Massive Event-Shape Distributions at N^{2}LL

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Outline

1. Event shapes
   - Massive schemes
   - Collinear limit
2. SCET
   - Jet function
   - P,E-scheme Thrust fixed order cross section
   - RG evolution
3. bHQET
   - Setup
   - Jet function
4. SCET Numerical analysis
   - Power corrections
   - Schemes comparison

Massive schemes

Generalization introduced in the definition of a given event shape for massive particles:

$$e = f(p_i)$$

E-scheme: $p_i = (p_i^0, \vec{p}_i) \rightarrow p_{i,E} = p_i^0\,(1, \vec{p}_i/|\vec{p}_i|)$
P-scheme: $p_i = (p_i^0, \vec{p}_i) \rightarrow p_{i,P} = |\vec{p}_i|\,(1, \vec{p}_i/|\vec{p}_i|)$
M-scheme: $p_i = (p_i^0, \vec{p}_i) = p_{i,M}$

* For massless particles: $p_E = p_P = p$
* $p_E^2 = p_P^2 = 0$ for massive and massless particles.
* No Lorentz covariant $\Rightarrow$ defined in the c.o.m. frame.
* Change the cross section sensitivity to the quark mass:

	τ	C	ρ
M-scheme	$1-\beta$	$12\hat{m}^2(1-\hat{m}^2)$	$\hat{m}^2$
P- and E- schemes	0	0	0

Table: Threshold position for various event shapes in the case of primary production of a stable quark-antiquark pair in different massive schemes. $\beta = \sqrt{1-4\hat{m}^2}$ is the velocity of the quarks at threshold in natural units.

Collinear limit

- In dijets:

$$e_{\text{dijet}} = \boxed{e_n} + e_{\bar{n}} + e_s \qquad\qquad e_{n,\bar{n},s} = \sum_{i\in n,\bar{n},s} f_{n,\bar{n},s}(p_i)$$

- For any scheme:
  - ◇ The momentum scaling of the collinear and soft particles remains the same.
  - ◇ Light-cone decomposition applies.

n-collinear limit: $n = (1,0,0,1); \quad \bar{n} = (1,0,0,-1); \quad p_n = (p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$

$$\left.\begin{aligned} p^0 &= (p^+ + p^-)/2 \simeq p^-/2 + \mathcal{O}(\lambda^2) \\ |\vec{p}\,| &= \sqrt{(p^0)^2 - m^2} \simeq p^-/2 + \mathcal{O}(\lambda^2) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{\begin{aligned} p^- &= p_E^- = p_P^- \\ p^\perp &= p_E^\perp = p_P^\perp \end{aligned}}$$

$$\left.\begin{aligned} p^+ &= p^0 - p_z = p^0 - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{p_\perp^2 + m^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_P^+ &= |\vec{p}\,| - p_z = |\vec{p}\,| - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{|\vec{p}_\perp|^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_E^+ &= p^0 - \frac{p^0}{|\vec{p}\,|}\, p_z = \frac{p^0}{|\vec{p}\,|}\, p_P^+ \simeq p_P^+ + \mathcal{O}(\lambda^4) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{p_P^+ = p_E^+ = p^+ - \frac{m^2}{p^-}}$$

At Leading Order in λ: $p_{n,P} = p_{n,E} \neq p_n \implies \boxed{e_n^P = e_n^E \neq e_n^M}$

Thrust, Hemisphere Jet Mass, C-parameter

	τ	C	ρ
E	$\frac{1}{Q}\min_{\hat{t}}\sum_i \frac{p_i^0}{\lvert\vec{p}_i\rvert}(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1 - \frac{1}{Q^2}\sum_{i,j}\frac{p_i^0 p_j^0(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert^2\lvert\vec{p}_j\rvert^2}\right]$	$\frac{1}{Q^2}\sum_{i,j\in \mathrm{h}}\frac{p_i^0 p_j^0(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert - \vec{p}_i\cdot\vec{p}_j)}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}$
P	$\frac{1}{Q_P}\min_{\hat{t}}\sum_i(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1 - \frac{1}{Q_P^2}\sum_{i,j}\frac{(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}\right]$	$\frac{1}{Q_P^2}\sum_{i,j\in \mathrm{h}}(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert - \vec{p}_i\cdot\vec{p}_j)$
M	$\frac{1}{Q}\min_{\hat{t}}\sum_i(p_i^0 - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[2 - \frac{1}{Q^2}\sum_{i\neq j}\frac{(p_i\cdot p_j)^2}{p_i^0 p_j^0}\right]$	$\frac{1}{Q^2}\left(\sum_{i\in h} p_i\right)^2$

Table: Thrust, C-parameter and hemisphere jet mass in the three massive schemes. In green, the original definitions. $Q_P \equiv \sum_i \lvert\vec{p}_i\rvert$. h is one of the hemispheres delimited by the plane normal to the thrust axis $\hat{t}$

	τ_n	C_n	ρ_n
E, P	$\tau_n^{E,P} = \frac{1}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$C_n^{E,P} = \frac{6}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$\rho_n^{E,P} = \frac{1}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$
M	$\tau_n^J = \frac{1}{Q}\sum_{i\in +} p_i^+$	$C_n^J = \frac{6}{Q}\sum_{i\in +} p_i^+$	$\rho_n = \frac{1}{Q}\sum_{i\in +} p_i^+$

Table: Thrust, C-parameter and hemisphere jet mass collinear limits.

SCET Jet function

Definition

Definition:

$$J_n(s,\mu) = \int \frac{\mathrm{d}\ell^+}{2\pi} \left[\frac{1}{4N_c} \mathrm{Tr} \int \mathrm{d}^d x \, e^{i\ell x} \, \langle 0| \bar{\not{n}} \chi_n(x) \, \delta(s - Q^2 \hat{e}_n) \, \overline{\chi}_{n,Q}(0) |0\rangle \right]$$

* $\chi_{n,Q}$ BARE Jet field $W_n^+ \xi_n$ with $p^- = Q$
* $\ell^- = Q$, $\vec{\ell}_\perp = 0$

$\Rightarrow$ for inclusive measurement computation through imaginary part of forward-scattering matrix element:

$$J_n(s,\mu) = \frac{-1}{4\pi N_c Q} \, \mathrm{Im} \left[i \int \mathrm{d}^d x \, e^{i\ell x} \, \langle 0| \, \mathrm{T}\{ \overline{\chi}_{n,Q}(0) \, \bar{\not{n}} \, \chi_n(x) \} |0\rangle \right]$$

* $\ell^+ = s/Q$

ONLY FOR M-SCHEME

Computational form

$$J_n(s,\mu) = \frac{(2\pi)^{d-1}}{N_C}\mathrm{Tr}\left[\frac{\bar{\not{n}}}{2}\langle 0|\chi_n(0)\delta^{(d-2)}(\vec{\mathcal{P}}_X^{\perp})\delta(\bar{\mathcal{P}}-Q)\delta(s-Q\hat{e}_n)\bar{\chi}_n(0)|0\rangle\right]$$

* $\vec{\mathcal{P}}_X^{\perp}$ perpendicular momentum operator
* $\bar{\mathcal{P}}$ operator for momentum in the minus direction

Insert the identity after the measurement delta's in the following way:

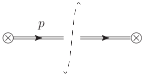
$$\sum_X |X\rangle\langle X| = \sum_{n=1}\sum_{\mathrm{spin}}\int\prod_{i=1}^{n}\frac{\mathrm{d}p_i^-\,\mathrm{d}^{d-2}\vec{p}_i^{\perp}\,\theta(p_i^-)}{(2\pi)^{d-1}(2p_i^-)}|X_n\rangle\langle X_n|$$

SCET Jet function

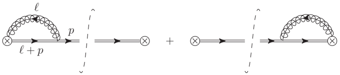
Feynman diagrams

Wave-function renormalization:

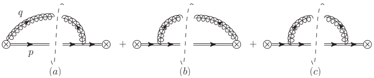
Tree-level:



One loop:



Virtual



Real

SCET Jet function

Computational issues in P,E-scheme

* Each real-radiation diagram diverges for $s \to m^2$. This divergence cancels out when summing all real-radiation contributions.
* Real-radiation from Wilson Line (diagrams **a** and **b**):

$$J^{\mathrm{real}}_{a,P}(s,\mu) = \frac{C_F \alpha_s\, e^{\varepsilon \gamma_E}}{2\pi m^2 \Gamma(1-\varepsilon)} \left(\frac{s}{\mu^2}\right)^{-\varepsilon} \int_0^1 \mathrm{d}x\, \frac{x^{2-\varepsilon}(1-x)^{-1-\varepsilon}}{1-x\left(1-\frac{s}{m^2}\right)}$$

In the limit $x \to 1$ the denominator in the integral goes as s/m^2 which combined with the prefactor gives $s^{-1-\varepsilon}$, and it leads to distributions when taking the epsilon expansion:

$$s^{-1+\varepsilon} = \frac{1}{\varepsilon}\delta(s) + \sum_{n=0} \frac{\varepsilon^n}{n!}\left[\frac{\log^n(s)}{s}\right]_+$$

⇒ Method 1:

1) $\Sigma(s_c) \equiv \int_0^{s_c} \mathrm{d}s\, J^{\mathrm{real}}_{a,P}(s,\mu)$
2) Use sector decomposition
3) $J^{\mathrm{real}}_{a,P}(s,\mu) = \mathrm{d}\Sigma(s)/\mathrm{d}s$ (taking into account: $\frac{\mathrm{d}}{\mathrm{d}x}[\theta(x)\log^n(x)] = n\left[\frac{\log^{n-1}(x)}{x}\right]_+$)

⇒ Method 2:

1) Solve the integral $\longrightarrow {}_2F_1\left(1, 3-\varepsilon, 3-2\varepsilon, 1-\frac{s}{m^2}\right)$
2) Apply Euler's identity $\longrightarrow \left(\frac{s}{m^2}\right)^{-1-\varepsilon} {}_2F_1\left(2-2\varepsilon, -\varepsilon, 3-2\varepsilon, 1-\frac{s}{m^2}\right)$
3) Write new hypergeometric function back as an integral: $\longrightarrow \int_0^1 \mathrm{d}x (1-x)^{2-\varepsilon} x^{-1-\varepsilon}\left[1-x\left(1-\frac{s}{m^2}\right)\right]^{-2+2\varepsilon}$

SCET Jet function

Results

Final result for the jet function [A. Bris, V. Mateu and M. Preisser, 2020]

$$J_n^{P,E}(s,\mu) = \delta(s) + \frac{\alpha_s C_F}{4\pi}\left\{\left[2\log\left(\frac{m}{\mu}\right) + 8\log^2\left(\frac{m}{\mu}\right) + 4 + \frac{\pi^2}{3}\right]\delta(s) + \frac{8}{\mu^2}\left[\frac{\log(s/\mu^2)}{s/\mu^2}\right]_+ \right.$$
$$\left. - \frac{4}{\mu^2}\left[1 + 2\log\left(\frac{m}{\mu}\right)\right]\left(\frac{\mu^2}{s}\right)_+ + \frac{s-7m^2}{(s-m^2)^2} - \frac{2s(2s-5m^2)}{(s-m^2)^3}\log\left(\frac{s}{m^2}\right)\right\}$$

Our direct computation of 2-jettiness (M-scheme) jet function agrees with Ref.[Fleming et al.]

$$J_n^J(s+m^2,\mu) = \delta(s) + \frac{\alpha_s C_F}{4\pi}\left\{\left[2\log\left(\frac{m}{\mu}\right) + 8\log^2\left(\frac{m}{\mu}\right) + 8 - \frac{\pi^2}{3}\right]\delta(s) + \frac{8}{\mu^2}\left[\frac{\log(s/\mu^2)}{s/\mu^2}\right]_+ \right.$$
$$\left. - \frac{4}{\mu^2}\left[1 + 2\log\left(\frac{m}{\mu}\right)\right]\left(\frac{\mu^2}{s}\right)_+ + \frac{s}{(m^2+s)^2} - \frac{4}{s}\log\left(1 + \frac{s}{m^2}\right)\right\}$$

* The two loop result is also known: [A.H. Hoang, C. Lepenik and M. Stahlhofen, 2019]

SCET Jet function

Massless limit

One loop Jet fuction structure:

$$J_n(\bar{s}+s_{\mathrm{min}},\mu) = \delta(\bar{s}) + \frac{\alpha_s(\mu)}{4\pi} C_F \left[J_{\mathrm{dist}}(\bar{s},\mu) + \frac{1}{m^2} J_{\mathrm{nd}}\left(\frac{\bar{s}}{m^2}\right)\right]$$

$$J_{\mathrm{dist}}(\bar{s},\mu) = \frac{1}{\mu^2} J_{m=0}\left(\frac{\bar{s}}{\mu^2}\right) + \frac{1}{m^2} J_m\left(\frac{\bar{s}}{m^2}\right)$$

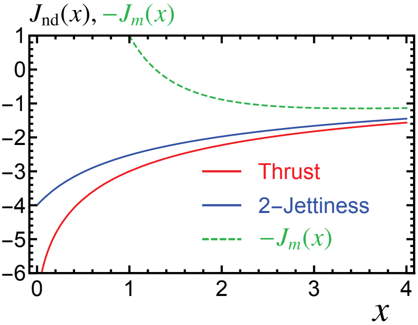


Figure: Massive corrections to the jet function. We show with solid lines the non-distributional functions J_{nd} for P- (red) and M- (blue) schemes. J_m function is shown multiplied -1 as a green dashed line (for $x > 0$ it is common to both schemes).

SCET

P,E-scheme Thrust fixed order cross section

SCET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}^{\mathrm{SCET}}_{\mathrm{FO}}}{\mathrm{d}\tau} = \delta(\tau) + \frac{\alpha_s(\mu)C_F}{4\pi}F_1^{\mathrm{SCET}}(\tau,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$F_1^{\mathrm{SCET}}(\tau,\hat{m}) = \delta(\tau)\left[\frac{10\pi^2}{3} - 8 + 4\log(\hat{m}) + 16\log^2(\hat{m})\right] - 8[1 + 2\log(\hat{m})]\left(\frac{1}{\tau}\right)_+$$
$$+ \frac{2(\tau - 7\hat{m}^2)}{(\tau-\hat{m}^2)^2} - \frac{4\tau(2\tau - 5\hat{m}^2)}{(\tau-\hat{m}^2)^3}\log\left(\frac{\tau}{\hat{m}^2}\right)$$
$$\equiv A^{\mathrm{SCET}}(\hat{m})\delta(\tau) + B^{\mathrm{SCET}}_{\mathrm{plus}}(\hat{m})\left(\frac{1}{\tau}\right)_+ + F^{\mathrm{SCET}}_{\mathrm{NS}}(\tau,\hat{m})$$

QCD: [C. Lepenik and V. Mateu, 2020][See talk by C. Lepenik].

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\hat{\sigma}^C_{\mathrm{FO}}}{\mathrm{d}\tau} = R_0^C(\hat{m})\,\delta(\tau) + C_F\frac{\alpha_s}{\pi}F_C^{\mathrm{QCD}}(\tau,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

$$F_C^{\mathrm{QCD}}(\tau,\hat{m}) = A^C(\hat{m})\delta(\tau) + B^C_{\mathrm{plus}}(\hat{m})\left(\frac{1}{\tau}\right)_+ + F^C_{\mathrm{NS}}(\tau,\hat{m})$$

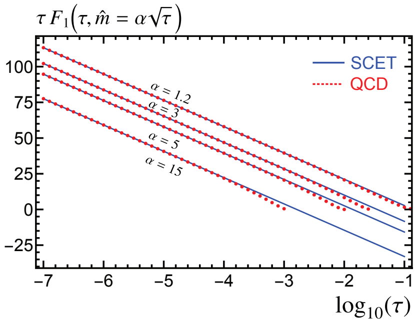


Figure: Comparison of the $\mathcal{O}(\alpha_s)$ correction to the differential cross sections in QCD $[F_{\mathrm{V}}^{\mathrm{QCD}}(\tau,\hat{m})]$ and SCET $[F_1^{\mathrm{SCET}}(\tau,\hat{m})]$.

- The running of the terms involving just distributions can be easily done in position space.
- Non-distributional cross section at N^{2}LL:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_{\rm nd}}{\mathrm{d}\tau} = \frac{C_F\alpha_s(\mu_J)}{2\pi Q^{1-\tilde{\omega}}}\frac{G(Q,\mu_i)}{\hat{m}^2\Gamma(-\tilde{\omega})}\int_{2s_{\rm min}}^{Q^2\tau}\mathrm{d}s\, J_{\rm nd}\left(\frac{s-2s_{\rm min}}{m^2}\right)(Q^2\tau - s)^{-1-\tilde{\omega}}$$
$$= \frac{C_F\alpha_s(\mu_J)}{2\pi}\frac{G(Q,\mu_i)}{Q\hat{m}^2}\left[\frac{1}{Q(\tau-\tau_{\rm min})}\right]^{\tilde{\omega}} I_{\rm nd}\left(\tilde{\omega},\frac{\tau-\tau_{\rm min}}{\hat{m}^2}\right)$$

- The running of the non-distributional term reduces to solving:

$$I_{\rm nd}(\tilde{\omega},y) = \frac{1}{\Gamma(-\tilde{\omega})}\int_0^1 \mathrm{d}z\,(1-z)^{-1-\tilde{\omega}} J_{\rm nd}(zy)$$

$$I^{J}_{\rm nd}(\tilde\omega, y) = \frac{1}{\Gamma(-\tilde\omega)} \int_0^1 \mathrm{d}z\,(1-z)^{-1-\tilde\omega}\left[\frac{zy}{(1+zy)^2} - \frac{4\log(1+zy)}{zy}\right]$$

$$I^{P,E}_{\rm nd}(\tilde\omega, y) = \frac{1}{\Gamma(-\tilde\omega)} \int_0^1 \mathrm{d}z(1-z)^{-1-\tilde\omega}\left[\frac{zy-7}{(1-yz)^2} + \frac{2zy(2zy-5)}{(1-yz)^3}\log(zy)\right]$$

1) **Analytical results:** Bring the previous expressions into the form of integral representations of hypergeometric functions:

$$\frac{\log(1+zy)}{zy} = \int_0^1 \mathrm{d}x\,\frac{1}{1+xzy}\ ; \qquad \log(zy) = \left[\log(y) + \frac{\mathrm{d}}{\mathrm{d}\varepsilon} z^{\varepsilon}\right]_{\varepsilon\to 0}$$

$\Rightarrow$ Despite the continuity, the cancellation between different terms divergent at $y \to 1$ ($s \to m^2$) in the P,E-scheme leads to troublesome expressions for numerical implementation.

- $\Rightarrow$ Using identities of hypergeometric functions different expressions can be found, each of them beeing more efficient for a given range of values. But they all have cancellations between divergent terms around $y \to 1$.
- $\Rightarrow$ Evaluation of hypergemetric functions is needed in both schemes which may be numerically expensive.

2) **Expansion series:** Expand the integrand or use the Mellin-Barnes representation and the converse mapping theorem:

$$\frac{1}{(1+X)^{\nu}} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}t\,(X)^{-t}\,\frac{\Gamma(t)\Gamma(\nu-t)}{\Gamma(\nu)}$$

The expansion around $X \gg 1$ ($X \ll 1$) is obtained integrating by residues the poles of the Mellin transform in the right (left) side of the fundamental strip $(0, \nu)$.

Different quickly convergent series for the regions:

- $\diamond$ Jettiness: $y \to 0$ and $y \to \infty$
- $\diamond$ Thrust P,E-scheme: $y \to 0$, $y \to 1$ and $y \to \infty$

which overlap with each other.

$\Rightarrow$ dramatic performance improvement in numerical implementation.

bHQET

framework

When $\tau - \tau_{\rm min} \ll \hat{m}^2$ large logs emerge in SCET Jet function $\Rightarrow$:

1) Integrate out the heavy quark and antiquark masses in their corresponding rest frames $\Rightarrow$ $2\times$ HQET's
2) Boost back to c.o.m. frame
3) Match onto SCET in order to account for global soft radiation.

	REST FRAMES		bHQET	
	DOF	Scalings	DOF	Scalings
HQET_1	quark: $p = mv + k$	$v = (1,1,0)$	quark: $p = mv_+ + k_+$	$v_+ = \left(\frac{m}{Q}, \frac{Q}{m}, 0\right)$
	soft: k	$k \sim \Gamma\,(1,1,1)$	n-ucollinear: k_+	$k_+ \sim \Gamma\left(\frac{m}{Q}, \frac{Q}{m}, 1\right)$
HQET_2	anti-quark: $p = mv + k$	$v = (1,1,0)$	anti-quark: $p = mv_- + k_-$	$v_- = \left(\frac{Q}{m}, \frac{m}{Q}, 0\right)$
	soft: k	$k \sim \Gamma\,(1,1,1)$	$\bar{n}$-ucollinear: k_-	$k_- \sim \Gamma\left(\frac{Q}{m}, \frac{m}{Q}, 1\right)$
			soft: q_s	$q_s \sim \frac{\Gamma m}{Q}\,(1,1,1)$

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{\rm bHQET}}{d\tau} = Q^2 H(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell\, B_\tau\left(\frac{Q^2(\tau - \tau_{\rm min}) - Q\ell}{m}, \mu\right) S_\tau(\ell,\mu)$$

$*$ For N^3LL 2-jettiness result, see poster by V.Mateu

bHQET Jet function

Computational form:

$$B_n(\hat{s}) = \frac{(2\pi)^{d-1} Q}{2m^2 N_C} \mathrm{Tr}\langle 0| W_{v_+}^\dagger(0) h_{v_+}(0) \delta\Big[\hat{s} - \frac{Q^2}{m}(\hat{e}_n - e_{\min})\Big] \delta^{(d-2)}(\vec{\mathcal{K}}^\perp)\delta(\mathcal{K}^-)\bar{h}_{v_+}(0) W_{v_+}(0)|0\rangle$$

* $\mathcal{K}$ residual momentum operator
* W_{v_+} Wilson lines with u-collinear gluons
* h_{v_+} Heavy quark field

Heavy quark Leading Order: $p = mv + k = (m^2/Q, Q, 0) + k$

→ On-shell condition:

$$v \cdot k = 0$$

→ Measurements:

$$Q(\tau_n^J - \hat{m}^2) = p^+ - \frac{m^2}{Q} = k^+ \,, \qquad Q\tau_n^{P,E} = p^+ - \frac{m^2}{p^-} = k^+ + \hat{m}^2 k^- = 0$$

→ Phase space:

$$\frac{\mathrm{d}p^+ \mathrm{d}p^- \mathrm{d}^{d-2}\vec{p}_\perp}{2(2\pi)^{d-1}} \delta\big[p^- p^+ - |\vec{p}_\perp|^2 - m^2\big]\theta(p^- + p^+) = \frac{\mathrm{d}k^- \mathrm{d}^{d-2}\vec{k}_\perp}{2Q(2\pi)^{d-1}}$$

bHQET Jet function

Diagrams:

- ⇒ Same diagrams as SCET but replacing $p \to k$ for heavy quark momenta.
- ⇒ Virtual diagrams are scaleless ⇒ Vanish in dim.reg.

Thrust P,E-scheme

$$
\begin{aligned}
mB_n^{P,E}(\hat{s},\mu) &= -\frac{\alpha_s \Gamma(2+\varepsilon) C_F\, e^{\varepsilon\gamma_E}}{\pi\mu\varepsilon}\left(\frac{\hat{s}}{\mu}\right)^{-1-2\varepsilon} \\
&= \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}+\frac{\pi^2}{6}\right)\delta(\hat{s}) - \frac{4}{\mu}\left(\frac{1}{\varepsilon}+1\right)\left(\frac{\mu}{\hat{s}}\right)_+ + \frac{8}{\mu}\left(\frac{\mu\log(\hat{s}/\mu)}{\hat{s}}\right)_+\right]
\end{aligned}
$$

2-Jettiness: [Fleming et al.]

$$
\begin{aligned}
mB_n^{J}(\hat{s},\mu) &= -\frac{\alpha_s \Gamma(2-\varepsilon) C_F\, e^{\varepsilon\gamma_E}}{\pi\mu\varepsilon\Gamma(2-2\varepsilon)}\left(\frac{\hat{s}}{\mu}\right)^{-1-2\varepsilon} \\
&= \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}+4-\frac{\pi^2}{2}\right)\delta(\hat{s}) - \frac{4}{\mu}\left(\frac{1}{\varepsilon}+1\right)\left(\frac{\mu}{\hat{s}}\right)_+ + \frac{8}{\mu}\left(\frac{\mu\log(\hat{s}/\mu)}{\hat{s}}\right)_+\right]
\end{aligned}
$$

- ∗ The two loop result is also known: [A. Jain, I. Scimemi and I.W. Stewart, 2008]

SCET Numerical analysis

Power corrections

- Mass and kinematic:

Mass power corrections are still kinematically singular:

$$\frac{\mathrm{d}\hat{\sigma}^{FO}_{\mathrm{QCD}}}{\mathrm{d}\tau} = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\mathrm{QCD}}}{\mathrm{d}\tau}(\tau,\hat{m}) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\mathrm{QCD}}}{\mathrm{d}\tau}(\tau,\hat{m})$$

$$\frac{\mathrm{d}\hat{\sigma}^{FO}_{\mathrm{SCET}}}{\mathrm{d}\tau} = H\,J\otimes S = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\mathrm{SCET}}}{\mathrm{d}\tau}(\tau,\hat{m}) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\mathrm{SCET}}}{\mathrm{d}\tau}(\tau,\hat{m}) = \frac{\mathrm{d}\hat{\sigma}^{dist}_{\mathrm{QCD}}}{\mathrm{d}\tau}(\tau,\hat{m}\to 0) + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\mathrm{SCET}}}{\mathrm{d}\tau}(\tau,\hat{m})$$

$\Rightarrow$ can absorb into SCET: (same approach as followed in Ref.[arXiv:1608.01318v1])

1) Modify hard and jet functions: $H \to \tilde{H} = H + \Delta H(\hat{m}),\;\; J \to \tilde{J} = J + \Delta J(\hat{m})$
2) Impose: $\tilde{H}\,\tilde{J}\otimes S|_{\mathrm{dist}} \equiv \frac{\mathrm{d}\hat{\sigma}^{dist}_{\mathrm{QCD}}}{\mathrm{d}\tau}(\tau,\hat{m})$.
3) One relation for two unknowns $\Rightarrow$ The uncertainty is encoded in one parameter variation.
4) Mass,kinematic-corrected cross section: $\boxed{\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \underbrace{U\otimes}_{\mathrm{resum.}}\,\tilde{H}\,\tilde{J}\otimes S + \frac{\mathrm{d}\hat{\sigma}^{nd}_{\mathrm{QCD}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\hat{\sigma}^{nd}_{\mathrm{SCET}}}{\mathrm{d}\tau}}$

- Hadronization: convolution with model function $F(p)$

$$\frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau} = \int_0^{Q\tau}\mathrm{d}p\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\left(\tau - \frac{p}{Q}\right)F(p)\,, \qquad F(p) = \frac{128\,p^3}{3\lambda^4}\,e^{-\frac{4p}{\lambda}}$$

SCET Numerical analysis

Differential cross sections

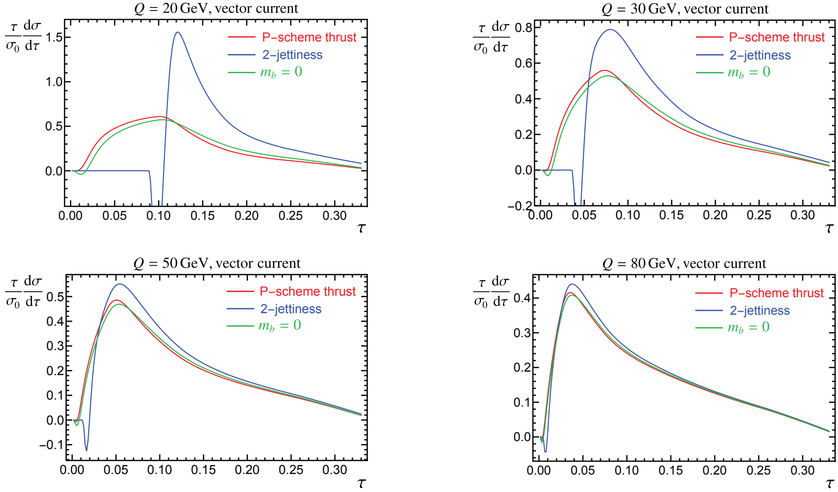
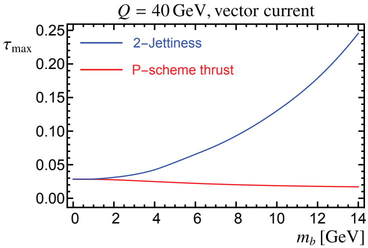


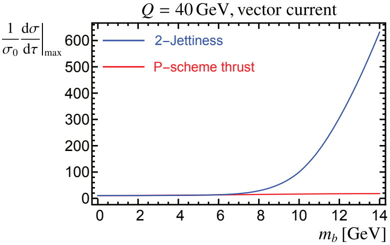
Figure: Differential cross section for massless quarks (green lines), 2-jettiness (blue lines) and P-scheme thrust (red lines) produced through the vector current.

SCET Numerical analysis

Mass dependence of the peak



(a) Peak position



(b) Peak height

Figure: Peak position and peak height for 2-jettiness (blue) and P-scheme thrust (red) massive cross section.

Conclusions

- $\rightarrow$ Variations on the massive scheme of an event shape allow for different mass sensitivity in the corresponding cross section.
- $\rightarrow$ P and E schemes are equivalent at leading order in collinear limit but they differ from Q scheme in general.
- $\rightarrow$ We computed the missing pieces for the SCET and bHQET cross sections in P,E-scheme at $\mathrm{N}^2\mathrm{LL} + \mathcal{O}(\alpha_s)$ accuracy, the jet functions.
- $\rightarrow$ We carried out RG evolution of non-distributional terms and studied its optimization for numerical implementation.
- $\rightarrow$ Mass, kinematic and hadronization power corrections were added for a final numerical analysis in SCET of 2-Jettiness and P-scheme thrust distributions.
- $\rightarrow$ Setup easily adaptable to other observables such as sum of hemisphere masses, heavy-jet-mass, C-parameter, etc.