make pQCD prediction.
  - Fixed order calculations, LO, NLO or NNLO
- Jet cross sections (in general pQCD cross sections) usually suffers on large logarithms and these logarithms need to be summed up all order.
  - Summing up logarithms analytically
  - Summing up logarithm numerically by parton shower algorithms.

$$\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\big(1\big|\mathcal{O}_J(\mu_J^2)\mathcal{D}^{(1)}(\mu^2)\big|\{p,f,\ldots\}_m\big) \sim \frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\log^2\frac{\mu^2}{\mu_J^2}$$