Exercises: Day #2: Stefan Prestel

Exercise:

Show that, for better or worse, multiple interactions do not change the inclusive cross section.
[NB: An analogous proof also applies to the parton shower.]

Assumptions

- The scattering will occur with probability $\sigma_{inelastic}$
- The probability of a hardest interaction at $p_{\perp 1}$ is

$$\Pi_{MPI}(p_{\perp 0}, p_{\perp 1}) < n(p_{\perp 1}) >$$

with

$$\Pi_{MPI}(p_{\perp 0}, p_{\perp 1}) = \exp\left(- \int_{p_{\perp 1}}^{p_{\perp 0}} dp_\perp < n(p_\perp) > \right)$$

- The probability of a second scattering at $p_{\perp 2} < p_{\perp 1}$ is

$$\Pi_{MPI}(p_{\perp 0}, p_{\perp 1}) < n(p_{\perp 1}) > \Pi_{MPI}(p_{\perp 1}, p_{\perp 2}) < n(p_{\perp 2}) >$$

- Assume that any $p_\perp > p_{\perp cut}$

Task: Show that the probability of having any partonic scattering is given by $\sigma_{inclusive} = \sigma_{inelastic} \cdot \langle n(p_\perp) \rangle$

Does an analogous proof hold for the parton shower?

Hints

- Exploit the properties of exponential functions
- Investigate the probability of n scatterings with $p_{\perp_n} < p_{\perp_{n-1}} < \dots < p_{\perp_1} < p_{\perp_0}$
- Use relations for nested integrals, e.g.

$$\int_{p_{\perp cut}}^{p_{\perp_0}} dp_{\perp_1} \int_{p_{\perp cut}}^{p_{\perp_1}} dp_{\perp_2} \langle n(p_{\perp_1}) \rangle \langle n(p_{\perp_2}) \rangle = \frac{1}{2} \left[\int_{p_{\perp cut}}^{p_{\perp_0}} dp_\perp \langle n(p_\perp) \rangle \right]^2$$

Exercises: Day #2: Dave Soper

Basics of QCD Perturbation Theory

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CTEQ/MCnet School, September 2021

Some discussion questions for the Tuesday QCD Intro lectures

1) In the lecture slides, calculation of the hard scattering $\hat \sigma$ at NLO was presented as using a cut $k_T^2 > \mu^2$ on emission of a gluon in the initial state. What do people really do?

2) Is the eikonal operator F in the operator definition of parton distribution functions really important?

3) What measurements are most important in limiting the errors in the determination of parton distribution functions.

4) The probability to find a quark at high momentum fraction x is small. If we look at a larger factorization scale, does this probability get larger or smaller?

5) How does the cross section to produce a Z boson depend on its transverse momentum q_T^2 for $q_T^2 \ll M_Z^2$ when calculated at first order?

6) Are there other kinds of factorization besides the one in the lectures?

7) Is there a version of a parton shower with a splitting operator calculated at order α_s^2?

8) Can a parton shower be matched to an NLO calculation of a hard scattering?

WITH HINTS: Some discussion questions for the Tuesday QCD Intro lectures

1) In the lecture slides, calculation of the hard scattering $\hat \sigma$ at NLO was presented as using a cut $k_T^2 > \mu^2$ on emission of a gluon in the initial state. What do people really do?

Hint: they subtract IR poles

2) Is the eikonal operator F in the operator definition of parton distribution functions really important?

Hint: Yes, if you want to do calculations in Feynman gauge, which is the most convenient method. Then the operator F absorbs unphysically polarized gluons, matching how partons absorb unphysically polarized gluons in calculating $\hat \sigma$.

3) What measurements are most important in limiting the errors in the determination of parton distribution functions.

Hint: This is for discussion. Maybe some of the students have experience with this.

4) The probability to find a quark at high momentum fraction x is small. If we look at a larger factorization scale, does this probability get larger or smaller?

Hint: Smaller.

5) How does the cross section to produce a Z boson depend on its transverse momentum q_T^2 for $q_T^2 \ll M_Z^2$ when calculated at first order?

Hint: It should produce log divergences, so $d\sigma/dq_T^2$ should be proportional to q/Q_T^2 times logs. (Actually, on log at first order).

6) Are there other kinds of factorization besides the one in the lectures?

Hint: Yes, the word "factorization" is used in lots of contexts, in which it is proposed that $A = B*C$. Sometimes a proposed factorization fails in QCD. The kind in the lectures is sometimes called collinear factorization.

7) Is there a version of a parton shower with a splitting operator calculated at order α_s^2?

Hint: No. That is a difficult problem.

8) Can a parton shower be matched to an NLO calculation of a hard scattering?

Hint: Yes. There has been a lot of work on that. The idea is to get the cross section of interest, for which the matching is designed, correct to NLO. At the same time, you get probabilities for complete multi-parton final states. However, cross sections other than the one for the matching is designed will not generally be correct to NLO.