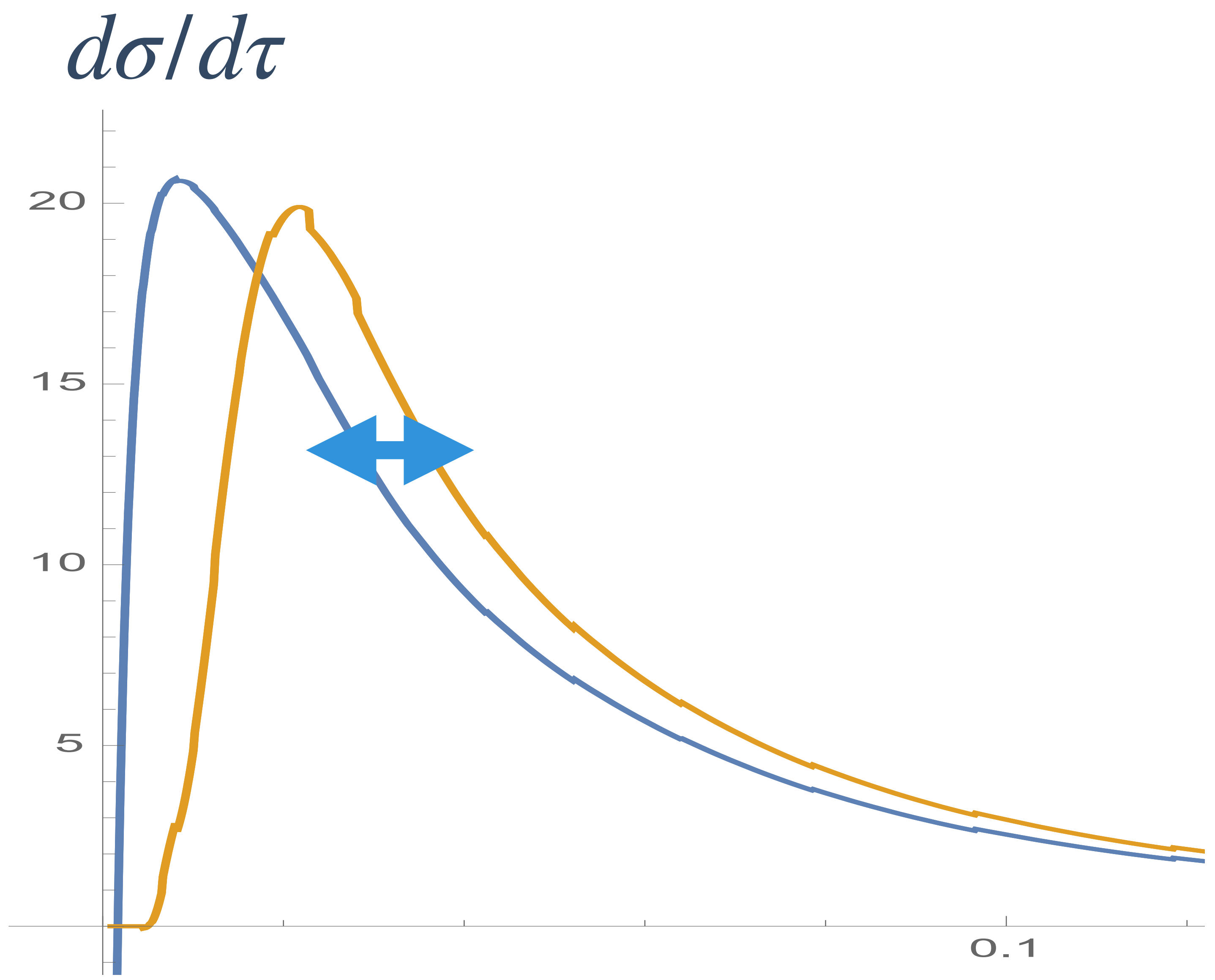
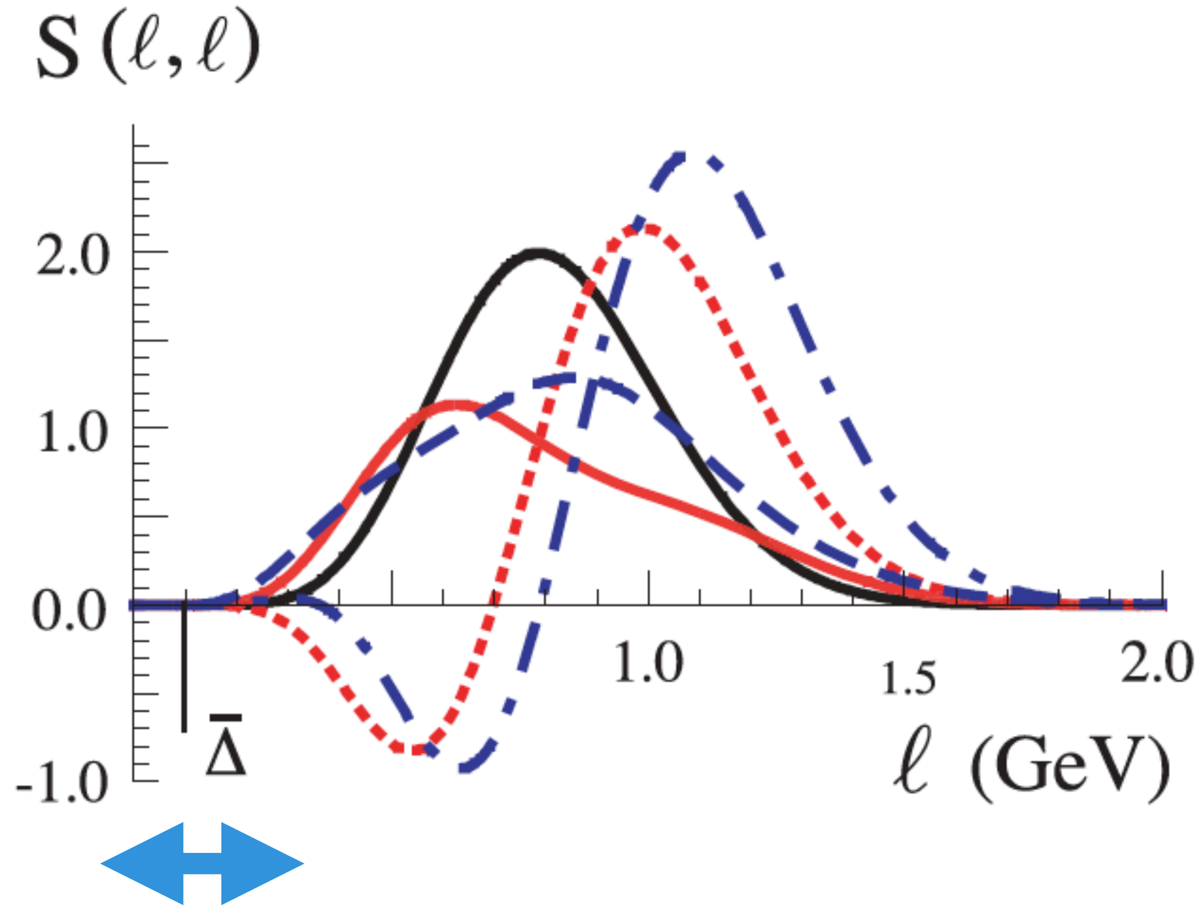
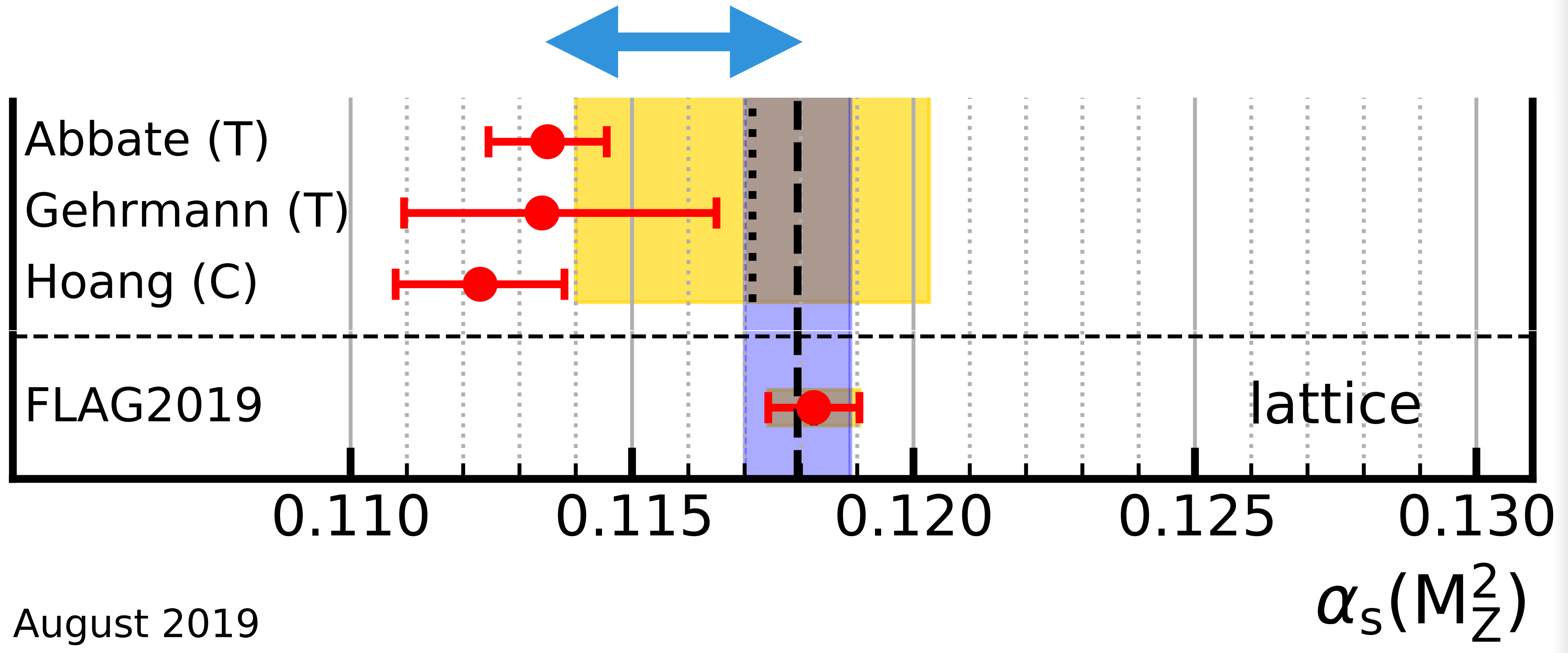
Angularities, gaps, and α_s





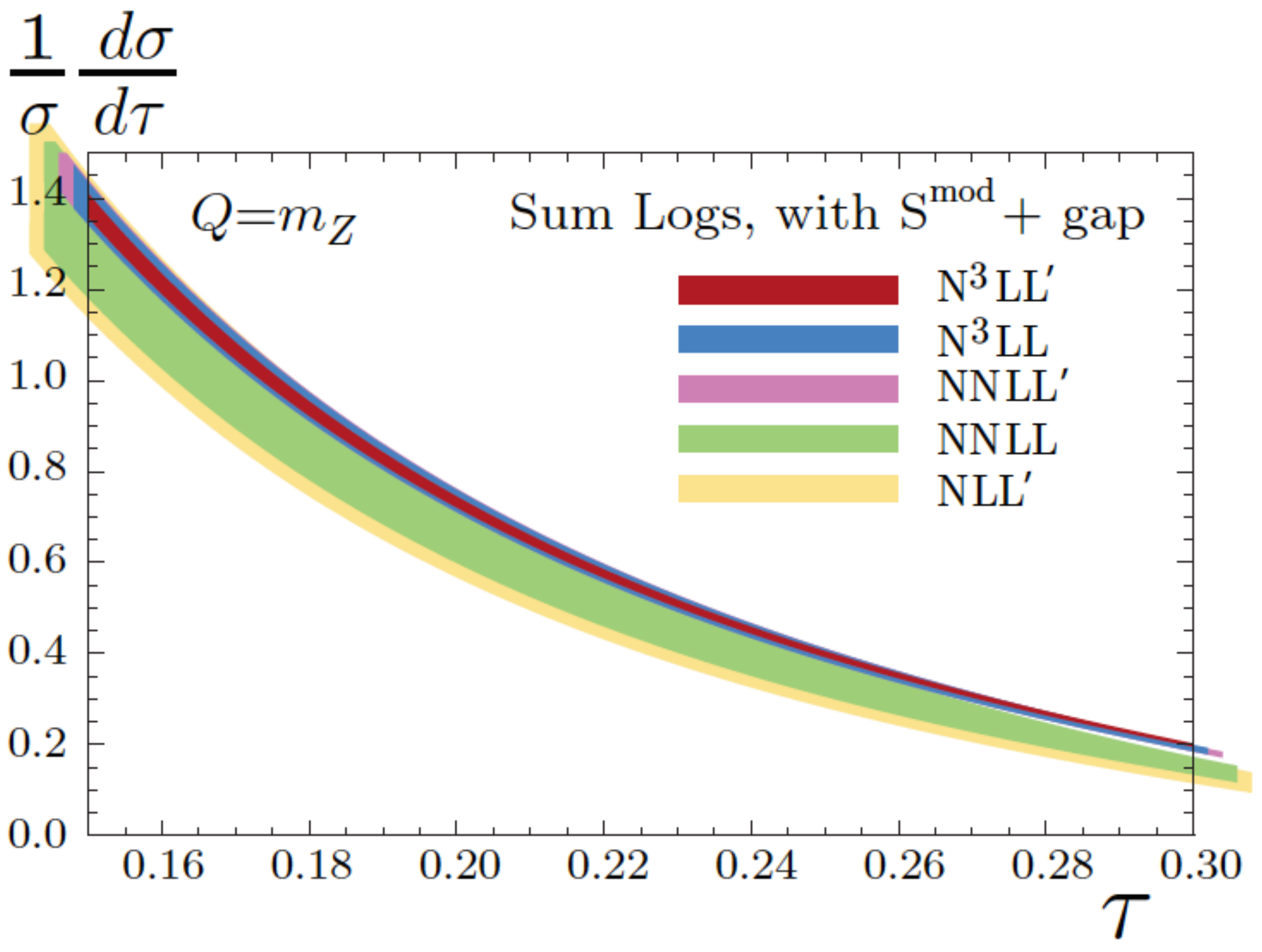


Christopher Lee (LANL)

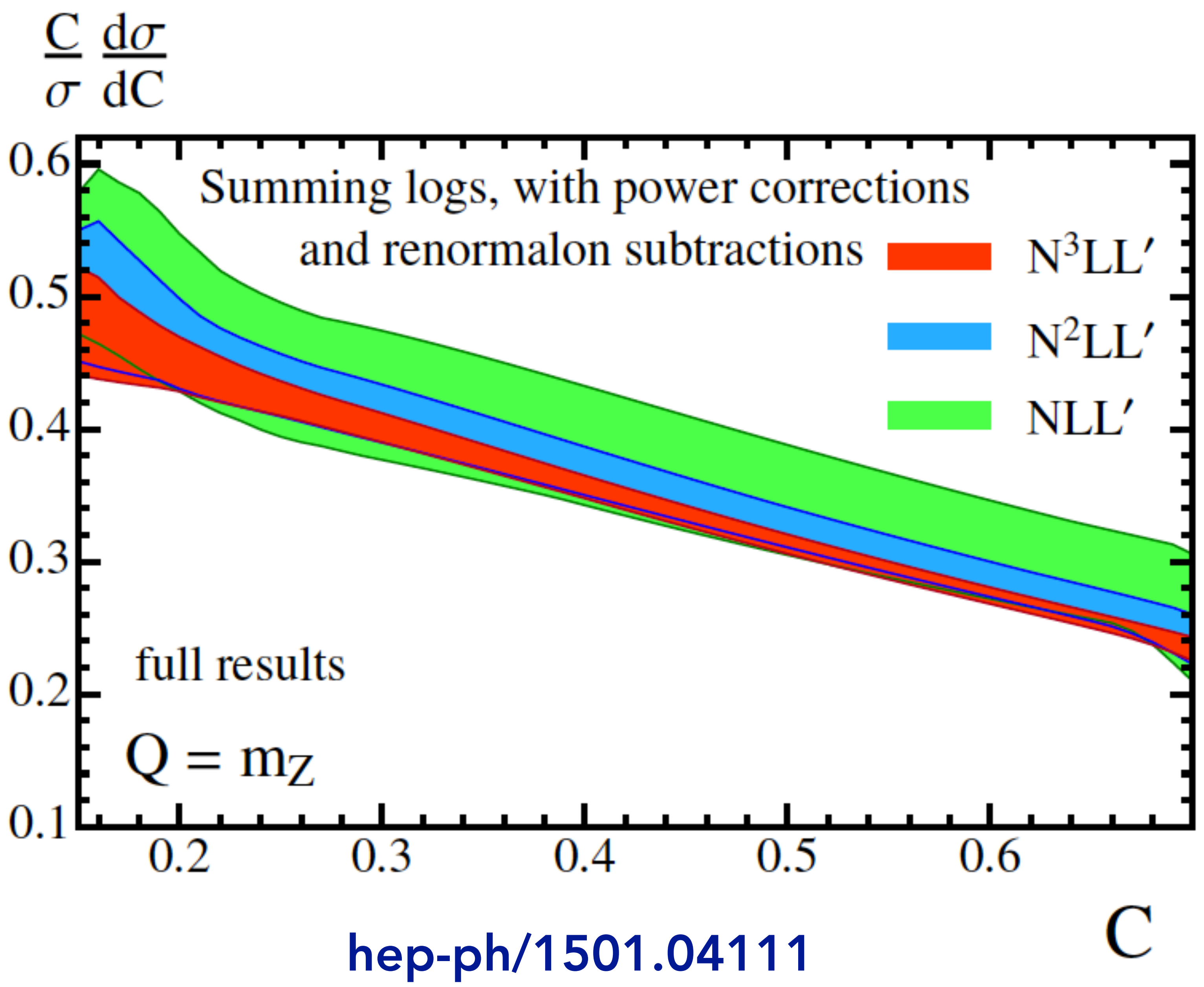
with G. Bell, Y. Makris, J. Talbert, B. Yan, B. Yoon

A triumph for SCET

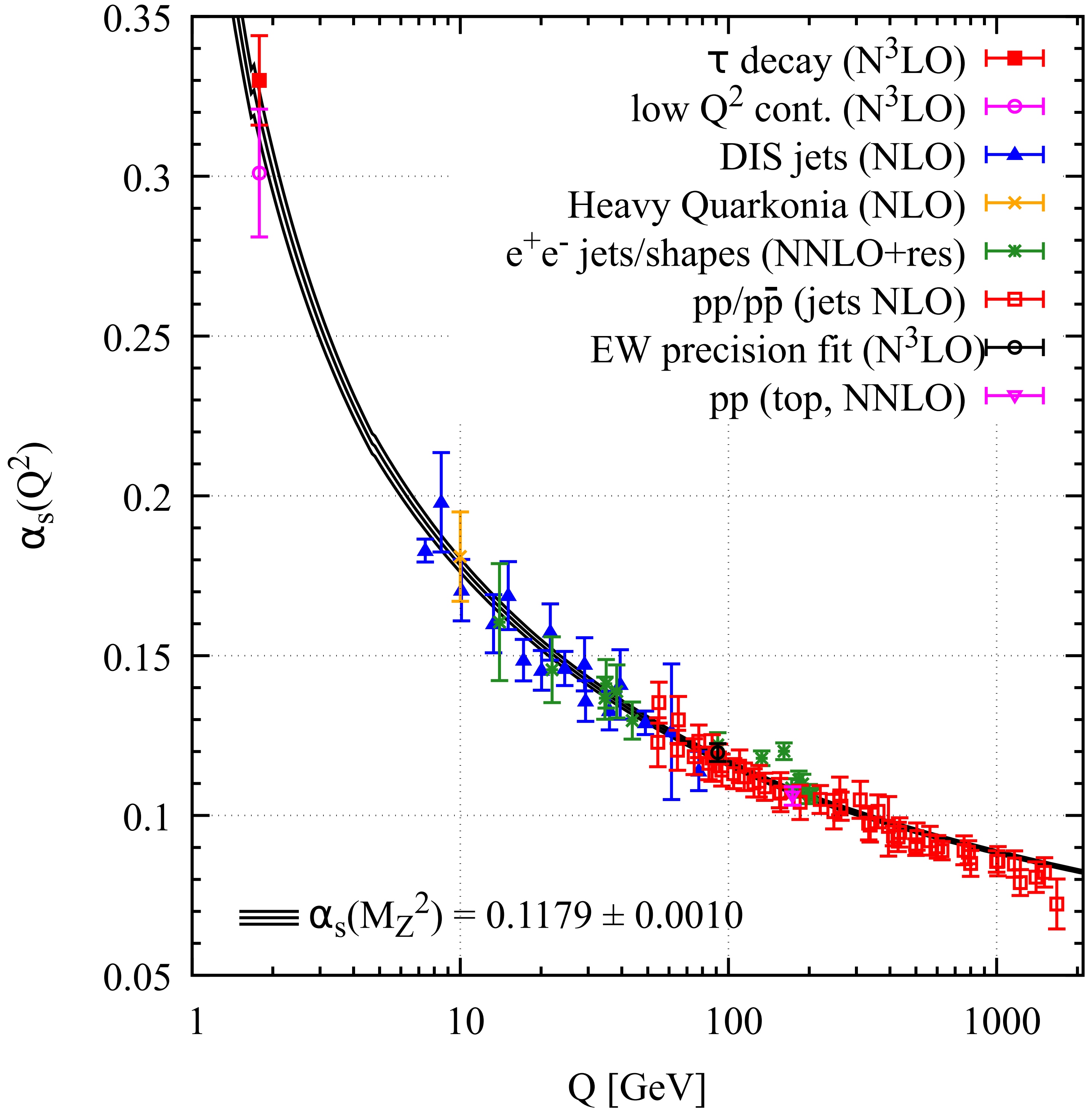
- **First N³LL′ resummed event shape distributions with state-of-the-art treatment of nonperturbative corrections, e.g.:**



hep-ph/1006.3080



hep-ph/1501.04111



Make e+e- event shapes some of the most precise ways to determine α_s

An anomaly?

2020 PDG world average: .1179 +- .0010

Thrust at N^{3}LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

Riccardo Abbate,[1] Michael Fickinger,[2] André H. Hoang,[3] Vicent Mateu,[3] and Iain W. Stewart[1]

hep-ph/1006.3080

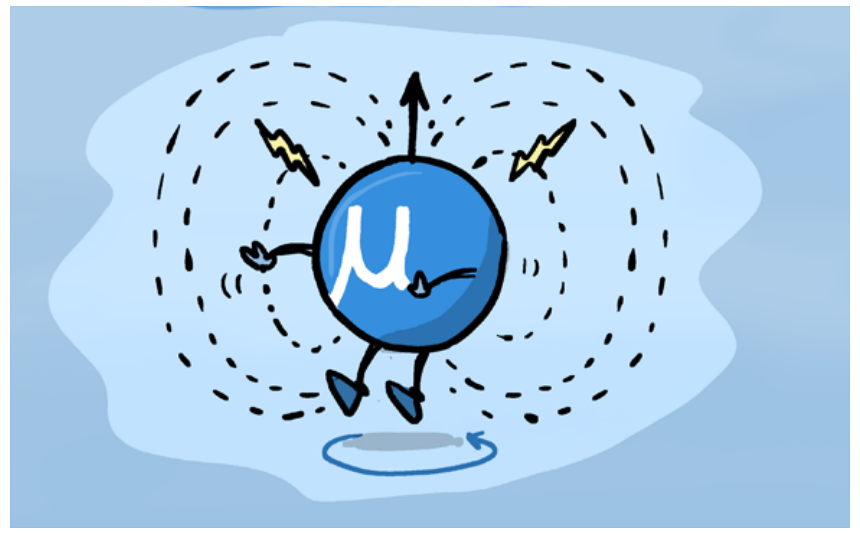
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

A Precise Determination of α_s from the C-parameter Distribution

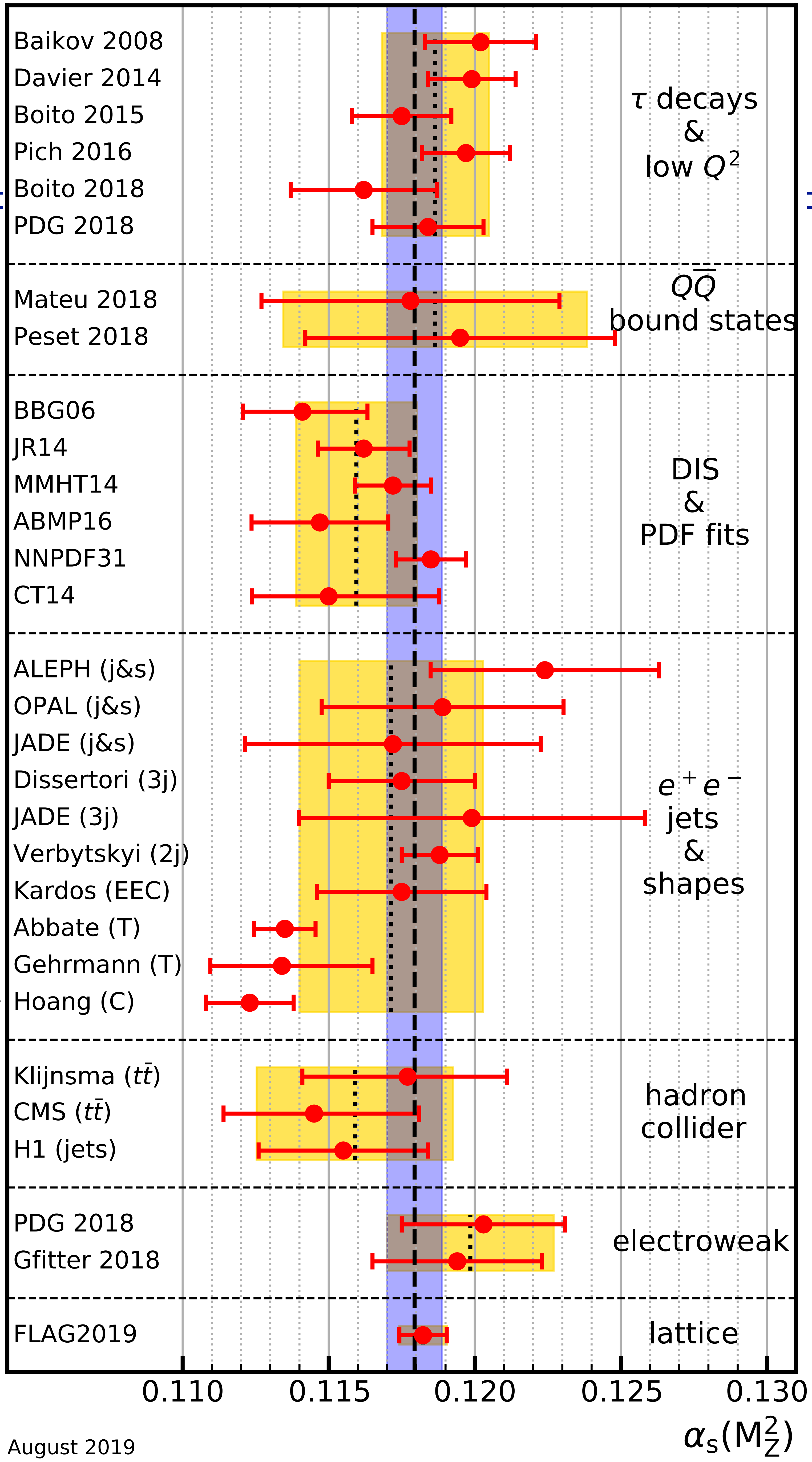
André H. Hoang,[1,2] Daniel W. Kolodrubetz,[3] Vicent Mateu,[1] and Iain W. Stewart[3]

hep-ph/1501.04111

$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\rm exp} \pm 0.0007_{\rm hadr} \pm 0.0014_{\rm pert}$$

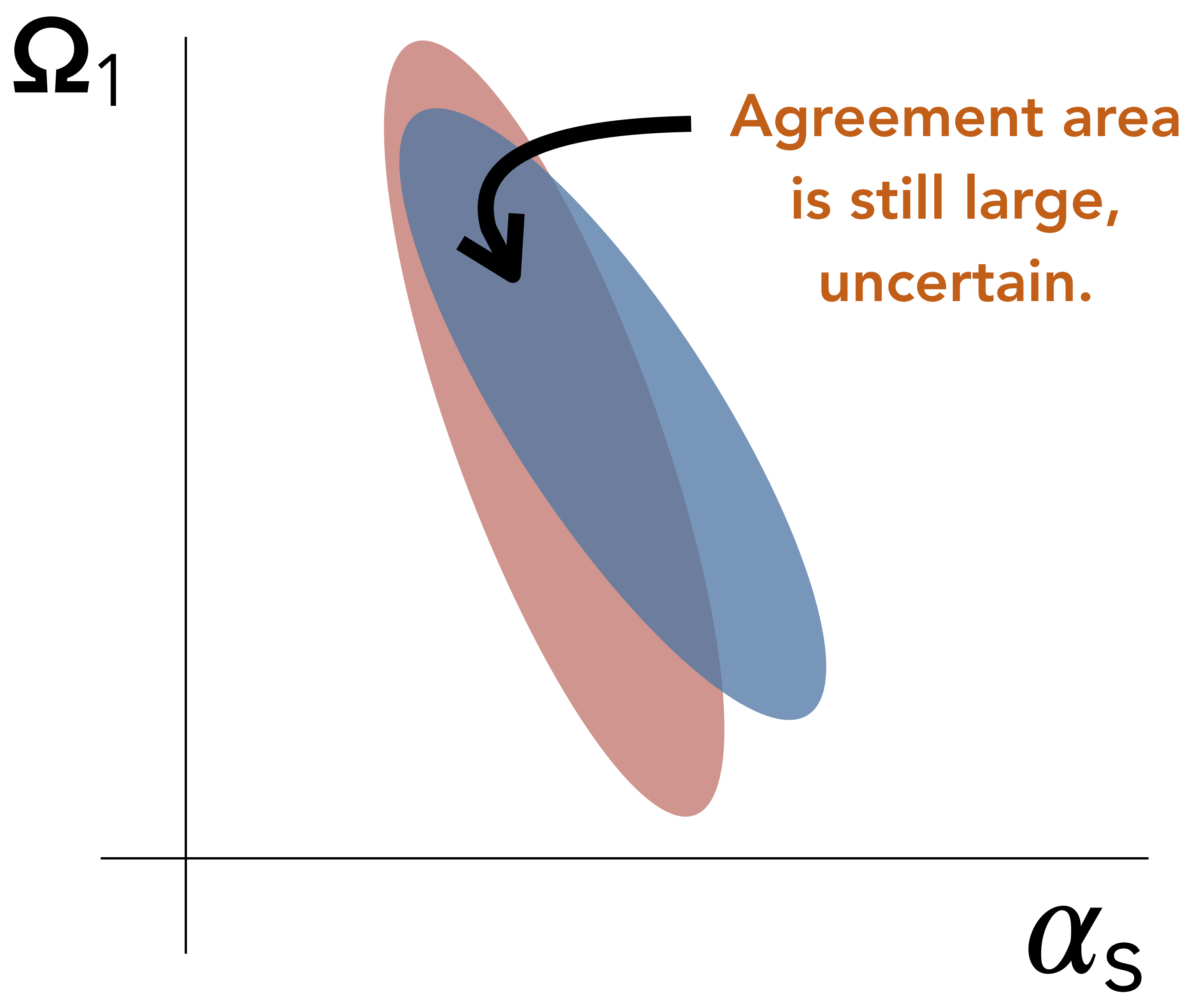


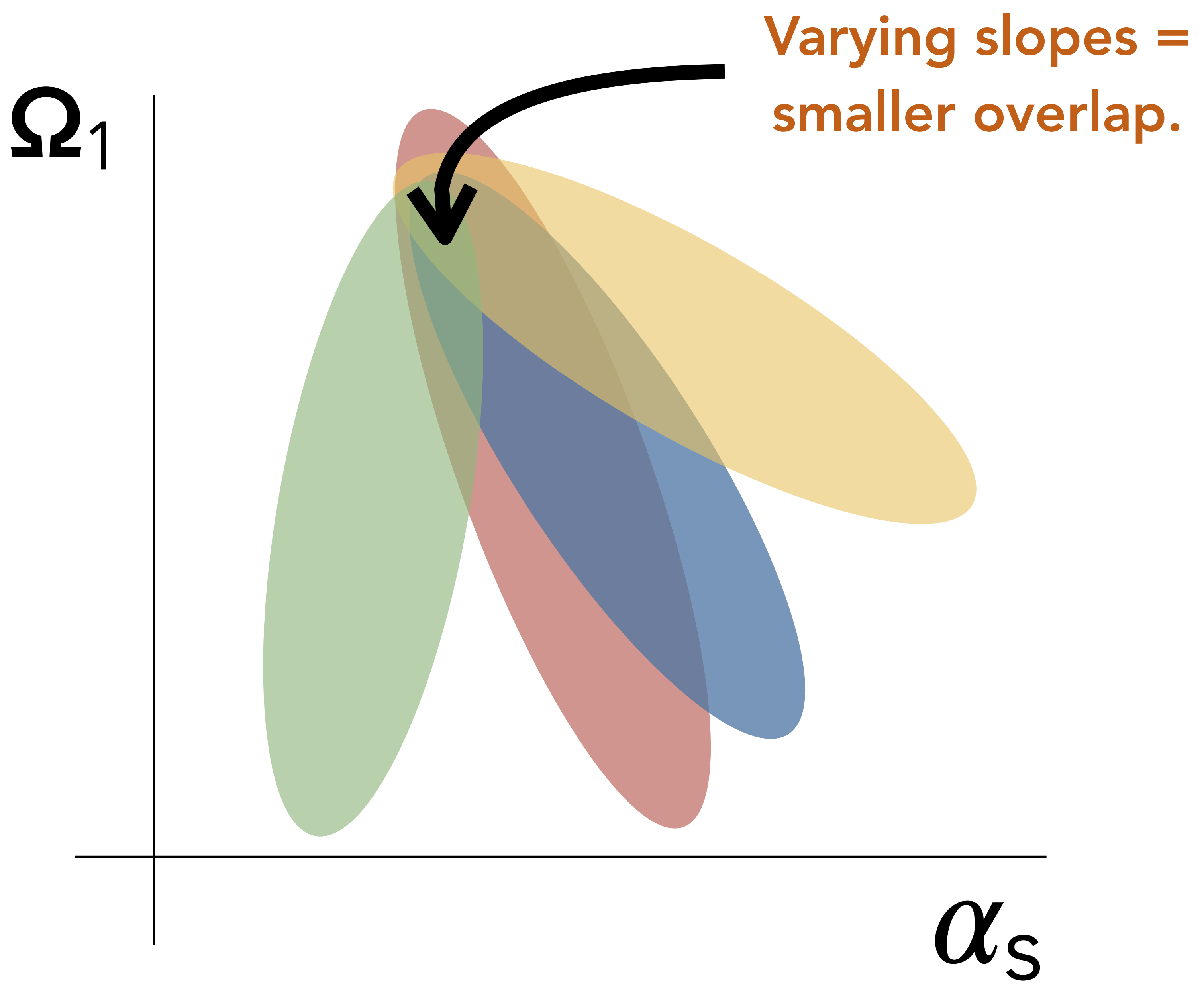
- ~4σ anomaly?



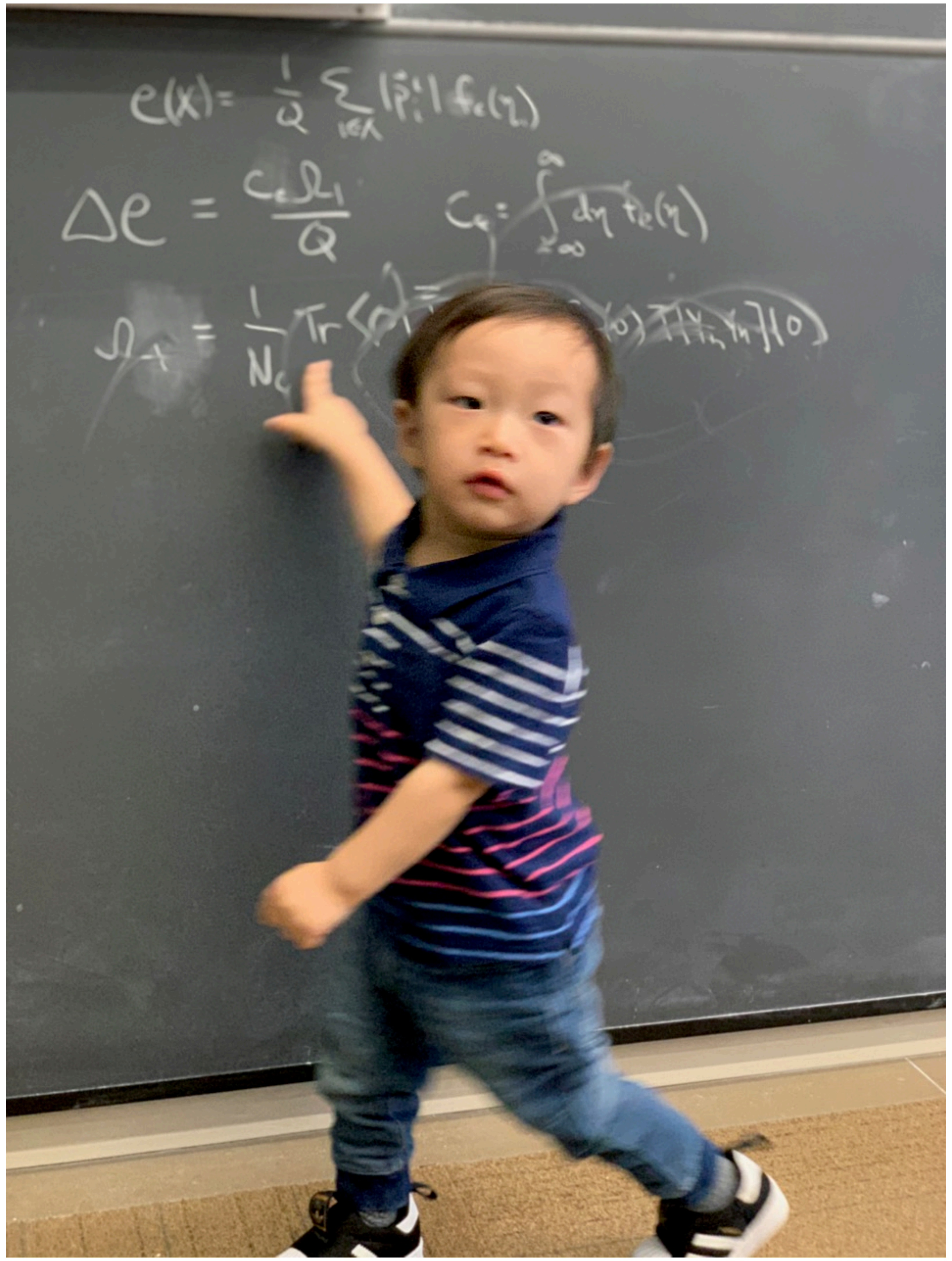
2-parameter fits to PT/NP effects: break degeneracies

- In "tail" region, leading nonperturbative effect is a shift by $c_e\Omega_1/Q$





Use different Q's.
Or different event shapes.



Caltech, March 2019

e^+e^- event shapes in SCET

- Consider *Angularities*, which can be defined in terms of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

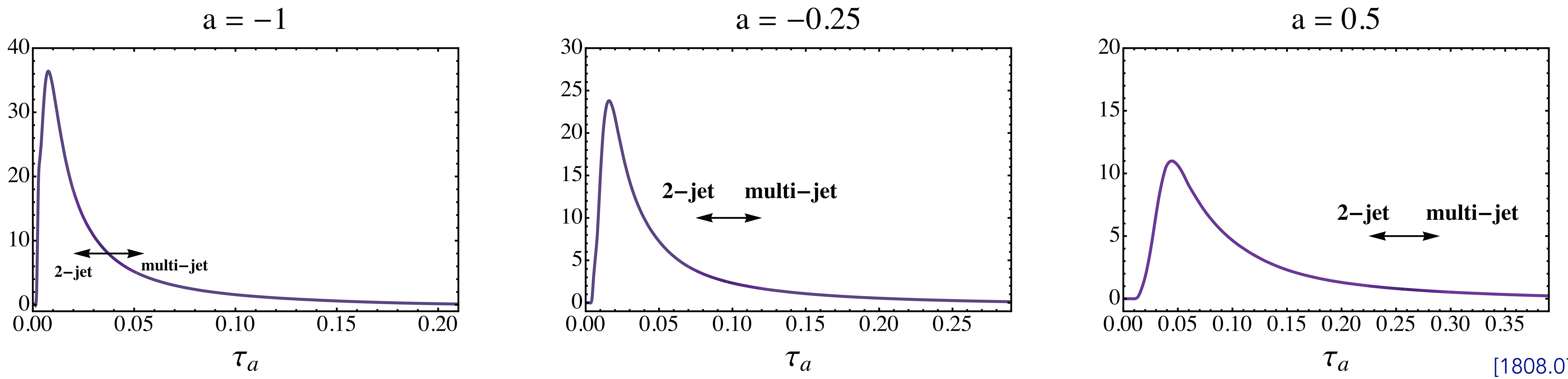
hep-ph/0303051

IR safe for a ∈ {-∞, 2}

$$\tau_a = \frac{1}{Q} \sum_i |\mathbf{p}_\perp^i| \, e^{-|\eta_i|(1-a)}$$

a = 0 <-> `Thrust'

a = 1 <-> `Jet Broadening' (for us a<1)



[1808.07867]

- Leading nonperturbative shift is $\dfrac{2\Omega_1}{Q(1-a)}$: changing *a* is like changing *Q*.

e⁺e⁻ event shapes in SCET

hep-ph/0303051
hep-ph/0801.4569
hep-ph/0901.3780

- An all-order dijet factorization theorem for the observable is easily derived in SCET:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \quad \overset{\text{RGE}}{\longleftrightarrow} \quad \frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln(\frac{Q^2}{\mu^2}) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

- Evolving all scales to/from their 'natural' settings, one arrives at the resummed cross section:

$$\frac{\sigma_{\text{sing}}(\tau_a)}{\sigma_0} = e^{K(\mu,\mu_H,\mu_J,\mu_S)} \left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)} \left(\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}\right)^{2\omega_J(\mu,\mu_J)} \left(\frac{\mu_S}{Q\tau_a}\right)^{\omega_S(\mu,\mu_S)} H(Q^2,\mu_H)$$

$$\times \widetilde{J}\left(\partial_\Omega + \ln\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}, \mu_J\right)^2 \widetilde{S}\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a}, \mu_S\right) \times \begin{cases} \frac{1}{\tau_a}\mathcal{F}(\Omega) & \sigma = \frac{d\sigma}{d\tau_a} \\ \mathcal{G}(\Omega) & \sigma = \sigma_c \end{cases}$$

$$\mathcal{F}(\Omega) = \frac{e^{\gamma_E \Omega}}{\Gamma(-\Omega)} \qquad \mathcal{G}(\Omega) = \frac{e^{\gamma_E \Omega}}{\Gamma(1-\Omega)}$$

- This predicts the singular component of the cross section. One must then match to QCD:

$$\frac{\sigma_c(\tau_a)}{\sigma_0} - \frac{\sigma_{c,\text{sing}}(\tau_a)}{\sigma_0} = r_c(\tau_a) = \theta(\tau_a)\left\{\frac{\alpha_s(Q)}{2\pi} r_c^1(\tau_a) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 r_c^2(\tau_a)\right\} + \ldots$$

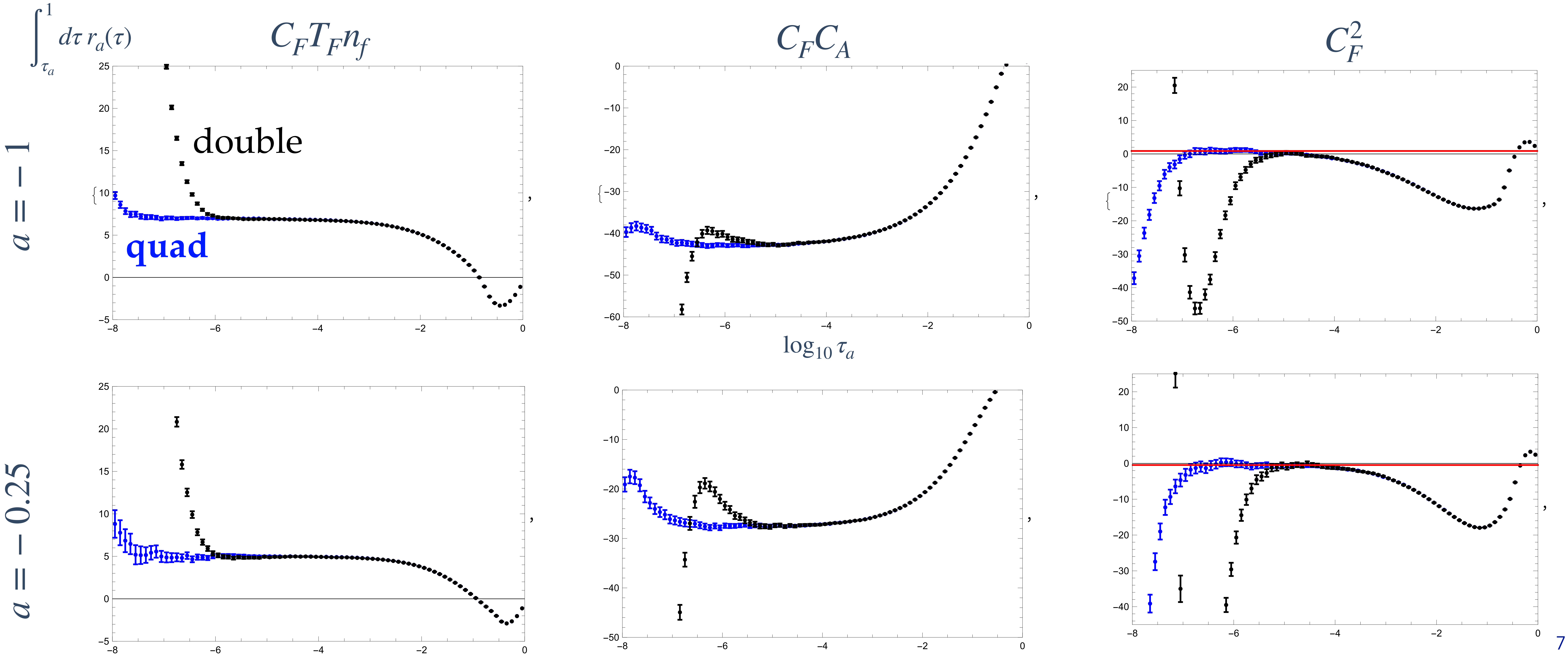
- Additionally, a treatment of non-perturbative effects is critical in *e⁺e⁻ -> hadrons*…

New fixed-order computations

LANL T-2 *Nuclei* cluster

- Improved determination of 2-loop singular constants from extrapolation of EVENT2 predictions using quad precision

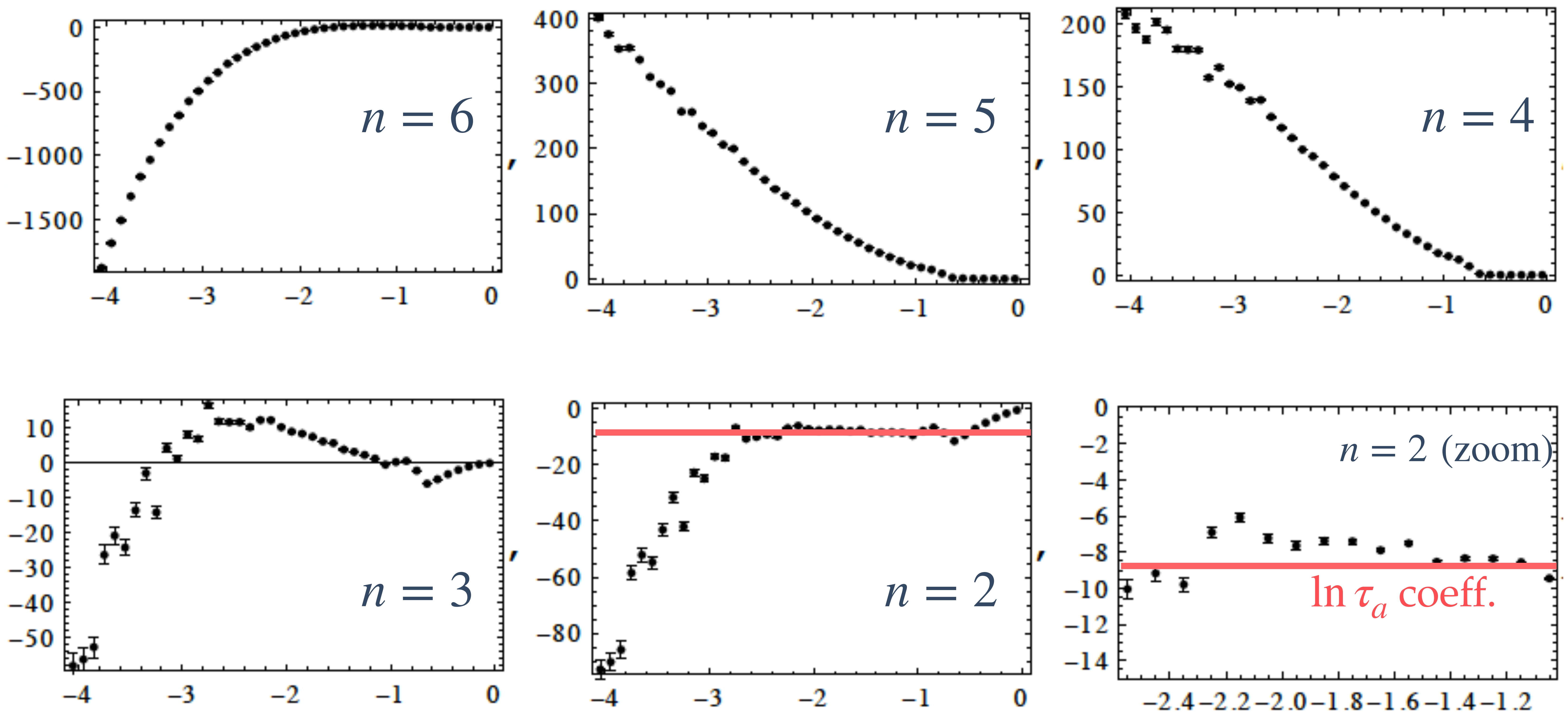
[see also next talk]



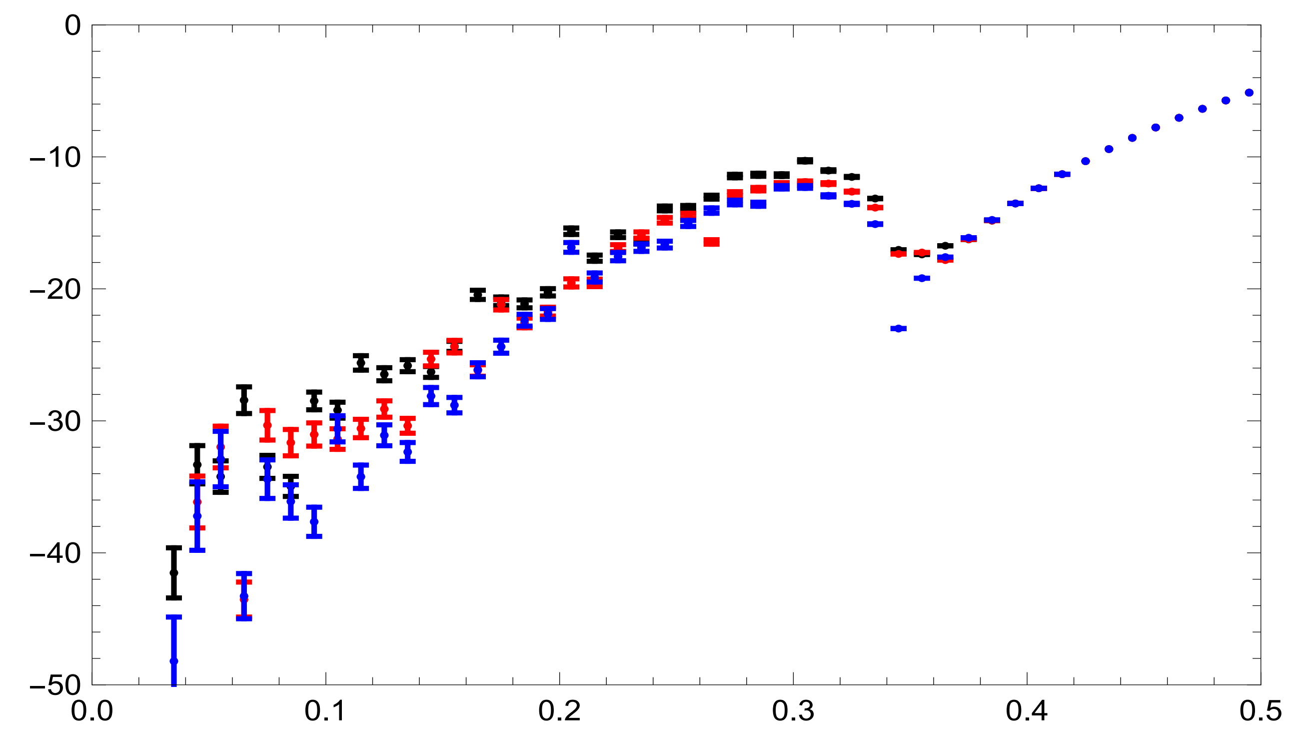
New remainder functions

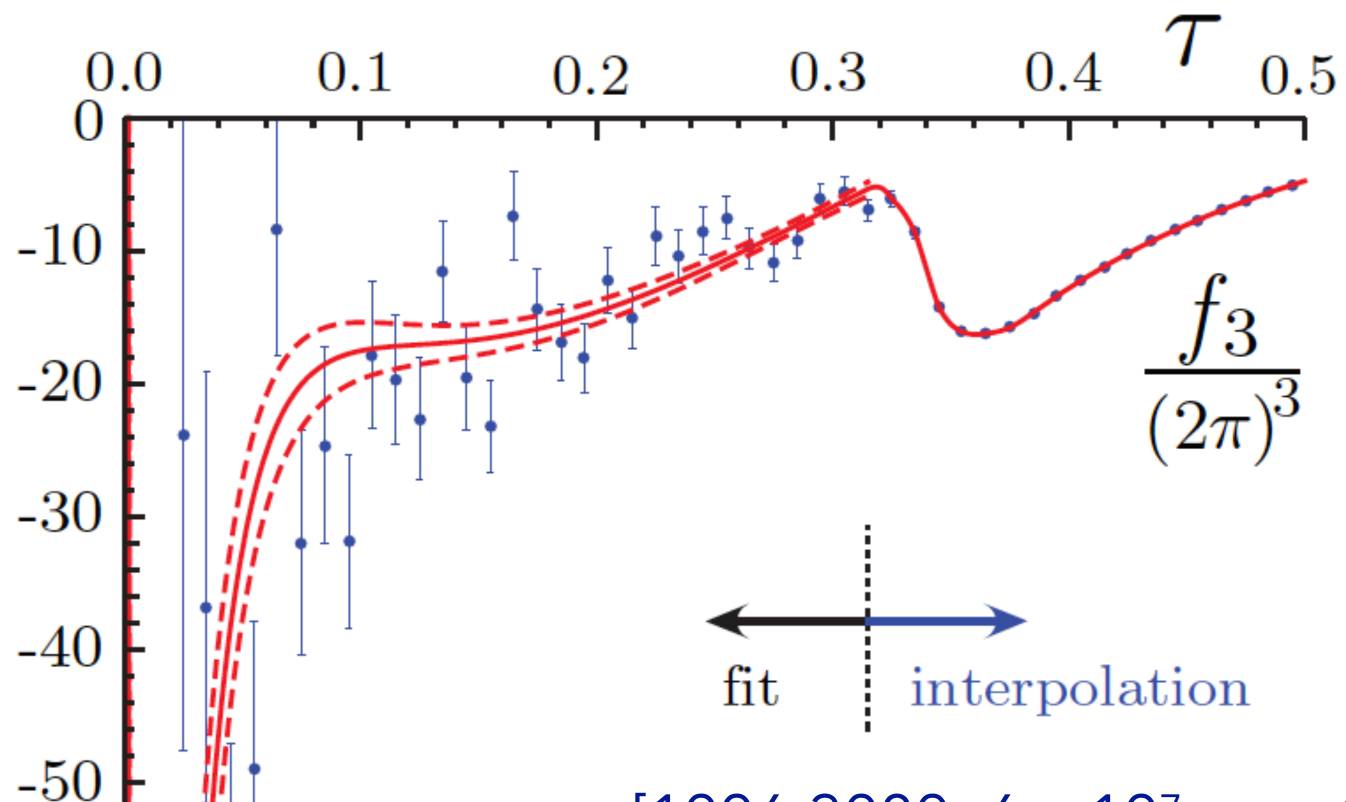
- Preliminary results for 3-loop fixed-order angularity distributions from EERAD3 (IR cutoff 10^{-7}, 1.5×10^{10} events)
- Unknown single log coefficient for nonzero *a*: extract from small τ_a region:

$\frac{1}{\sigma}\frac{d\sigma}{d\ln\tau}$ minus $\ln^n \tau_a$ terms: e.g. for $a = -1$:



- Finite remainder functions, e.g. *a=0*:





[1006.3080, 6 x 10⁷ events]

- (3-loop results not yet included in cross section predictions presented in this talk)

Non-perturbative effects and gapped soft function

- When dominant power corrections come from the soft function, NP effects can be parameterized into a shape function f_{mod}:

'Gap' parameter accounting for parton -> hadron acceptance

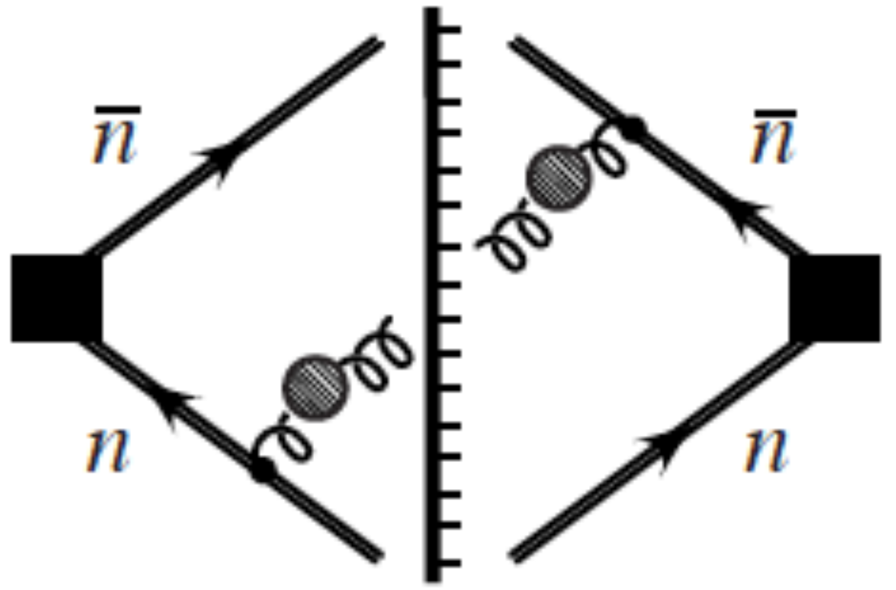
$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k' - 2\overline{\Delta}_a) \qquad f_{\rm mod}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} b_n\, f_n\left(\frac{k}{\lambda}\right)\right]^2$$

[0709.3519]
[0807.1926]

λ constrained by first moment of the shape function

complete orthonormal basis

- However, both the perturbative soft function and gap parameter suffer renormalon ambiguities.



- $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity in gap $\overline{\Delta}_a$

- Subtract a series with the same/canceling ambiguity from both PT and NP pieces:

$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \xrightarrow{\text{Laplace space}} \widetilde{S}(\nu,\mu) = \left[e^{-2\nu\Delta_a(\mu)}\widetilde{f}_{\rm mod}(\nu)\right]\left[e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)\right]$$

R_{gap} scheme

[0803.4214]
[0806.3852]

- Choosing the ***R*$_{gap}$ scheme** to cancel the leading renormalon,

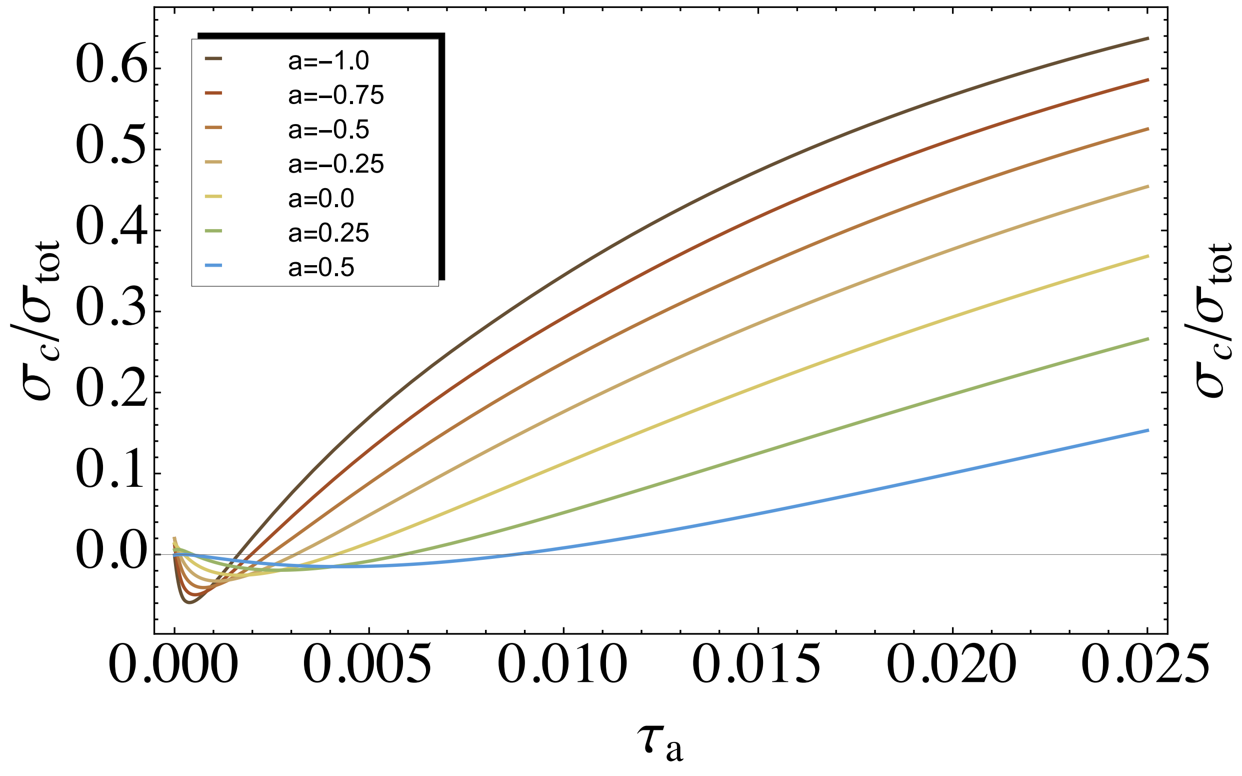
$$R e^{\gamma_E} \frac{d}{d\ln\nu}\left[\ln \widehat{S}_{\rm PT}(\nu,\mu)\right]_{\nu=1/(Re^{\gamma_E})} = 0 \longrightarrow \delta_a(\mu,R) = \frac{1}{2} R e^{\gamma_E} \frac{d}{d\ln\nu}\left[\ln \widetilde{S}_{\rm PT}(\nu,\mu)\right]_{\nu=1/(Re^{\gamma_E})},$$

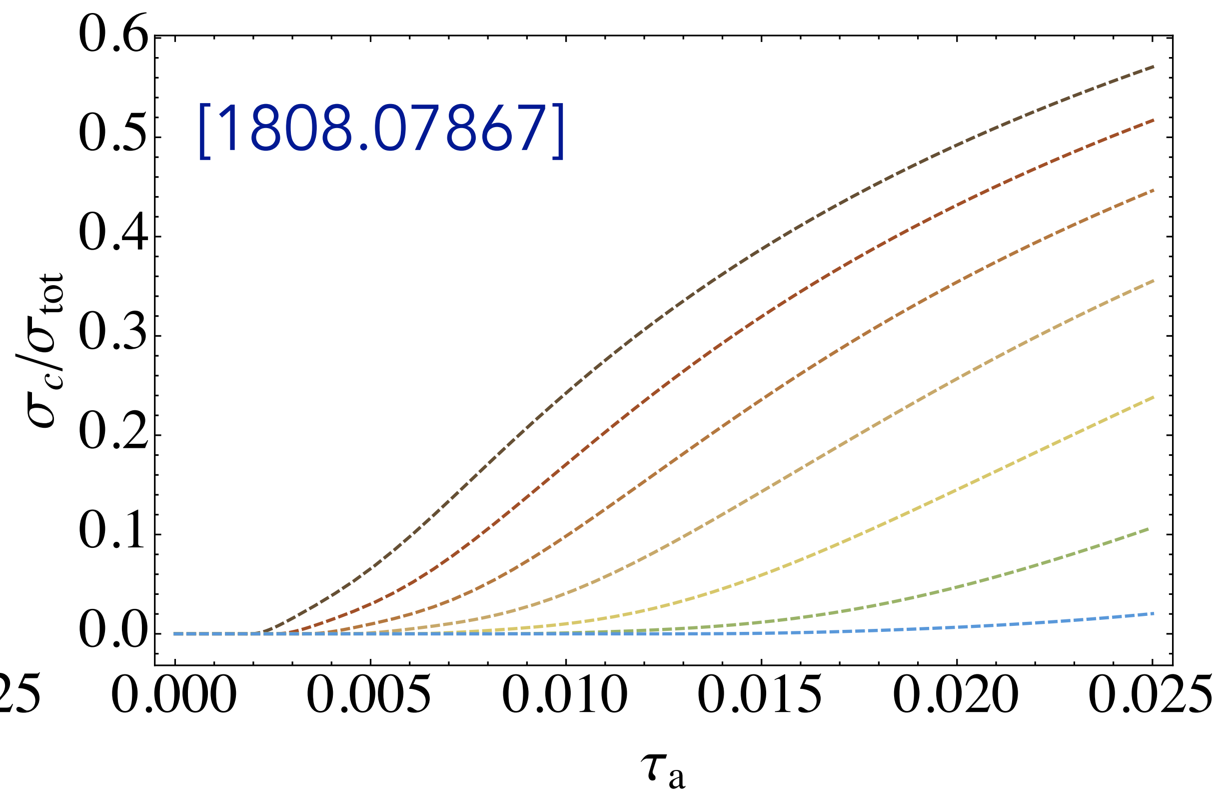
$$\widehat{S}_{\rm PT}(\nu,\mu) = e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)$$

Gapped and renormalon free soft function $\quad S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\left[e^{-2\delta_a(\mu,R)\frac{d}{dk'}} f_{\rm mod}(k'-2\Delta_a(\mu,R))\right]$

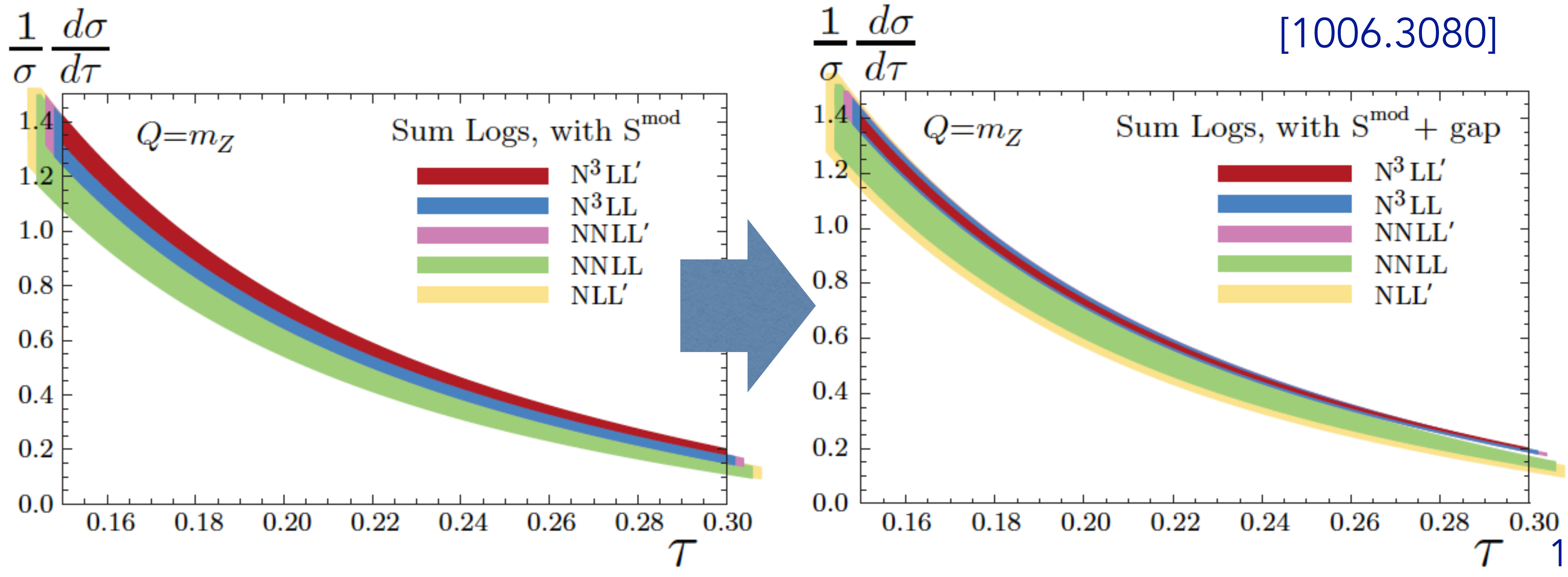
Final cross section is expanded order-by-order in bracketed term $\quad \frac{1}{\sigma_0}\sigma(\tau_a) = \int dk\, \sigma_{\rm PT}\left(\tau_a - \frac{k}{Q}\right)\left[e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\rm mod}(k-2\Delta_a(\mu_S,R))\right]$

- Improves small τ_a behavior and perturbative convergence:





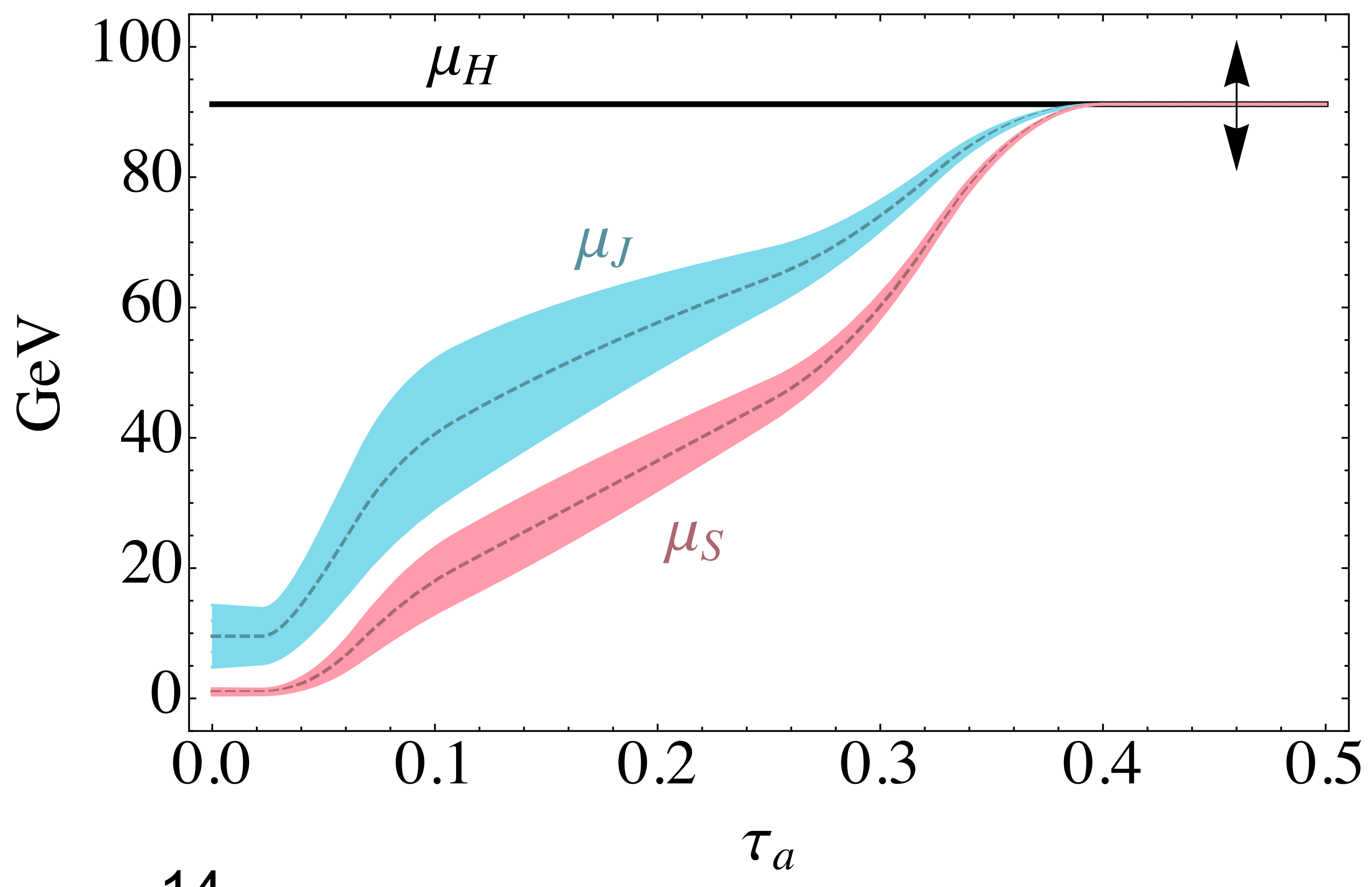
[1006.3080]



R-evolution

[0801.0743]
[0908.3189]

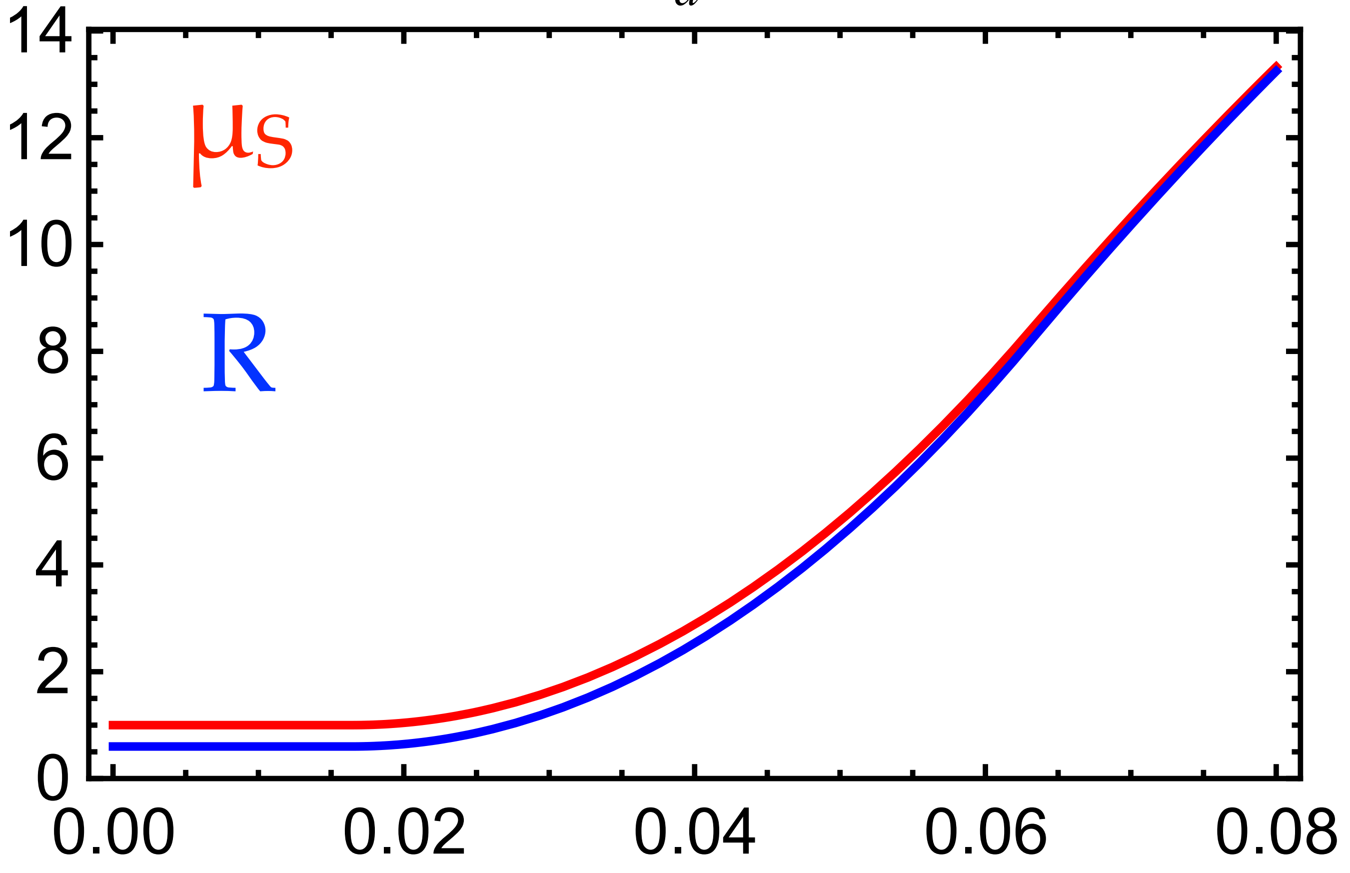
- Want to keep R near IR scales, but also avoid large logs $\ln \frac{\mu_S}{R}$ in subtraction terms



- but μ_S grows to be as large as Q:

- Sum logs by μ and R evolution: $$\mu \frac{d}{d\mu} \Delta_a(\mu, R) = -\mu \frac{d}{d\mu} \delta_a(\mu, R) \equiv \gamma_\Delta^\mu[\alpha_s(\mu)]$$

$$\frac{d}{dR} \Delta_a(R, R) = -\frac{d}{dR} \delta_a(R, R) \equiv -\gamma_R[\alpha_s(R)]$$



- Anomalous dimensions:

$$\gamma_\Delta^\mu[\alpha_s(\mu)] = -Re^{\gamma_E}\Gamma_S[\alpha_s(\mu)]$$

$$\gamma_R[\alpha_s(R)] = \sum_{n=0}^{\infty} \left(\frac{\alpha_s(R)}{4\pi}\right)^{n+1} \gamma_R^n \qquad \gamma_R^0 = 0\,, \quad \gamma_R^1 = \frac{e^{\gamma_E}}{2}\left[\gamma_S^1(a) + 2c_{\tilde{S}}^1 \beta_0\right]$$

Effective non-perturbative shifts

- Before considering gapped renormalons, the leading-order NP effect is a constant shift:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\Omega_1}{Q}\right) \qquad c_{\tau_a} = \frac{2}{1-a} \qquad \Omega_1 = \frac{1}{N_C}\mathrm{Tr}\,\langle 0|\,\overline{Y}_{\bar n}^{\dagger} Y_n^{\dagger}\mathcal{E}_T(0)\,Y_n \overline{Y}_{\bar n}\,|0\rangle$$

Note: this is only valid in the tail region!

- Define an 'effective shift' of the distribution in the R_{gap} scheme:

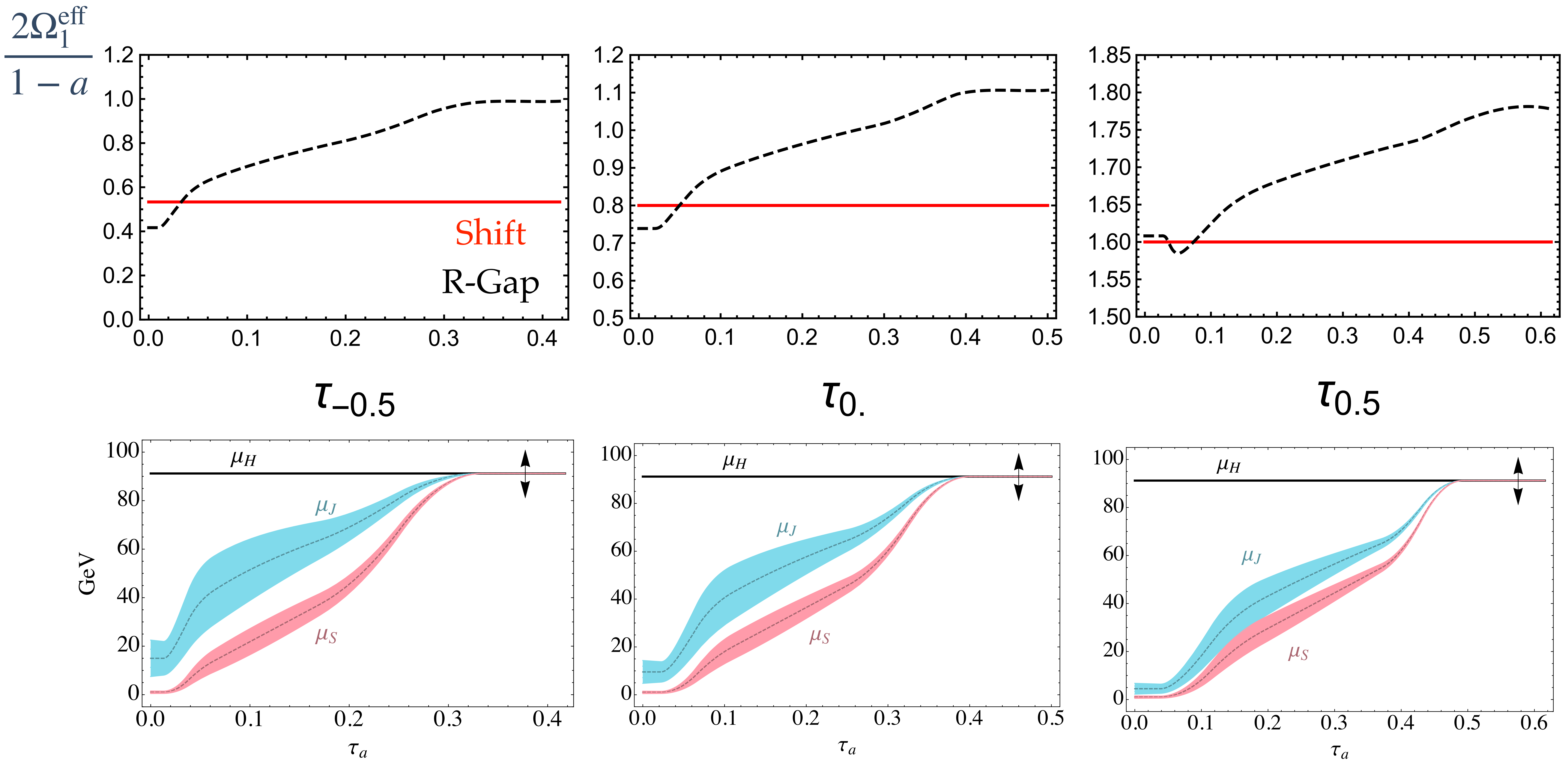
$$\int dk\, k\, e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\text{mod}}\left(k - 2\Delta_a\left(\mu_S, R\right)\right) = \int dk\, k \left[\sum_i f_{\text{mod}}^{(i)}\left(k - 2\Delta_a\left(\mu_S, R\right)\right)\right] \equiv \frac{2}{1-a}\Omega_1^{\text{eff}}$$

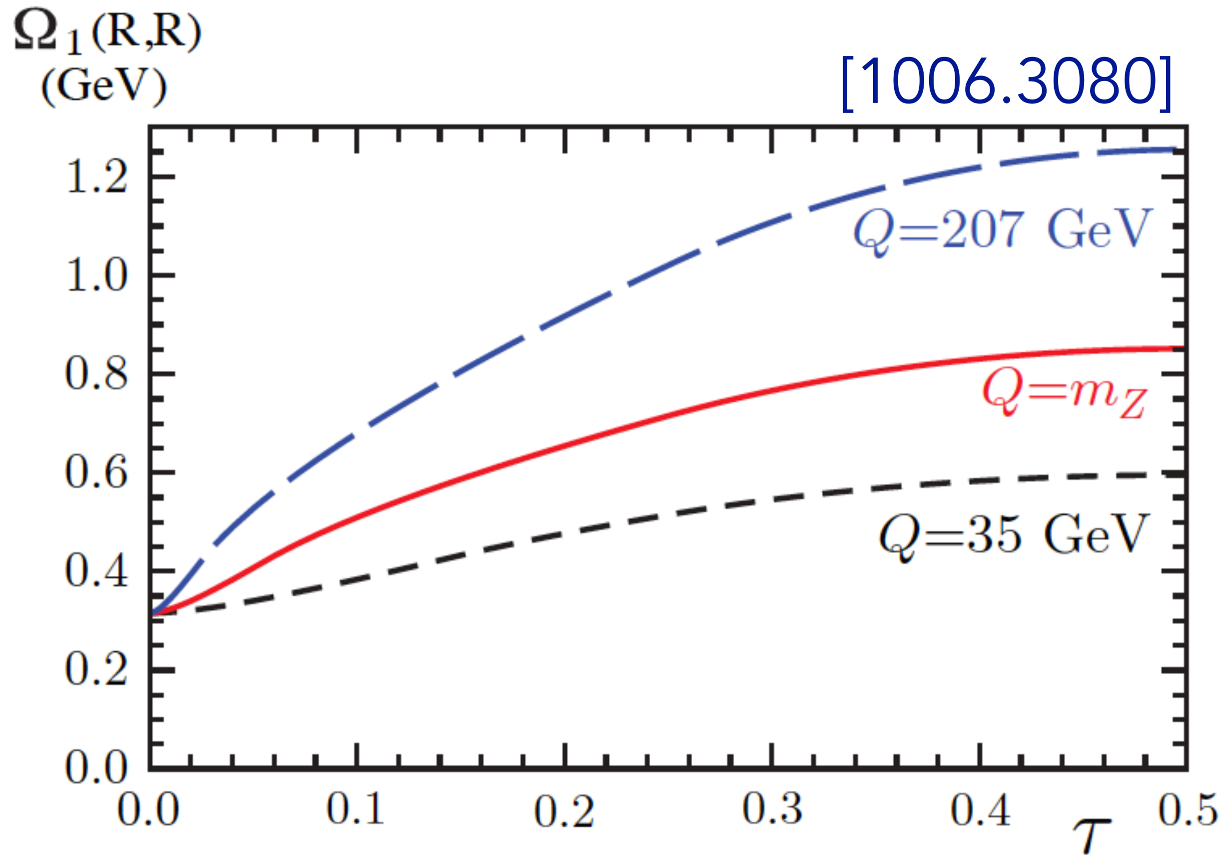
- Shape function expanded order-by-order depending on logarithmic accuracy:

$$\begin{aligned}
f_{\text{mod}}^{(0)}(k - 2\Delta_a(\mu_S, R)) &= f_{\text{mod}}(k - 2\Delta_a(\mu_S, R))\,, \\
f_{\text{mod}}^{(1)}(k - 2\Delta_a(\mu_S, R)) &= -\frac{\alpha_s(\mu_S)}{4\pi}\, 2\delta_a^1(\mu_S, R) R e^{\gamma_E} f'_{\text{mod}}(k - 2\Delta_a(\mu_S, R))\,, \\
f_{\text{mod}}^{(2)}(k - 2\Delta_a(\mu_S, R)) &= \left(\frac{\alpha_s(\mu_S)}{4\pi}\right)^2 \Big[-2\delta_a^2(\mu_S, R) R e^{\gamma_E} f'_{\text{mod}}(k - 2\Delta_a(\mu_S, R)) \\
&\qquad + 2(\delta_a^1(\mu_S, R) R e^{\gamma_E})^2 f''_{\text{mod}}(k - 2\Delta_a(\mu_S, R))\Big]\,,
\end{aligned}$$

Growing shifts

- Distributional shifts at NNLL′ accuracy (central profile scales):





- Effectively, we shift the distribution to the right by *larger* amounts as we move from the 2-jet region out to the multi-jet tail. What might be the effect on extracting α_s?

A scheme to limit the growth of the shift

- Can we find a way to cut off the growth of this shift? i.e. turn off R-evolution above some $\tau = \tau_{\max}$:

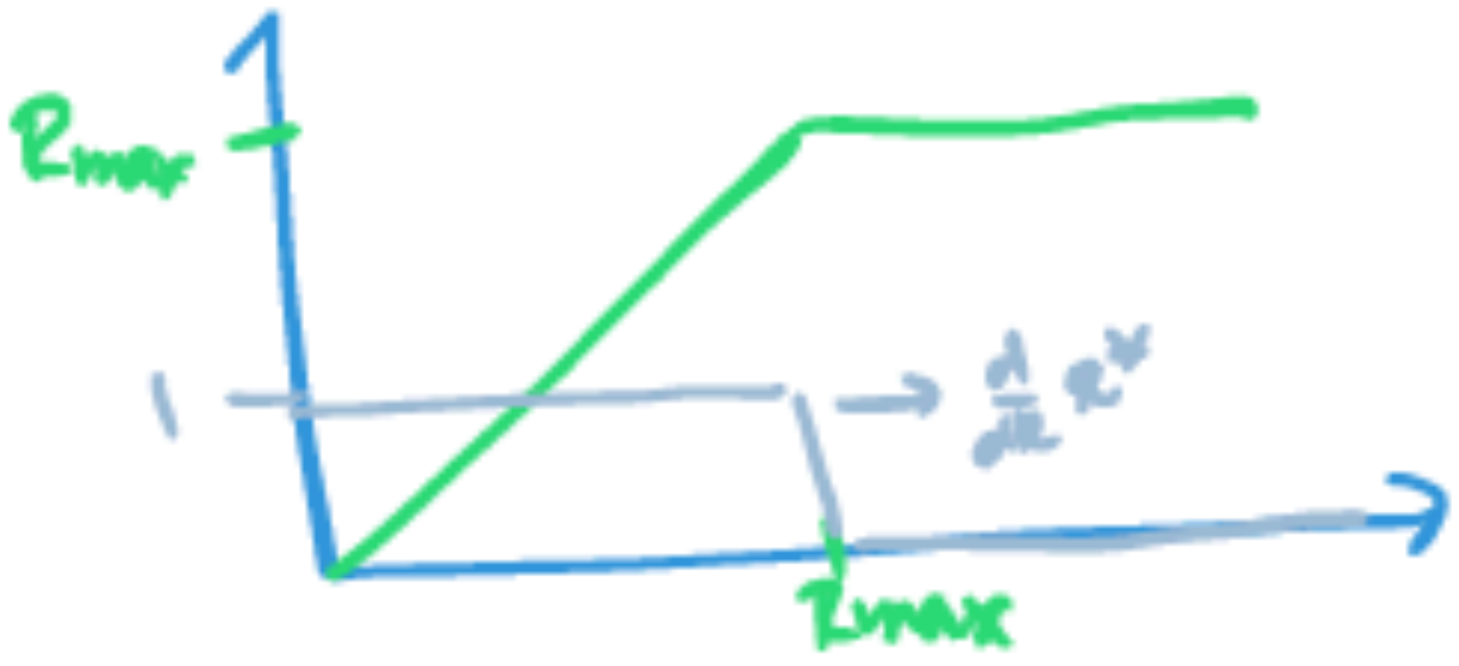
$$\gamma_R \to \theta(R_{\max} - R)\gamma_R \qquad\qquad R = R(\tau)$$

need: $$\frac{d}{dR}\delta_a(R,R) = \gamma_R[\alpha_s(R)]\theta(R_{\max} - R)$$

recall: $$\delta_a(R,R) = Re^{\gamma_E}\left[\frac{\alpha_s(R)}{4\pi}\delta_a^1(R,R) + \left(\frac{\alpha_s(R)}{4\pi}\right)^2\delta_a^2(R,R) + \cdots\right]$$

to the order we need,
just change the R in front to:

$$R^* \equiv \begin{cases} R & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases}$$



$$\delta_a^1(\mu,R) = \Gamma_S^0 \ln\frac{\mu}{R},$$

$$\delta_a^2(\mu,R) = \Gamma_S^0\beta_0 \ln^2\frac{\mu}{R} + \Gamma_S^1 \ln\frac{\mu}{R} + \frac{\gamma_S^1(a)}{2} + c_S^1(a)\beta_0$$

A scheme to limit the growth of the shift

"R^* scheme"

$$\delta_a^*(\mu, R) = \frac{1}{2} R^* e^{\gamma_E} \frac{d}{d\ln\nu} \left[\ln S_{\rm PT}(\nu, \mu)\right]_{\nu = 1/(Re^{\gamma_E})}$$

To the order we work:

$$\frac{d}{dR}\delta_a^*(R, R) = \theta(R_{\max} - R) e^{\gamma_E} \delta_a(R, R) + \mathcal{O}(\alpha_s^3)$$

R-evolution:

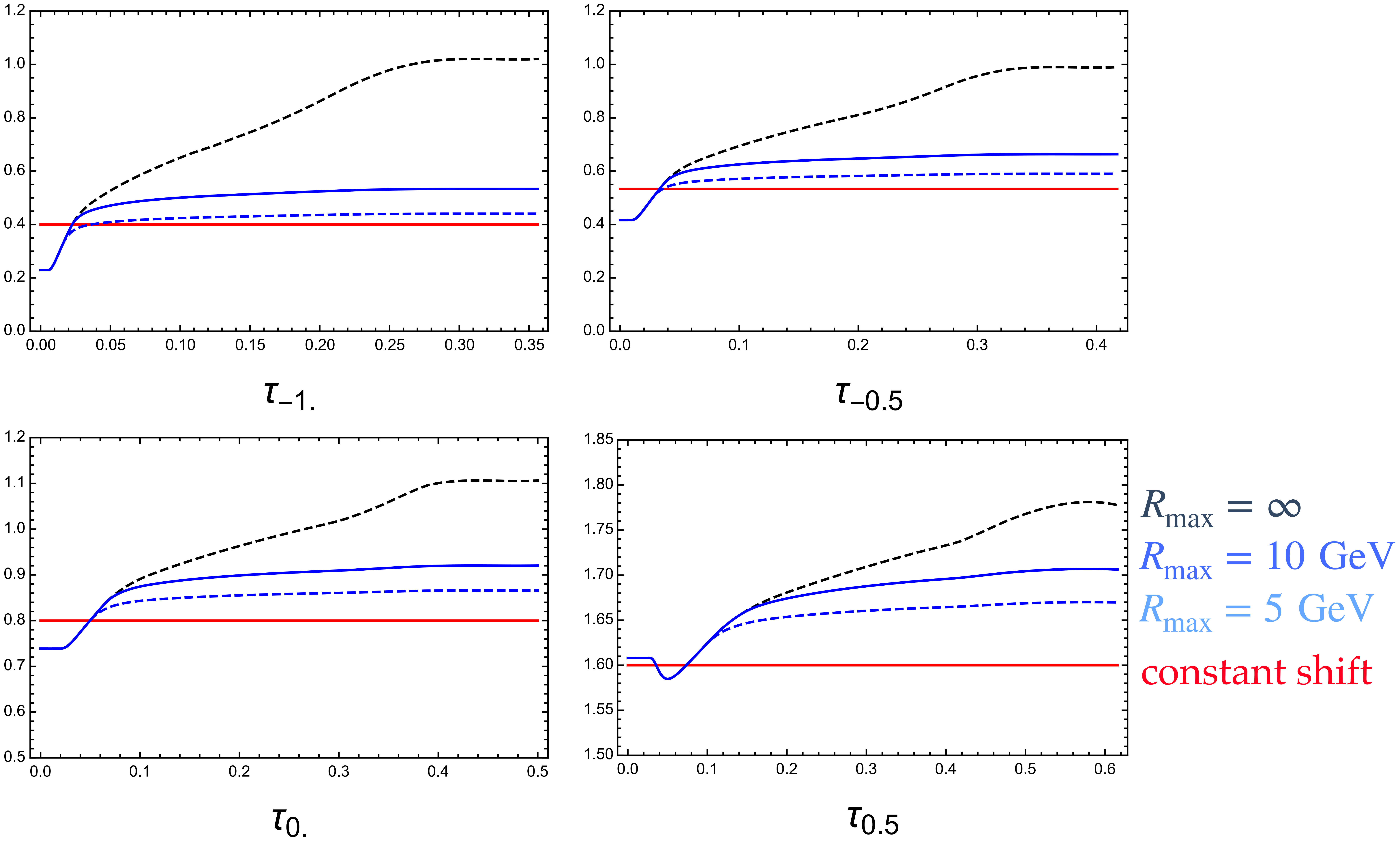
$$\gamma_R^* = \theta(R_{\max} - R) e^{\gamma_E} \left[\frac{\alpha_s(R)}{4\pi} \cdot 0 + \left(\frac{\alpha_S(R)}{4\pi}\right)^2 \gamma_R^1 + \mathcal{O}(\alpha_s^3)\right]$$

μ-evolution:

$$\gamma_\Delta[\alpha_s(\mu)] = -R^* e^{\gamma_E} \Gamma_S[\alpha_s(\mu)]$$

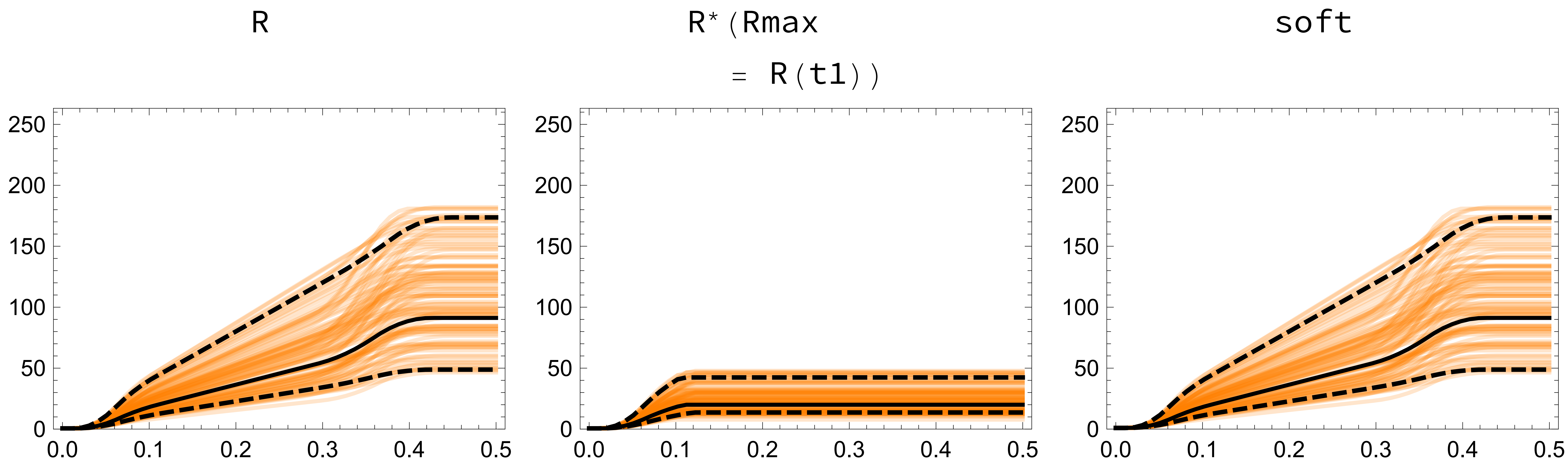
- Nothing fancy. Just one way to freeze growth of effective shift for large τ_a in event shapes.

Frozen shifts



R vs R* profiles

- In our results, we let R^* grow until we hit $\tau_a = t_1(a)$, where we finish transitioning from "shape function" region to "resummation region" in profile functions:

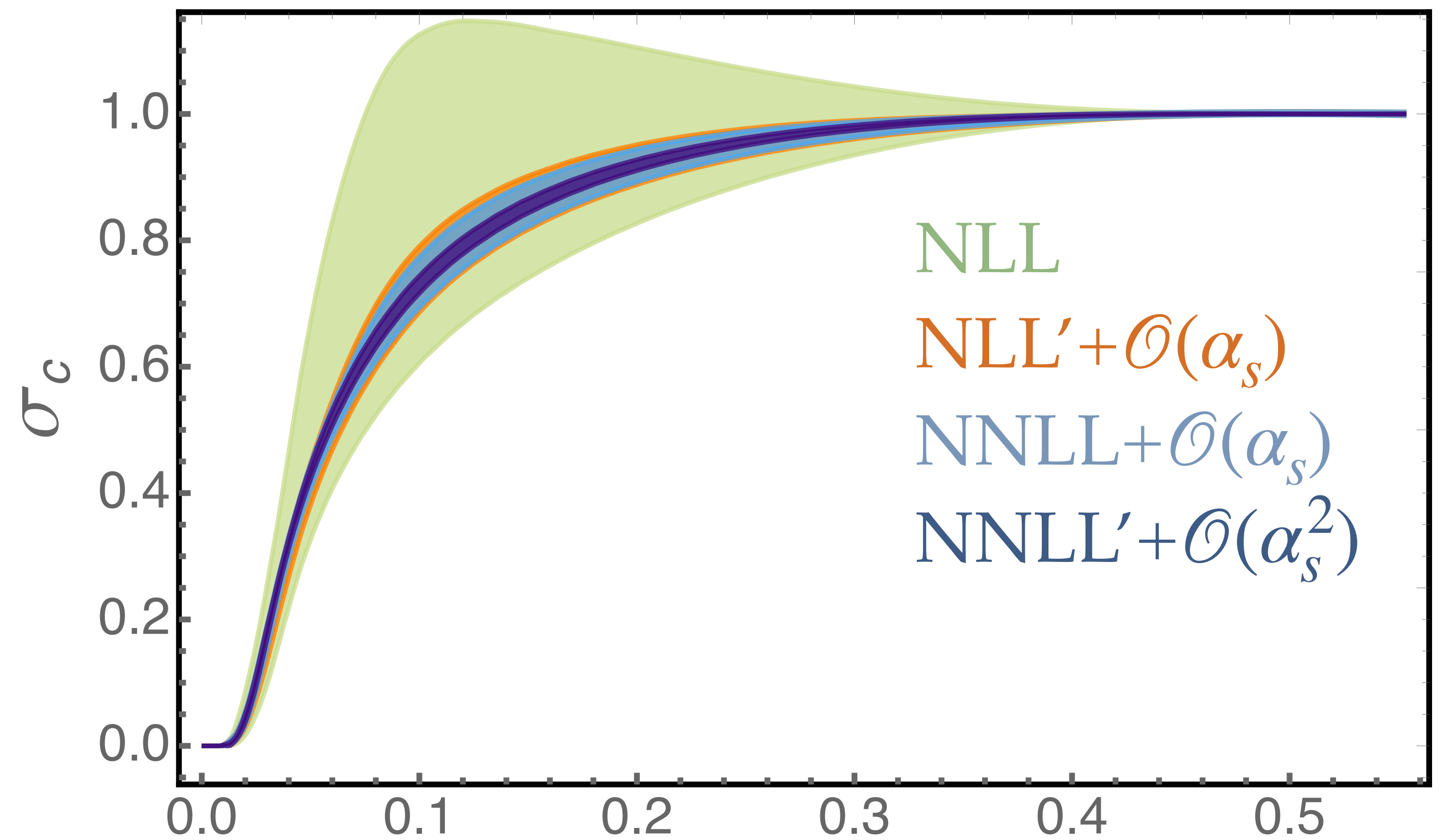


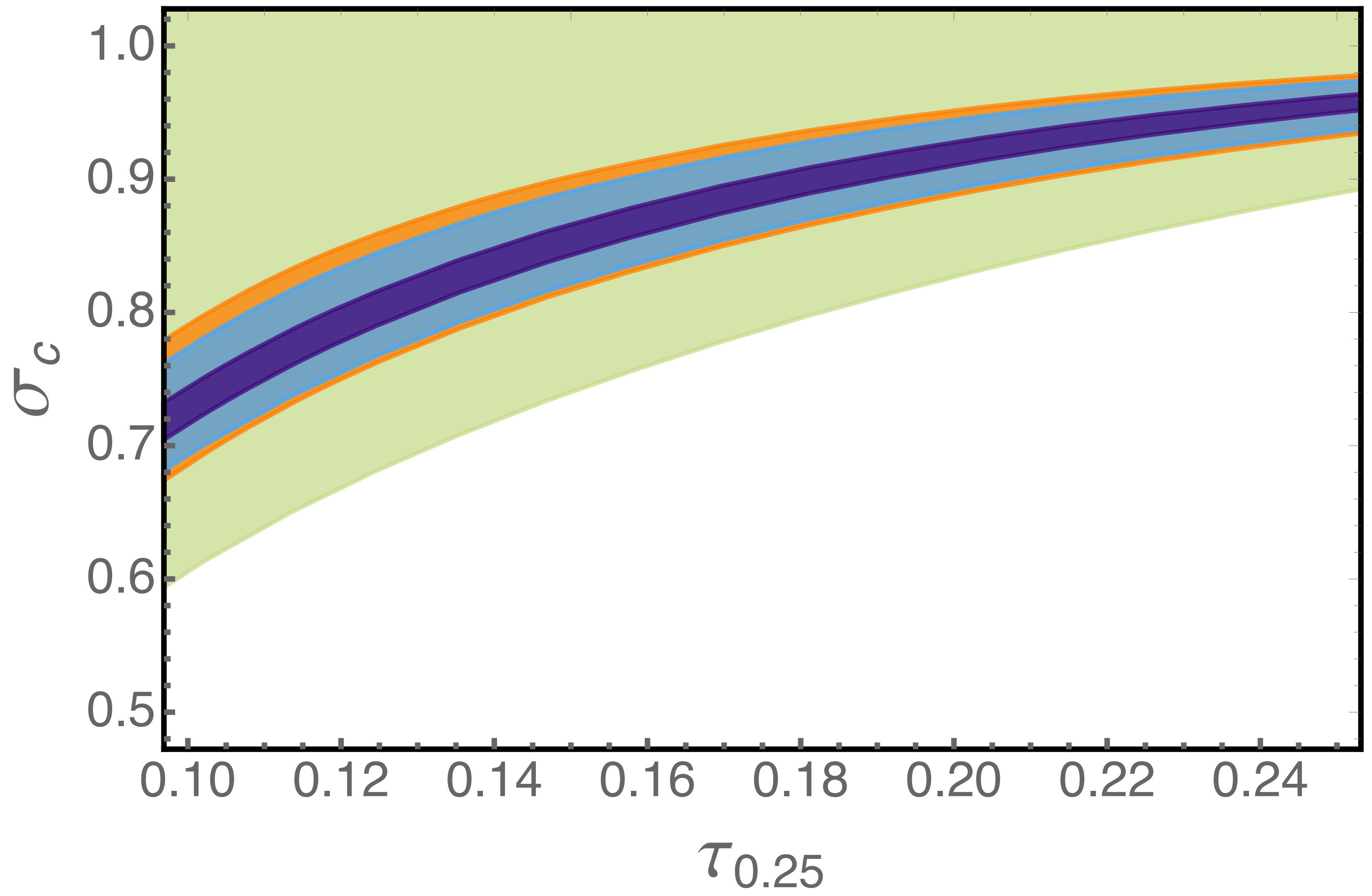
- Different R_{max} values are probed in tandem with variation of the t_1 profile parameter

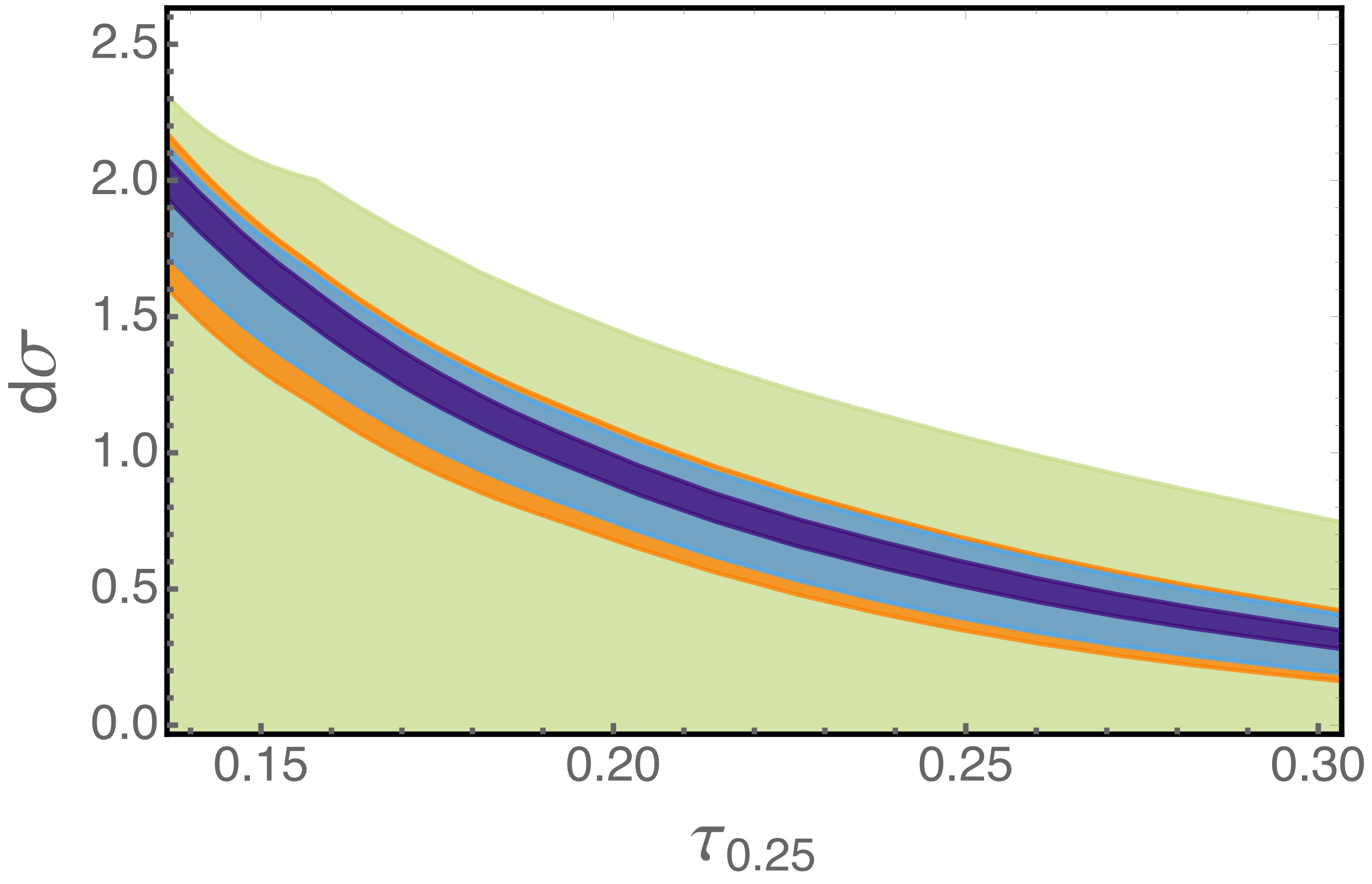
Convergence in R vs R* schemes

$Q = M_Z,\ a = 0.25$

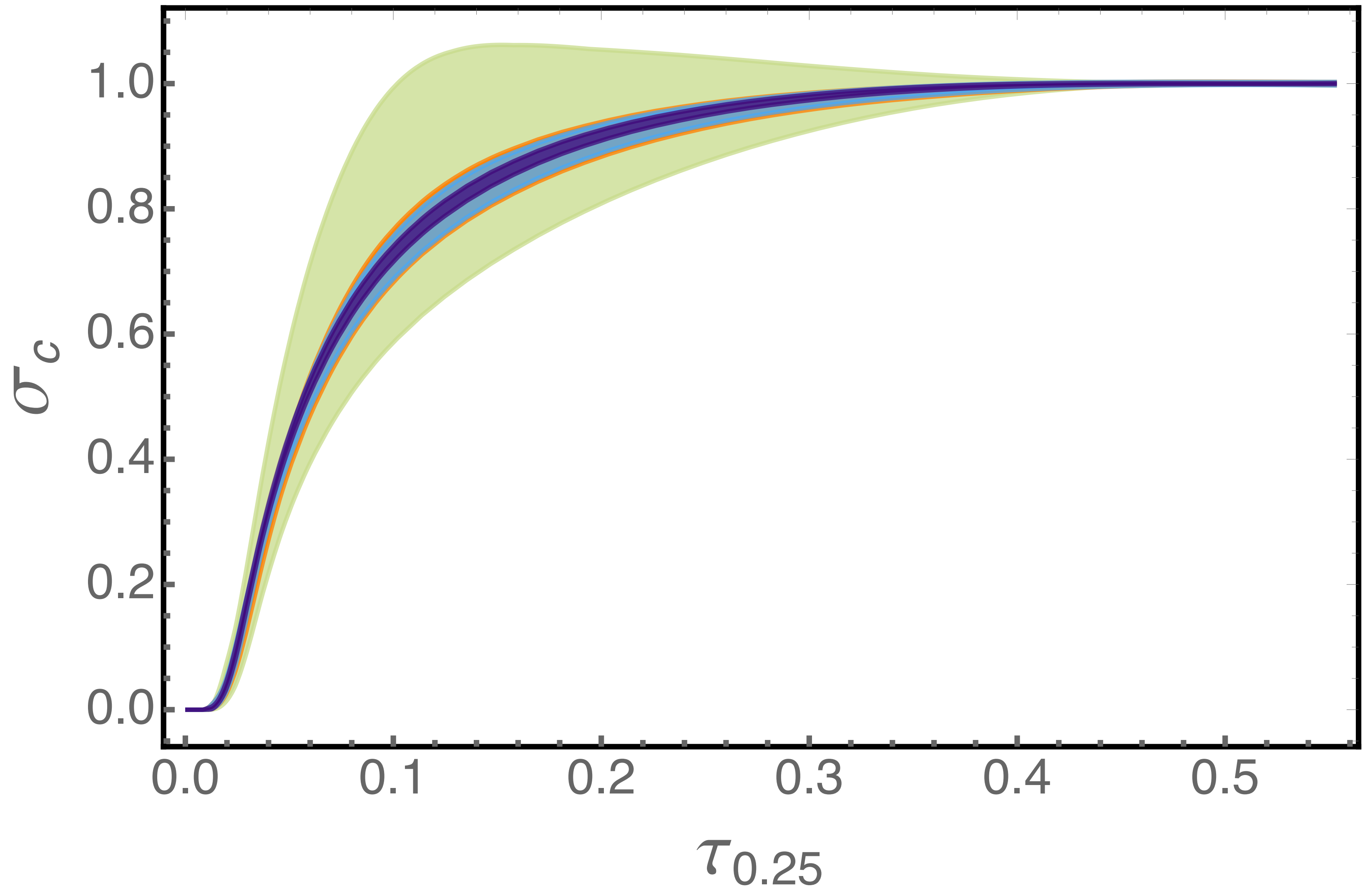
R_{gap} scheme:

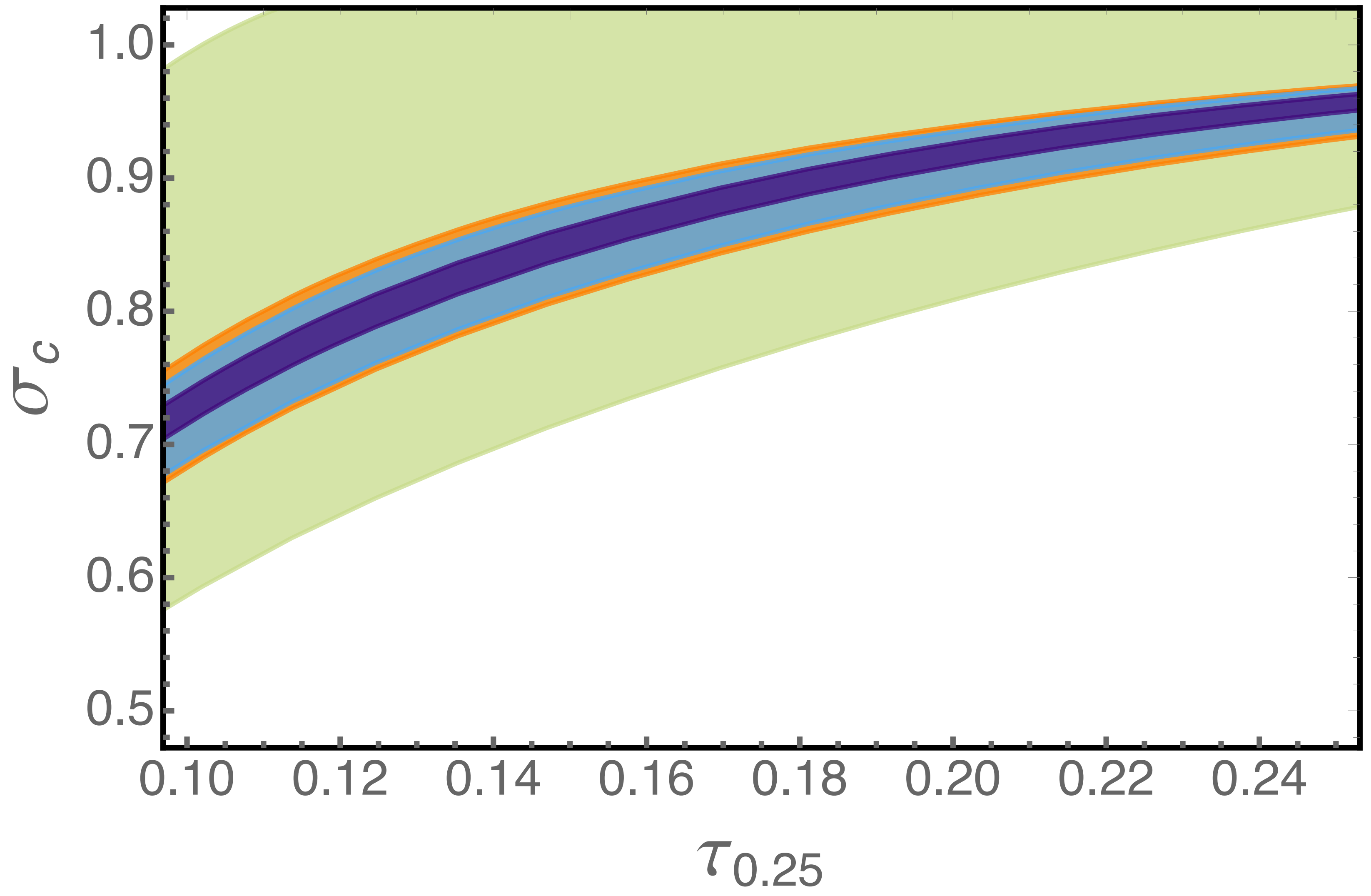


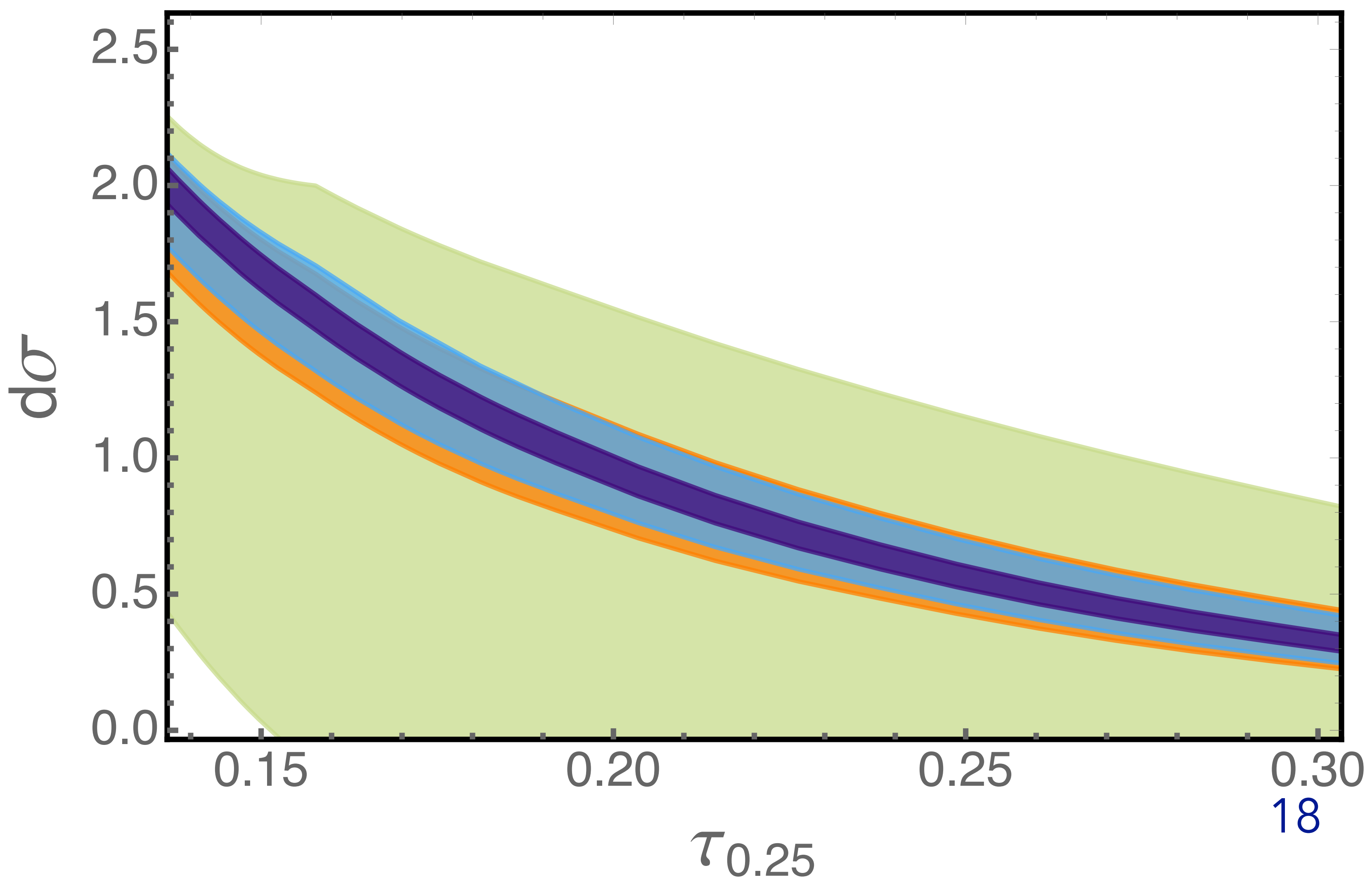




R scheme:*



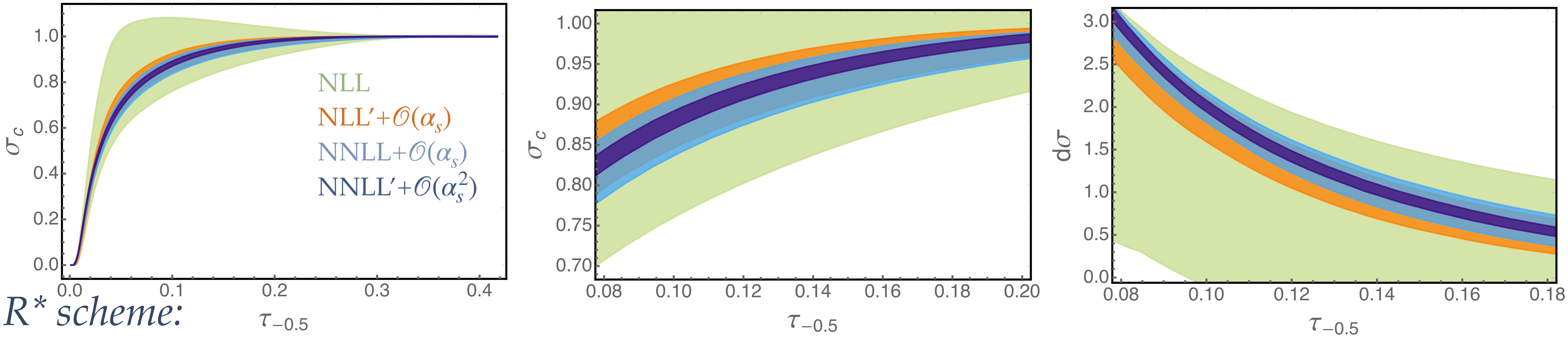




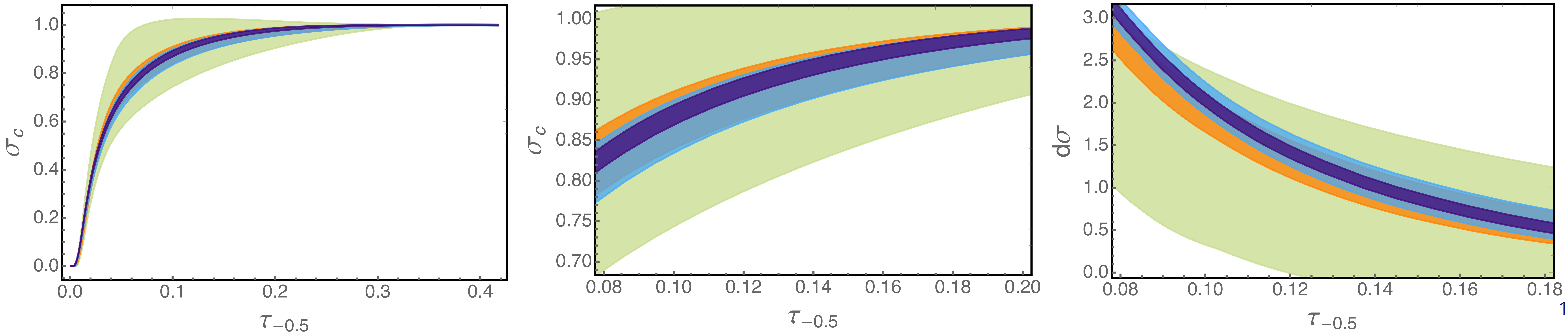
Convergence in R vs R* schemes

$Q = M_Z,\ a = \text{-}0.5$

R_{gap} scheme:

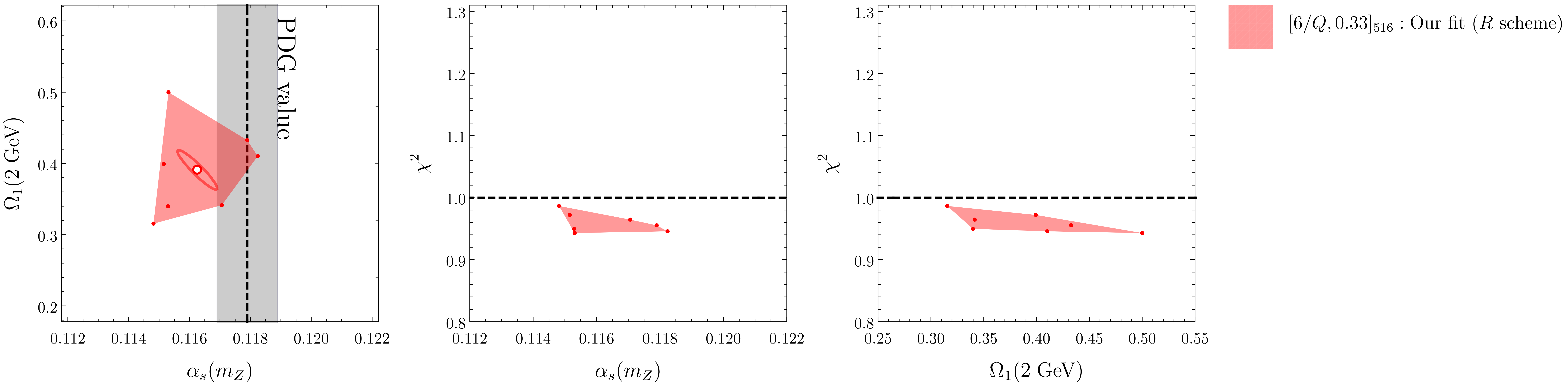


R^* scheme:

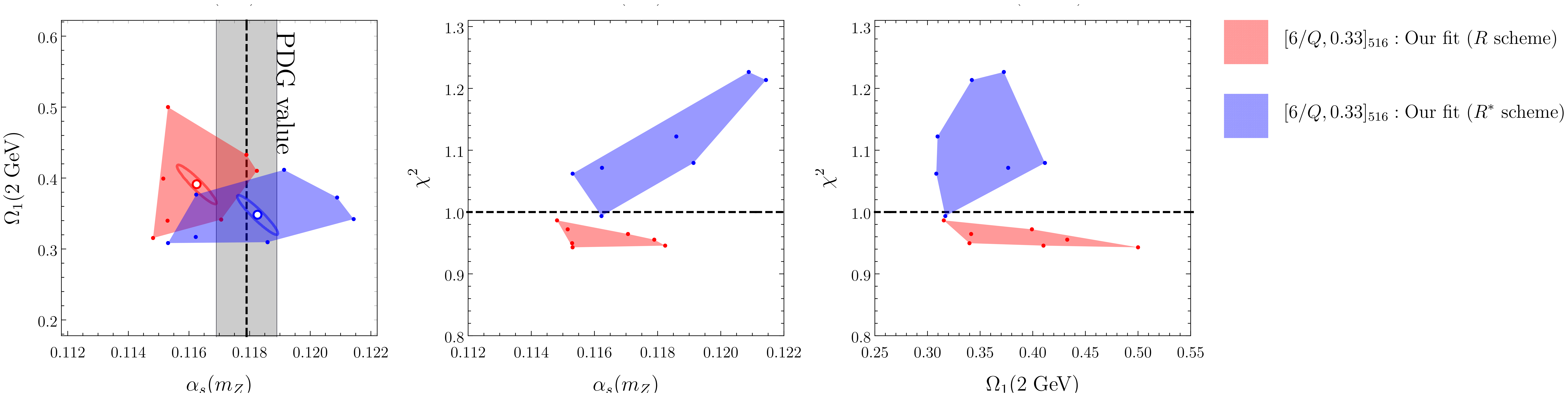


Effect on thrust fits

[NNLL′+$\mathcal{O}(\alpha_s^2)$]



Effect on thrust fits $[\mathrm{NNLL}'+\mathcal{O}(\alpha_s^2)]$



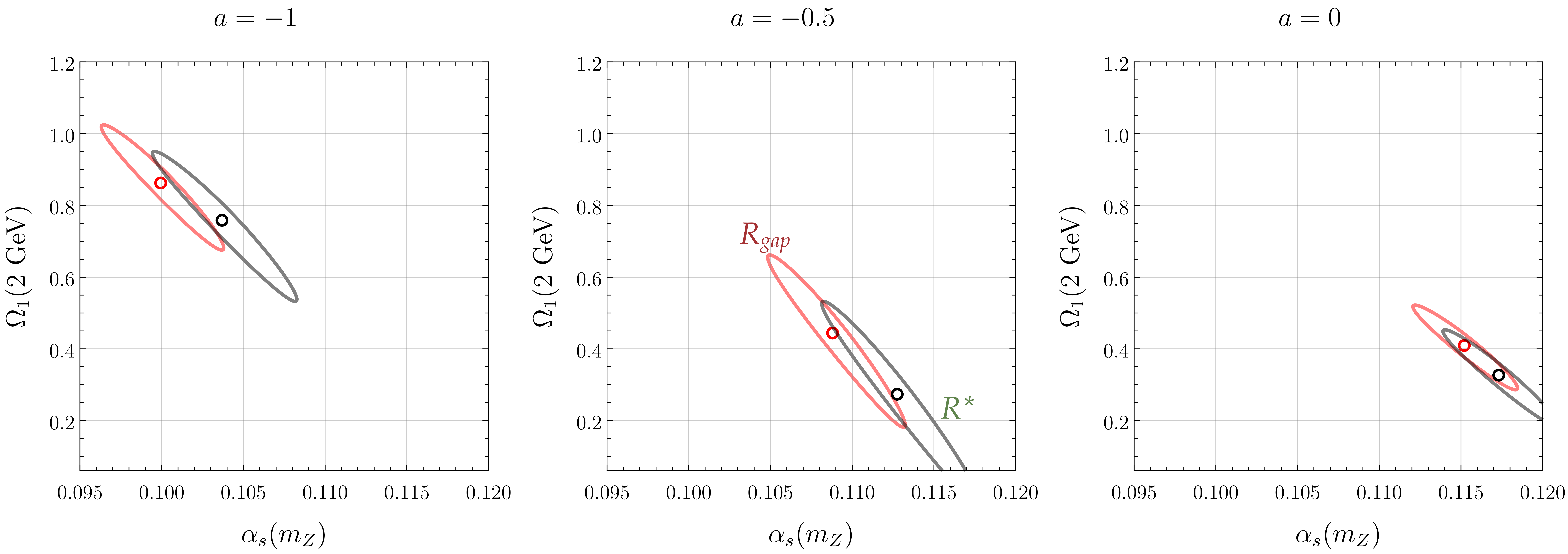
Effect on thrust fits [NNLL′+$\mathcal{O}(\alpha_s^2)$]

green -> red : several other systematics, e.g. profile functions, no b-mass or QED corrections (for us), slightly different data sets/bins, scale setting in bins…

Effect on angularity fits (all *a*) [NNLL′+$\mathcal{O}(\alpha_s^2)$]

small ellipses: experimental error
larger ellipses: theory error
shaded: combined error

Effect on angularity fits (single a's) $[\text{NNLL}'+\mathcal{O}(\alpha_s^2)]$



Consistent shift from using R*

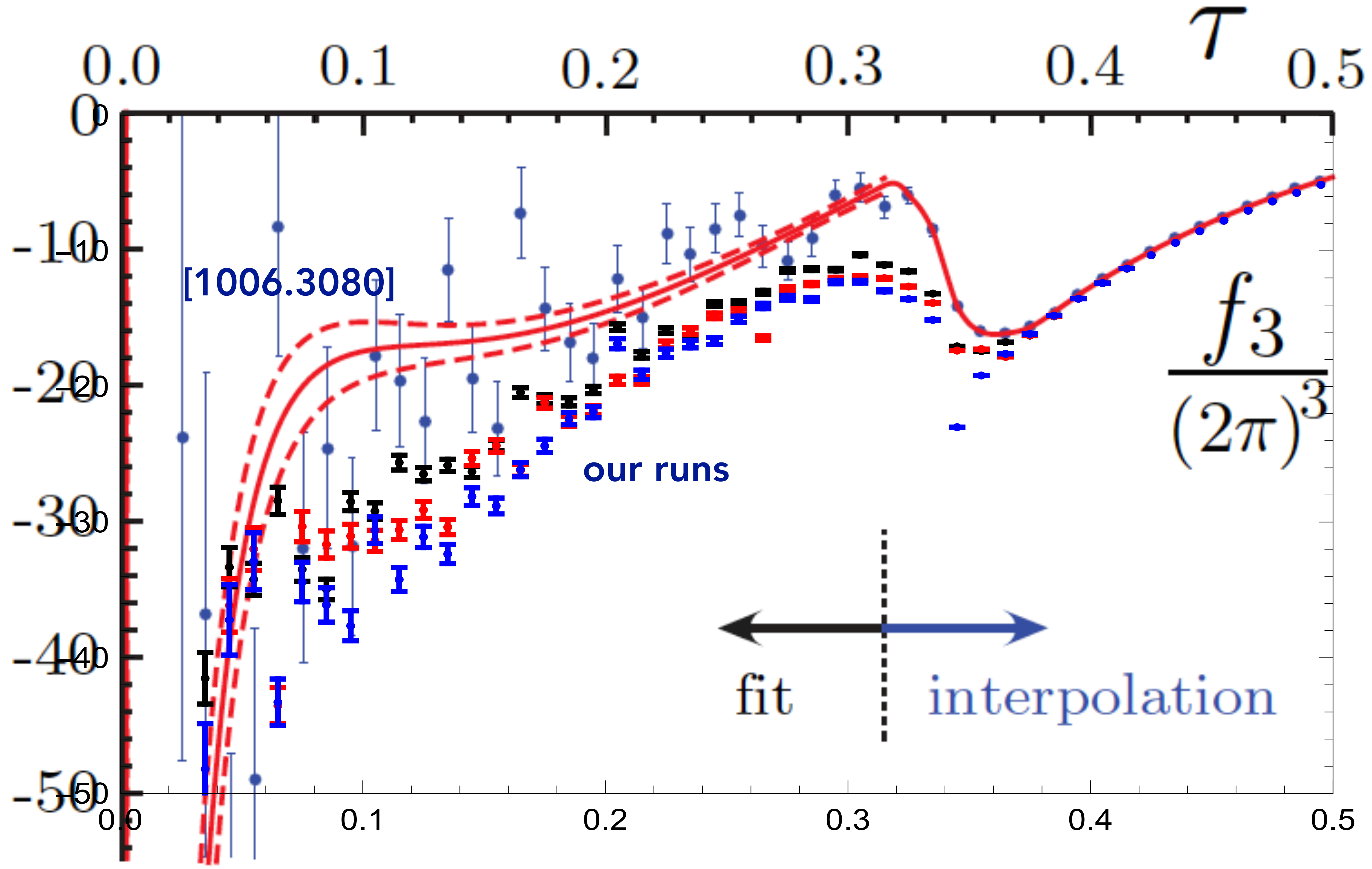
- There are still lots of systematics to consider: fitting regions, choice of profile functions, data sets, scale choice inside bins, etc. **Our illustrations are based on NNLL′+$\mathcal{O}(\alpha_s^2)$ predictions only, so far.**
- **You (and we!) are not allowed to quote a value of α_s or Ω_1 coming from this talk!!**
- What seems consistent is, when controlling on other systematic choices, a shift in α_s of about a **few percent** when switching from standard R_{gap} to R* scheme.
- Shifted values are within uncertainties, but might alleviate tension with PDG value.
  - Similar conclusion, from different considerations, as G. Luisoni, P. Monni, G. Salam [2012.00622] who tried varying size of nonperturbative shift in *C*-parameter distribution as function of *C* (smaller shifts for large *C* ⇒larger values of α_s by a few percent)

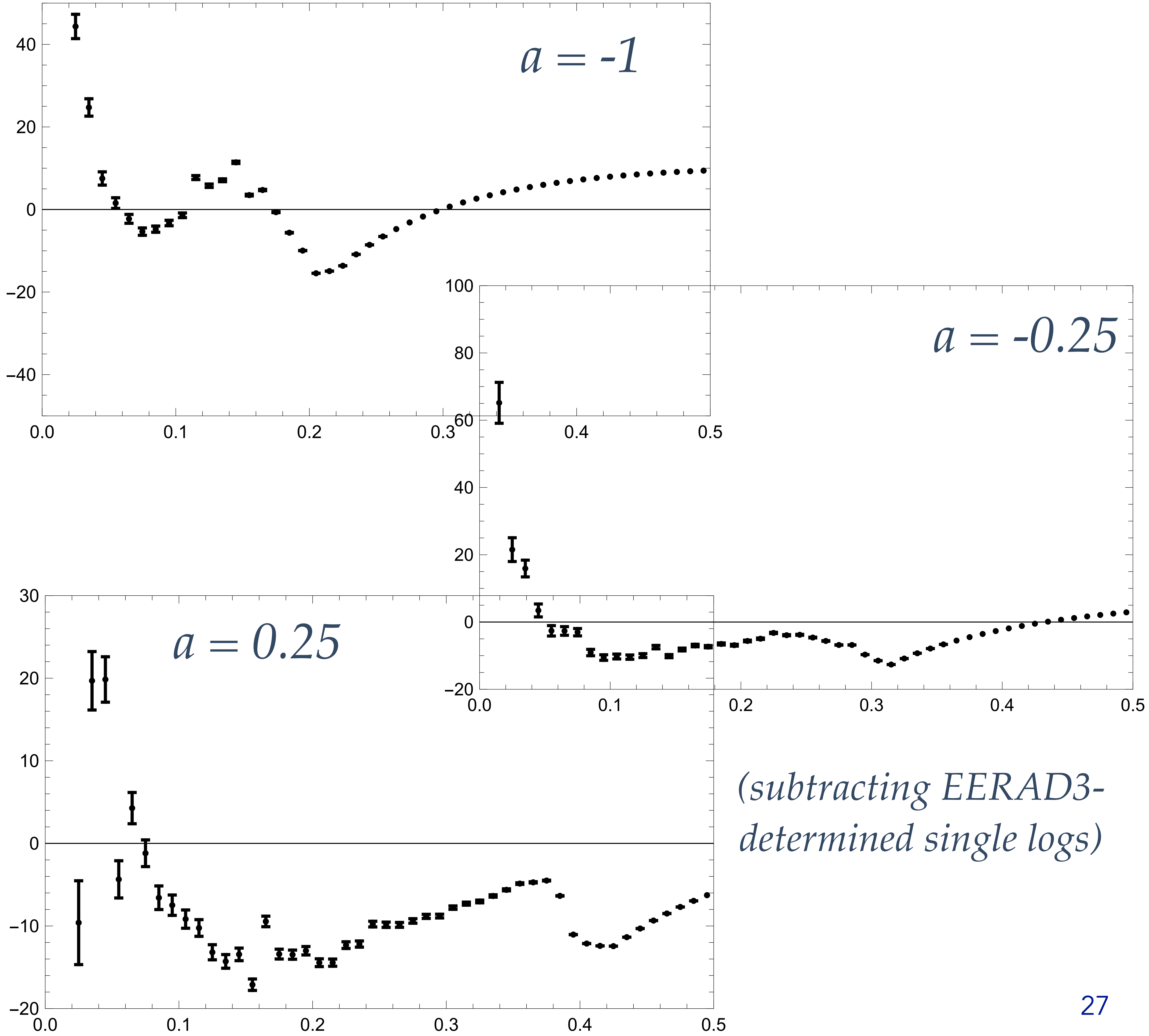
Backups

New remainder functions from EERAD3

using LANL Institutional Computing

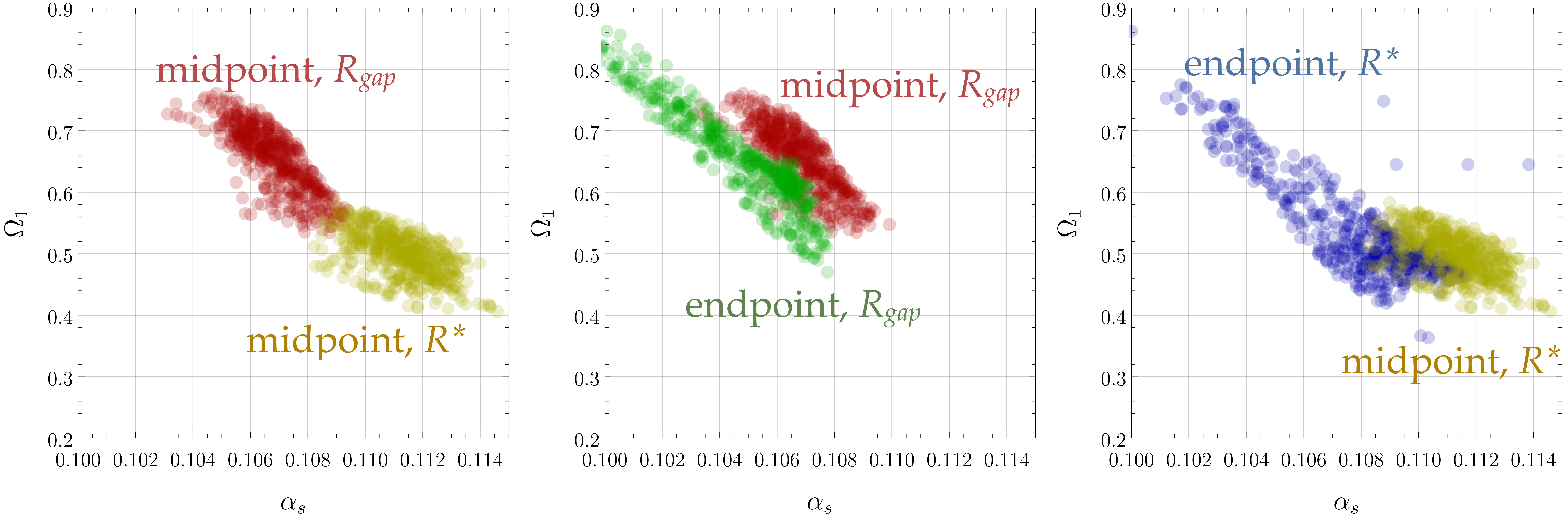
- a=0:





(subtracting EERAD3-determined single logs)

Effect of scale setting & R-scheme on angularity fits



- "midpoint" scales

$$\sigma^i_{\text{bin}} = \sigma_c(\tau_{i+1}, \mu_{J,S}(\tau_{\text{mid}})) - \sigma_c(\tau_i, \mu_{J,S}(\tau_{\text{mid}}))$$

may not preserve total cross section (cf. 1401.4460)

- "endpoint" scales

$$\sigma^i_{\text{bin}} = \sigma_c(\tau_{i+1}, \mu_{J,S}(\tau_{i+1})) - \sigma_c(\tau_i, \mu_{J,S}(\tau_i))$$

"spurious" uncertainties (cf. 1006.3080)

- worth exploring "total integral"-preserving scale profiles of Bertolini, Solon, Walsh [1701.07919]

Data sets

- For thrust:

```
ALEPH-2004: 133. GeV (7)
ALEPH-2004: 161. GeV (7)
ALEPH-2004: 172. GeV (7)
ALEPH-2004: 183. GeV (7)
ALEPH-2004: 189. GeV (7)
ALEPH-2004: 200. GeV (6)
ALEPH-2004: 206. GeV (8)
ALEPH-2004: 91.2 GeV (26)
AMY-1990: 55.2 GeV (5)
DELPHI-1999: 133. GeV (7)
DELPHI-1999: 161. GeV (7)
DELPHI-1999: 172. GeV (7)
DELPHI-1999: 89.5 GeV (11)
DELPHI-1999: 93. GeV (12)
DELPHI-2000: 91.2 GeV (12)
DELPHI-2003: 183. GeV (14)
DELPHI-2003: 189. GeV (15)
DELPHI-2003: 192. GeV (15)
DELPHI-2003: 196. GeV (14)
DELPHI-2003: 200. GeV (15)
DELPHI-2003: 202. GeV (15)
DELPHI-2003: 205. GeV (15)
DELPHI-2003: 207. GeV (15)
DELPHI-2003: 45. GeV (5)
DELPHI-2003: 66. GeV (8)
DELPHI-2003: 76. GeV (9)
JADE-1998: 35. GeV (5)
JADE-1998: 44. GeV (7)
L3-2004: 130.1 GeV (11)
L3-2004: 136.1 GeV (10)
L3-2004: 161.3 GeV (12)
L3-2004: 172.3 GeV (12)
L3-2004: 182.8 GeV (12)
L3-2004: 188.6 GeV (12)
L3-2004: 194.4 GeV (12)
L3-2004: 200. GeV (11)
L3-2004: 206.2 GeV (12)
L3-2004: 41.4 GeV (5)
L3-2004: 55.3 GeV (6)
L3-2004: 65.4 GeV (7)
L3-2004: 75.7 GeV (7)
L3-2004: 82.3 GeV (8)
L3-2004: 85.1 GeV (8)
L3-2004: 91.2 GeV (10)
OPAL-1997: 161. GeV (7)
OPAL-2000: 172. GeV (8)
OPAL-2000: 183. GeV (8)
OPAL-2000: 189. GeV (8)
OPAL-2005: 133. GeV (6)
OPAL-2005: 177. GeV (8)
OPAL-2005: 197. GeV (8)
OPAL-2005: 91. GeV (5)
SLD-1995: 91.2 GeV (6)
TASSO-1998: 35. GeV (4)
TASSO-1998: 44. GeV (5)

------ Summary ------
Totlal: 516
Q > 95 : 345
Q < 88 : 89
Q ~ MZ : 82
```

- For angularities:

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\textbf{GeV}$

L3 Collaboration

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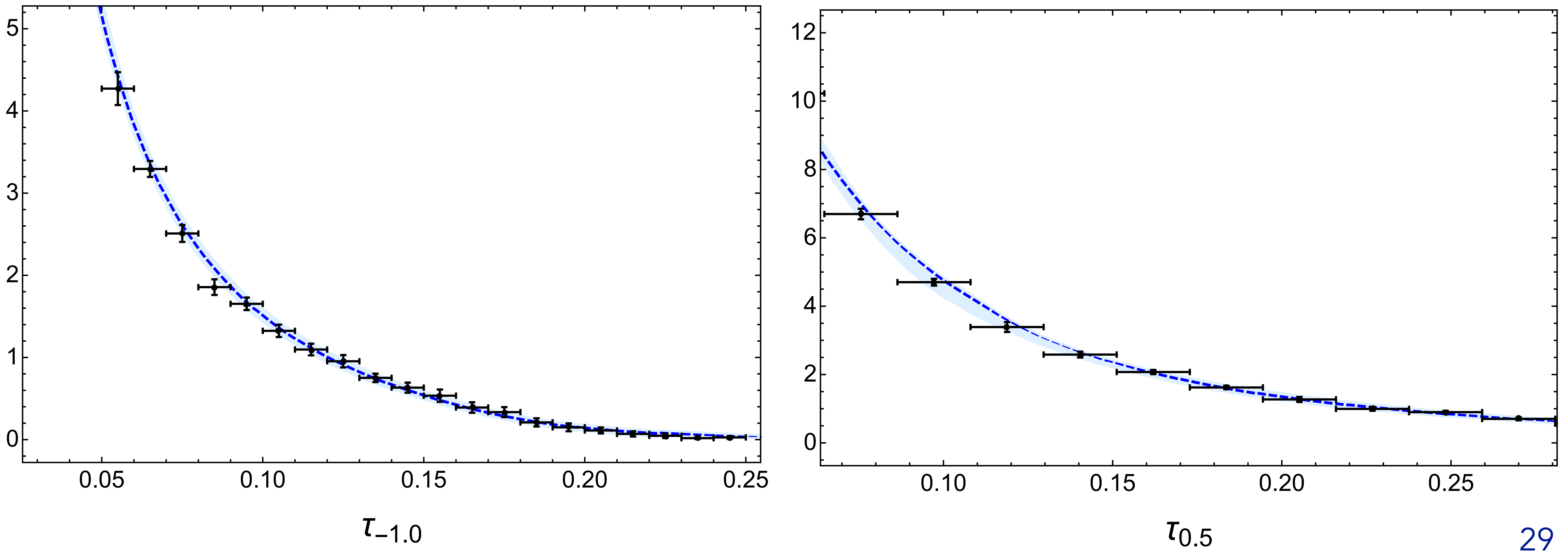
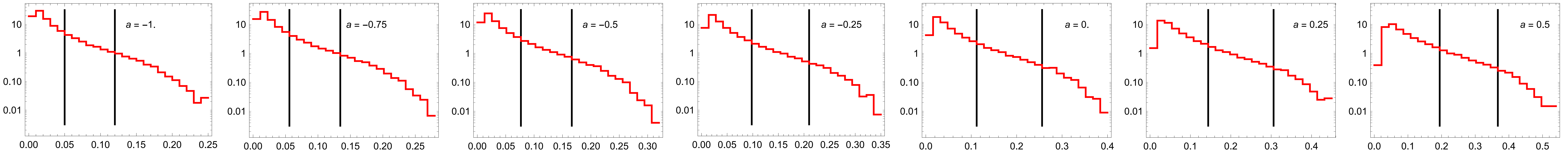
Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- e.g. a = -1 and 0.5, Q = 91.2 GeV, compared to our NNLL' prediction:

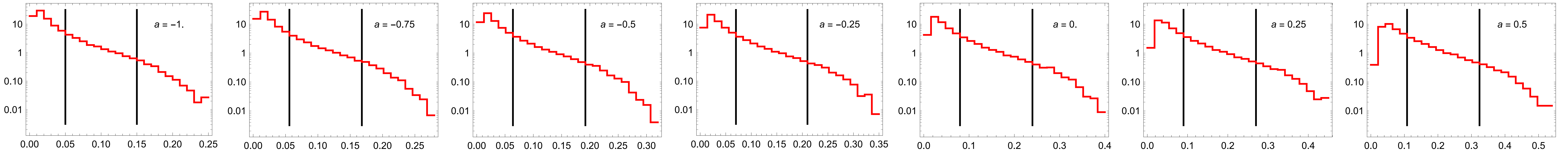


Fitting regions currently used

- For all-angularity fit:



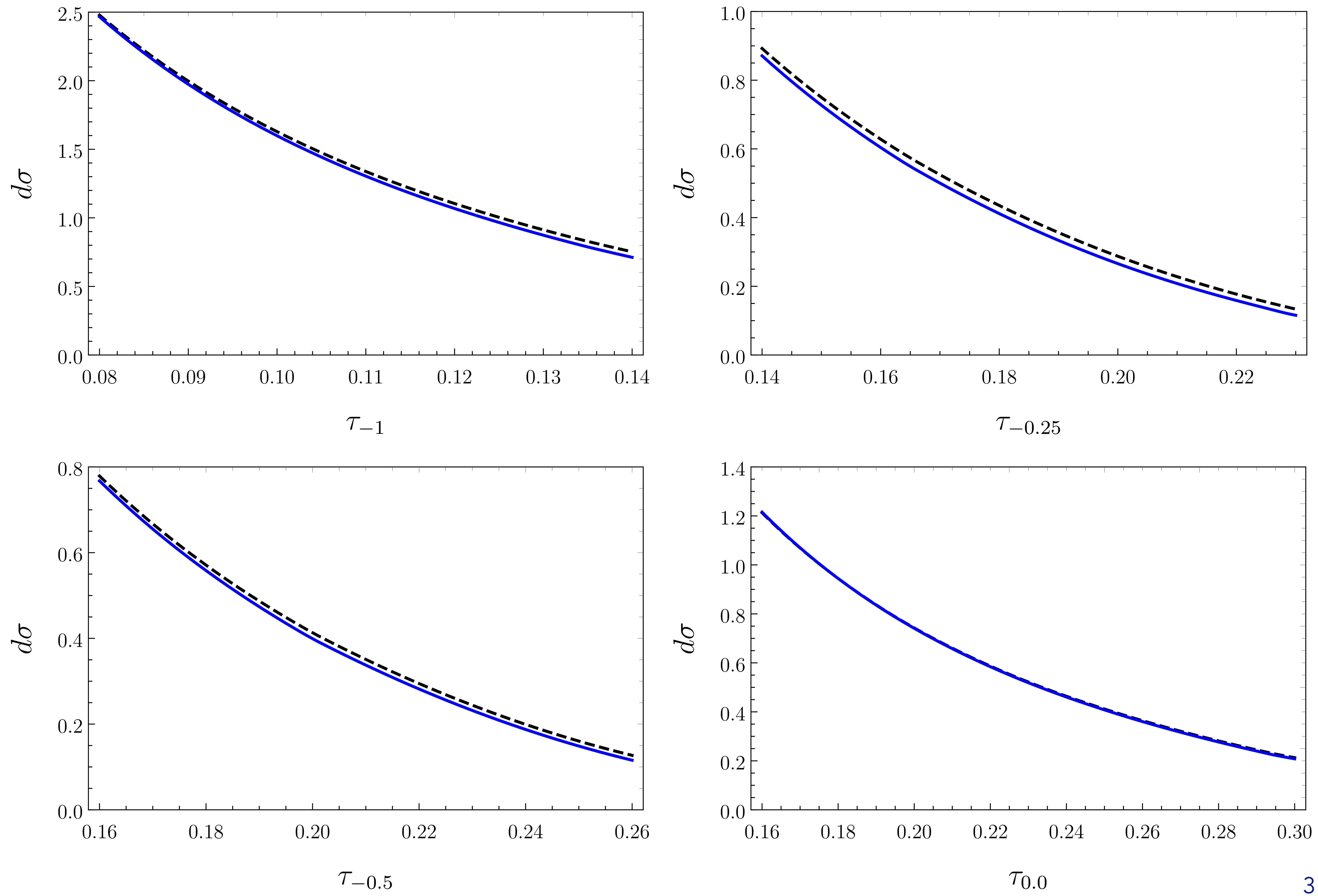
- For single-angularity fits:



Shift in distributions

- from R_{gap} to R^* scheme, are actually quite small:

- Note these shifts will also grow for larger Q



Correlations among angularities

Estimated from Pythia simulations:

i^{th} bin $\qquad$ k^{th} experiment

$$\boldsymbol{T}^k = (\tau^{(1)}_{a=-1}, \tau^{(2)}_{a=-1}, \ldots \tau^{(i)}_{a=-1}, \ldots \tau^{(1)}_{a=-0.75}, \ldots \tau^{(1)}_{a=-0.5}, \ldots)^k$$

$$k = 1, 2, 3, \ldots, 10^5$$

each experiment containing 4.93×10^5 events

$$\mathcal{V}_{ij} = \frac{1}{10^5}(\boldsymbol{T} - \mu_{\boldsymbol{T}})^T(\boldsymbol{T} - \mu_{\boldsymbol{T}}) =$$