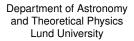


# **Soft Modelling and Heavy Ions (2)**





CTEQ-MCnet School Dresden 2021-09-14



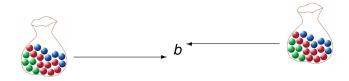
#### **Outline**

- ► The Glauber model(s)
- Nuclear effect in the initial state
- Collective effect in the final state
- Heavy Ions in PYTHIA8



#### The Glauber formalism

- ► How do we model the geometrical distribution of nucleons in colliding nuclei?
- How do we determine which nucleon interacts with which nucleon?
- How do they interact?





### Distributing nucleons in a nuclei

There are advanced models for the shell-structure of nuclei — we will not be that advanced.

Assume a simple density of nucleons based on the (spherically symmetric) Woods–Saxon potential

$$\rho(r) = \frac{\rho_0(1 + wr^2/R^2)}{1 + \exp((r - R)/a)}$$

- R is the radius of the nucleus
- a is the skin width
- w can give a varying density but is typically = 0



For a nucleus (Z, A), we simply generate A nucleon positions randomly according to

$$P(\vec{r}_i) = \rho(r_i)d^3\vec{r}_i$$

The Woods–Saxon parameters are tuned to measurements of (low enegry) charge distributions assuming some charge distribution of each nucleon (proton).

We normally assume iso-spin invariance ( $p \approx n$ ).

There are absolutely no correlations between the nucleons.

What happens if two nucleons end up in the same place.

Leif Lönnblad

We can get some correlations if we assume that nucleons have a hard core,  $R_h$ , and require  $\Delta r_{ij} > 2R_h$ 

If you generate a nucleon which is too close to a previously generated nucleon you could either

- generate a new position for the last one (efficient, but may give a bias)
- throw away everything and start over (inefficient, unbiased)



There are many implementations of this, and most experiments have their own. Typical parameters for A > 16 are (from the GLISSANDO program):

$$R$$
 (fm)  $a$   $w$   $R_h$  (1.120 $A^{1/3} - 0.860A^{-1/3}$ ) 0.540 0 0 (1.100 $A^{1/3} - 0.656A^{-1/3}$ ) 0.459 0 0.45



We can estimate the AA section assuming the nuclei are like black disks,

$$\sigma^{AA} = \int_{-\infty}^{\infty} d^2 \vec{b} \frac{d\sigma^{AA}(b)}{d^2 \vec{b}} = 4\pi R^2$$

where

$$\frac{d\sigma^{AA}(b)}{d^2\vec{b}} = \begin{cases} 1 : & b < 2R \\ 0 : & b > 2R \end{cases}$$



We can also look at the positions of the individual nucleons:

$$\frac{d\sigma^{AA}(b)}{d^2\vec{b}} = 1 - \prod_{i,j} \int d^2\vec{r}_i d^2\vec{r}_j \left( 1 - \frac{d\sigma^{NN}(b_{ij})}{d^2\vec{b}} \right) \rho(\vec{r}_i) \rho(\vec{r}_j)$$

where 
$$b_{ij} = \left| \vec{b} + \vec{r}_i - \vec{r}_j \right|$$
.

But we have to think about which cross section we are talking about. Total? Non-difractive? Inelastic?

#### Interactions between nucleons

Let's assume that a projectile with some kind of internal structure interacts with a structureless target. The projectile can have different mass-eigenstates,  $\Psi_i$ , and these can be different from the eigenstates of the (diffractive) interaction,  $\Phi_{\kappa}$ .

$$\Psi_i = \sum_k c_{ik} \Phi_k$$
 with  $\Psi_0 = \Psi_{in}$ .

With an elastic amplitude  $T_k$  for each interaction eigenstate we get the elastic cross section for the incoming state

$$\frac{d\sigma_{\rm el}(b)}{d^2\vec{b}} = |\langle \Psi_0 | T | \Psi_0 \rangle|^2 = \left(\sum_k |c_{0k}|^2 T_k\right)^2 = \langle T \rangle^2.$$

For a completely black target and projectile, we know from the optical theorem that the elastic cross section is the same as the absorptive cross section and

$$\sigma_{\rm el} = \sigma_{\rm abs} = \sigma_{\rm tot}/2$$

but with substructure and fluctuations we have also diffractive scattering with the amplitude

$$\langle \Psi_i \, | \, T | \, \Psi_0 
angle = \sum_k c_{ik} \, T_k c_{0k}^*$$

and

$$\frac{d\sigma_{\rm diff}(b)}{d^2\vec{b}} = \sum_i \langle \Psi_0 | T | \Psi_i \rangle \langle \Psi_i | T | \Psi_0 \rangle = \langle T^2 \rangle.$$



#### The importance of fluctuations

We see now that diffractive excitation to higher mass eigenstates is given by the fluctuations

$$\frac{d\sigma_{\rm dex}(b)}{d^2\vec{b}} = \frac{d\sigma_{\rm diff}(b)}{d^2\vec{b}} - \frac{d\sigma_{\rm el}(b)}{d^2\vec{b}} = \langle T^2(b) \rangle - \langle T(b) \rangle^2$$

When looking at AA interactions we may assume that the state of each nucleon is frozen during the interaction according to the eikonal approximation.

We also assume the elastic nucleon scattering amplitude is purely imaginary and  $T(b) \equiv -iA(b)$  giving  $0 \le T \le 1$  from unitarity.

We can now also write down the total and absorptive (aka. non-diffractive) cross section, and we can look at the situation where both the projectile and target nucleon has a sub-structure:

$$\frac{d\sigma_{\text{tot}}^{\text{NN}}(b)}{d^{2}\vec{b}} = 2\langle T(b) \rangle 
\frac{d\sigma_{\text{abs}}^{\text{NN}}(b)}{d^{2}\vec{b}} = 2\langle T(b) \rangle - \langle T^{2}(b) \rangle 
\frac{d\sigma_{\text{el}}^{\text{NN}}(b)}{d^{2}\vec{b}} = \langle T(b) \rangle^{2} 
\frac{d\sigma_{\text{dex}}^{\text{NN}}(b)}{d^{2}\vec{b}} = \langle T^{2}(b) \rangle - \langle T(b) \rangle^{2}$$



We can also divide the diffractive excitation depending on whether the target or projective nucleon is excited.

$$\begin{array}{lcl} \frac{d\sigma_{\mathrm{Dp}}^{\mathrm{NN}}(b)}{d^{2}\vec{b}} & = & \langle\langle T(b)\rangle_{t}^{2}\rangle_{p} - \langle\langle T(b)\rangle_{t}\rangle_{p}^{2} \\ \frac{d\sigma_{\mathrm{Dt}}^{\mathrm{NN}}(b)}{d^{2}\vec{b}} & = & \langle\langle T(b)\rangle_{t}^{2}\rangle_{p} - \langle\langle T(b)\rangle_{p}\rangle_{t}^{2} \\ \frac{d\sigma_{\mathrm{DD}}^{\mathrm{NN}}(b)}{d^{2}\vec{b}} & = & \langle\langle T(b)^{2}\rangle_{t}\rangle_{p} - \langle\langle T(b)\rangle_{p}^{2}\rangle_{t} - \langle\langle T(b)\rangle_{t}^{2}\rangle_{p} + \langle\langle T(b)\rangle_{t}\rangle_{p}^{2} \end{array}$$



We note in particular that the probability of a target nucleon being wounded is given by

$$\frac{d\sigma_{\text{Wt}}^{\text{NN}}(b)}{d^{2}\vec{b}} = \frac{d\sigma_{\text{abs}}^{\text{NN}}(b)}{d^{2}\vec{b}} + \frac{d\sigma_{\text{DD}}^{\text{NN}}(b)}{d^{2}\vec{b}} + \frac{d\sigma_{\text{Dt}}^{\text{NN}}(b)}{d^{2}\vec{b}} 
= \frac{d\sigma_{\text{tot}}^{\text{NN}}(b)}{d^{2}\vec{b}} - \frac{d\sigma_{\text{el}}^{\text{NN}}(b)}{d^{2}\vec{b}} - \frac{d\sigma_{\text{Dp}}^{\text{NN}}(b)}{d^{2}\vec{b}} 
= 2\langle T(b)\rangle_{tp} - \langle T(b)\rangle_{tp}^{2} \rangle_{p}$$

and thus only depends on the fluctuations in the projectile, but only on average properties of the target itself.

Introducing the *S*-matrix, S(b) = 1 - T(b) we see that the individual absorbtive and wounded cross sections factorises for pA

$$\frac{d\sigma_{abs}^{pA}(b)}{d^{2}\vec{b}} = 1 - \prod_{j} \left( 1 - \frac{d\sigma_{abs}^{NN}(b_{j})}{d^{2}\vec{b}} \right) = 1 - \prod_{j} \langle S^{2}(b_{j}) \rangle_{tp}$$

$$\frac{d\sigma_{Wt}^{pA}(b)}{d^{2}\vec{b}} = 1 - \prod_{j} \left( 1 - \frac{d\sigma_{Wt}^{NN}(b_{j})}{d^{2}\vec{b}} \right) = 1 - \prod_{j} \langle \langle S(b_{j}) \rangle_{t}^{2} \rangle_{p}$$



#### The standard (naive) Glauber implementation

Estimate the distribution in number of participants in a pA or AA collision.

- Distribute the nucleons randomly according to Woods–Saxon
- Monte-Carlo the *b*-distributions (typically in a square with side  $\sim 4R$ ).
- ▶ Count the number of nucleons in the target that is within a distance  $d = \sqrt{\sigma/2\pi}$  from any of the projectile nucleons. (Gives you  $N_{\rm coll}$  and  $N_{\rm part}$ .)

Normally no fluctuations, but includes diffractively wounded nucleons by using  $\sigma = \sigma_{\rm abs}^{\rm NN} + \sigma_{\rm dex}^{\rm NN} = \sigma_{\rm tot}^{\rm NN} - \sigma_{\rm el}^{\rm NN}$ .

#### A more sofisticated Glauber implementation

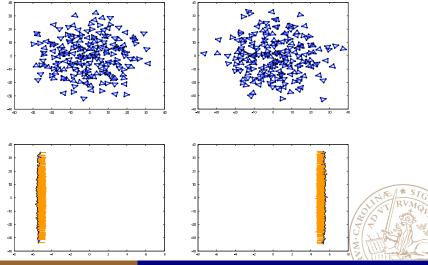
Assume a fluctuating NN cross section

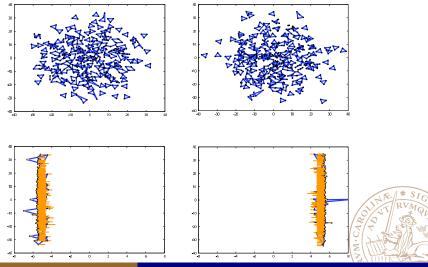
$$P(\sigma) = \rho \frac{\sigma}{\sigma + \sigma_0} \exp\left\{-\frac{(\sigma/\sigma_0 - 1)^2}{\Omega^2}\right\}$$

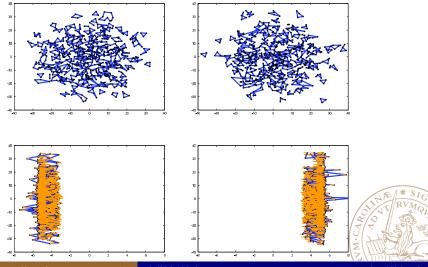
with

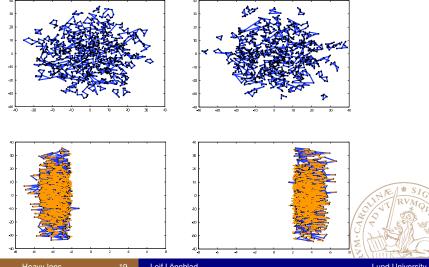
$$T(b,\sigma)\propto \exp\left(-cb^2/\sigma\right).$$

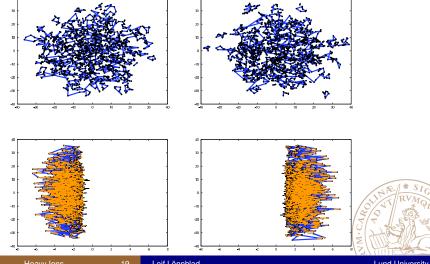
For pA this gives a longer tail out to a large number of wounded nucleons.

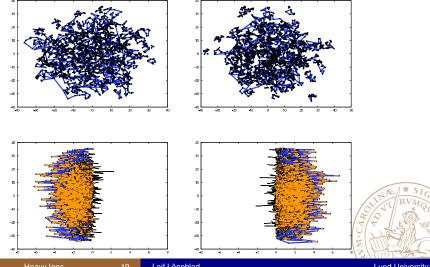


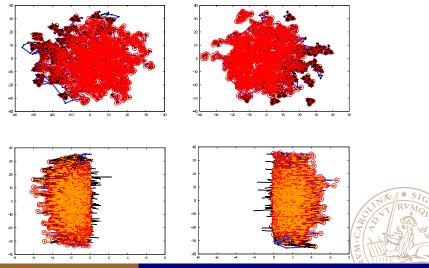


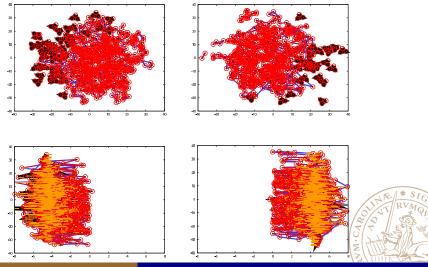


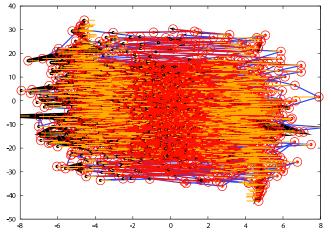




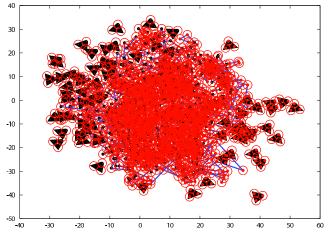














#### **Color Glass Condesate (CGC)**

A mean-field statistical approach to the density of gluons.

- Color: It's American, and yes, it's QCD
- Glass: Solid on short timescales, amorphous on long.
- Condensate: There are a lot of gluons.

Includes Saturation of gluons.

In standard DGLAP the gluon density increases rapidly with decreasing x. Also in BFKL. Somewhere it has to stop,  $g+g \rightarrow g=$  Saturation

$$Q_{\rm sat} = Q_0^2 \left(\frac{x}{x_0}\right)^{\lambda}$$

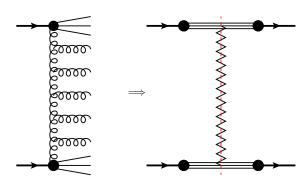


CGC starts with an initial gluon density at some  $x \sim 0.01$ , and evolves it to smaller x using a (non-linear) renormalization group equation (JIMWLK  $\sim$  BFKL + Saturation)

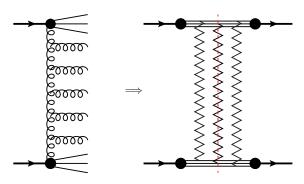
The initial density is folded with the nucleon distribution in b.

IP-Glasma model is similar but uses DGLAP + Saturation.

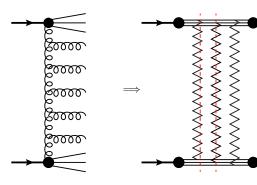




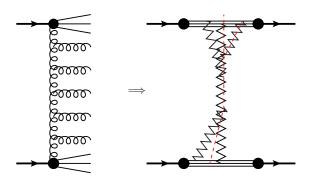








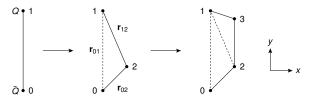




Each cut pomeron will give rise to a string (or two) spanned between two colliding nucleons, or between a nucleon another Pomeron.

#### The DIPSY model

More or less same ingredients as the CGC, but generating each gluon explicitly using the (mueller) dipole model.



- Mueller's formulation of BFKL
- Dipoles in impact parameter space, evolved in rapidity
- Builds up virtual Fock-states of the proton

#### The interaction

#### Dipole-dipole interaction:

$$ightharpoonup F = \sum_{ij} f_{ij} \qquad f_{(12)(34)} \propto \alpha_s^2 \ln^2 \left( \frac{r_{13}r_{24}}{r_{14}r_{23}} \right)$$

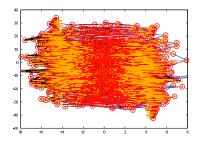
▶ Unitarize to get saturation effects  $T = 1 - e^{-F}$ 

#### Saturation in the evolution with the Swing model

- Colour reconnection
- Two dipoles with the same colour may reconnect.
- ▶ Does not reduce the number of dipoles, but smaller dipoles are favoured, and these have weaker interactions.
- Also reconnections between different nucleons in a nuclei

Models all kinds of fluctuations and correlations.

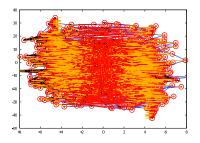
### **The Final State**



Model each gluon/dipole individually?



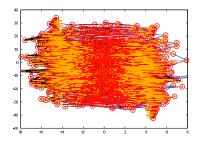
### **The Final State**



Model each gluon/dipole individually?
Or give up and use statistical methods?



### **The Final State**



Model each gluon/dipole individually? Or give up and use statistical methods? Or both?



#### **Quark-Gluon Plasma**

Construct the the energy momentum density and the flavour flow vector for all point in space at an initial proper time  $\tau = \tau_0$ :

$$T^{\mu\nu}(x) = \sum_{i} \frac{\delta p_{i}^{\mu} \delta p_{i}^{\nu}}{\delta p_{i}^{0}} g(x - x_{i})$$

$$N_q^{\mu}(x) = \sum_i \frac{\delta p_i^{\mu}}{\delta p_i^0} q_i g(x - x_i)$$

- $ightharpoonup q_i = u, d, s$
- $\triangleright$   $\delta p$  is the momentum of the parton (or string segment)
- ightharpoonup g(x) is a smoothing kernel with some assumed width

# Relativistic hydrodynamics

The individual flavour flow is a conserved current

$$\partial_{\nu}N_{q}^{\nu}=0$$

So is the energy-momentum tensor

$$\partial_{\nu} T^{\mu\nu} = 0$$

Typically divide up in small cells, get the velocity vector  $u^{\nu}$  in the restframe of each cell (comoving frame) and evolve.

but we have four only equations for  $T^{\mu\nu}$  so we need to have extra assumptions.

#### Ideal fluid

#### In the comoving frame:

- ▶  $T^{00} = \varepsilon$ : energy density
- $ightharpoonup T^{0i} = 0$ : no energy flow
- $ightharpoonup T^{i0} = 0$ : no no momentum
- $T^{ij} = \delta_{ij}p$ : isotropic pressure

But it s also possible to include viscous effects...



# Freezeout = Hadronisation and Rescattering

After the evolution we convert  $T^{\mu\nu}$  and  $N_q^{\mu}$  back into particles (Hadrons). This happens at some given hypersurface.

There is still a fairly high density of hadrons, and we expect some rescattering:

$$h_1 + h_2 \to h'$$
 or  $h_1 + h_2 \to h'_1 + h'_2$ 



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Sorry, I don't understand this enough myself

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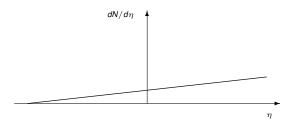
$$h_1 + h_2 \to h'$$
 or  $h_1 + h_2 \to h'_1 + h'_2$ 

c.f. the model in PYTHIA



A simple model by Białas and Czyż, implemented in Fritiof

Each wounded nucleon contributes with hadrons according to a function  $F(\eta)$ . Fitted to data, and approximately looks like



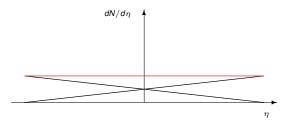
$$\frac{dN}{d\eta} = F(\eta)$$

(single wounded nucleon)

[Nucl.Phys.B111(1976)461, J.Phys.G35(2008)044053, Nucl.Phys.B281(1987)289.]

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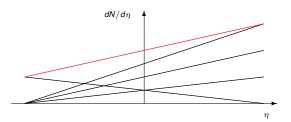
$$\frac{dN}{d\eta} = F(\eta) + F(-\eta)$$
 (pp)



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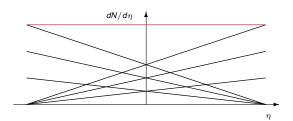
$$\frac{dN}{d\eta} = w_t F(\eta) + F(-\eta)$$
 (pA)

, [Nucl.Phys.B111(1976)461, J.Phys.G35(2008)044053, Nucl.Phys.B281(1987)289.]



A simple model by Białas and Czyż, implemented in Fritiof

Each wounded nucleon contributes with hadrons according to a function  $F(\eta)$ . Fitted to data, and approximately looks like



$$\frac{dN}{d\eta} = w_t F(\eta) + w_p F(-\eta) \tag{AA}$$

[Nucl.Phys.B111(1976)461, J.Phys.G35(2008)044053, Nucl.Phys.B281(1987)289.]



In Fritiof this was modelled by stretching out a string from each wounded nucleon with an invariant mass distributed as  $dm_X/m_X$ , which reproduces  $F(\eta) \propto \eta - \eta_0$ .

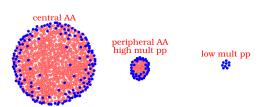
Note that there are no collective effects here. But nevertheless Fritiof reproduced most data: No conclusive evidence for QGP until the late nineties.



## Core - Corona

The EPOS generator uses a *Core–Corona* model:

- Start with the Pomeron picture.
- Create strings
- Divide up:
  - Core: If the density of strings is high, chop them up and use relativistic hydrodynamics.
  - Corona: For lower densities, allow for hard interactions and perturbative ISR/FSR/MPI evolution





#### HIJING

(one of the standard HI generators)

- Inspired by Fritiof
- Hard scatterings with nuclear PDFs + Shadowing
- Soft radiation with ARIADNE
- String fragmentation



#### **AMPT**

(Another standard HI generator with collective effects)

- Same initial state as HIJING
- String melting
  - String fragmetation
  - Convert back to qq̄
  - Evolve in time with elastic scattering
  - Nearest neighbour recombination into hadrons
- Hadron rescattering



# Heavy Ions in PYTHIA8



- Glauber model with advanced fluctuation treatment
- Divides NN interactions into absorptive, single or double diffractive.
- Also differentiates absorptive interactions:
  - ► Primary: is modelled as a PYTHIA non-diffractive pp event.
  - Secondary: an interaction with a nucleon that has already had an interaction with another. Modelled as a (modified) diffractive excitation event (with dm<sub>X</sub>/m<sub>X</sub> as in Fritiof).
- All sub-events generated on parton level and merged together into a consisten pA or AA event and then hadronised.

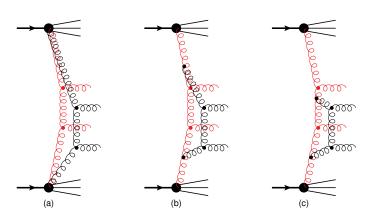
(No string interactions vet.

# Heavy Ions in PYTHIA8

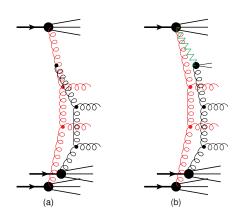


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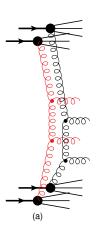
  (No string interactions yet.)

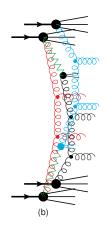


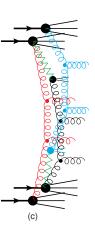














The Initial State
The Final State
Heavy Ions in PYTHIA8

Angantyr Comparison to data String Interactions

projectile	targe



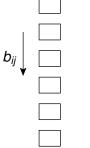
projectile	collisions	target



#### projectile



#### collisions

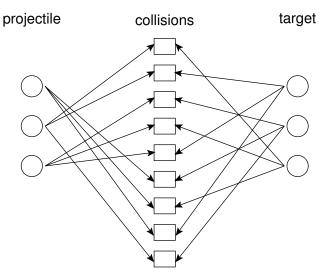


#### target



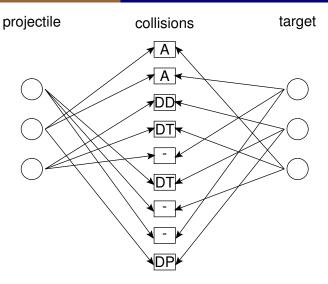




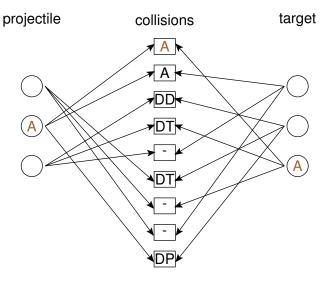




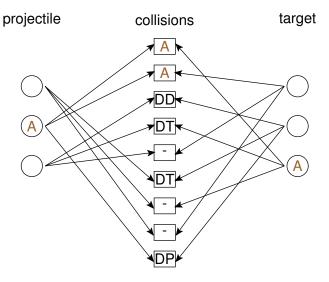
#### Angantyr Comparison to data String Interactions



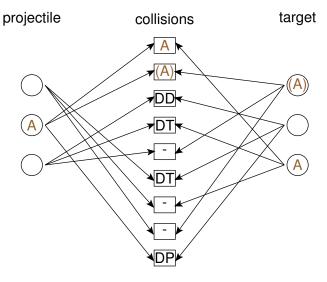




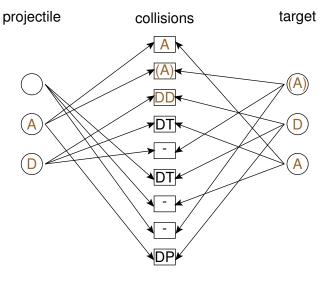




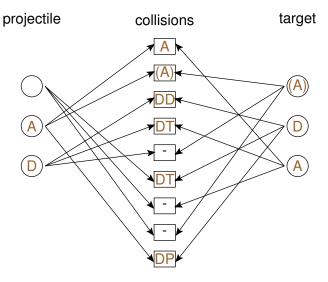








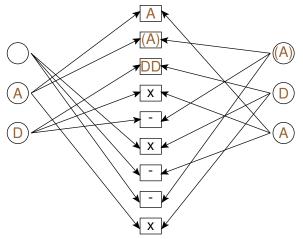






#### Angantyr Comparison to dat String Interactions

# projectile sub-events target





# Signal processes

Not only min-bias. Rather than just generating non-diffractive events, The first absorptive sub-event can be generated using any hard process in PYTHIA8, giving the final event a weight  $N_A\sigma_{hard}/\sigma_{ND}$ .

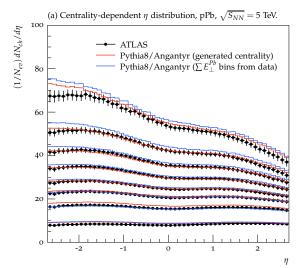


# Comparison to data

Several parameters in addition to the pp PYTHIA8 ones.

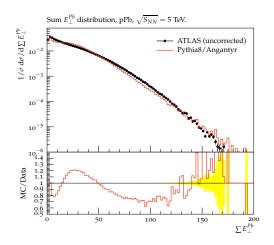
- Nucleon distributions can in principle be measured independently.
- NN cross section fluctuations are fitted to (semi-) inclusive pp cross sections (total, non-diffractive, single and double diffractive, elastic, and elastic slope) for given  $\sqrt{s_{NN}}$ .
- Diffractive parameters for secondary absorptive collisions, "tuned" to non-diffractive PYTHIA.
- ►  $M_X$  distribution:  $dM_X^2/M_X^{2(1+\epsilon)}$ , could be tuned (to pA), but we choose  $\epsilon = 0$ .
- Few other choices concerning energy momentum conservation which do not have large impact.

# **Eta distribution in pPb**



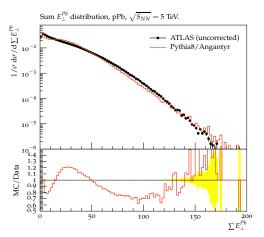


# Centrality in pPb





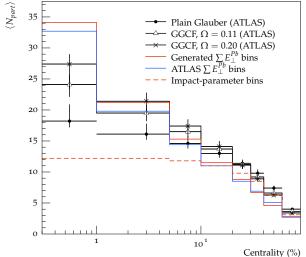
## Centrality in pPb



What was actually measured in the previous slide is a correlation between the  $\eta$ -distribution and the forward activity.

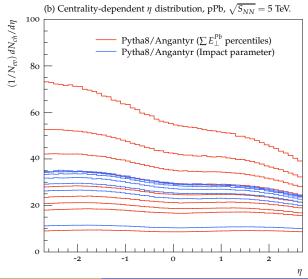
# p-Pb number of participants

Number of wounded nucleons



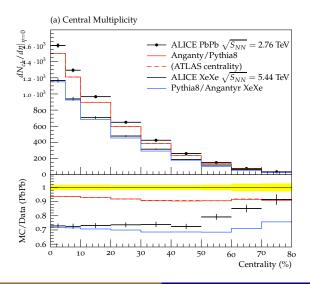


### p–Pb $\eta$ -distribution

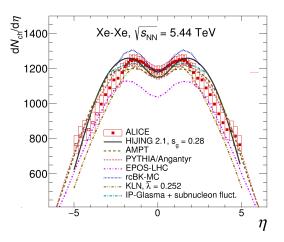




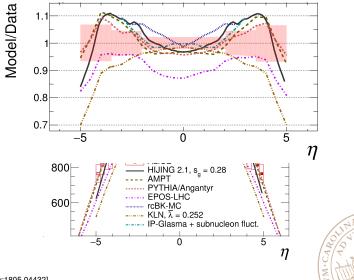
#### Central multiplicity in PbPb



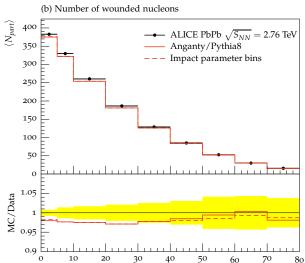








# Pb-Pb number of participants





Centrality (%)

#### Go generate yourself!

```
pythia.readString("Beams:idA = 1000822080");
pythia.readString("Beams:idB = 1000822080");
pythia.readString("Beams:eCM = 2760.0");
```



So far there are no collective effects in Angantyr

(but we are working on it).

- Colour reconnections between individual sub-collisions.
- Overlapping strings may repel each other.
- Overlapping strings may increase the string tension
- Final-state hadrons may collide



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   Swing
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- Colour reconnections between individual sub-collisions.
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- ► Overlapping strings may repel each other. shoving → flow
- ► Overlapping strings may increase the string tension Rope hadronization → strangeness enhancement
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- Colour reconnections between individual sub-collisions.
   Swing
- ► Overlapping strings may repel each other. shoving → flow
- ► Overlapping strings may increase the string tension Rope hadronization → strangeness enhancement
- Final-state hadrons may collide Rescattering (already in PYTHIA)



#### **Summary**

- Heavy-Ion collisions are messy
- Not just overlayed NN collisions
- Initial state effects (saturation, fluctuations, ...)
- Final state effects (QGP, hydrodynamics, string interactions, flow, jet-quenching, rescattering, ...)



#### **Final Comments**

#### By tradition HI and HEP have been separate communities

- LHC brought them together
- There are collective effects in pp
- There are jets in AA
- We can (and need to) learn from each other

