Automating the calculation of jet and beam functions

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World SCET 2021

- Starting point: Typical factorization theorems in SCET

$$d\sigma \simeq H(\mu_F) \cdot \prod_i B_i(\mu_F) \otimes \prod_j J_j(\mu_F) \otimes S(\mu_F)$$

- Calculated perturbatively at their own characteristic scale
- Evolved to a common reference scale via RGE

  $\hookrightarrow$ Requires extraction of renormalized quantities

  $\hookrightarrow \left\{\Gamma_{\text{Cusp}}, \gamma_{\text{H}}, c_{\text{H}}, \gamma_{\text{J}}, \gamma_{\text{S}}, c_{\text{J}}, c_{\text{S}}\right\}$

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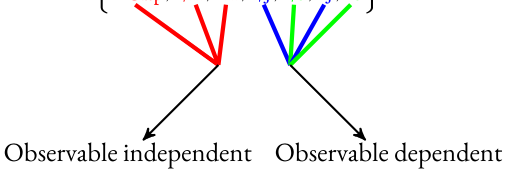


- Hard anomalous dimensions are know to three-loop order

  [Becher, Neubert 09; Almelid, Duhr, Gardi 15]

- Jet- and soft function quantities were previously calculated on a case-by-case basis

SoftSERVE

Current version(v.1.0.1):

- Automated framework of NNLO dijet soft functions
- Applies to SCET I and SCET II observables
- Computes all colour structures
- Automated renormalisation scripts
- publically available at https://softserve.hepforge.org/

[Bell,Rahn,Talbert,18,20]

In progress:

- Extension to arbitrary number of lightlike directions
- Focus on SCET I observables that obey NAE

[Bell,Dehnadi, Mohrmann, Rahn, in progress]

SoftSERVE

Current version(v.1.0.1):
e^+e^- event shapes

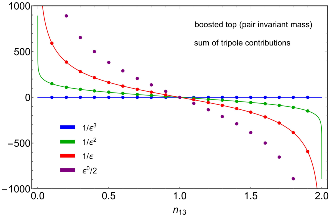
- Thrust
- C-Parameter
- Angularities
- Hemisphere masses

Hadron-collider observables

- Threshold resummation
- p_T-resummation
- Jet veto resummation

In progress:
N-Jet soft functions

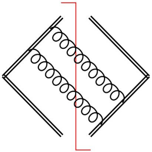
- N-jettiness
  [Bell,Dehnadi, Mohrmann, Rahn,18]
- N-angularities
- pair invariant mass distribution of boosted top quarks

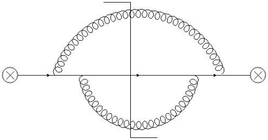


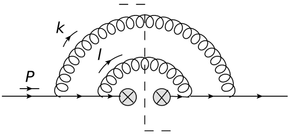
Differences between soft- jet- and beam-function

- Differences at two loop order :

	Particle phase space	Renormalization
soft function	• 2-particle final state	• direct renormalization
jet function	• 3-particle final state ↪ complicated divergence structure	• direct renormalization ↪ similar to soft function
beam function	• 2-particle final state ↪ similar to soft function	• additional matching step ↪ more complicated







Definition:

$$J_q(\tau,\mu) \sim \sum_{i\in X} \delta(\mathrm{Q} - \textstyle\sum_i \bar{n}\cdot k_i)\,\delta^{(\mathrm{d}-2)}(\textstyle\sum_i k_\perp^i)\,\mathcal{M}(\tau,\{k_i\})\,\langle 0|\,\chi\,|X\rangle\,\langle X|\,\bar{\chi}\,|0\rangle$$

- collinear field operators $\chi = W^\dagger \frac{\not{\bar{n}}\not{n}}{4}\psi$
- generic measurement function $\mathcal{M}(\tau,\{k_i\})$

Status:

- automated formalism exists for NLO calculation ✓

[KB Master Thesis,18; Basdew-Sharma et al, 20]

- formalism extended to NNLO real-virtual interference ✓
- currently working on NNLO real-real contribution

NLO calculation

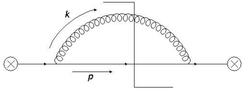
- Matrix element: LO splitting function $P^0_{qg}(z_k)$

$$P^0_{qg}(z_k) \sim \frac{(1-\epsilon)z_k^2+2(1-z_k)}{z_k}$$

[Altarelli,Parisi,77]

- Phase space parametrization that factorizes all divergences

$$k_T = \sqrt{k_+k_-}, \quad z_k = \frac{k_-}{Q}, \quad t_k = \frac{1-\cos(\theta_k)}{2}$$



- Generic parametrization of measurement function in Laplace space

$$\mathcal{M}_1(\tau,k,p) = exp\Big(-\tau k_T\Big(\frac{k_T}{z_kQ}\Big)^n f(z_k,t_k)\Big)$$

- Example : Thrust, Angularities

$$n=1, \quad f(z_k,t_k) = \frac{1}{1-z_k}$$

$$n = 1-A, \quad f(z_k,t_k) = 1 + \left(\frac{z_k}{1-z_k}\right)^{1-A}$$

- Master formula for NLO jet function

$$J_q^1(\tau,\mu) \sim \Gamma\left(\frac{-2\epsilon}{1+n}\right)\int_0^1 \mathrm{d}z_k\, z_k^{-1-2\frac{n}{1+n}\epsilon}\,\bar{z}_k^{-2\frac{n}{1+n}\epsilon}\left(z_k P_{qg}^0(z_k)\right) \times \int_0^1 \mathrm{d}t_k\,\left(4t_k\bar{t}_k\right)^{-1/2-\epsilon} f(z_k,t_k)^{\frac{2\epsilon}{1+n}}$$

- Collinear divergence ($k_T \to 0$): $\Gamma\left(\frac{-2\epsilon}{1+n}\right)$
- $\bar{n}$-collinear divergence ($z_k \to 0$): $z_k^{-1-2\frac{n}{1+n}\epsilon}$
- Soft divergence ($\{z_k \to 0\} + \{k_T \to 0\}$)

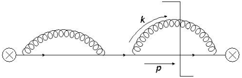
$\Rightarrow$ All singularities are factorized

NNLO real-virtual interference

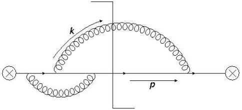
- Matrix element: NLO splitting function $P^1_{qg}(z_k)$

[Furmanski, Petronzio, 80]

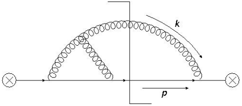
$$J_q^{2,RV}(\tau,\mu) \sim f(\epsilon)\Gamma\left(\frac{-4\epsilon}{1+n}\right)\int_0^1 \mathrm{d}z_k \int_0^1 \mathrm{d}t_k\, z_k^{-1-4\frac{n}{1+n}\epsilon}\,\mathcal{W}(z_k,t_k)$$



  - Virtual corrections: $f(\epsilon) \sim \frac{1}{\epsilon^2} + \mathcal{O}(\epsilon^{-1})$
  - Collinear divergence ($k_T \to 0$): $\Gamma\left(\frac{-4\epsilon}{1+n}\right)$



  - $\bar{n}$-collinear divergence ($z_k \to 0$): $z_k^{-1-4\frac{n}{1+n}\epsilon}$
  - Soft divergence ($\{z_k \to 0\} + \{k_T \to 0\}$)

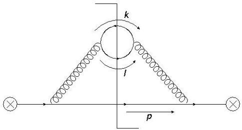


$\Rightarrow$ Both phase space singularities are factorized

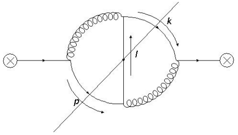
NNLO real-real contribution

- Matrix element: LO triple collinear splitting functions

$$P^{0}_{q'\bar{q}'q},\ P^{0,\mathrm{id}}_{q\bar{q}q},\ P^{0,C_F^2}_{ggq},\ P^{0,C_FC_A}_{ggq}$$ [Catani,Grazzini,99]

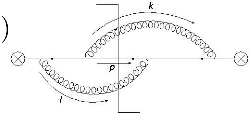


- Complicated divergence structure
  - $P^{0}_{q'\bar{q}'q} \sim \frac{1}{s_{123}^2 s_{12}^2 (z_1+z_2)^2}$
  - $P^{0,\mathrm{id}}_{q\bar{q}q} \sim \frac{1}{s_{123}^2 s_{12}},\ \frac{1}{s_{123}s_{12}(1-z_2)(1-z_3)},\ \frac{1}{s_{13}s_{12}(1-z_2)(1-z_3)}$
  - $P^{0,C_F^2}_{ggq} \sim \frac{1}{s_{123}^2 s_{13}},\ \frac{1}{s_{123}s_{13}(z_1)(z_2)},\ \frac{1}{s_{13}s_{23}(z_1)(z_2)}$



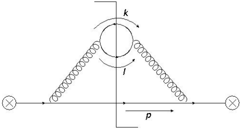
$$s_{123} = s_{12} + s_{13} + s_{23},\ s_{12} = (2k\cdot l),\ s_{13} = (2k\cdot p),\ s_{23} = (2l\cdot p)$$

$$z_1 = \frac{k_-}{Q},\ z_2 = \frac{l_-}{Q},\ z_3 = \frac{p_-}{Q}$$



NNLO real-real contribution: CF TF nf

- Divergence structure
  - $P^0_{q'\bar{q}'q} \sim \frac{1}{s_{123}{}^2 s_{12}{}^2 (z_1+z_2)^2}$



- Phase space parametrization

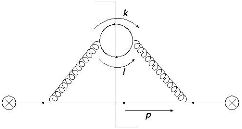
$$z_{12} = \frac{k_- + l_-}{Q}, \quad w = \frac{k_+}{l_+},$$
$$a = \frac{k_- l_T}{k_T l_-}, \quad t_{kl} = \frac{1 - \cos(\theta_{kl})}{2},$$
$$q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$

- Generic parametrization of measurement function in Laplace space

$$\mathcal{M}_2(\tau, k, l, p) = exp\left(- \tau q_T \left(\frac{q_T}{z_{12} Q} \right)^n F(z_{12}, a, w, t_{kl}, t_l, t_k) \right)$$

- Remap the integration region onto the unit hypercube

$$J^{2,RR}_{q'\bar{q}'q}(\tau,\mu) \sim \Gamma\left(\frac{-4\epsilon}{1+n}\right)\int_0^1 \mathrm{d}z_{12}\int_0^1 \mathrm{d}u\, z_{12}^{-1-4\frac{n}{1+n}\epsilon}\, u^{-1-2\epsilon} \times \int_0^1 \mathrm{d}w \int_0^1 \mathrm{d}v\, \mathcal{W}(z_{12},u,v,w,t_l,t_k)$$



- ▶ Triple collinear divergence $(q_T \to 0)$: $\Gamma\left(\frac{-4\epsilon}{1+n}\right)$
- ▶ Double collinear divergence $(u \to 0)$: $u^{-1-2\epsilon}$
- ▶ $\bar{n}$-collinear divergence $(z_{12} \to 0)$: $z_{12}^{-1-4\frac{n}{1+n}\epsilon}$

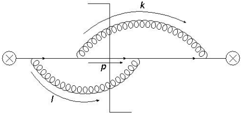
⇒ All singularities factorized

NNLO real-real contribution: CF^2

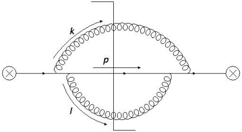
- Similar phase space parametrization

  $\hookrightarrow \{z_{ij}, w_{ij}, a_{ij}, t_{ij}, q_T^{ij}\}$

  $\rightarrow$ ij dependents on occurring invariant masses



- Many overlapping divergences in matrix element
- Additional complications related to measurement function
  - Selector function
  - Sector decomposition
  - Non-linear transformations



$\Rightarrow$ We managed to factorize all divergences ($\mathcal{O}(60)$-Regions C_F^2 contribution)

$\Rightarrow$ $C_F C_A$ structure in progress

SCET I renormalization

- For SCET I the jet function fulfills the RG equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} J(\tau,\mu) = \left[\frac{2(1+n)}{n}\Gamma_{\text{Cusp}}(\alpha_s)\ln\left(\frac{\mu\bar{\tau}^{\frac{1}{1+n}}}{Q^{\frac{n}{1+n}}}\right) + 2\gamma_{\text{J}}(\alpha_s)\right] J(\tau,\mu)$$

$$\bar{\tau} = \tau e^{\gamma_E}$$

Two-loop solution

$$J(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1+n}{n}\right)\Gamma_0 L^2 + 2\gamma_0 L + c_1\right] + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{1}{2}\left(\frac{1+n}{n}\right)^2\Gamma_0^2 L^4 + \left(\frac{2}{3}\left(\frac{1+n}{n}\right)\beta_0\Gamma_0 + 2\left(\frac{1+n}{n}\right)\Gamma_0\gamma_0\right)L^3 + \left(2\gamma_0^2 + \left(\frac{1+n}{n}\right)\Gamma_0 c_1 + 2\gamma_0\beta_0 + \left(\frac{1+n}{n}\right)\Gamma_1\right)L^2 + \left(2\gamma_0 c_1 + 2\beta_0 c_1 + 2\gamma_1\right)L + c_2\right] + \mathcal{O}(\alpha_s^3)$$

$\hookrightarrow$ Extraction of $\{\Gamma_0, \Gamma_1, \gamma_0, \gamma_1, c_1, c_2\}$

Preliminary thrust results

- We implemented our formula in `PySecDec` [Heinrich,et al.,17]
- Derived in few hours on an single 8 core Laptop

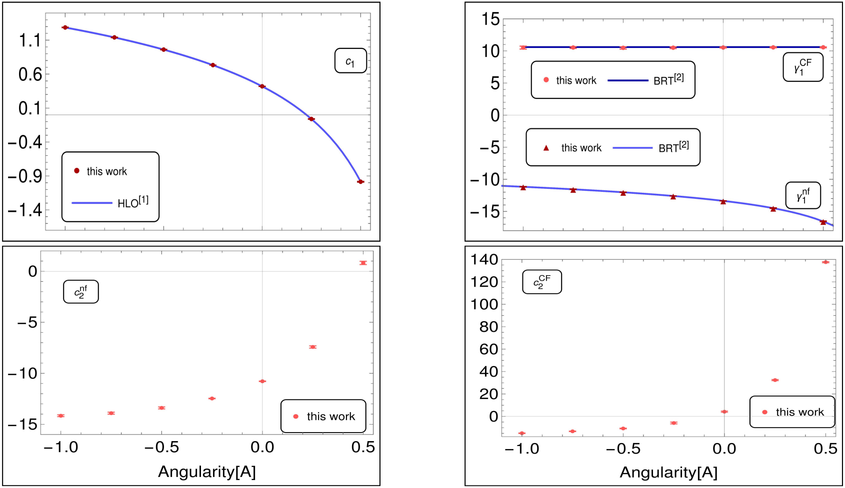
C_F	analytic[1]	this work
Γ_0	4	3.9999(1)
γ_0	3	2.9997(3)
c_1	0.4203	0.4201(8)

$C_F T_F n_F$	analytic[1]	this work
Γ_1	−8.8889	−8.8893(21)
γ_1	−13.349	−13.350(9)
c_2	−10.787	−10.783(26)

C_F^2	analytic[1]	this work
Γ_1	0	−0.034(37)
γ_1	10.610	10.562(114)
c_2	4.6551	4.531(325)

[1]:[Becher,Neubert, 06]

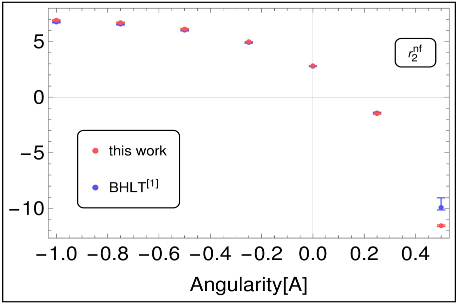
Preliminary angularity results

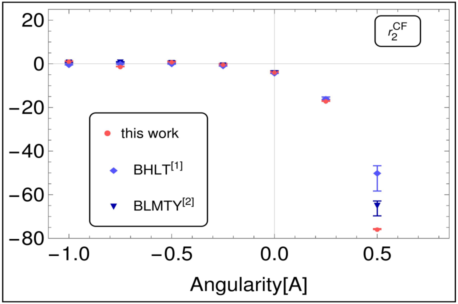


A	−1	−0.75	−0.5	−0.25	0	0.25	0.5
c_2^{nf}	−14.16(9)	−13.91(9)	−13.39(9)	−12.47(1)	−10.78(3)	−7.42(11)	0.81(16)
c_2^{CF}	−14.93(22)	−13.25(23)	−10.73(28)	−5.87(78)	4.22(53)	32.42(49)	137.56(51)

[1]:[Hornig,Lee,Ovanesyan, 09], [2]:[Bell,Rahn,Talbert,19]

Preliminary angularity results





A	−1	−0.75	−0.5	−0.25	0	0.25	0.5
$r_2^{nf}[1]$	$6.76^{+0.08}_{-0.03}$	$6.57^{+0.08}_{-0.03}$	$6.03^{+0.07}_{-0.03}$	$4.92^{+0.06}_{-0.02}$	$2.78^{+0.03}_{-0.02}$	$-1.42^{+0.02}_{-0.06}$	$-9.92^{+0.23}_{-0.87}$
this work	6.92(4)	6.70(4)	6.13(5)	4.99(2)	2.82(1)	−1.48(6)	−11.56(8)

A	−1	−0.75	−0.5	−0.25	0	0.25	0.5
$r_2^{CF}[1]$	$-0.16^{+0.37}_{-0.28}$	$0.19^{+0.34}_{-0.24}$	$0.24^{+0.35}_{-0.18}$	$-0.66^{+0.36}_{-0.29}$	$-4.03^{+0.38}_{-0.27}$	$-15.9^{+0.4}_{-0.7}$	$-49,9^{+3.2}_{-8.4}$
$r_2^{CF}[2]$	$0.89^{+0.17}_{-0.22}$	$0.90^{+0.14}_{-0.09}$	$0.67^{+0.29}_{-0.59}$	$-0.45^{+0.28}_{-0.64}$	$-3.98^{+0.31}_{-0.78}$	$-16.7^{+0.3}_{-0.4}$	$-64.48^{+1.9}_{-4.9}$
this work	1.24(11)	1.21(11)	0.97(14)	−0.19(39)	−3.77(26)	−16.98(25)	−75.77(26)

[1]:[Bell,Hornig,Lee,Talbert, 19], [2]:[Bell, Lee, Makris, Talbert, Yan unpublished]

Definition:

$$\mathcal{B}_{qq}(\tau,\mu,x) \sim \sum_{i\in X_c} \delta((\bar{n}\cdot P)(1-x) - \bar{n}\cdot k_{X_c})\mathcal{M}(\tau,\{k_i\}) \langle P|\,\bar{\chi}\,|X_c\rangle \frac{\not{\bar{n}}}{2} \langle X_c|\,\chi\,|P\rangle$$

$$\mathcal{B}_{qq}(\tau,\mu,x) = \sum_{k} \mathcal{I}_{qk}(\tau,\mu,x) \otimes \phi_{kq}(\mu,x)$$

$\hookrightarrow$ Distribution valued in x

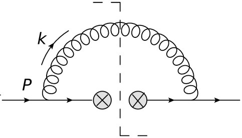
Status:

- automated formalism exists for NLO calculation ✓ [KB Master Thesis,18]
- formalism extended to NNLO real-virtual interference ✓
- currently working on NNLO real-real contribution

NLO calculation

- Matrix element: Related by crossing symmetry to LO splitting function $P^0_{qg}(z_k)$
- Phase space parametrization that factorizes all divergences

$$k_T = \sqrt{k_+ k_-}, \quad k_- = (1-x)P_-, \quad t_k = \frac{1-\cos(\theta_k)}{2}$$



- Generic parametrization of measurement function in Laplace space

$$\mathcal{M}_1(\tau, k) = exp\Big(-\tau k_T \Big(\frac{k_T}{(1-x)P_-}\Big)^n f(t_k)\Big)$$

- Example : $\mathrm{p_T}$-resummation, $\mathrm{p_T}$-veto

$$n = 0, \quad f(t_k) = -2i(1-2t_k)$$
$$n = 0, \quad f(t_k) = 1$$

- Master formula for NLO beam function

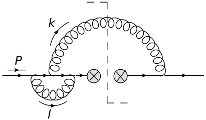
$$B^1_{qq}(\tau,\mu,x) \sim \Gamma\left(\frac{-2\epsilon}{1+n}\right)(1-x)^{-1-\frac{2n\epsilon}{1+n}-\alpha}\left[(1-\epsilon)(1-x)^2+2x\right] \times \int_0^1 \mathrm{d}t_k\,\left(4t_k\bar{t}_k\right)^{-1/2-\epsilon} f\left(t_k\right)^{\frac{2\epsilon}{1+n}}$$

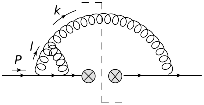
  - ▶ Collinear divergence ($k_T \to 0$): $\Gamma\left(\frac{-2\epsilon}{1+n}\right)$
  - ▶ $\bar{n}$-collinear divergence ($x \to 1$): $(1-x)^{-1-\frac{2n\epsilon}{1+n}-\alpha}$
  - ▶ Soft divergence ($\{x \to 1\} + \{k_T \to 0\}$)

$\Rightarrow$ All singularities are factorized

NNLO real-virtual interference

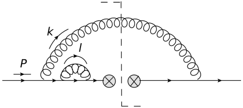
- Matrix element: Related by crossing symmetry to NLO splitting function $P^1_{qg}(z_k)$





$$B^{2,RV}_{qq}(\tau,\mu,x) \sim f(\epsilon)\Gamma\left(\frac{-4\epsilon}{1+n}\right)(1-x)^{-1-\frac{4n\epsilon}{1+n}-\alpha}\mathcal{W}(x)$$
$$\times \int_0^1 \mathrm{d}t_k\, (4t_k\bar{t}_k)^{-1/2-\epsilon} f(t_k)^{\frac{4\epsilon}{1+n}}$$

  - Virtual corrections: $f(\epsilon) \sim \frac{1}{\epsilon^2} + \mathcal{O}(\epsilon^{-1})$
  - Collinear divergence ($k_T \to 0$): $\Gamma\left(\frac{-2\epsilon}{1+n}\right)$
  - $\bar{n}$-collinear divergence ($x \to 1$): $(1-x)^{-1-\frac{4n\epsilon}{1+n}-\alpha}$
  - Soft divergence ($\{x \to 1\} + \{k_T \to 0\}$)

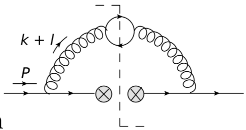


$\Rightarrow$ Both phase space singularities are factorized

NNLO real-real contribution: CF TF nf

- Phase space parametrizations

$$x_{12} = \frac{k_- + l_-}{P_-}, \quad b = \frac{k_T}{l_T},$$
$$a = \frac{l_T k_-}{k_T l_-}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$



- Generic parametrization of measurement function in Laplace space

$$\mathcal{M}_2(\tau, k, l) = exp\left(- \tau q_T \left(\frac{q_T}{x_{12} P_-} \right)^n F(x_{12}, a, b, t_{kl}, t_l, t_k) \right)$$

NNLO real-real contribution: C_F^2

- Two different phase space parametrizations

$\hookrightarrow$ $\frac{1}{s_{12}s_{13}z_1z_2}$:

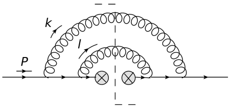
$$x_1 = \frac{k_-}{P_-}, \quad x_2 = \frac{l_-}{P_-},$$

$$b = \frac{k_T}{l_T}, \quad q_T = k_T + l_T$$

$\hookrightarrow$ Other structures:

$$x_{12} = \frac{k_- + l_-}{P_-}, \quad r = \frac{k_-}{l_-},$$

$$\tilde{a} = \frac{k_T l_-}{l_T k_-}, \quad q_T = \sqrt{(k_- + l_-)(k_+ + l_+)}$$



- Overlapping divergences remain in certain structures

SCET II renormalization

- Perform Mellin-transformation
- SCET-II renormalization based on collinear-anomaly approach [Becher,Neubert, 10]

$$\left[\mathcal{S}(\bar{\tau},\mu,\nu)\,\mathcal{I}_{qq}(\mathbf{N}_1,\bar{\tau},\mu,\nu)\,\bar{\mathcal{I}}_{\bar{q}\bar{q}}(\mathbf{N}_2,\bar{\tau},\mu,\nu)\right]_{q^2} \stackrel{\alpha=0}{=} \left(\bar{\tau}^2 q^2\right)^{2F^B_{q\bar{q}}(\bar{\tau},\mu)} I_{qq}(N_1,\bar{\tau},\mu)\bar{I}_{\bar{q}\bar{q}}(N_2,\bar{\tau},\mu)$$

- Anomaly coefficient $F^B_{q\bar{q}}(\bar{\tau},\mu)$ and remainder function $I_{qq}(N_1,\bar{\tau},\mu)$ fulfill RGEs

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}F^B_{q\bar{q}}(\bar{\tau},\mu) = -\,\Gamma_{\text{cusp}}(\alpha_s)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}I_{qq}(N_1,\bar{\tau},\mu) = \left[2\Gamma_{cusp}L + 2\gamma_I\right]I_{qq}(N_1,\bar{\tau},\mu) - 2\sum_k I_{qk}(N_1,\bar{\tau},\mu)\cdot P_{kq}(N_1,\mu)$$

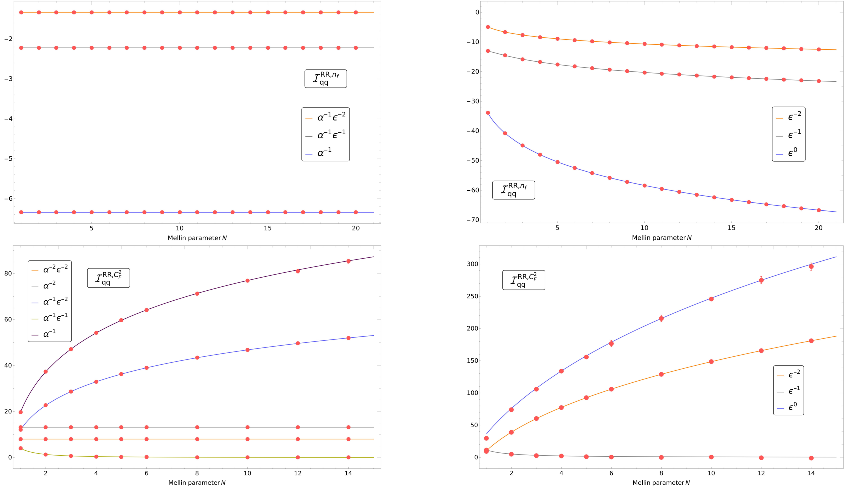
$$L = \ln(\mu\bar{\tau})$$

Two-loop solution anomaly coefficient

$$F^B_{q\bar{q}}(\bar{\tau},\mu) = -\frac{\alpha_s}{4\pi}\left[\Gamma_0 L - d_1^B\right] - \left(\frac{\alpha_s}{4\pi}\right)^2\left[\beta_0\Gamma_0 L^2 + \Gamma_1 L + \beta_0 d_1^B L - d_2^B\right]$$

$\hookrightarrow$ Extraction of $\{d_1^B, d_2^B\}$

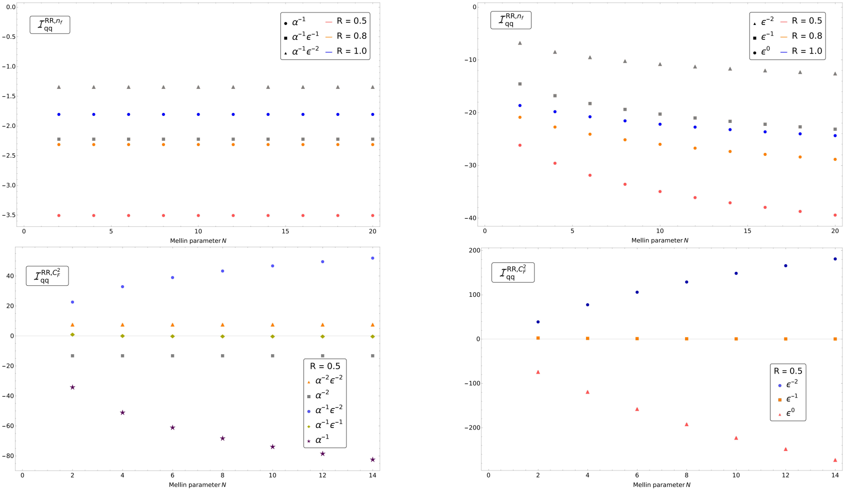
Preliminary p_T-resummation results



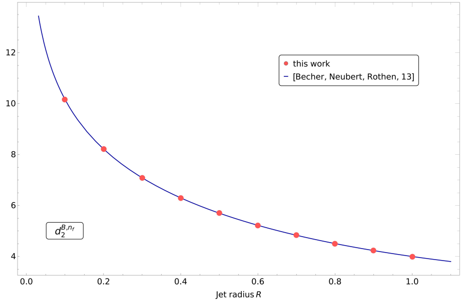
d_2^B	analytic[1]	this work
$d_2^{B,nf}$	4.148	4.147(4)
$d_2^{B,CF}$	0	0.02(11)

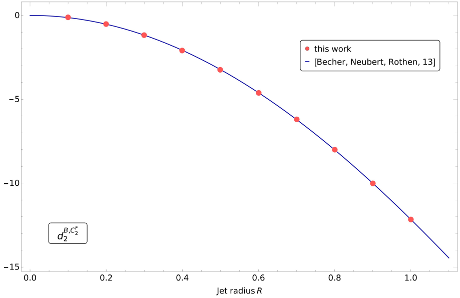
[1]: [Gehrmann, Lübbert, Yang, 14]

Preliminary p_T-veto results



Preliminary p_T-veto results





Conclusion

- We are aiming at developing an automated approach for NNLO jet-and beam functions
- Jet function:
  - Divergence structure of double real-emission contribution more complicated than for soft functions
  - Preliminary results for thrust and angularities jet function
- Beam function:
  - Two out of three color structures of the NNLO-RR contribution implemented
  - Preliminary results for p_T-resummation, p_T-veto beam function

Conclusion

- We are aiming at developing an automated approach for NNLO jet-and beam functions

<u>Outlook</u>

- Jet function:
  - Last NNLO color structure in progress
  - SCET II observables
  - Implementation of gluon jet function
- Beam function:
  - Last NNLO color structure in progress
  - SCET I observables
  - Implementation of other matching kernels

Thank You