NLO Massive Event-Shape Differential and Cumulative Distributions

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FWF Der Wissenschaftsfonds.

$\int dk\Pi$ Doktoratskolleg Particles and Interactions

in collaboration with **Vicent Mateu**

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Outline

- Intro & Motivation
- NLO Massive Event-Shape Cross Section: General Form
- Calculation Details
- Results and Cross Checks
- Summary and Outlook

Intro & Motivation

- Event-shape cross sections for e^+e^-:
  - Very precise predictions due to resummation/factorization

- Event shapes with massless quarks:
  - $\mathrm{N^2LL}$, or even $\mathrm{N^3LL}$ [Hornig (2009), Becher (2012), Bell (2019)] [Becher (2008), Chien (2010), Hoang (2015), Moult (2018)]
  - FO (some event shapes):
    - $\mathcal{O}(\alpha_s)$ analytically,
    - $\mathcal{O}(\alpha_s^{1,2,3})$ numerically [Catani (1997), Dixon (2018)] [Gehrmann (2007, 2014), Weinzierl (2008, 2009), Del Duca (2016)]

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  - FO (some event shapes):
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    - $\mathcal{O}(\alpha_s^{1,2,3})$ numerically

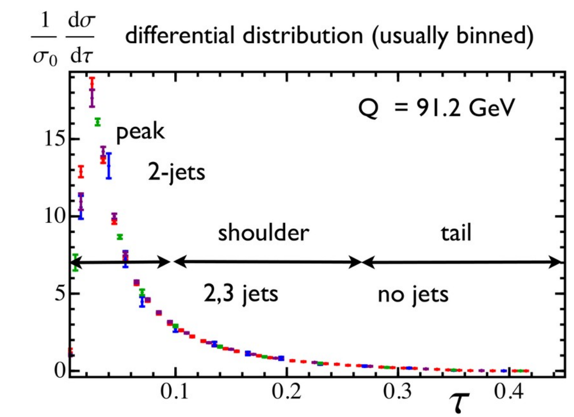
- Event shapes with massive quarks:
  - $\mathrm{N^2LL}$ often available $\mathrm{N^3LL}$ for 2-jettiness
    [Jain (2008), Gritschacher (2013), Pietrulewicz (2014), Hoang (2015, 2019)]
  - FO (some event shapes):
    - $\mathcal{O}(\alpha_s^{1,2})$ numerically
      [Nason (1998), Bernreuther (1997), Rodrigo (1999)]
  - This talk: General method to compute FO analytically at $\mathcal{O}(\alpha_s)$
    - $\mathrm{N^2LL}$ for many event shapes

Intro & Motivation

- Why masses?
  - Mass-specific studies
    - Top quark mass measurements at future colliders
    - Monte Carlo mass calibration [Butenschoen (2016)]
    - ...
  - More and more precise measurements → smaller and smaller corrections
  - General insights, e.g. on mass sensitivity
    - Massive schemes → following talk by Alejandro Bris

Intro & Motivation

- Why FO?
  - Want to have a complete description of massive event shapes at $\mathrm{N^2LL}$
    - Peak: SCET + bHQET
    - (Far) tail: matching to FO prediction



[Vicent Mateu]

- Not-so-boosted heavy quarks:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma_{\mathrm{NS}}}{\mathrm{d}\tau}$$

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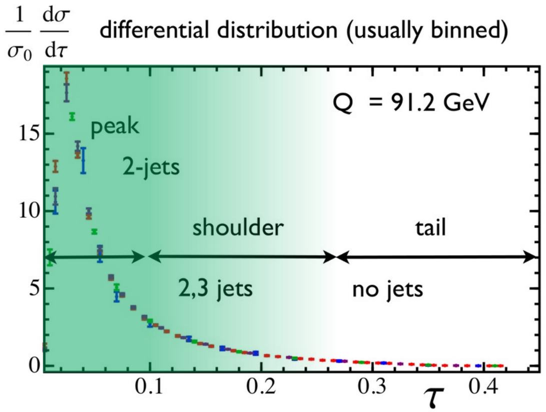

[Vicent Mateu]

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma_{\mathrm{NS}}}{\mathrm{d}\tau}$$

- Factorized, resummed

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}\tau} = Q^2 H(Q,\mu)\int_0^{Q(\tau-\tau_{\mathrm{min}})} \mathrm{d}\ell\, J_\tau(Q^2\tau - Q\ell,\mu) S_\tau(\ell,\mu)$$

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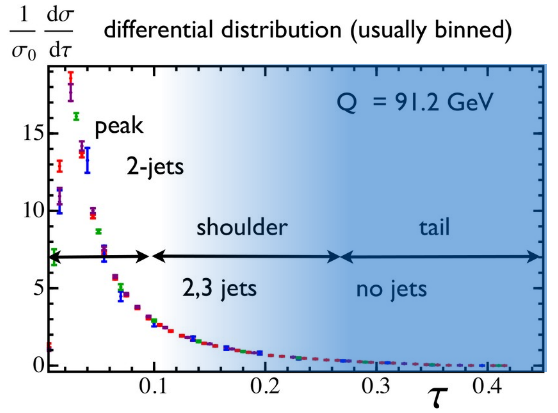

[Vicent Mateu]

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma_{\mathrm{NS}}}{\mathrm{d}\tau}$$

- "Non-singular": contains two types of power corrections:
  - Kinematic: the "usual ones"
  - Mass: can be absorbed into SCET distributions (see following talk)

NLO massive event-shape cross section

- Main topic of this talk:
  - General form of massive FO event-shape cross sections at $\mathcal{O}(\alpha_s)$

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma_C}{\mathrm{d}e} = R_C^0(\hat{m})\,\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}A_e^C(\hat{m})\delta(e-e_{\min})$$
$$+ C_F\frac{\alpha_s}{\pi}B_{\mathrm{plus}}^C(\hat{m})\left[\frac{1}{e-e_{\min}}\right]_+ + C_F\frac{\alpha_s}{\pi}F_e^{\mathrm{NS}}(e,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

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- $C=\{V,A\}$: Vector and axial-vector currents
- $\hat{m}\equiv m/Q$: Reduced mass
- e_{min}: Minimal event shape value (zero or $\hat{m}$ dependent)

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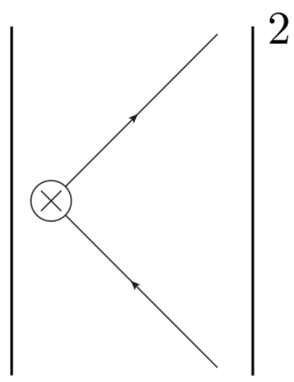
- Born-normalized cross section
  - Known analytically



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- Dirac-delta coefficient
  - New: simple analytic formula
  - Event-shape dependent
  - Mass power corrections – absorbable into SCET factorization

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- Plus-distribution coefficient
  - New: universal, computed analytically
  - Mass power corrections – absorbable into SCET factorization

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- Non-singular terms
  - New: (quite) simple formula, algorithm to determine precisely numerically
  - Depends on event shape
  - Mass and kinetic power corrections – not absorbable

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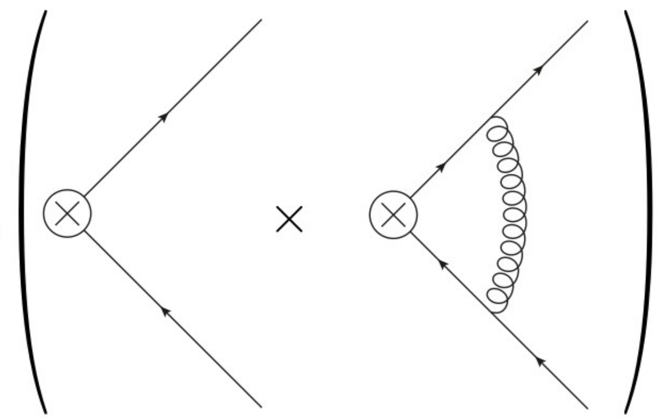
- Valid for event shapes that are
  - IR & collinear safe
  - Linearly sensitive to soft dynamics (all popular ones)

Calculation Details

- Virtual:
  - Standard computation
  - Pure Dirac-delta contribution
  - Universal
  - IR divergences

$2\,\mathrm{Real}\Big($



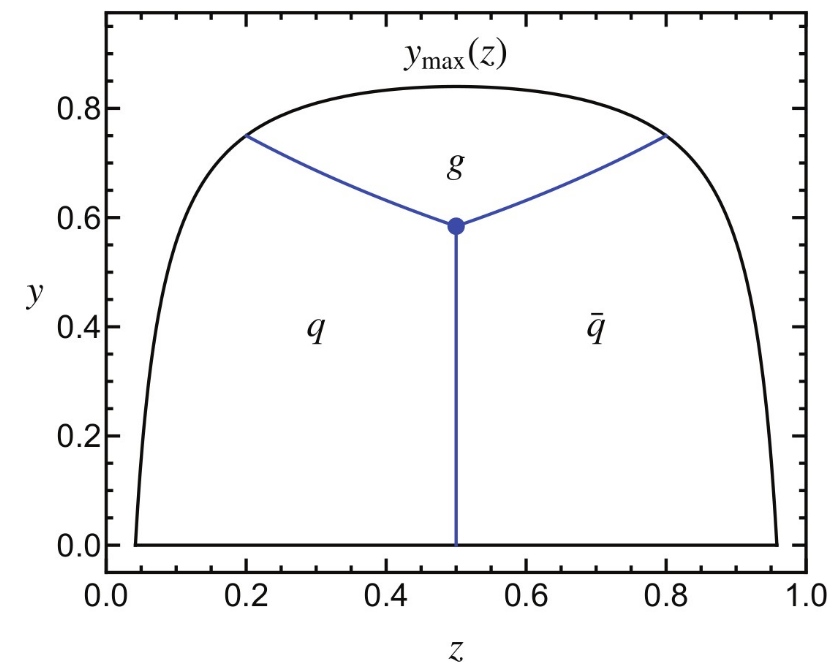
$\Big) \times \Phi_2$

$$= C_F \frac{\alpha_s}{\pi} R_C^{\mathrm{virt}}(\hat{m})\, \delta(e - e_{\min})$$

Calculation Details

- Real:
  - Quark masses screen collinear divergences → only soft
  - Symmetric phase-space parametrization → soft singularities at $y \to 0$

$$E_g = \frac{yQ}{2} \qquad E_q = \frac{Q}{2}(1 - zy)$$



Calculation Details

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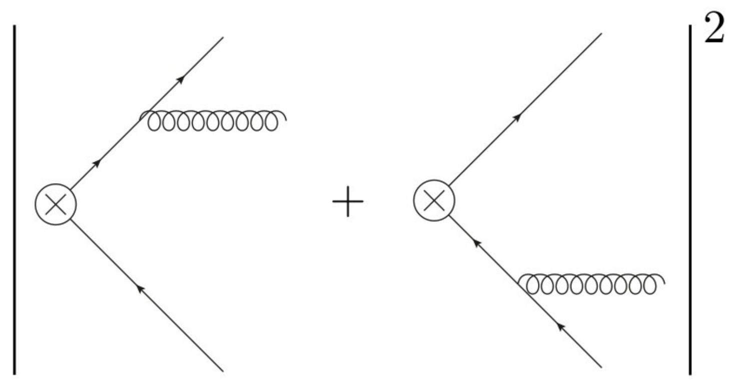
$$E_g = \frac{yQ}{2} \qquad E_q = \frac{Q}{2}(1 - zy)$$

  - d-dimensional phase space → expand in ε, distributional identities

$$x^{-1+\varepsilon} = \frac{1}{\varepsilon}\delta(x) + \sum_{i=0}^{\infty} \frac{\varepsilon^n}{n!} \left[\frac{\log^n(x)}{x}\right]_+$$



$$\frac{|\sum_{\text{spin}} \mathcal{M}_C|^2}{4\sigma_0^C} = \frac{256\pi^2 \alpha_s \tilde{\mu}^{2\varepsilon} C_F}{y^2} M_C(y, z, \hat{m}, \varepsilon)$$

$$M_C(y, z, \hat{m}, \varepsilon) = M_C^0(z, \hat{m}) + \varepsilon M_C^1(z, \hat{m}) + y M_C^{\text{hard}}(y, z) + \mathcal{O}(\varepsilon^2)$$

Calculation Details

- After some analytic manipulations...

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma_C}{\mathrm{d}e} = R_C^0(\hat{m})\,\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}A_e^C(\hat{m})\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}B_{\rm plus}^C(\hat{m})\left[\frac{1}{e-e_{\min}}\right]_+ + C_F\frac{\alpha_s}{\pi}F_e^{\rm NS}(e,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$A_e^C(\hat{m}) = A_e^{\rm real}(\hat{m}) + R_C^{\rm virt}(\hat{m})$$

$$A_e^{\rm real}(\hat{m}) = -\frac{P(Q,\varepsilon)}{2}\int\mathrm{d}z\left\{M_C^1(z,\hat{m}) + M_C^0(z,\hat{m})\left[\frac{1}{\varepsilon} + 2\log\left(\frac{\mu}{Q}\right) - \log\left(\frac{z(1-z)-\hat{m}^2}{[f_e(z)]^2}\right)\right]\right\}$$

$$B_{\rm plus}(\hat{m}) = \int\mathrm{d}z\,M_C^0(z,\hat{m})$$

$$F^{\rm NS} = \int\mathrm{d}z\,\mathrm{d}y\left\{M_C^{\rm hard}(y,z)\delta[e-\hat{e}(y,z)] + \frac{M_C^0(z,\hat{m})}{y}\Big[\delta[e-\hat{e}(y,z)] - \Theta[y-y_{\max}(z)]\,\delta[e-\bar{e}(y,z)] - \delta[e-\bar{e}(y,z)]\Big]\right\}$$

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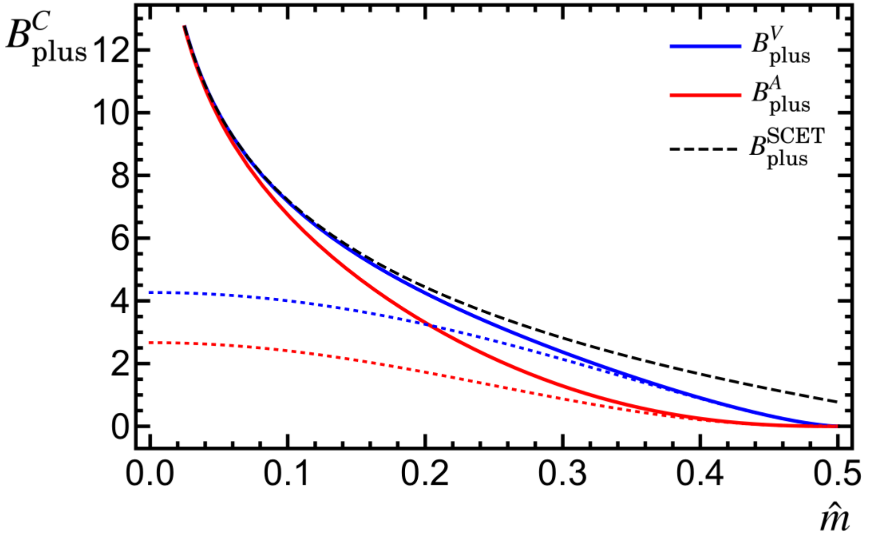
- Plus-distribution coefficient:
  - Independent of event shape
  - Solve integral analytically

$$B_{\rm plus}(\hat{m}) = \left(\frac{3-v^2}{2\,v^2}\right)\left[(1+v^2)L_v - v\right]$$



$$v = \sqrt{1-4\hat{m}^2}$$

$$L_v = \log\left(\frac{1+v}{2\hat{m}}\right)$$

$$A_e^C(\hat m) = A_e^{\rm real}(\hat m) + R_C^{\rm virt}(\hat m)$$

$$A_e^{\rm real}(\hat m) = -\frac{P(Q,\varepsilon)}{2}\int {\rm d}z\biggl\{M_C^1(z,\hat m) + M_C^0(z,\hat m)\biggl[\frac{1}{\varepsilon} + 2\log\Bigl(\frac{\mu}{Q}\Bigr) - \log\biggl(\frac{z(1-z)-\hat m^2}{[f_e(z)]^2}\biggr)\biggr]\biggr\}$$

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- Dirac-delta coefficient:
  - IR divergences cancel here
  - Solve event-shape independent integral analytically

$$v = \sqrt{1-4\hat m^2}$$

$$L_v = \log\left(\frac{1+v}{2\hat m}\right)$$

$$A_e^V(\hat m) = (1+2\hat m^2)\biggl\{(1-2\hat m^2)\biggl[{\rm Li}_2\biggl(-\frac{v(1+v)}{2\hat m^2}\biggr) - 3\,{\rm Li}_2\biggl(\frac{v(1-v)}{2\hat m^2}\biggr) + 2\log^2(\hat m) + \pi^2 - 2\log^2\biggl(\frac{1+v}{2}\biggr)\biggr] + 2v\bigl[\log(\hat m)-1\bigr] - 2I_e(\hat m)\biggr\} + (4+v^2-16\hat m^4)L_v$$

$$A_e^A(\hat m) = v^2\biggl\{(4+v^2)L_v + 2v\bigl[\log(\hat m)-1\bigr] - 2I_e(\hat m) + (1-2\hat m^2) \times\biggl[{\rm Li}_2\biggl(-\frac{v(1+v)}{2\hat m^2}\biggr) - 3\,{\rm Li}_2\biggl(\frac{v(1-v)}{2\hat m^2}\biggr) + \pi^2 + 2\log^2(\hat m) - 2\log^2\biggl(\frac{1+v}{2}\biggr)\biggr]\biggr\},$$

$$A_e^C(\hat{m}) = A_e^{\text{real}}(\hat{m}) + R_C^{\text{virt}}(\hat{m})$$

$$A_e^{\text{real}}(\hat{m}) = -\frac{P(Q,\varepsilon)}{2}\int \mathrm{d}z \left\{ M_C^1(z,\hat{m}) + M_C^0(z,\hat{m})\left[\frac{1}{\varepsilon} + 2\log\left(\frac{\mu}{Q}\right) - \log\left(\frac{z(1-z)-\hat{m}^2}{[f_e(z)]^2}\right)\right]\right\}$$

$$B_{\text{plus}}(\hat{m}) = \int \mathrm{d}z\, M_C^0(z,\hat{m})$$

$$F^{\text{NS}} = \int \mathrm{d}$$

$$I_e(\hat{m}) = \int_{(1-v)/2}^{1/2} \mathrm{d}z\, \frac{(1-z)z-\hat{m}^2}{(1-z)^2 z^2}\log[f_e(z)]$$

$$\hat{e}(z,y) = e_{\text{min}} + y f_e(z) + \mathcal{O}(y^2)$$

Analytic form found in all cases

- Dirac-delta coe…
  - IR divergences cancel here
  - Solve event-shape independent inte… …nalytically

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$$A_e^A(\hat{m}) = v^2\left\{(4+v^2)L_v + 2v[\log(\hat{m})-1] - 2I_e(\hat{m}) + (1-2\hat{m}^2) \times\left[\mathrm{Li}_2\left(-\frac{v(1+v)}{2\hat{m}^2}\right) - 3\,\mathrm{Li}_2\left(\frac{v(1-v)}{2\hat{m}^2}\right) + \pi^2 + 2\log^2(\hat{m}) - 2\log^2\left(\frac{1+v}{2}\right)\right]\right\},$$

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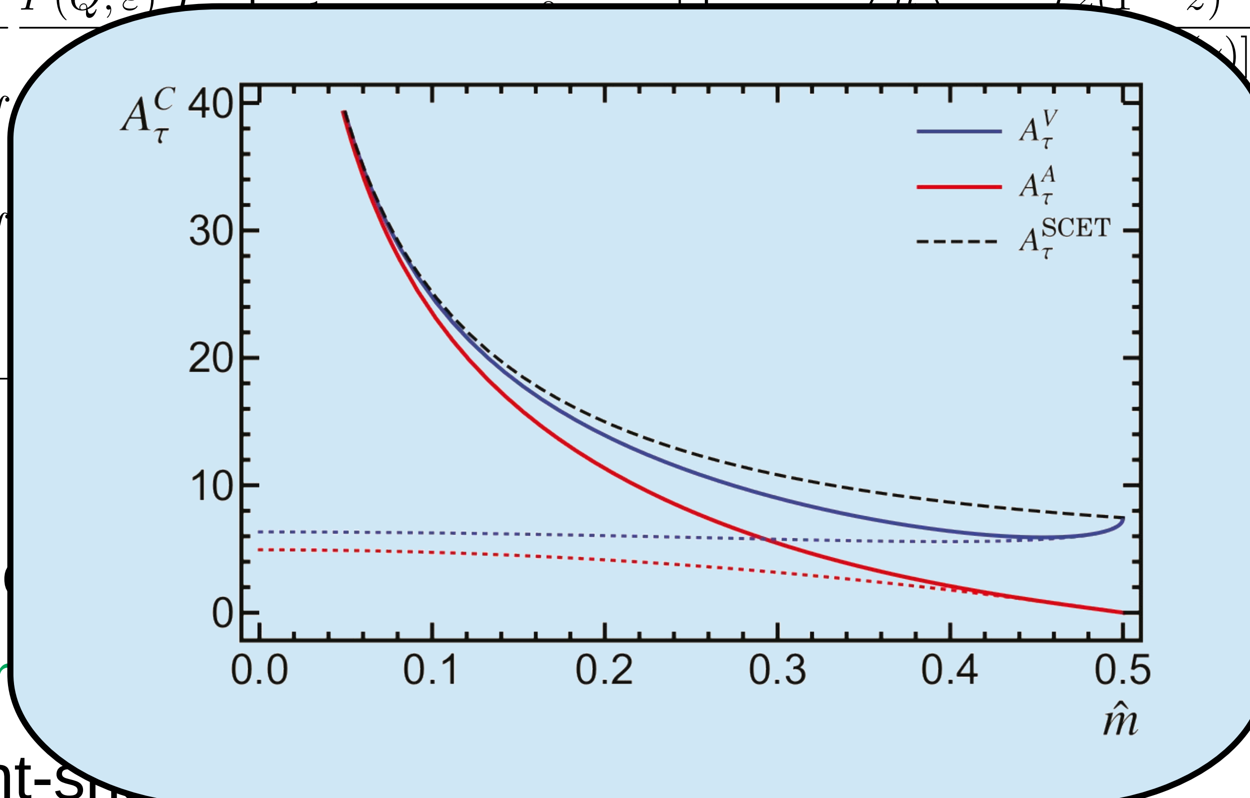
$$A_e^{\rm real}(\hat{m}) = -\frac{P(Q,\varepsilon)}{\ldots} \ldots \frac{z(1-z)-\hat{m}^2}{\ldots)]^2}\Big)\Big]\Big\}$$

$$B_{\rm plus}(\hat{m}) = \int \ldots$$

$$F^{\rm NS} = \int \ldots$$



- Dirac-delta co…
  - IR divergen…
  - Solve event-sh…

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$$- 2\log^2\Big(\frac{1+v}{2}\Big)\bigg] + 2v\big[\log(\hat{m}) - 1\big] - 2I_e(\hat{m})\bigg\} + (4+v^2-16\hat{m}^4)L_v$$

$$A_e^A(\hat{m}) = v^2\bigg\{(4+v^2)L_v + 2v\big[\log(\hat{m})-1\big] - 2I_e(\hat{m}) + (1-2\hat{m}^2)$$
$$\times\bigg[\mathrm{Li}_2\Big(-\frac{v(1+v)}{2\hat{m}^2}\Big) - 3\,\mathrm{Li}_2\Big(\frac{v(1-v)}{2\hat{m}^2}\Big) + \pi^2 + 2\log^2(\hat{m}) - 2\log^2\Big(\frac{1+v}{2}\Big)\bigg]\bigg\},$$

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$$B_{\rm plus}(\hat{m}) = \int \mathrm{d}z\, M_C^0(z,\hat{m})$$
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- Non-Singular:
  - Sometimes possible to solve analytically
  - In general: numeric strategy
  - Full radiative tail:

$$F_e(e,\hat{m}) \equiv \frac{B_{\rm plus}(\hat{m})}{e-e_{\rm min}} + F_e^{\rm NS}(e,\hat{m}) = 2\int_{z_{\rm min}(e)}^{1/2} \mathrm{d}z \sum_{y=y_\pm(e,z)} \frac{M_C(y,z)}{y\left|\frac{\mathrm{d}\hat{e}(y,z)}{\mathrm{d}y}\right|}$$

Non-Singular: Numeric Strategy

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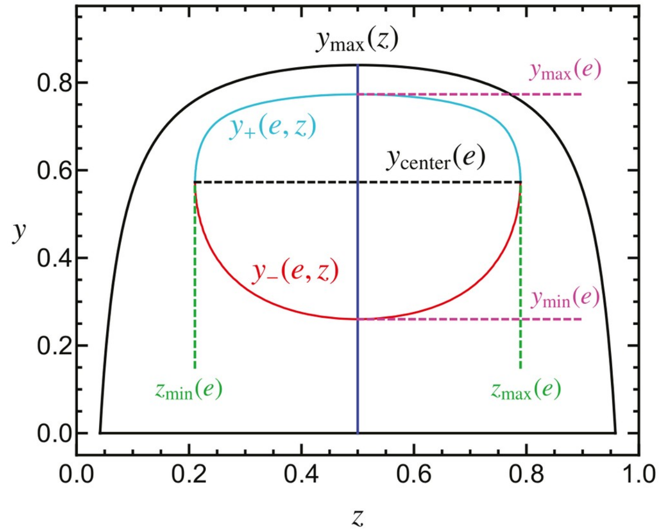
- Requirements: event shape measurement in y, z variables, first derivative
- Finding all boundaries and the integration itself can be automatized
  - E.g. Python's scipy module: scipy.optimize, scipy.integrate



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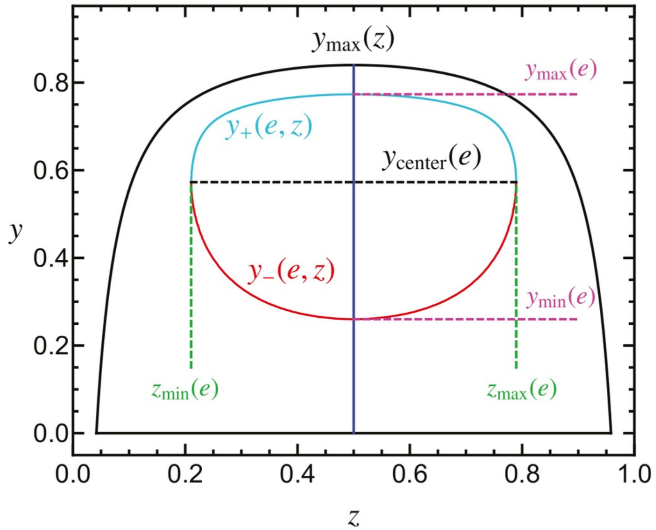
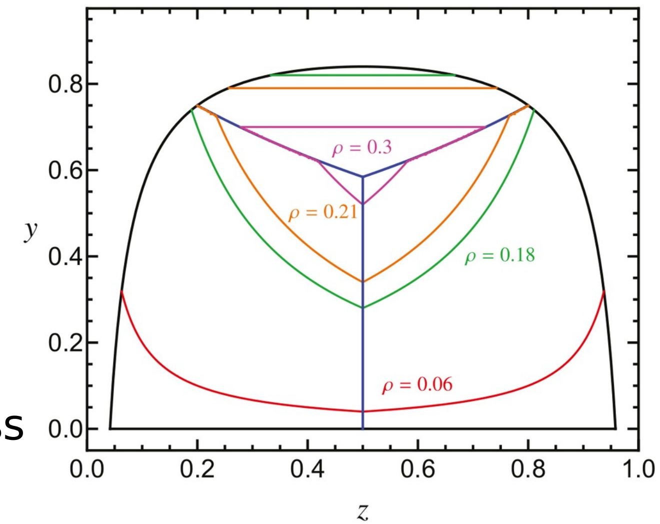


C-parameter

Heavy jet Mass



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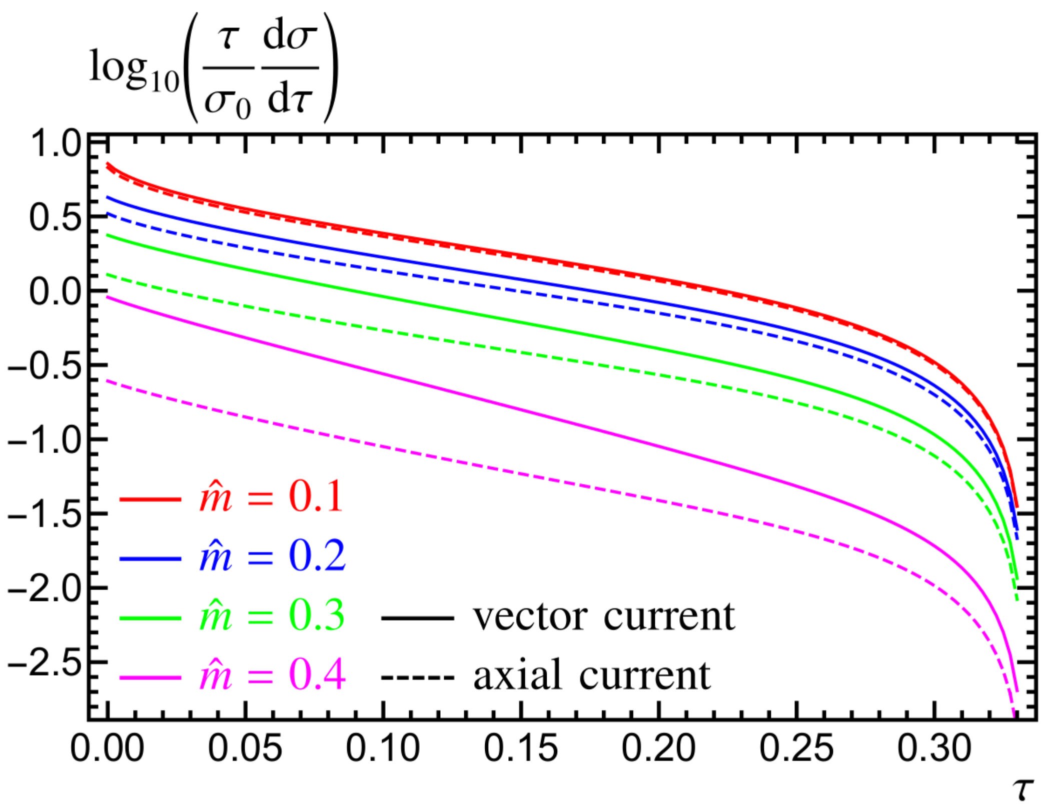
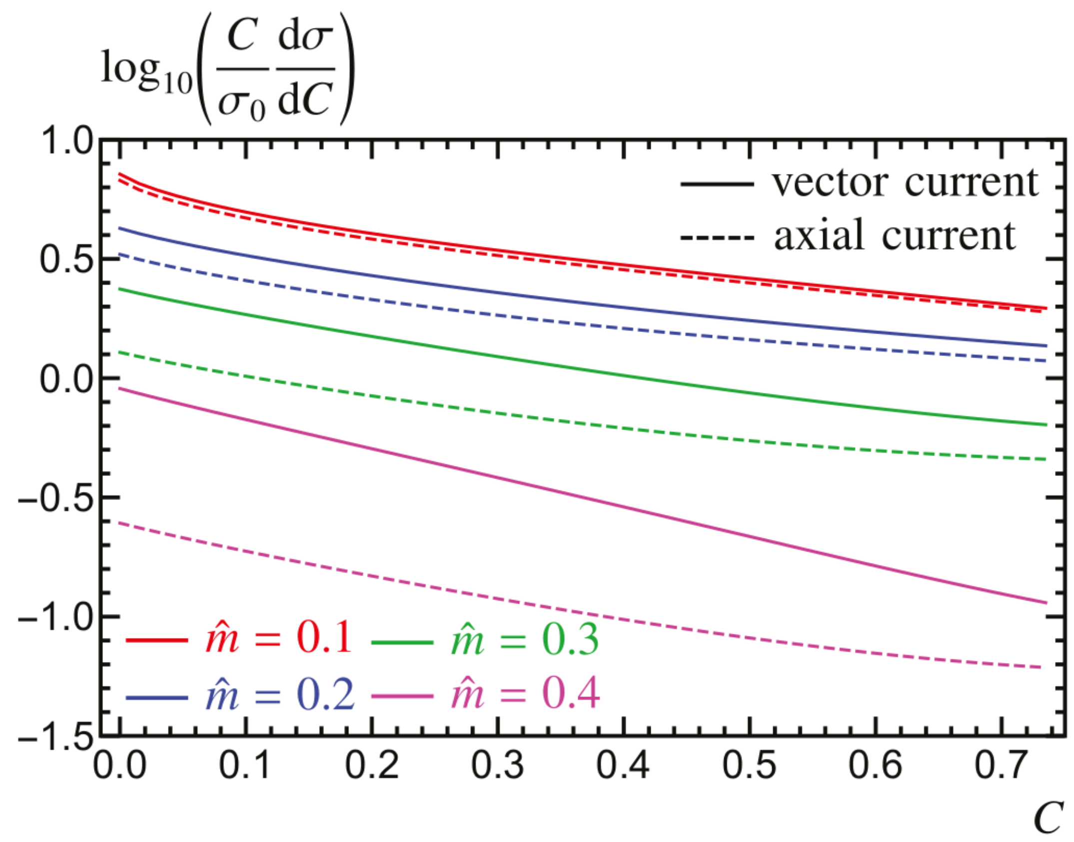
- Much faster and more accurate than binning + MC methods
- F_e can be determined with high precision, $B_{\rm plus}$ known analytically $\rightarrow F_e^{\rm NS}$ with high precision
- Can by used to determine cumulative distribution

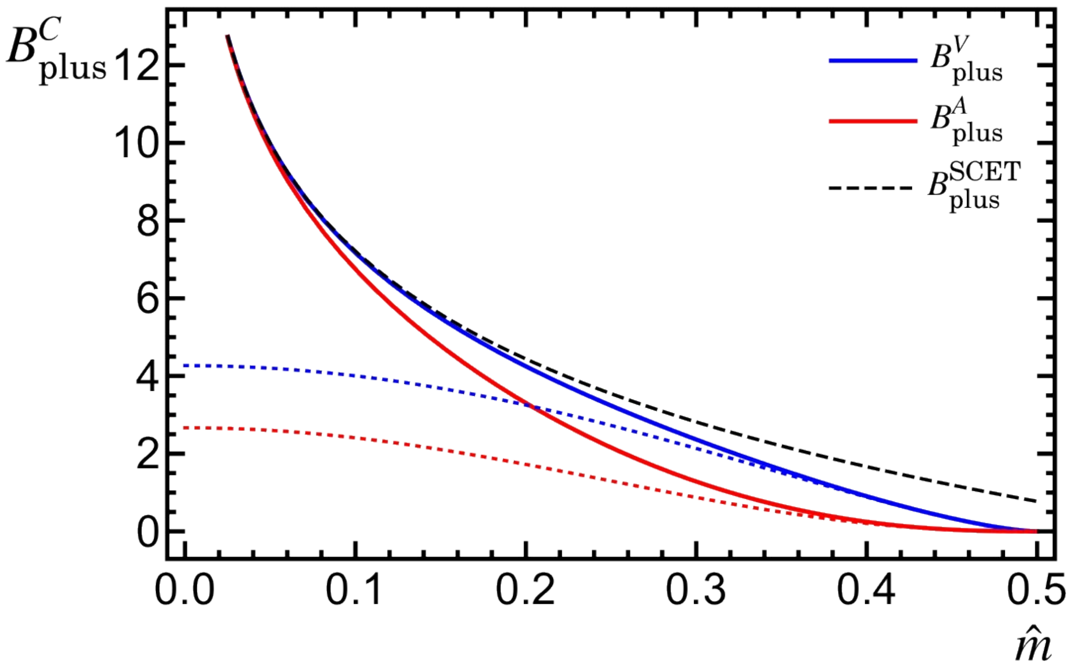
Non-Singular: Numeric Strategy

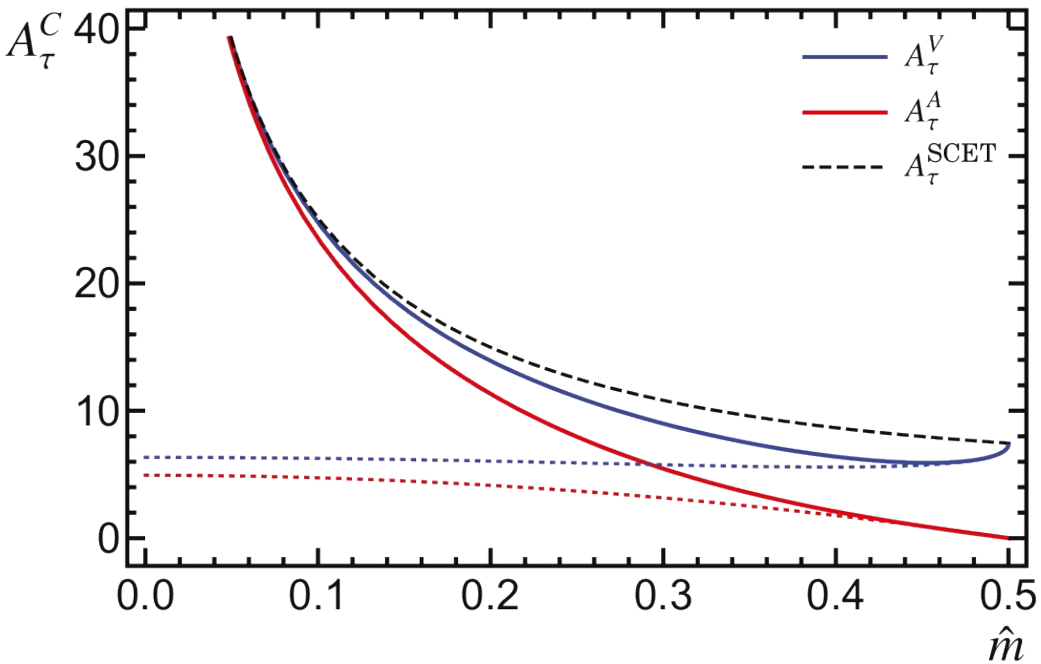




Cross Checks

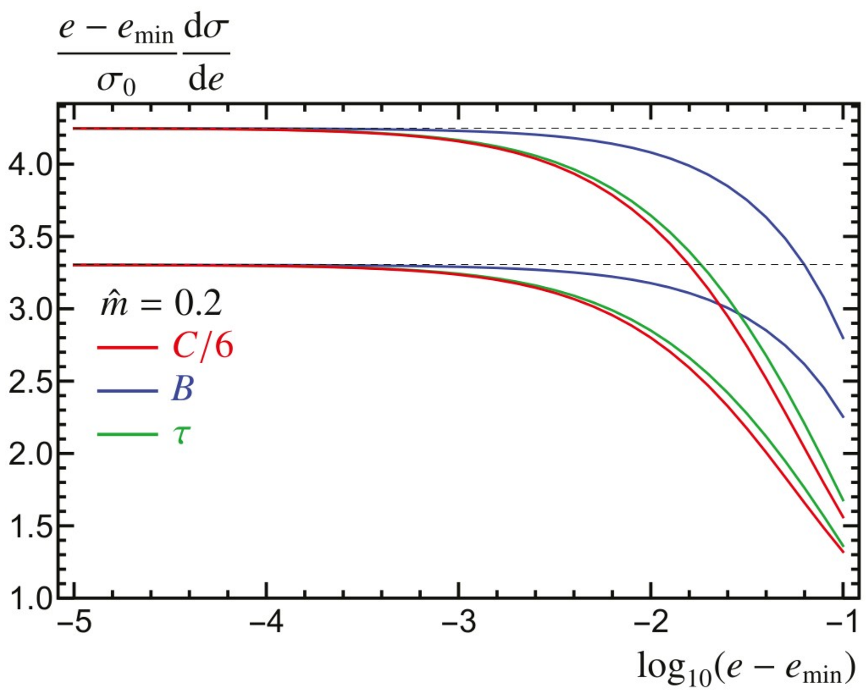
- Delta and plus coefficients in SCET limit ✓





Cross Checks

- Delta and plus coefficients in SCET limit ✓
- Check universality of plus-distribution coefficient numerically ✓



$$B_{\text{plus}} = \lim_{e \to e_{\min}} (e - e_{\min}) F_e(e, \hat{m})$$

Cross Checks

- Delta and plus coefficients in SCET limit ✓
- Check universality of plus-distribution coefficient numerically ✓
- Various analytic and numeric cross checks ✓
- Various consistency checks ✓

Summary & Outlook

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  - Accurately computed differential and cumulative massive event-shape cross section at NLO
  - All singular terms analytically
    - Plus-distribution coefficient: universal
    - Delta coefficient: analytically + event-shape dependent 1D integral
      - Analytic results given in article: HJM, Thrust, C-parameter, Broadening, Angularities (different massive schemes)
  - Non-singular terms: numerical algorithm
    - Efficient and accurate, even in dijet region

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  - Top mass measurements at future colliders

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Thank you for your attention!