Electroweak corrections I

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Introduction

In this two-lecture course we will inspect the anatomy of the electroweak part of the Standard Model. This will already point us towards the phenomenological impact higher-order electroweak effects can have.

We will then look at how to construct infrared-safe observables (what and what not to do) and how this impacts measurements.

Besides electroweak corrections at next-to-leading order, we will outline how they can be resummed and examine the phenomenological input.

Finally, we will explore how we can incorporate higher-order electroweak corrections in event generation.

Literature

Peskin, Schroeder, *An introduction to Quantum Field Theory*

Böhm, Denner, Joos, *Gauge Theories of the Strong and Electroweak Interaction*

Denner, Dittmaier, *Electroweak Radiative Corrections for Collider Physics*, arXiv:1912.06823

Electroweak corrections I

The electroweak Standard Model

The electroweak Standard Model

To understand the structure of the electroweak part of the Standard Model we need to first examine the Lagrangian after its electroweak gauge symmetry is broken.

If we think of the Lagrangian of the Standard Model, we typically think of

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\begin{aligned}
\mathcal{L} = &-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
&+ i \bar{\psi} \not{D} \psi + \text{h.c.} \\
&+ \psi_i y_{ij} \psi_j \phi + \text{h.c.} \\
&+ |D_\mu \phi|^2 - V(\phi)
\end{aligned}
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but, while compact, it hides the true structure of the EW sector

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The electroweak Standard Model

Let us consider the **Lagrangian** of the unbroken Standard Model

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\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = & -\tfrac{1}{4}\, G^a_{\mu\nu} G_a^{\mu\nu} - \tfrac{1}{4}\, W^a_{\mu\nu} W_a^{\mu\nu} - \tfrac{1}{4}\, B_{\mu\nu} B^{\mu\nu} \\
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Covariant derivative (induces gauge interactions)

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D_\mu = \partial_\mu - \tfrac{i}{2}\, g_s\, t^a G^a_\mu - \tfrac{i}{2}\, g\, \sigma^a W^a_\mu - \tfrac{i}{2}\, g' Y B^\mu
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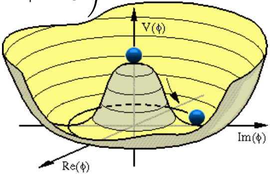

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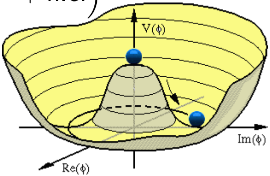

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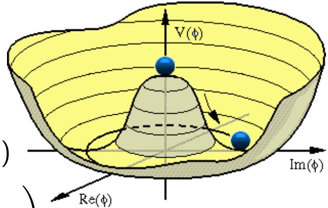

Let us now break the symmetry and expand the Higgs field around its minimum (choose $\Phi_0 = \binom{0}{v}$)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left(v + h + \chi\right) \end{pmatrix}$$

where h, $\phi^\pm$ and χ have vanishing vacuum expection values.

The $\phi^\pm$ and χ are the goldstone bosons, they are gauge dependent and thus unphysical. In the *unitary* gauge they form the longitudinal modes of the emerging massive gauge bosons.

The h field forms the physical Higgs field with the Higgs particle as an excitation around the physical minimum.

The $SU(2)_L \times U(1)_Y$ symmetry is still there, just not manifest in Φ_0. However, a new symmetry is explicitly manifest in Φ_0: $U(1)_{QED}$.

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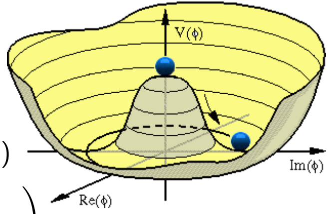

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Still very compact, but not in terms of physical fields.
Plenty of **mass terms** created (bilinear in the fields),
but rarely as bilinears of the same field.

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The electroweak Standard Model

Introduce the **physical** EW gauge bosons

$$W^\pm = \tfrac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right) \qquad \text{and} \qquad \begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} ,$$

with the electroweak mixing angle $s_w^2 \equiv \sin^2\theta_w$ as short-hand for

$$s_w^2 = \frac{g'^2}{g^2 + g'^2} \qquad \text{and} \qquad c_w^2 = \frac{g^2}{g^2 + g'^2} \; .$$

This diagonalises the mass matrices and gives rise to the physical gauge bosons masses

$$m_W = \tfrac{1}{2}\, gv \; , \qquad m_Z = \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v \qquad \text{and} \qquad m_A = 0 \; .$$

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The electroweak Standard Model

The coupling of the newly emerging unbroken (massless) gauge field A is

$$e = \frac{g\,g'}{\sqrt{g^2 + g'^2}} = \frac{2\,m_W\,s_w}{v}\;.$$

The corresponding gauge group is $\mathrm{U}(1)_{\mathrm{QED}}$.

The fine structure constant is

$$\alpha = \frac{e^2}{4\pi}\;.$$

The electroweak Standard Model

The Yukawa interactions provide mass terms for quarks and leptons after diagonalisation, which gives rise to the CKM matrix and quark generational mixing.

Even though, the remaining Higgs-fermion interactions are of $\mathcal{O}(y_i)$ and have nothing to do with the electroweak gauge interactions, they are commonly taken as part of what are called $\mathcal{O}(\alpha)$ corrections.

Parameters of the electroweak Standard Model

Couplings:

g, g', α

Higgs potential:

v, μ, λ

v is commonly substituted for G_F, the muon decay constant

Masses:

m_h, m_W, m_Z, m_ℓ, m_q

Mixing angles:

$\sin\theta_w$

⚠ **Not all of them are independent!** ⚠

Parameters of the electroweak Standard Model

Commonly used input parameter schemes:

$\alpha(0)$ scheme	$\{\alpha(0), m_W, m_Z\}$	real photon couplings
G_μ scheme	$\{G_F, m_W, m_Z\}$	W-interactions with fermion currents
$\alpha(m_Z)$ scheme	$\{\alpha(m_Z), m_W, m_Z\}$	generic high-energy interactions
$\sin\theta_w$ scheme	$\{G_F, m_Z, \sin\theta_w\}$	precision measurements on Z pole

$+(m_h, m_\ell, m_q)$

Mixed schemes are possible for well-defined gauge-invariant sets of EW diagrams/couplings.

Parameters of the electroweak Standard Model

Consequently, there are different EW input parameter schemes.

Generally, **only input parameters are free parameters** of the model, derived parameters are only short-hands to keep the notation tidy.

Therefore, when confronting experimental data with a theoretical calculation, only input parameters of that calculation can be extracted from data.

Example:
One cannot measure/extract m_W, m_Z and $\sin\theta_w$ from data independently.

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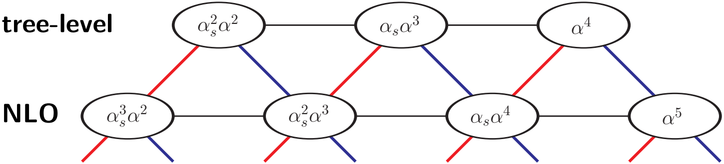
The electroweak Standard Model at higher orders

Definition of higher-order electroweak corrections

QCD and EW corrections are only defined through power counting.

Example: *Vjj* production



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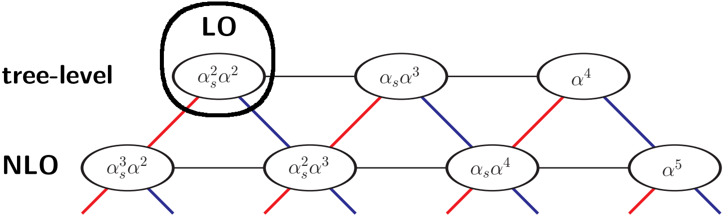

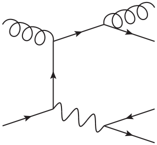

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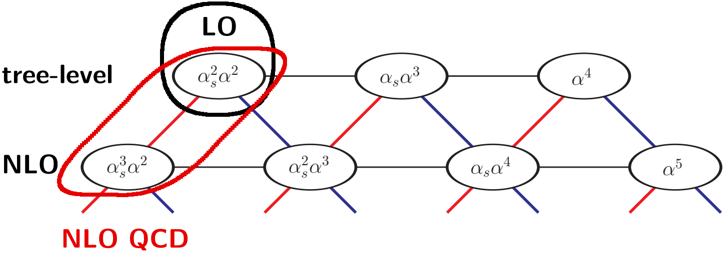

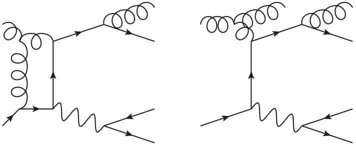

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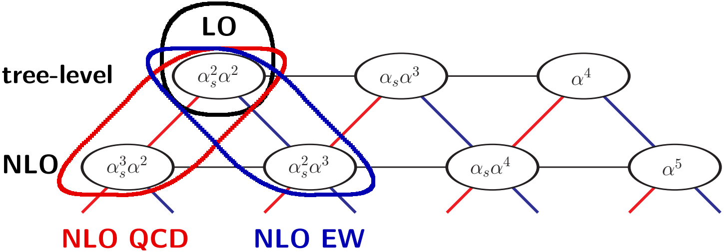

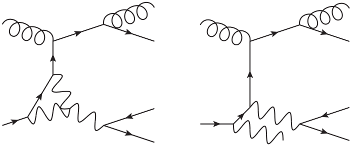

Definition of higher-order electroweak corrections

QCD and EW corrections are only defined through power counting.

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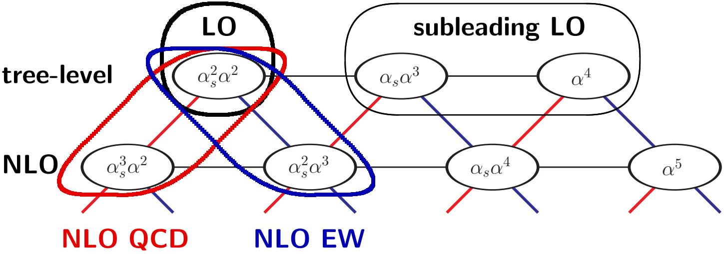

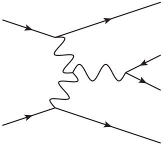

$\mathcal{O}(\alpha^4)$ contains VBF

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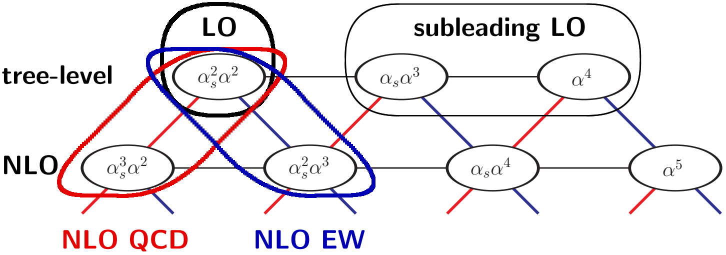

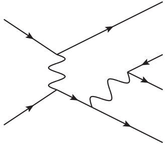

$\mathcal{O}(\alpha^4)$ contains VBF, V strahlung

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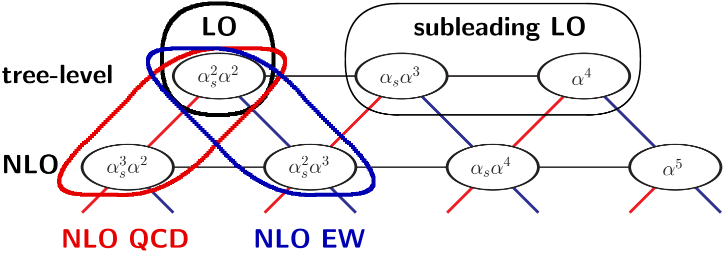

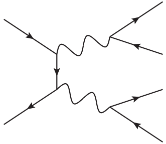

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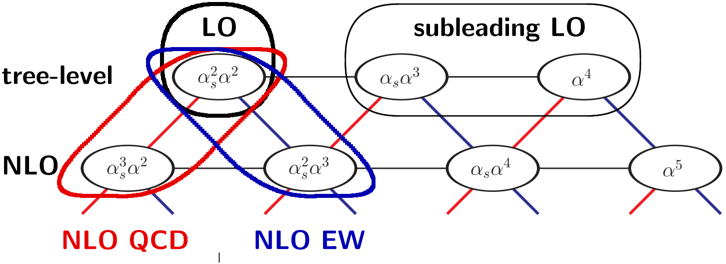

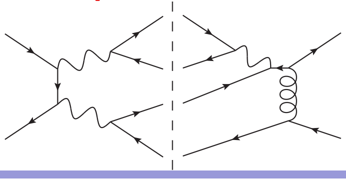

$\mathcal{O}(\alpha_s\alpha^3)$ is an interference contrib
not $|\mathcal{M}|^2$, but $\mathcal{R}\mathrm{e}(\mathcal{M}_1\mathcal{M}_2^*)$

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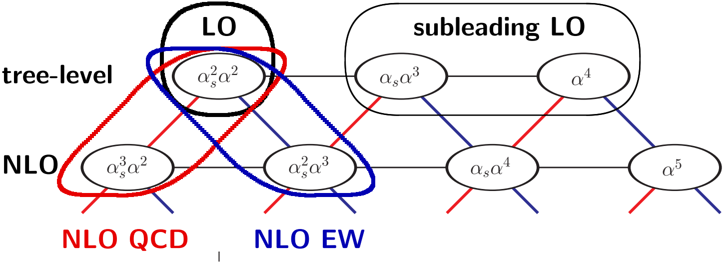

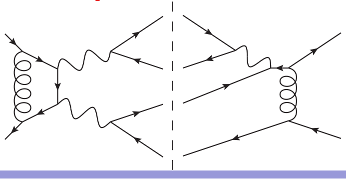

Thus: **NLO EW** contains a piece that is a QCD correction to a subleading Born contribution

Definition of higher-order electroweak corrections

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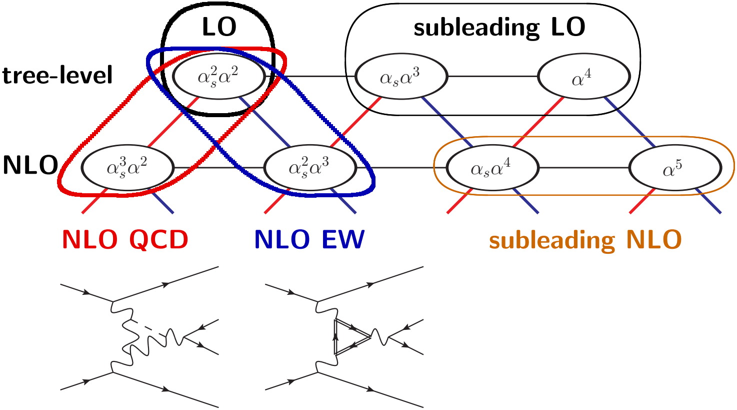


Short-distance and long-distance objects

Differentiate:
The objects in our calculations are short-distance objects (partons), they are not observable.
What are observable are long-distance objects, states that travel macroscopic distances to a detector.
QCD: (quarks, gluons) $\leftrightarrow$ (hadrons)

Problem:
Language, since there is strongly coupled regime in the IR, both are quanta of the same fields, but at different energy scales.

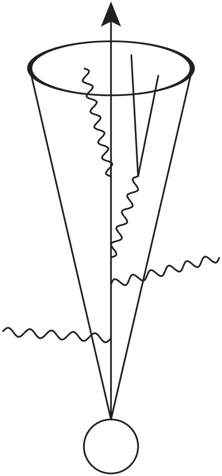
Be aware:
A photon that participates in a hard interaction is not the same as a photon that hits the detector. Similarly for leptons.

Infrared-safe observables

The presence of massless gauge bosons (γ) and (possibly) massless fermions necessitates the restriction to infrared-safe observables.

As the emission probability of infinitely soft and/or collinear photons and photons splitting into collinear fermion-antifermion pairs divergence, observables must be insensitive to such emissions.

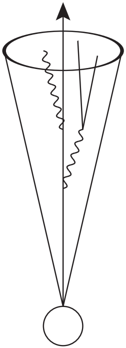
Collinearly unsafe observables can be defined with the help of fragementation functions, transfering short-distance objects into long-distance ones.



Definition of physical objects

What is a jet?

- photons and leptons must be part of a jet, but to what extent?
- **democratic:**
  - $+$ straight forward, always well defined
  - $-$ many contributions
  - $\rightarrow$ single photons constitute a jet
  - $\rightarrow$ single leptons constitute a jet
- **anti-tagging jets with certain flavour content:**
  - $+$ fewer contributions
  - $-$ needs a lot of care to be well-defined at all contributing orders
  - $\rightarrow$ anti-tag jets with too large photon content
  - $\rightarrow$ anti-tag jets with net lepton content
- which approach is closer to experiment depends on analysis, general anti-tagging must proceed through fragmentation functions



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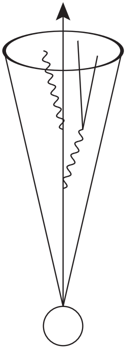

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Definition of physical objects

Example: dijet production at the LHC

- define jets completely democratically,
  incl. all massless visible particles of the SM (q, g, γ, ℓ)

- anti-tag jets against leptons
  exclude jets with net lepton number within lepton acceptance
  care: jet acceptance and lepton acceptance may differ

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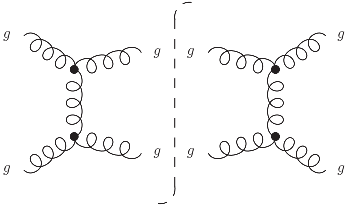

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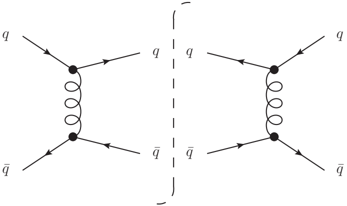

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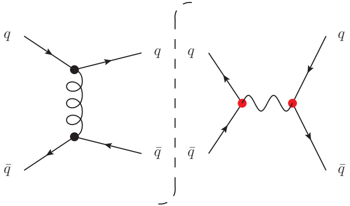

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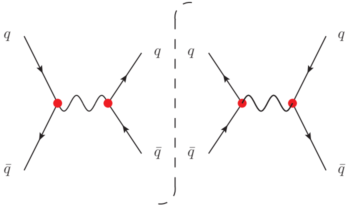

hadronic DY

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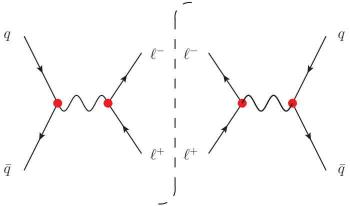

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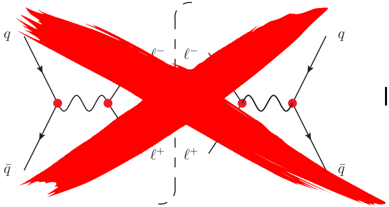

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Definition of physical objects

What is a photon?

- differentiate: short-distance photon (photon as parton),
  long-distance photon (identified, measurable photon)
- identify through fragmentation function

$$D_\gamma^\gamma(z,\mu) = \frac{\alpha(0)}{\alpha_{\text{sd}}}\,\delta(1-z) + \mathcal{O}(\alpha^2)$$

$\Rightarrow$ leads to $\alpha(0)$-scheme for identified photons

Definition of physical objects

What is a lepton?

- in principle, again differentiate between short-distance parton and long-distance identified and measurable object
- simplified as leptons not gauge bosons, thus

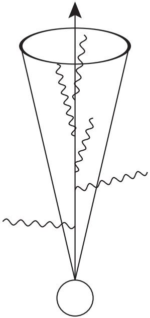
$$D_\ell^\ell(z,\mu) = \delta(1-z) + \mathcal{O}(\alpha)$$

$$D_\ell^\gamma(z,\mu) = \mathcal{O}(\alpha)$$

problematic in processes with leptons
and unresolved photons in Born

$$D_\ell^q(z,\mu) = \mathcal{O}(\alpha^2)$$

$$D_\ell^g(z,\mu) = \mathcal{O}(\alpha_s\alpha^2)$$



- dressed lepton: masseless leptons must be dressed for IR safety
- bare lepton: massive leptons may be measured bare
- Born lepton: not an infrared-safe concep

Renormalisation schemes

Leptons and photons are asymptotic states, and the vector bosons produce clear Breit-Wigner-shaped resonances.

Thus, typically EW sector typically renormalised on-shell such that the location of the propagator pole does not receive higher-order corrections. But this is only possible for input parameters.

There is no renormalisation scale as a free parameter of the continuous set of $\overline{\mathrm{MS}}$ renormalisation schemes. Scheme variations are discrete.

Renormalisation schemes

Example 1:

$\alpha(0)$ scheme, input $\{\alpha(0), m_W, m_Z\}$.
The position of the W and Z propagator pole is given at LO and does not receive higher-order corrections.
This time, the photon coupling in the Thomson limit is fixed to be the fine structure constant $\alpha(0)$ to all orders.
$\sin\theta_w$ is again dependent parameter.
Every EW short-distance interaction (at the EW scale or higher) receives significant higher-order corrections from fermion loops.

Renormalisation schemes

Example 2:

G_μ scheme, input $\{G_F, m_W, m_Z\}$.
The position of the W and Z propagator pole is given at LO and does not receive higher-order corrections.
Similarly, the muon decay is fully determined at LO and does not receive higher-order corrections, absorbing all its higher-order corrections into the dependent coupling parameter α.
$\sin\theta_w$ is a dependent parameter. If to be determined from the Drell-Yan forward-backward asymmetry, it receives higher-order corrections.

Renormalisation schemes

Example 3:

$\sin\theta_w$ scheme, input $\{G_F, m_Z, \sin\theta_w\}$.
The value of $\sin\theta_w$ is fixed and can be extracted from a measurement. The position of the W propagator, however, is not fixed to its LO value and receives higher-order corrections. Thus, the Lagrangian parameter m_W is not the W pole mass.

Summary

- The EW Standard Model is somewhat more complex than QCD, sadly $\alpha < \frac{\alpha}{s_w^2} < \alpha_s$
- EW input parameter schemes and related renormalisation scheme play a key role and need to match the intended data comparison
- infrared safety somewhat more complex than in QCD, but otherwise plays similarly central role in defining observables

Questions:

1. In which ways does the electroweak part of the Standard Model differ from its QCD part?
2. What role does the input parameter scheme play?
3. What role does the renormalisation scheme play?
4. How are higher-order corrections defined?
5. What characterises IR safety in the EW sector? What are its consequences?
6. What are the consequences of the fact that the EW sector of the Standard Model is a broken symmetry? How does it impact NLO calculations?