TMD factorization for dijet and heavy meson pair production in DIS

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Outline

Dijet production

- EIC coverage
- Cross-section factorization
- New TMD Soft Function
- Consistency AD check

Heavy meson pair production

- Cross-section factorization
- Soft Function AD up to three-loops

ζ-prescription & angle dependence

Based on the work published by

Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris & Ignazio Scimemi

https://arxiv.org/abs/2008.07531v4

Introduction

- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect.

For example: Higgs production

Gutierrez-Reyez, Leal-Gómez, Scimemi, Vladimirov, 2019

- We consider two processes which are presently attracting increasing attention

$$\ell + h \to \ell' + J_1 + J_2 + X \qquad\qquad \ell + h \to \ell' + H + \bar{H} + X$$

Dijet

Heavy-meson

Working in the Breit frame

Dominguez, Xiao, Yuan, 2013

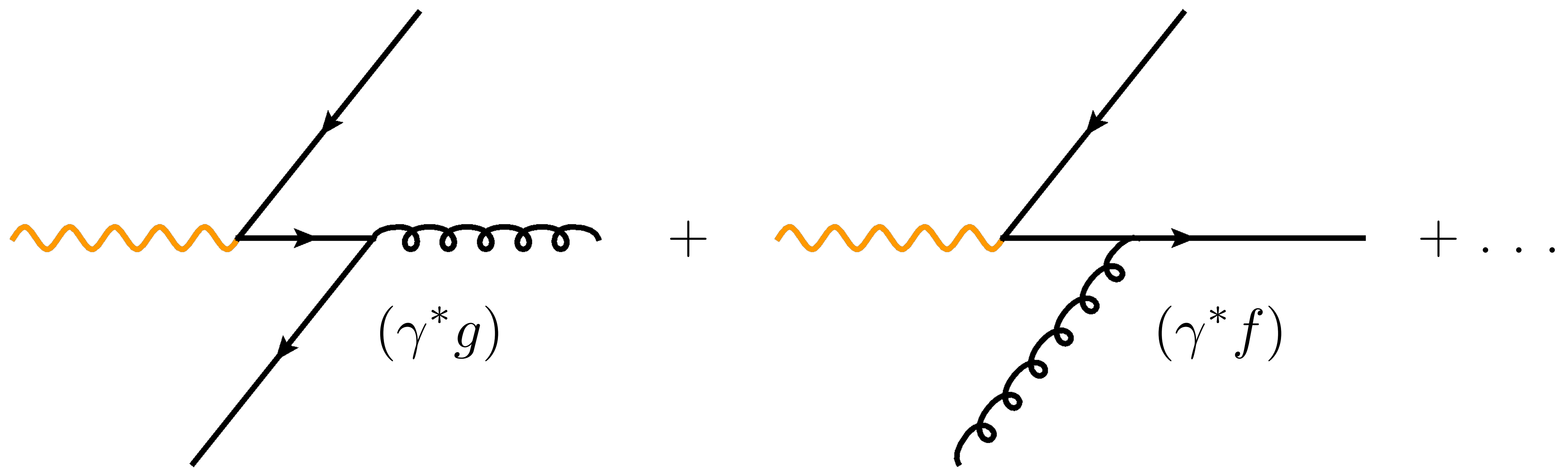
Boer, Brodsky, Mulders, Pisano, 2011

Zhang, 2017

Dijet production

$$\ell + h \to \ell' + J_1 + J_2 + X$$

dijet LO process:



- Sensitive of polarized and unpolarized TMDPDFs
- Experimental observation should be possible in the future EIC Page, Chu, Aschenauer, 2020
- Jets here described have $p_T \in [2, 20]$ GeV and are found in the central rapidity region
- Factorization within SCET

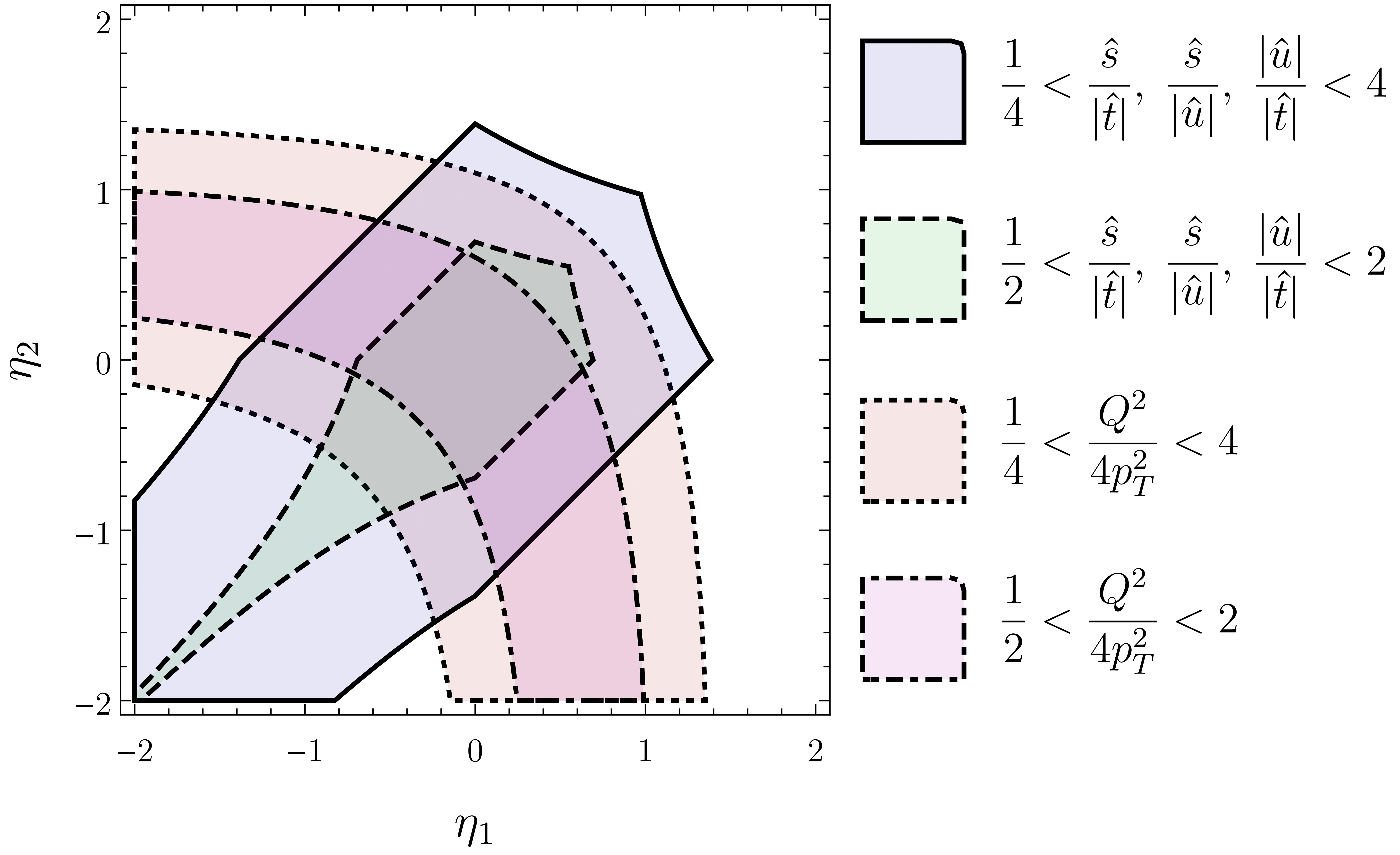
Kinematic region

Dijet production

$$\boldsymbol{r}_T = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$$

$$p_T = \frac{|\boldsymbol{p}_{1T}| + |\boldsymbol{p}_{2T}|}{2}$$

$$|\boldsymbol{r}_T| \ll p_T$$



Factorization holds for $|\boldsymbol{r}_T| \ll p_T$ and for the central rapidity region

Cross-section factorization

Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T}$$

We measure over

- x parton momentum fraction
- η_i jet pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$(\gamma^* g)$

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sum_f H^{\mu\nu}_{\gamma^* g\to f\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_{g,\mu\nu}(\xi,\boldsymbol{b},\mu,\zeta_1) \times S_{\gamma g}(\boldsymbol{b},\eta_1,\eta_2,\mu,\zeta_2)\Big(\mathcal{C}_f(\boldsymbol{b},R,\mu)J_f(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\Big)$$

$(\gamma^* f)$

$$\frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\boldsymbol{r}_T} = \sigma_0^{fU} \sum_f H^U_{\gamma^* f\to g\bar{f}}(\hat{s},\hat{t},\hat{u},\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T) F_f(\xi,\boldsymbol{b},\mu,\zeta_1) \times S_{\gamma f}(\boldsymbol{b},\zeta_2,\mu)\Big(\mathcal{C}_g(\boldsymbol{b},R,\mu)J_g(p_T,R,\mu)\Big)\Big(\mathcal{C}_{\bar{f}}(\boldsymbol{b},R,\mu)J_{\bar{f}}(p_T,R,\mu)\Big)$$

New soft function

n - incoming beam direction

v_1 - jet 1 direction

v_2 - jet 2 direction

Soft function

$$\hat{S}_{\gamma g}(\boldsymbol{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\boldsymbol{b}, -\infty)_{ca'} \mathrm{Tr}\Big[S_{v_2}(+\infty, \boldsymbol{b}) T^{a'} S_{v_1}^\dagger(+\infty, \boldsymbol{b}) \times S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \Big] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

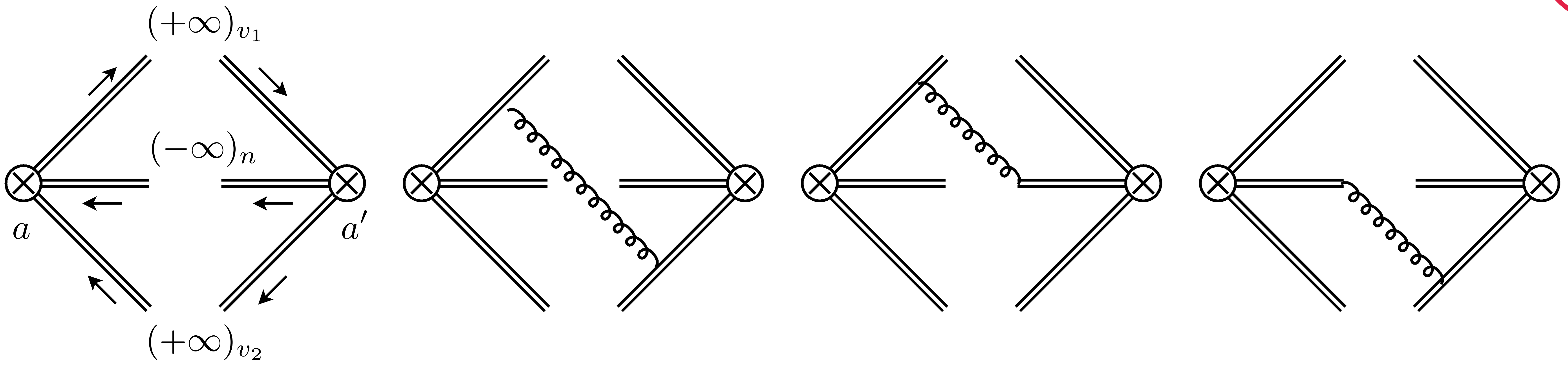
Wilson lines

$$S_v(+\infty, \xi) = P\exp\Big[-ig \int_0^{+\infty} d\lambda \; v \cdot A(\lambda v + \xi) \Big] \qquad S_{\bar{v}}^\dagger(+\infty, \xi) = P\exp\Big[ig \int_0^{+\infty} d\lambda \; \bar{v} \cdot A(\lambda \bar{v} + \xi) \Big]$$

$$S_n(+\infty, \xi) = \lim_{\delta^+ \to 0} P\exp\Big[-ig \int_0^{+\infty} d\lambda \; n \cdot A(\lambda n + \xi) e^{-\delta^+ \lambda} \Big]$$

δ - regulator !!!

Echevarría, Scimemi, Vladimirov, 2016



+ virtual diagrams at one-loop order...

New soft function

Finite result

$(\gamma^* g)$ - channel

$$\hat{S}^{\text{finite}}_{\gamma g}(\boldsymbol{b}) = 1 + a_s \Big\{ C_A \Big[\ln\big(B\,\mu^2 e^{2\gamma_E}\big) \Big(\ln\big(B\mu^2 e^{2\gamma_E}\big) + 4 \ln\Big(\frac{\sqrt{2}\,\delta^+}{\mu}\Big) + 2\ln(2A_n) \Big) - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{6} - 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] + C_F \Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] \Big\}$$

with... $A_{\boldsymbol{b}} = \dfrac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\dfrac{\hat{s}}{4\,p_T^2\,c_b^2}$ $\qquad A_n = \dfrac{(v_1 \cdot v_2)}{2\,(v_1 \cdot n)(v_2 \cdot n)}$

Zero-bin subtraction

- We need to subtract the zero-bin from the TMD beam function
- The zero-bin corresponds to the two-direction back-to-back soft function (the one used in Drell-Yan or SIDIS). Here, we use the subtraction as done in Echevarría, Idilbi, Scimemi, 2013.
- We can reorganize the zero-bin to obtain rapidity divergence-free function as expressed in the cross-section factorization
- This leads to the universal TMDPDF and a rapidity divergence-free new TMD soft function and the introduction of the scale ζ

$$B_i^{\text{un.}}(\xi, \boldsymbol{b}, \mu, k^-/\delta^-) \quad \longrightarrow \quad F_i(\xi, \boldsymbol{b}, \mu, \zeta_1)$$

$$\hat{S}_{\gamma i}(\boldsymbol{b}, \mu, \sqrt{A_n}\,\delta^+) \quad \longrightarrow \quad S_{\gamma i}(\boldsymbol{b}, \mu, \zeta_2)$$

Zero-bin subtraction

Due to the rapidity divergencies structure of the two-direction soft function (zero-bin)

can be split as

$$S(\boldsymbol{b},\mu,\sqrt{\delta^+\delta^-}) = S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)$$

ν arbitrary positive number

$$\left(S_i^{\text{bare}}(\boldsymbol{b},\delta)\right)^{\frac{1}{2}} = 1 + a_s C_i\left\{-\frac{2}{\epsilon^2}+\frac{4}{\epsilon}\ln\left(\frac{\sqrt{2}\,\delta}{\mu}\right)+\ln\left(B\mu^2 e^{2\gamma_E}\right)\left[4\ln\left(\frac{\sqrt{2}\,\delta}{\mu}\right)+\ln\left(B\mu^2 e^{2\gamma_E}\right)\right]+\frac{\pi^2}{6}\right\}$$

In this way we defined the rapidity divergence-free objects

Universal TMDPDF

$$F_i(\xi,\boldsymbol{b},\mu,\zeta_1) = \left.\frac{B_i^{\text{un.}}(\xi,\boldsymbol{b},\mu,k^-/\delta^-)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^-/\nu)}\right|_{\sqrt{2}\,k^-/\nu\to\sqrt{\zeta_1}}$$

Rapidity divergence-free new soft function

$$S_{\gamma i}(\boldsymbol{b},\mu,\zeta_2) = \left.\frac{\hat{S}_{\gamma i}(\boldsymbol{b},\mu,\sqrt{A_n}\,\delta^+)}{S^{\frac{1}{2}}(\boldsymbol{b},\mu,\delta^+\nu)}\right|_{\nu/\sqrt{2A_n}\to\sqrt{\zeta_2}}$$

ζ scale associated with the δ-regulator and zero-bin split

Zero-bin subtraction

ζ scale

The scale can be removed from the final result by introducing the constrain

$$\zeta_1\,\zeta_2 = \frac{(k^-)^2}{A_n} = \frac{\hat{u}\,\hat{t}}{\hat{s}}$$

In the Breit-frame this leads to

$$\zeta_1\,\zeta_2 = p_T^2$$

Notice that
- ζ_1 has square mass dimension
- ζ_2 is dimensionless

→ $\zeta_1 = p_T^2$, $\zeta_2 = 1$: natural way of choosing the scale

Procedure totally analogous to the one used in Drell-Yan or SIDIS

This allows to use ζ-prescription for TMDPDF and SF evolution

Scimemi, Vladimirov, 2020

Zero-bin subtraction

Subtracted soft function, finite result

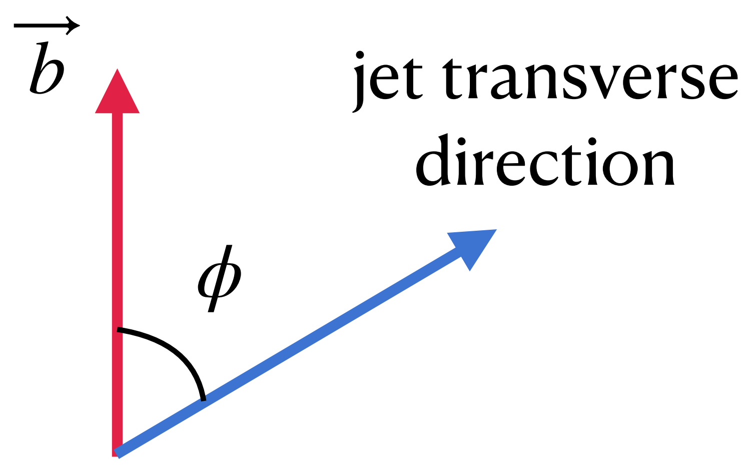
$(\gamma^* g)$ - channel

$$S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2) = 1 + a_s \Big\{ C_F \Big[\frac{\pi^2}{3} + 2\ln^2\Big(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\Big) + 4\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big]$$
$$+ C_A \Big[-2\ln\big(B\mu^2 e^{2\gamma_E}\big) \boxed{\ln \zeta_2} - \ln^2(-A_{\boldsymbol{b}}) - \frac{\pi^2}{3} - 2\mathrm{Li}_2(1 + A_{\boldsymbol{b}}) \Big] \Big\} + \mathcal{O}(a_s^2)$$

with... $$A_{\boldsymbol{b}} = \frac{(v_1 \cdot v_2)}{2\,(v_1 \cdot \hat{b})\,(v_2 \cdot \hat{b})} = -\frac{\hat{s}}{4\,p_T^2\, c_b^2}$$

Consistency check

Dijet-production



$$\frac{d}{d\ln\mu}G(\mu) = \gamma_G(\mu)\,G(\mu)$$

$(\gamma^* g)$-channel $\quad \gamma_{H_{\gamma g}} + \gamma_{S_{\gamma g}} + \gamma_{F_g} + 2\gamma_{J_f} + \gamma_{\mathcal{C}_1} + \gamma_{\mathcal{C}_2} + \gamma_\alpha = 0$

$(\gamma^* f)$-channel $\quad \gamma_{H_{\gamma f}} + \gamma_{S_{\gamma f}} + \gamma_{F_f} + \gamma_{J_f} + \gamma_{J_g} + \gamma_{\mathcal{C}_f} + \gamma_{\mathcal{C}_g} + \gamma_\alpha = 0$

The sum of all anomalous dimensions should cancel for each channel

ζ-logs

ϕ-logs

$$\gamma^{[1]}_{S_{\gamma g}} = 4\Big\{ - C_A \ln\zeta_2 + 2C_F\Big[\ln\big(B\mu^2\, e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big]\Big\},$$

$$\gamma^{[1]}_{S_{\gamma f}} = 4\Big\{(C_F + C_A)\Big[\ln\big(B\mu^2 e^{2\gamma_E}\big) - \ln\hat{s} + \ln p_T^2 + \ln\big(4c_{\boldsymbol{b}}^2\big)\Big] + (C_F - C_A)\Big[\ln\Big(\frac{\hat{t}}{\hat{u}}\Big) - \kappa(v_f)\Big] - C_F\ln\zeta_2\Big\}$$

$$\gamma^{[1]}_{F_i} = 4C_i\Big[-\ln\Big(\frac{\zeta_1}{\mu^2}\Big) + \gamma_i\Big],$$

$$\gamma^{[1]}_{\mathcal{C}_g} = 4C_A\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_g)\Big]$$

$$\gamma^{[1]}_{\mathcal{C}_i} = 4C_F\Big[-\ln\Big(B\mu^2\, e^{2\gamma_E}\Big) + \ln R^2 - \ln\big(4c_{\boldsymbol{b}}^2\big) + \kappa(v_i)\Big]$$

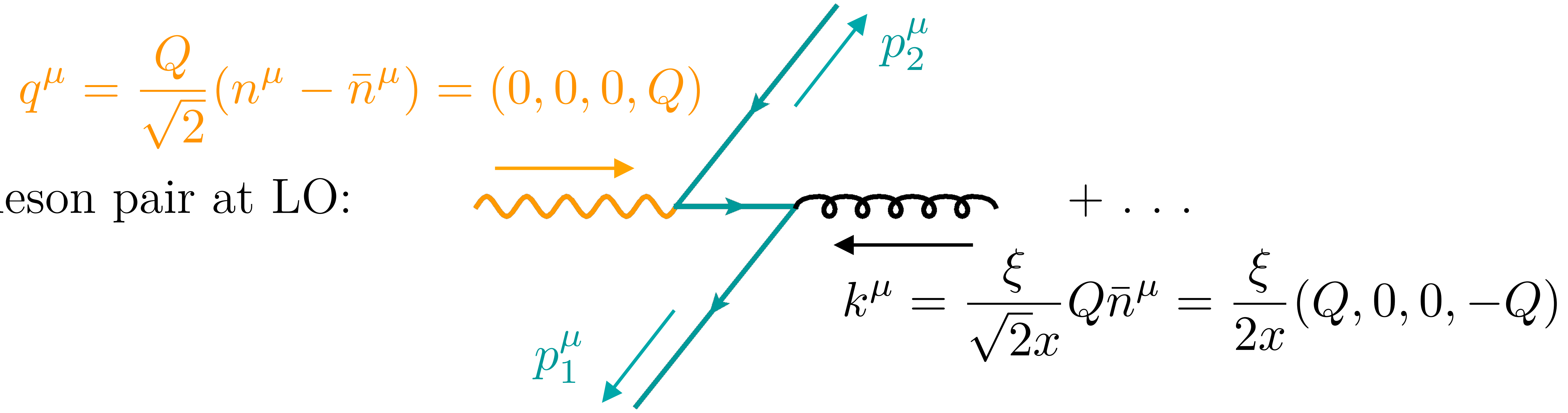
$$\kappa(v_f) = -\kappa(v_{\bar{f}}) = -\kappa(v_g) = i\pi\,\mathrm{sign}(c_{\boldsymbol{b}})$$

They cancel !!!

Heavy-meson pair production

$$\ell + h \to \ell' + H + \bar{H} + X$$

heavy meson pair at LO:



- Experimentally more challenging
- Observation of charmed mesons could be possible

Arratia, Furletova, Hobbs, Olness, Nguyen et al. 2020

Li, Liu, Vitev, 2020

Chudakov, Higinbotham, Hyde, Furletov, Furletova, Nguyen, 2016

Cross-section factorization

Heavy meson pair production

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\boldsymbol{r}_T}$$

We measure over

- x parton momentum fraction
- η_H, $\eta_{\bar{H}}$ heavy meson pseudorapidity
- p_T transverse momentum
- $\boldsymbol{r}_T$ transverse momentum imbalance

$$\frac{d\sigma(\gamma^* g)}{dx d\eta_H d\eta_{\bar{H}} dp_T d\boldsymbol{r}_T} = H^{\mu\nu}_{\gamma^* g \to Q\bar{Q}}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} \exp(i\boldsymbol{b}\cdot\boldsymbol{r}_T)\, F_{g,\mu\nu}(\xi, \boldsymbol{b}, \mu, \zeta_1)$$
$$\times\, S_{\gamma g}(\boldsymbol{b}, \mu, \zeta_2)\, J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu)\, J_{\bar{Q}\to\bar{H}}(\boldsymbol{b}, p_T, m_Q, \mu)$$

Fickinger, Fleming, Kim, Mereghetti, 2016

Region sensitive to TMD	$\lvert\boldsymbol{r}_T\rvert \ll p_T^{H,\bar{H}}$
Factorization for highly boosted heavy mesons	$p_T^H \gg m_H$
We have a new scale	m_Q

Refactorization of heavy-quark fragmentation

Heavy meson pair production

We use heavy-quark jet function to describe the fragmentation of heavy mesons from heavy quarks. In the limit $r_T \ll p_T$ there are two scales that need to be resummed

$$\mu_+ = m_Q, \qquad \text{and} \qquad \mu_{\mathcal{J}} = m_Q \frac{r_T}{p_T}$$

To do this we use **bHQET (boosted heavy quark effective theory)** to factorize the jet function into a hard matching coefficient and a TMD matrix element

$$J_{Q\to H}(\boldsymbol{b}, p_T, m_Q, \mu) = H_+(m_Q,\mu)\mathcal{J}_{Q\to H}\left(\boldsymbol{b}, \frac{m_Q}{p_T}, \mu\right)$$

Appears for the first time

Known up to two-loops

Refactorization of heavy-quark fragmentation

Heavy meson pair production

Up to one loop order we find

$$H_+(m_Q,\mu) = 1 + \frac{\alpha_s}{4\pi}C_F\left\{\ln\left(\frac{\mu^2}{m_Q^2}\right) + \ln^2\left(\frac{\mu^2}{m_Q^2}\right) + 8 + \frac{\pi^2}{6}\right\}$$

$$\gamma_+ = \frac{\alpha_s C_F}{\pi}\left\{\frac{1}{2} - \ln\left(\frac{m_Q^2}{\mu^2}\right)\right\}$$

$$\mathcal{J}^{\text{bare}}_{Q\to Q}\left(\boldsymbol{b},\frac{m_Q}{p_T}\right) = 1 + \frac{\alpha_s C_F}{\pi}\left\{-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\left[1 - 2\ln\mathcal{R}\right] + \ln\mathcal{R} - \ln^2\mathcal{R} - \frac{5\pi^2}{24}\right\}$$

$$\gamma_{\mathcal{J}} = \frac{\alpha_s C_F}{\pi}\left\{1 - 2\ln\mathcal{R}\right\}$$

with... $\mathcal{R} = -\dfrac{i\,p_T\mu\,e^{\gamma_E}(\boldsymbol{v}\cdot\boldsymbol{b})}{m_Q|\boldsymbol{v}|}$

We have separated scales and can now be resummed

Anomalous dimension are consistent $\gamma_{\mathcal{J}} + \gamma_+ = \gamma_J + \gamma_{\mathcal{C}_f}$ → Consistent !!!

Connection to the fragmentation shape function

Heavy meson pair production

We can see that the shape function is related to the bHQET jet function

$$\mathcal{J}_{Q\to H}(\boldsymbol{b}) = \frac{m_H}{\sqrt{2}\,p_H^-}\tilde{S}_{Q\to H}\left(\tau \to \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\sqrt{2}}\right)$$

- We can check our NLO calculation (finite terms)
- We can get the jet function AD up to two loops

Fickinger, Fleming, Kim, Mereghetti, 2016

This sum is known up to three-loops...

$$\gamma_{S_{\gamma g}} = -\left(\gamma_{H_{\gamma g}} + \gamma_{F_g} + \gamma_\alpha + \gamma_{\mathcal{J}}(\boldsymbol{v}_1) + \gamma_{\mathcal{J}}(\boldsymbol{v}_2) + 2\gamma_+\right)$$

Soft function anomalous dimension

$$\gamma_{S_{\gamma g}} = \gamma_{\text{cusp}}\left[2C_F \ln\left(\frac{B\mu^2 e^{2\gamma_E}}{-A_{\boldsymbol{b}}}\right) - C_A \ln\zeta_2\right] + \delta\gamma_{S_{\gamma g}}$$

$$\delta\gamma^{[1]}_{S_{\gamma g}} = 0$$

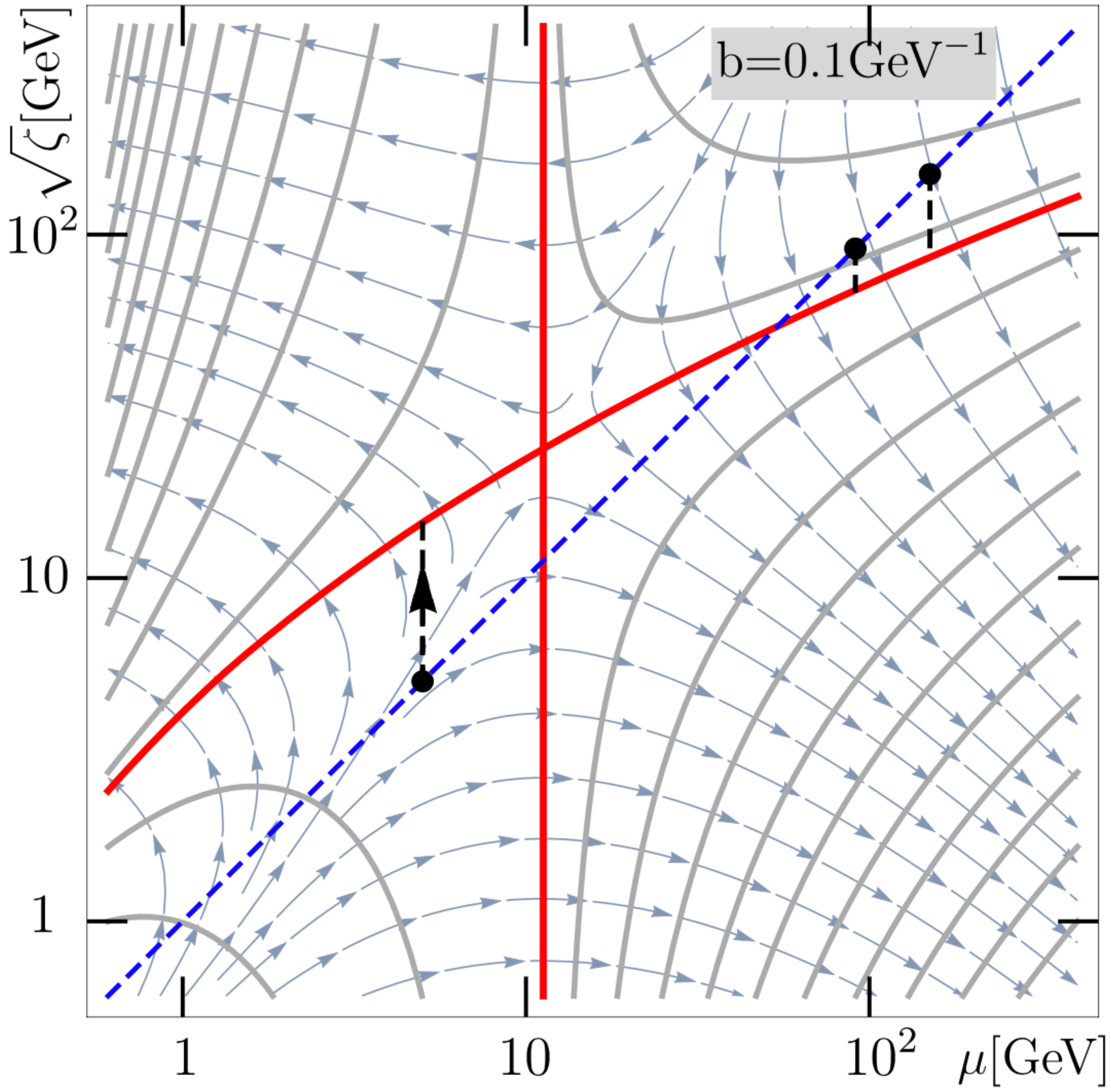
$$\delta\gamma^{[2]}_{S_{\gamma g}} = C_F\left[C_A\left(\frac{1616}{27} - \frac{22}{9}\pi^2 - 56\,\zeta_3\right) + n_f T_F\left(-\frac{448}{27} + \frac{8}{9}\pi^2\right)\right]$$

$$\delta\gamma^{[3]}_{S_{\gamma g}} = \ldots$$

Evolution, ζ-prescription

Scimemi, Vladimirov, 2018
Scimemi, Vladimirov, 2020

fixed μ evolution



Evolution kernel is given by

Saddle point

$$S(\boldsymbol{b};\mu_f,\zeta_{2,f}) = \exp\left[\int_P (\gamma_S(\mu,\zeta_2)\,d\ln\mu - \mathcal{D}_S(\mu,b)d\ln\zeta_2)\right] S(\boldsymbol{b};\overbrace{\mu_0,\zeta_{2,0}})$$

$$\left.\begin{aligned}\frac{d}{d\ln\mu}S(\boldsymbol{b};\mu,\zeta) &= \gamma_S(b;\mu,\zeta)S(\boldsymbol{b};\mu,\zeta)\\ \frac{d}{d\ln\zeta}S(\boldsymbol{b};\mu,\zeta) &= -\mathcal{D}_S(b,\mu)S(\boldsymbol{b};\mu,\zeta)\end{aligned}\right\} \longrightarrow \boxed{\boldsymbol{\nabla}F = \boldsymbol{E}F}$$

$$\boldsymbol{E} = (\gamma_S(b,\mu,\zeta), -\mathcal{D}_S(b,\mu))$$

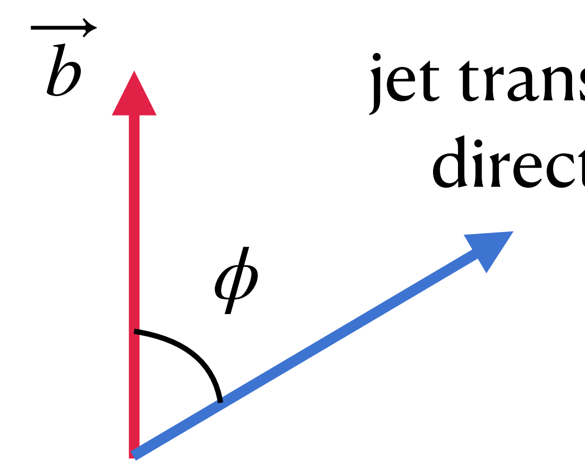
Equipotential (null-evolution) line is given by $\gamma_S = 2\mathcal{D}_S \dfrac{d\ln\zeta_\mu}{d\ln\mu^2}$

gluon channel solution $\quad \zeta_{2,\mu}^{\gamma * g}(\boldsymbol{b},\mu) = \left(\dfrac{\mu}{\mu_0}\right)^{\frac{2C_F}{C_A}} \zeta_{2,0}e^{v_S(\boldsymbol{b},\mu)}$

perturbative

$$\boxed{R_S((\mu_0,\zeta_{2,0}) \to (\mu_f,\zeta_f)) = \left(\frac{\zeta_f}{\zeta_{2,\mu}(\boldsymbol{b},\mu_f)}\right)^{-D_S(\boldsymbol{b},\mu_f)}}$$

ϕ-dependent part



SF (and CSF) depends on the angle between the transverse plane and the jet direction ϕ

Our approach is to separate the ϕ-dependent part of the evolution and integrate it

$$\gamma_S(b,\mu,\zeta,\phi) = \bar{\gamma}_S(b,\mu,\zeta) + \gamma_S^\phi(\phi)$$

$$S(\boldsymbol{b};\mu_f,\zeta_{2,f}) = \exp\left[\int_{\mu_0}^{\mu_f}\left(\gamma_S^\phi(\phi)d\ln\mu\right)\right]\exp\left[\int_P\left(\bar{\gamma}_S(\mu,\zeta_2)d\ln\mu - \mathcal{D}_S(\mu,b)d\ln\zeta_2\right)\right]S(\boldsymbol{b};\mu_0,\zeta_{2,0})$$

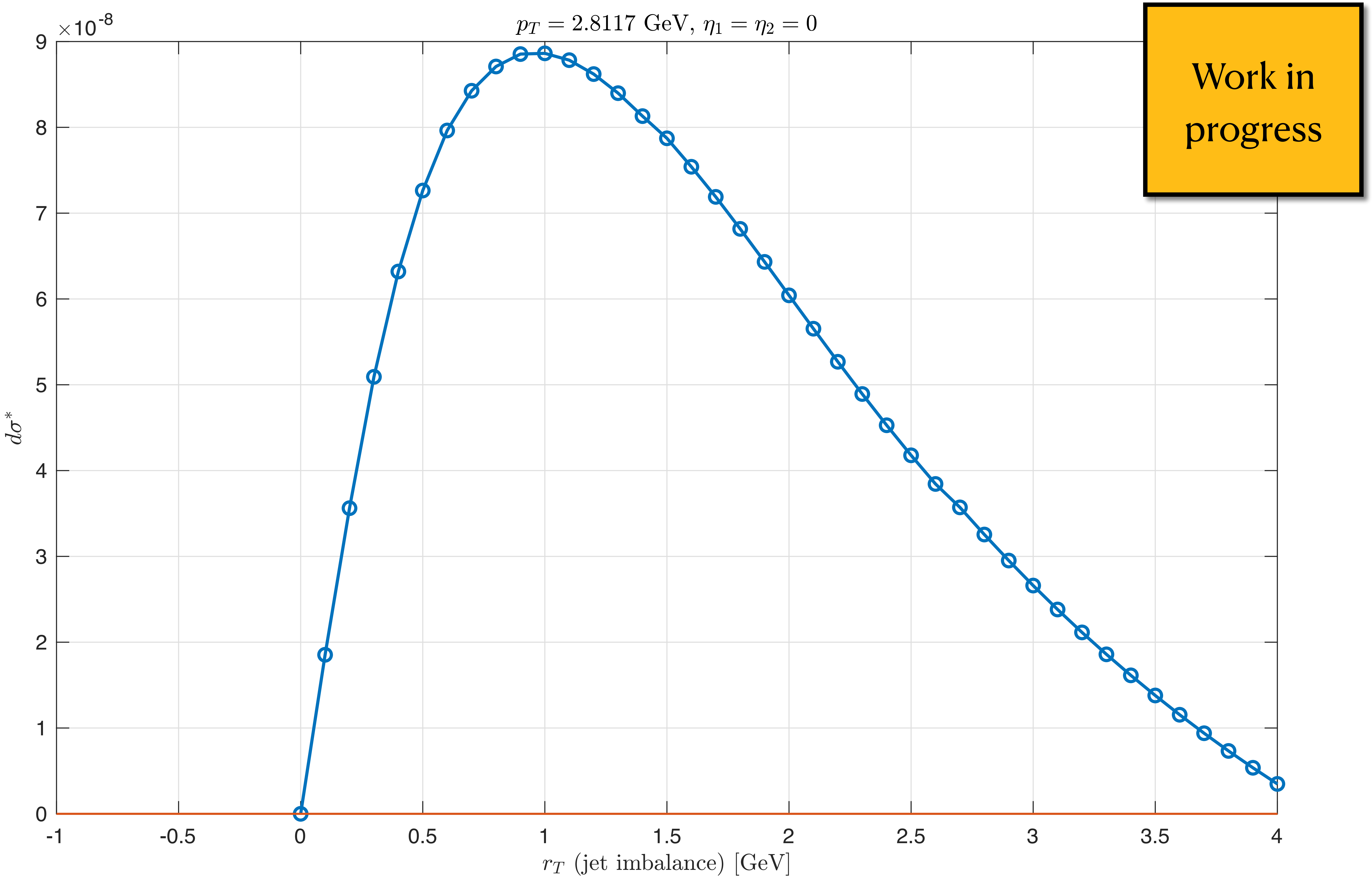
R_S^ϕ → Integrate over ϕ

R_S → ζ-prescription

...and we do the same for CSF

This ϕ-dependent factors should be integrated along with the ϕ-dependent terms of the perturbative result

Dijets gluon channel (preliminary) plot



arTeMiDe has been used to get these plots

https://teorica.fis.ucm.es/artemide/
https://github.com/vladimirovalexey/artemide-public."

New functions and evolution kernels has been included

Conclusion

- We have established factorization for the dijet and heavy meson production
- Both cases can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the ζ-prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- We are working on codes for the phenomenology of these processes