Heavy quarks and new scales: Understanding subtleties of QCD

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Outline

1. **Heavy quarks**
   - Charm, beauty and truth
   - What is a heavy quark?
2. **Using heavy quarks to understand QCD**
   - Going beyond DIS and Drell Yan
   - Interpreting the initial state
   - Matrix elements
   - Interpreting the final state
3. **Summary**

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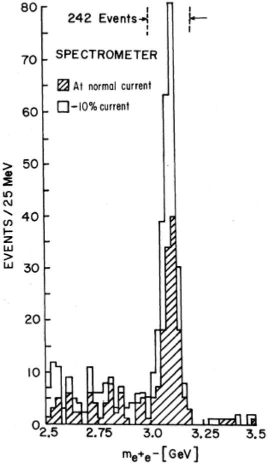
A charming discovery



The first heavy quark, charm was discovered in 1974 in $p\bar{p}$ collisions at BNL and e^+e^- at SLAC

The observations were published together:
PRL 33, 1404 (1974); PRL 33, 1406 (1974)

The J/ψ was recognized as a $c\bar{c}$ bound state

$\Rightarrow m_c \sim 1.5$ GeV

The existence of a 4th quark confirmed the Glashow-Iliopoulos-Maiani explanation for why FCNC decays ($s \to d\nu\bar{\nu}$) did not occur.

— And it loosened the shackles of $\mathrm{SU(3)}_{\mathrm{flavor}}$, Gell-Mann's "Eightfold way"

A charming crisis

While the J/ψ was clearly a quark bound state,
it had an extremely narrow width of 88 keV.

This caused a minor crisis in the fledgling QCD...

After all how could a strongly interacting state be narrow?
$\Gamma_\rho \sim 150$ MeV, $\Gamma_\omega \sim 8.5$ MeV, $\Gamma_\phi \sim 4.3$ MeV, $\Gamma_{J/\psi} \sim 88$ keV

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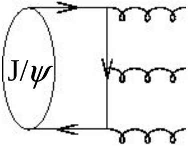
An explanation was found by Appelquist and Politzer, PRL 34, 43 (75).

Write the width as
$\Gamma(^3S_1 \to 3\ \mathrm{gluons}) = |R(0)|^2 |M(q\bar{q} \to ggg)|^2$

Following the model of positronium, solve the Schroedinger Eqn. for $R(r) = \frac{2}{a_0^{3/2}} e^{-r/a_0}$,
where $a_0 = \frac{1}{\alpha_s m_c/2}$.

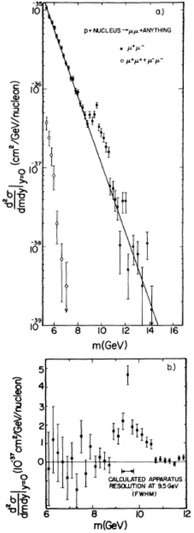


$|M(q\bar{q} \to ggg)|^2 \sim \alpha_s^3$ — one power for each gluon

$\Rightarrow \Gamma(^3S_1 \to 3\ \mathrm{gluons}) \sim 0.2\ \alpha_s^6\ m_c \sim 90\ \mathrm{keV}; \alpha_s \approx 0.26$

Consider: Why do we not see $J/\psi \to gg$?

A beautiful discovery

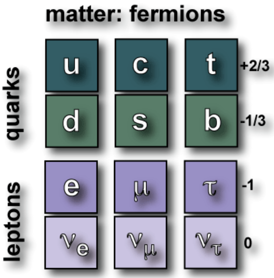


In 1975 the τ was discovered and led to the search for other 3rd-generation particles.

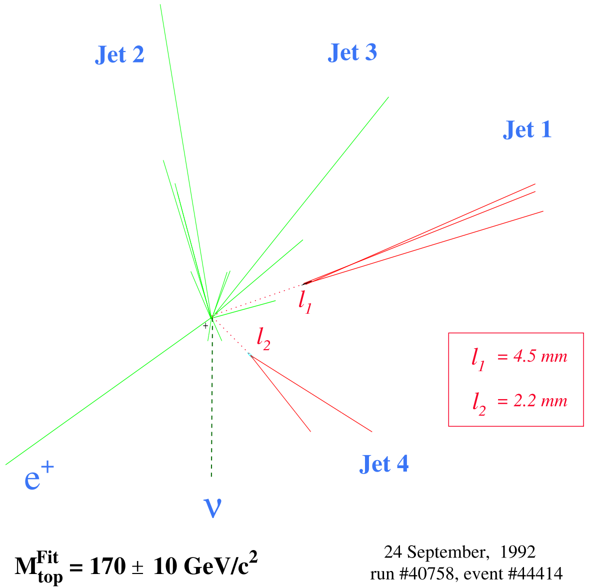
In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron. PRL 39, 252 (1977)

(The Upsilon is also very narrow.)

Once the bottom quark was found it was clear that a sixth quark was needed to complete the family structure.



"This is the top quark!"



What is a "heavy quark?"

Usual definition: A heavy quark is a quark with $m_q \gg \Lambda_{\mathrm{QCD}}$.

	Pole mass M	$\overline{\mathrm{MS}}$ mass $\overline{m}(\overline{m})$
Charm	1.67 ± 0.02 GeV	1.27 ± 0.02 GeV
Bottom	4.78 ± 0.06 GeV	4.18 ± 0.03 GeV
Top	172.76 ± 0.3 GeV (?)	$162.5^{+2.1}_{-1.5}$ GeV

PDG (6/1/21)

Top: TEVEWWG: $174.30 \pm 0.35 \pm 0.54$ GeV, ATLAS: $172.64 \pm 0.25 \pm 0.55$ GeV
CMS: $172.26 \pm 0.07 \pm 0.61$ GeV

Pole Mass: $\sim \frac{1}{\not{p}-M}$

$\overline{\mathrm{MS}}$ Mass: Related to pole mass by

$$\frac{M}{\overline{m}(\overline{m})} = 1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left(-1.0414 \ln(M^2/\overline{m}^2) + 13.4434\right) + \ldots$$

It seems kind of funny to list 2 different masses...

c and b masses are best written in $\overline{\mathrm{MS}}$ scheme.

t mass is given in pole-mass scheme.

What is a heavy quark mass?

Answer 1: A parameter of the Lagrangian $\mathcal{L} \sim m_t \bar{t} t$

A weak answer, but if the number is big enough we can expand in inverse powers of the mass to create a convergent series. (E.g., HQET)

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Answer 2: An effective (Yukawa) coupling between t–t–h

$m_t = Y_t/(2\sqrt{2}G_F)^{1/2}$, $Y_t \approx 1.00$ in the SM

This is better, as the Standard Model predicts that quark masses are not fundamental, but rather an artifact of dynamical interactions.

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Answer 3: The kinematic mass seen by the experiments

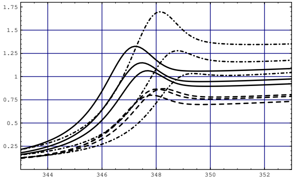
Right after the discovery of the top quark, Martin Smith and Scott Willenbrock asked this question about the "pole mass" of the top quark. They showed that a renormalon (the closest pole of the Borrel transform) induced an ambiguity of $\mathcal{O}(\Lambda_{QCD})$ in the definition of the pole mass.

Top mass from $t\bar{t}$ threshold at a linear collider

There is a subtle question when you try to make a precision measurement of QCD: What mass do you use?

The pole mass is not defined beyond Λ_{QCD}.

In fact it is not well-defined at all, since there are no free quarks.



Yakovlev,Groote PRD63, 074012(01)

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Solution: Use the 1S mass (pseudo bound state)

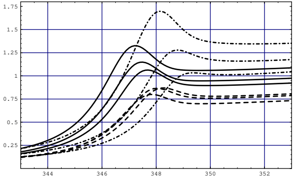
There are large non-relativistic corrections

$$\sigma_{t\bar{t}} \propto v \sum \left(\frac{\alpha_s}{v}\right) \times \left\{ \begin{array}{c} 1 \\ \sum(\alpha_s \ln v) \end{array} \right\} \times \left\{ \begin{array}{l} \mathrm{LO}(1) + \mathrm{NLO}(\alpha_s, v) + \mathrm{NNLO}(\alpha_s^2, \alpha_s v, v^2) \\ \mathrm{LL} + \mathrm{NLL} + \mathrm{NNLL} \end{array} \right\}$$

Normalization changes, but peak stable.

$\delta\sigma_{t\bar{t}}$ is $\pm 6\%$ before ISR/beamstrahlung

$\delta m_t \sim 100$ MeV is attainable



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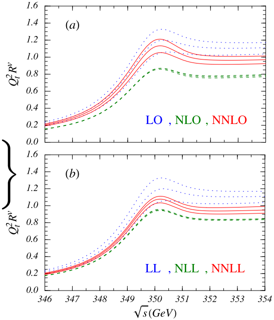

Hoang, et al. PRD69, 034009(04)

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This led to the recommended use of the $\overline{\mathrm{MS}}$ mass for top quarks.

We theorists are good at setting standards that make our life easier ... most perturbative calculations use the $\overline{\mathrm{MS}}$ mass for simplicity.

Of course mass is NOT measured directly. Instead, it affects the distribution of events that are measured, and that distribution is used to INFER the mass by matching to a calculation. . . .

What is a heavy quark mass?

Answer 4: A new scale in the problem.

This will both complicate our calculations and lead to new insights into the meaning of QCD structures that are hidden when we ignore quark masses.

The key is context.

Depending on the other scales in the problem, a heavy quark mass may teach us something deep about the physics, or be completely irrelevant.

E.g., most mass corrections go like $\mathcal{O}(m^2/\mu^2)$

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Consider: Show in the top quark width $\Gamma(t \to bW)$, dropping m_b loses terms of $\mathcal{O}(m_b^2/m_t^2) \sim 1\%$.

In the rest of this lecture we will concentrate on what we learn from corrections that go like $\mathcal{O}[\ln(m_t^2/m_b^2)]$.

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Structure of an observable cross section

cf. lectures by Soper and Nagy

$$\sigma_{\rm obs.} = \int f_1(x_1,\mu_1) f_2(x_2,\mu_2) \otimes \overline{|M|^2} \otimes d\mathrm{P.S.} \otimes D_i(p_i)\ldots D_n(p_n)$$

Theorists factorize (break) the cross section into:

- Initial-state IR singularities swept into parton distribution "functions".
  These are not physical, but include scheme dependent finite terms:
  $\overline{\rm MS}$ — the current standard
  DIS — ambiguous in modern PDF sets, could be fixed, but why?

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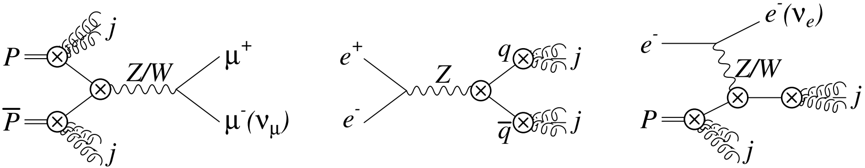
  $\Rightarrow$ Exclusive cross sections (jet counting), angular correlations
- Fragmentation functions or jet definitions.
  These provide the coarse graining to hide final-state IR singularities.

Drell-Yan and DIS

cf. lectures by Bertone and Schienbein

The traditional testbed of perturbative QCD have been restricted to Drell-Yan production, e^+e^- to jets, or deeply inelastic scattering (DIS).

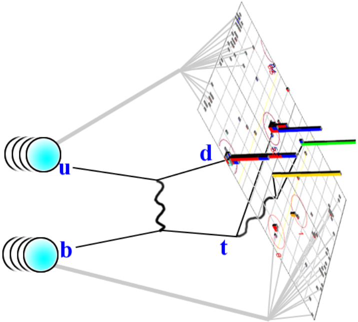


A key property that all three processes share is a complete factorization of QCD radiation between different parts of the diagrams.

- Drell-Yan $\rightarrow$ Initial-state (IS) QCD radiation only.
- $e^+e^- \rightarrow$jets $\rightarrow$ Final-state (FS) QCD radiation only.
- DIS $\rightarrow$ Proton structure and fragmentation functions probed. Simple color flow.

A heavy quark testbed for QCD: single top

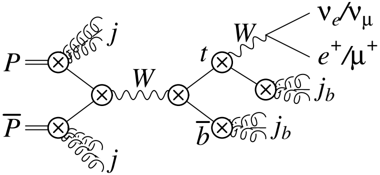
Experimentalist: Single top quark production is the observation of $b\,\ell^{\pm}\,\not{E}_T$ that reconstruct to a top quark mass, plus an extra jet (or two).



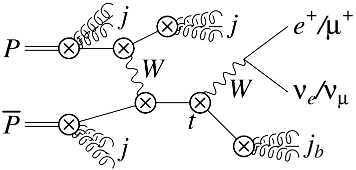
Theorist: Single top quark production is a playground in which we refine our understanding of perturbative QCD in the presence of heavy quarks.

s-/t-channel single-top-quark production (A generalized Drell-Yan and DIS)

A perfect factorization through next-to-leading order (NLO) makes single-top-quark production mathematically *identical*[†] to DY and DIS!



Generalized Drell-Yan.
IS/FS radiation are independent.



Double-DIS (DDIS) w/ 2 scales:
$\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

† Massive forms: m_t, m_b, and m_t/m_b are relevant.

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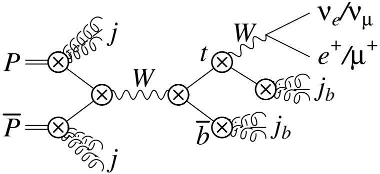

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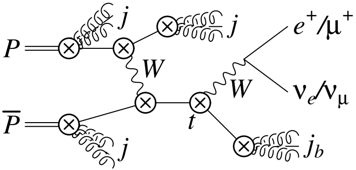


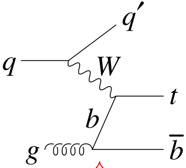
Double-DIS (DDIS) w/ 2 scales:
$\mu_l = Q^2$, $\mu_h = Q^2 + m_t^2$

Color conservation forbids the exchange of just 1 gluon between the independent fermion lines.

† Massive forms: m_t, m_b, and m_t/m_b are relevant.

Rethinking the initial state:
W-gluon fusion $\rightarrow$ t-channel single-top

W-gluon fusion (circa 1996)



$\Uparrow$

$$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$$

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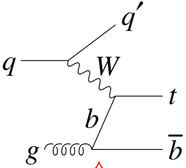

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$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$

Look at the internal b.

The propagator is

$$\frac{1}{(P_g - P_{\bar{b}})^2 - m_b^2} = \frac{1}{-2P_g \cdot P_{\bar{b}}}$$

$$P_g = E_g(1,0,0,1),\ P_{\bar{b}} = (E_b, \vec{p}_T, p_z)$$

$$P_g \cdot P_{\bar{b}} = E_g\left(p_z\sqrt{1 + \frac{p_T^2+m_b^2}{p_z^2}} - p_z\right)$$

$$\approx E_g p_z\left(\frac{p_T^2+m_b^2}{2p_z^2}\right) \sim (p_T^2 + m_b^2)$$

$$\int_{p_{T\,\mathrm{cut}}} \frac{\mathrm{d}p_T^2}{p_T^2+m_b^2} \to \ln\left(\frac{1}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$$

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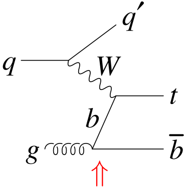

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The same procedure for the W leads to the massive formula for DIS.

$$\sigma \sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$$

We now have multiple scales entering the problem: $Q, m_t, m_b, p_{T\,\mathrm{cut}}$

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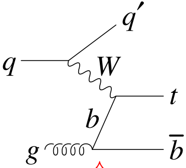

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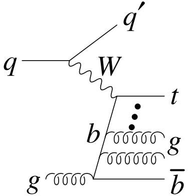
$$\sim \alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right) + \mathcal{O}(\alpha_s)$$

$m_t \approx 35 m_b$! $\alpha_s \ln \sim .7\text{-}.8$



Each order adds

$$\frac{1}{n!}\left[\alpha_s \ln\left(\frac{Q^2+m_t^2}{m_b^2}\right)\right]^n$$

Looks bad for perturbative expansion. . .

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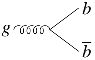
Resummation of large logs and b PDF

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation sums large logs in (almost) collinear singularities in gluon splitting.

$$\frac{db(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{bg} \otimes g + \cancel{\frac{\alpha_s}{2\pi} P_{bb} \otimes b}; \qquad b \ll g$$



$$P_{bg}(z) = \tfrac{1}{2}[z^2 + (1-z)^2]$$

$$b(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}(z) g\left(\frac{x}{z},\mu^2\right)$$

Barnett, Haber, Soper, NPB 306, 697 (88)
Olness, Tung, NPB 308, 813 (88)
Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94)

The procedure is the same for c or t.

Resummation of large logs and b PDF

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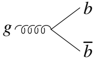
$b \propto \alpha_s \ln(\mu^2/m_b^2) \times g$

$$\frac{db(\mu^2)}{d\ln(\mu^2)} \approx \frac{\alpha_s}{2\pi} P_{bg} \otimes g + \cancel{\frac{\alpha_s}{2\pi} P_{bb} \otimes b}; \qquad b \ll g$$

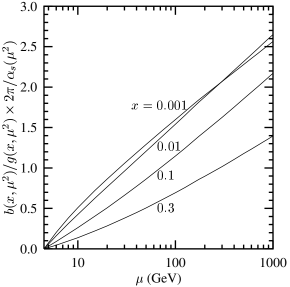


$$P_{bg}(z) = \tfrac{1}{2}[z^2 + (1-z)^2]$$

$$b(x,\mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{\mu^2}{m_b^2}\right) \int_x^1 \frac{dz}{z} P_{bg}(z) g\left(\frac{x}{z},\mu^2\right)$$

Barnett, Haber, Soper, NPB 306, 697 (88)
Olness, Tung, NPB 308, 813 (88)
Aivazis, Collins, Olness, Tung, PRD 50, 3102 (94)

The procedure is the same for c or t.



Stelzer, ZS, Willenbrock, PRD 56, 5919 (1997)

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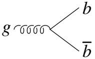

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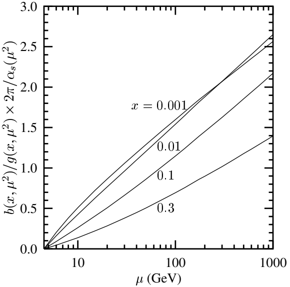

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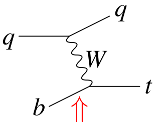
The procedure is the same for c or t.

Aside: In the $\overline{\mathrm{MS}}$ scheme, $b(\mu \leq m_b) \equiv 0$.
DIS scheme is not uniquely defined for heavy quarks.
Do you choose $F_2 \equiv 0$ (traditional) or define w.r.t. $\overline{\mathrm{MS}}$?
The first attempt to calculate single-top failed because the DIS scheme was used. Bordes, van Eijk, NPB435, 23 (95)

Remove 1 scale (m_b) w/improved perturbation theory

New Leading Order



$$b \sim \alpha_s \ln\left(\frac{\mu^2}{m_b^2}\right) \times g$$

The t-channel W exchange
naturally lead to
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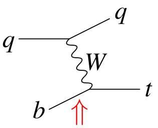

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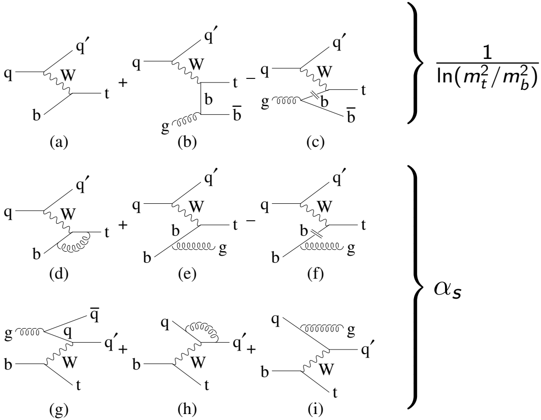
The t-channel W exchange naturally lead to the nomenclature of t-channel production



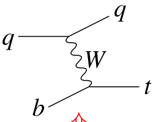
NLO: Terms that generated large logs are already resummed.

⇒ Must subtract overlap to avoid double-counting (general issue)

⇒ Reorders PT into 2 types of corrections: α_s and $\frac{1}{\ln(m_t^2/m_b^2)}$ w.r.t. LO

New nomenclature and classification

New Leading Order



$(P_W^2 < 0)$

$\Uparrow$

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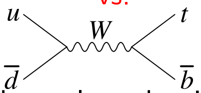
t-channel production
Named for the "*t*-channel" exchange of a W boson.

vs.



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New nomenclature and classification

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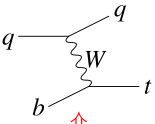

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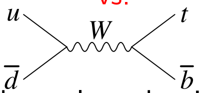

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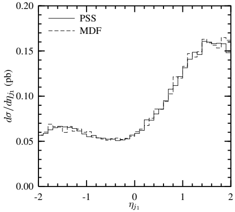
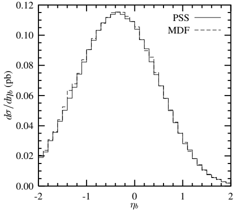
Classifying processes by analytical structure leads directly to kinematic insight:

Jets from *t*-channel processes are more forward than those from *s*-channel.

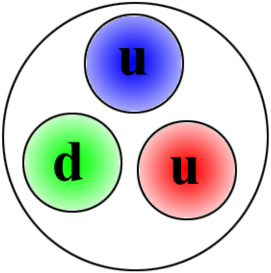


jet from *t*-channel



b jet from *s*-channel

Rethinking the proton



Using DGLAP was NOT just a math trick!

This "valence" picture of the proton is not complete.

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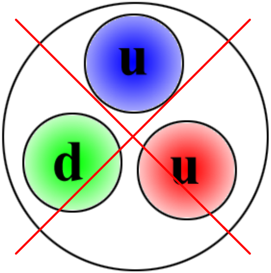
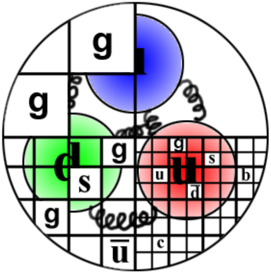


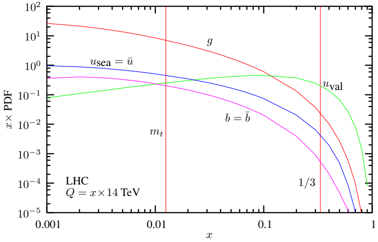


Using DGLAP was NOT just a math trick!

This "valence" picture of the proton is not complete.

Larger energies resolve smaller structures.

The probability of finding a particle inside the proton is given by PDFs (Parton Distribution Functions)



b (and c) quarks are full-fledged members of the proton structure.

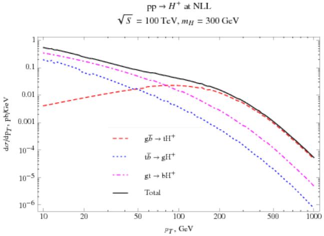
The top quark as a parton

In general, we do not consider the top quark when discussing proton structure.

The reason is simple: We do not tend to measure at scales far enough above m_t to ignore its mass.

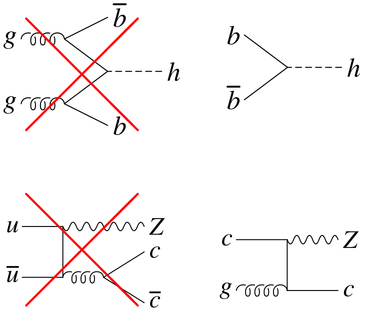
Dawson, Ismail, and Low (PRD90, 014005(14)) revisited this issue and demonstrated it was indeed not sensible for inclusive cross sections at 100 TeV.

However, the p_T distributions for some processes, such as $H^+ + X$ production need a top PDF to get the correct result.



Rethinking several physical processes

Why is this important?

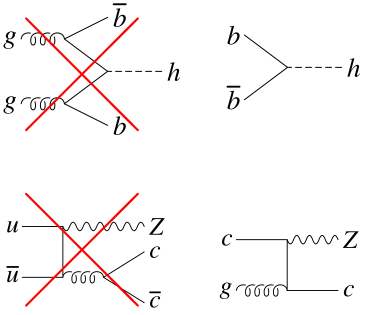


Starting with a c/b gives us:

$b\bar{b} \to h$	Largest SUSY Higgs cross section
Zb/Zc	Affects LHC luminosity monitor
Zbj/Zcj	Higgs background
Wbj	Largest single-top background

etc.

Rethinking several physical processes



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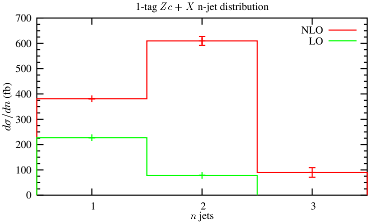
Zc at Tevatron



Parton luminosity and large logs can be more important than counting powers of α_s!

This can be exaggerated at LHC

$Z \approx Z + 1\,\text{jet} \approx Z + 2\,\text{jets!}$

(w/ reasonable cuts)

What is LO when 0/1/2 jets are all the same?

Rethinking the matrix element: A practical problem for experiments

The same large logs that lead to a reordered perturbation for t-channel single-top, implied a potentially large uncertainty in measurable cross sections when cuts were applied.

Recall: t-channel and s-channel are distinguished by the number of b-jets.

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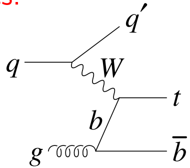
Real problem: Is the b contamination 20%, 30%, 10%?



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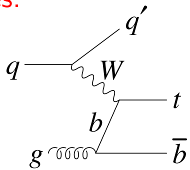

Large $\ln\left(\frac{Q^2+m_t^2}{p_{T\,\mathrm{cut}}^2+m_b^2}\right)$ terms return

Another problem: To distinguish from $t\bar{t}$, the cross section in the $W+2$ jet bin has to be known.

Counting jets is IDENTICAL to performing a jet veto.

Inclusive cross sections are not enough, we need to calculate exclusive cross sections

Fully Differential NLO Techniques

- In 2001, there were few matrix-element techniques or calculations that could deal IR singularities in processes with massive particles.
- Experiments were mostly stuck using LO matrix elements to predict semi-inclusive or exclusive final states.
- We needed methods to provide the 4-vectors, spins, and corresponding weights of exclusives final-state configurations.

These needs led to work on 3 techniques:

- Phase space slicing method with 2 cutoffs.

  L.J. Bergmann, Ph.D. Thesis, FSU (89)
  cf. H. Baer, J. Ohnemus, J.F. Owens, PRD 40, 2844 (89)
  B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

- Phase space slicing method with 1 cutoff.

  W.T. Giele, E.W.N. Glover, PRD 46, 1980 (92)
  cf. W.T. Giele, E.W.N. Glover, D.A. Kosower, NPB 403, 633 (93)
  E. Laenen, S. Keller, PRD 59, 114004 (99)

- Massive dipole formalism (a subtraction method) coupled with a helicity-spinor calculation. Invented to solve single-top production.

  cf. L. Phaf, S. Weinzierl, JHEP 0104, 006 (01)
  S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi, NPB 627,189 (02)

Phase Space Slicing Method (2 cutoffs)

B.W. Harris, J.F. Owens, PRD 65, 094032 (02)

The essential challenge of NLO differential calculations is dealing with initial- and final-state soft or collinear IR divergences.

$$\sigma_{\text{obs.}} \sim \int \frac{1}{s_{ij}} \sim \int \frac{dE_i dE_j d\cos\theta_{ij}}{E_i E_j (1 - \cos\theta_{ij})}$$

If $E_{i,j} \to 0$ "soft" singularity
If $\theta_{ij} \to 0$ "collinear" singularity

IDEA: Introduce arbitrary cutoffs (δ_s, δ_c) to remove the singular regions...

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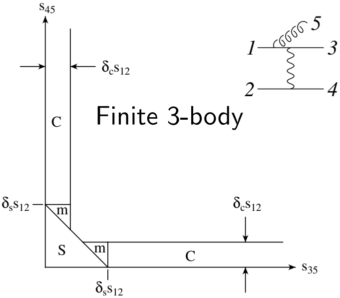
Divide phase space into 3 regions:

1. soft: $E_g \leq \delta_s\sqrt{\hat{s}}/2$ gluons only
2. collinear: $\hat{s}_{35}, \hat{s}_{45}, \ldots < \delta_c \hat{s}$;
3. hard non-collinear: (finite, particles well separated, $E > 0$)

Phase space plane (s_{35}, s_{45})



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We traded dependence on physical observables (energy, angles) for logarithmic dependence on arbitrary parameters $(\ln\delta_s, \ln\delta_c)$

When a massive quark radiates, $(1-\beta\cos\theta_{ij})$ has no collinear singularity

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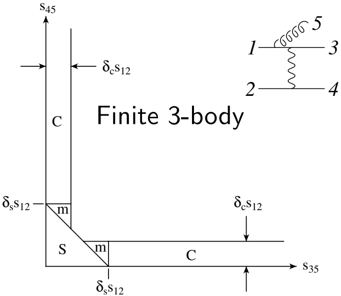
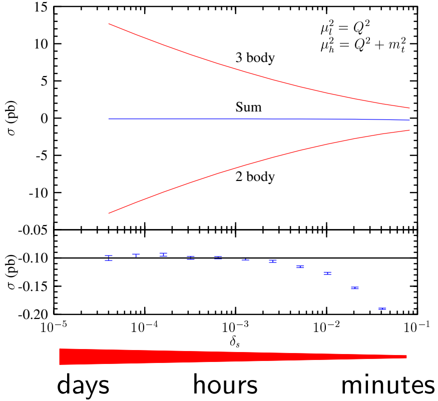


Cut-off dependence of NLO correction

Each term is logarithmically divergent for small δ_s, δ_c

Logarithmic dependence on the cutoffs cancels in any IR-safe observable at the histogramming stage.



At the end we take δ_s and δ_c to zero via numerical computation. This can take a long time. . .

Massive Dipole Formalism (subtraction)

$$\begin{aligned}\sigma_{NLO} &= \int_{n+1} d\sigma^{\text{Real}} + \int_n d\sigma^{\text{Virtual}} \\ &= \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)\end{aligned}$$

- $d\sigma^A$ is a sum of color-ordered dipole terms.
  - $d\sigma^A$ must have the same point-wise singular behavior in D dimensions as $d\sigma^R$.
    $\Rightarrow$ $d\sigma^A$ is a local counterterm for $d\sigma^R$.
  - $\int_1 d\sigma^A$ is analytic in D dimensions, and reproduces the soft and collinear divergences of $d\sigma^R$.
- Some advantages over Phase Space Slicing are:
  - You can easily project out spin eigenstates.
    $\Rightarrow$ Explicitly test different spin bases at NLO after cuts.
  - Event generators use color-ordered matrix elements.
- Both methods have some contribution to n-body final states from $n+1$ phase-space. Hence, you must do 2 separate integrations.

Subtraction vs. phase space slicing

In practical terms, the difference in methods is in how to integrate in the presence of infrared singularities.

$$I = \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

Subtraction: Add and subtract $F(0)$ under the integral

$$\begin{aligned} I &= \lim_{\epsilon \to 0^+} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon \left[F(x) - F(0) + F(0) \right] - \frac{1}{\epsilon} F(0) \right\} \\ &= \int_0^1 \frac{dx}{x} \left[F(x) - F(0) \right], \quad \text{finite up to machine precision} \end{aligned}$$

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PSS: Integration region divided into two parts $0 < x < \delta$ and $\delta < x < 1$, with $\delta \ll 1$. A Maclaurin expansion of $F(x)$ yields

$$\begin{aligned} I &= \lim_{\epsilon\to 0^+}\left\{\int_0^\delta \frac{dx}{x} x^\epsilon F(x) + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon}F(0)\right\} \\ &= \int_\delta^1 \frac{dx}{x} F(x) + F(0)\ln\delta + \mathcal{O}(\delta), \quad \text{take } \lim_{\delta\to 0} \text{ numerically} \end{aligned}$$

Remaining $\ln\delta$ singularities removed by summing all integrals I_i.

Rethinking jet definitions and phase space: Experiments need exclusive $t+1$ jet at NLO

ZTOP, Z.S., PRD 70, 114012 (2004)

Tevatron	# b-jets	tj (Wbj)	tjj ($Wbjj$)
s-channel	$=2$	0.620 pb $^{+13}_{-11}\%$	0.168 pb $^{+24}_{-19}\%$
	$=1$	0.022 pb $^{+24}_{-19}\%$	(NNLO)
t-channel	$=1$	0.950 pb $^{+16}_{-15}\%$	0.152 pb $^{+17}_{-14}\%$
	$=2$	0.146 pb $^{+21}_{-16}\%$	0.278 pb $^{+21}_{-16}\%$

Cuts: $p_{Tj} > 15$ GeV, $|\eta_j| < 2.5$, no cuts on t
Jet definition: $\Delta R_{k_T} < 1.0$ ($\approx \Delta R_{\mathrm{cone}} < 0.74$)

Breakdown of *shape-independent* uncertainties

Process		$\times\delta m_t$(GeV)	$\mu/2$–2μ	PDF	b mass	$\alpha_s(\delta_{\mathrm{NLO}})$
s-channel	$p\bar{p}$	$^{-2.33}_{+2.71}\%$	$^{+5.7}_{-5.0}\%$	$^{+4.7}_{-3.9}\%$	$< 0.5\%$	$\pm1.4\%$
	pp	$^{-1.97}_{+2.26}\%$	$\pm2\%$	$^{+3.3}_{-3.9}\%$	$< 0.4\%$	$\pm1.2\%$
t-channel	$p\bar{p}$	$^{-1.6}_{+1.75}\%$	$\pm4\%$	$^{+11.3}_{-8.1}\%$	$< 1\%$	$\pm0.01\%$
	pp	$^{-0.73}_{+0.78}\%$	$\pm3\%$	$^{+1.3}_{-2.2}\%$	$< 1\%$	$\pm0.1\%$

Consider: Why do we vary scales to estimate higher order corrections?

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Process		$\times\delta m_t$(GeV)	$\mu/2$–2μ	PDF	b mass	$\alpha_s(\delta_{\rm NLO})$
s-channel	$p\bar{p}$	$^{-2.33}_{+2.71}\%$	$^{+5.7}_{-5.0}\%$	$^{+4.7}_{-3.9}\%$	$< 0.5\%$	$\pm 1.4\%$
	pp	$^{-1.97}_{+2.26}\%$	$\pm 2\%$	$^{+3.3}_{-3.9}\%$	$< 0.4\%$	$\pm 1.2\%$
t-channel	$p\bar{p}$	$^{-1.6}_{+1.75}\%$	$\pm 4\%$	$^{+11.3}_{-8.1}\%$	$< 1\%$	$\pm 0.01\%$
	pp	$^{-0.73}_{+0.78}\%$	$\pm 3\%$	$^{+1.3}_{-2.2}\%$	$< 1\%$	$\pm 0.1\%$

Every number here, even the concept of t-channel single-top, required a new or revised understanding of QCD.

- b PDFs $\rightarrow$ t-channel
- PDF uncertainties
- multiple scales: m_t/m_b
- 2 expansions: α_s, $1/\ln$
- Fully differential NLO jet calculations

...

Consider: Why do we vary scales to estimate higher order corrections?

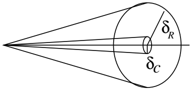
Thinking about the final state: How do we interpret exclusive NLO calculations?

Z.S., PRD 70, 114012 (2004)

"Paradigm of jet calculations"

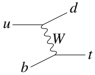
- We are calculating extensive objects, i.e., jets not "improved quarks."
- Unlike inclusive NLO calculations, exclusive NLO calculations are only well-defined in the presence of a jet definition or hadronization function. ($D_i(p_i)$)

  $\Rightarrow$ The mathematics of quantum field theory tells us we cannot resolve the quarks inside of these jets!

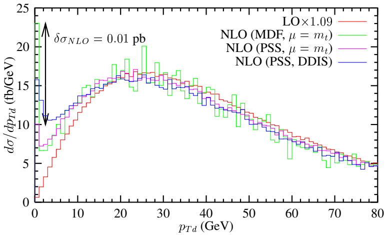


- "Bad things" happen if you treat jets as NLO partons. . .

At LO, a d-quark recoils against the top quark in t-channel.



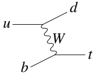
NLO "d-jet" (no cuts)



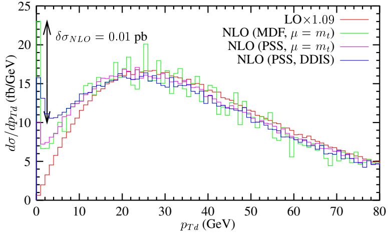
- Perturbation theory is not terribly stable at low p_{Td} (or even high p_{Td}).
- This is not what we want.
  Be careful what you ask for!

Transverse momenta distributions at NLO

At LO, a d-quark recoils against the top quark in t-channel.



NLO "d-jet" (no cuts)



- Perturbation theory is not terribly stable at low p_{Td} (or even high p_{Td}).
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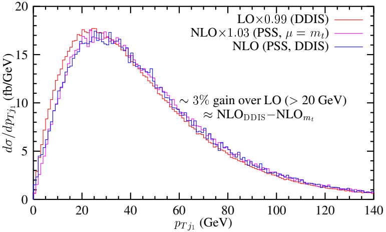
We measure the highest p_T jet



The highest p_T jet recoils against the top. The measurable change in shape is comparable to the scale uncertainty.

Fixed-order theory predicts jets not quarks: MC requires matching


THEORY
Experiment


THEORY
Event Generators
Experiment

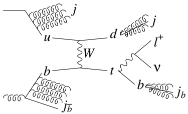

THEORY
Experiment

Event generators vs. NLO t-channel $t\bar{b}$ ($Wb\bar{b}$)

Z.S., PRD 70, 114012 (2004)

Initial-state radiation (ISR) is generated by backward evolution of angular-ordered showers.

$\Rightarrow$ The jet containing the extra $\bar{b}$ comes from soft ISR.



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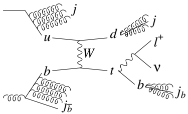
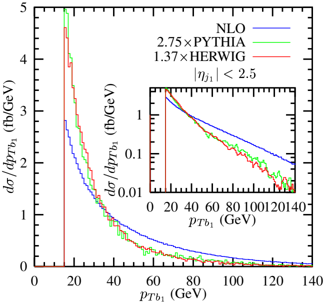
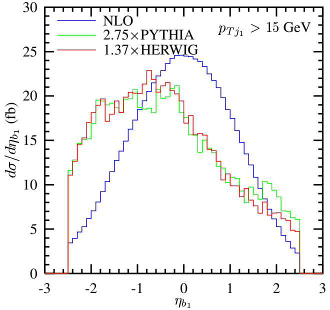






- Lesson: n-jets+showers $\neq n+1$ jets. $\Rightarrow$ Need NLO matching. (Schemes have since proliferated: cf. lectures by Hoche and Platzer)
- Showering can be improved: Including b mass effects in splitting kernels for shower helps (Nagy, Soper JHEP 06(14)179)
  NLO Implementation for ttj is promising (Czakon et al. JHEP 06(15)033)

Summary

"Heavy quarks" (c, b, and t) are interesting because their mass adds a new scale to any problem.

$$\sigma \sim \alpha_s \ln\left(\frac{\mu^2}{p_{T\,\mathrm{cut}}^2 + m_Q^2}\right)$$

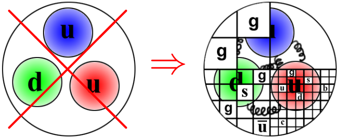
These terms can appear in the initial or final state, and need to be resummed.

When you see logs of this type, it is often a hint there is something deeper to be learned.

Consider: Are there other (perhaps kinematic) ratios of scales that arise that lead to logarithms that need to be resummed?

Single-top-quark production is the "new" DIS and DY

$$\sigma_{\mathrm{obs.}} = \int f_1(x_1, \mu_1) f_2(x_2, \mu_2) \otimes \overline{|M|^2} \otimes d\mathrm{P.S.} \otimes D_i(p_i) \ldots D_n(p_n)$$



— b/c PDFs are inside the proton
— Radiation off heavy quarks requires modified ME techniques
— HQ jets are not like light jets How?

Questions for discussion

Consider: Why do we not see $J/\psi \to gg$?

Consider: Show in the top quark width $\Gamma(t \to bW)$, dropping m_b loses terms of $\mathcal{O}(m_b^2/m_t^2) \sim 1\%$.

Consider: Why do we vary scales to estimate higher order corrections?

Consider: Are there other (perhaps kinematic) ratios of scales that arise that lead to logarithms that need to be resummed?

Consider: How are heavy-quark jets not like light-quark jets?