Electroweak and Finite-Lifetime Effects in Boosted Top Production

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Doktoratskolleg Particles and Interactions

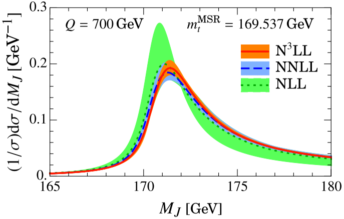
Der Wissenschaftsfonds.

Boosted top jets

- High-precision determinations of top mass from event shape distributions
- SCET + bHQET factorization theorem for thrust/invariant-mass distribution
  [Fleming et al, 07]

$$\text{peak}: \quad \frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H_Q * H_t * J_{B,t}^{(\Gamma_t)} \otimes J_{B,\bar{t}}^{(\Gamma_t)} \otimes S$$

$$\text{tail}: \quad \frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H_Q * H_t * J_t \otimes J_{\bar{t}} \otimes S$$



[Hoang et al, 20]

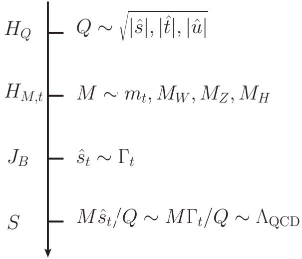
- State-of-the-art:
  - NLL QCD + LO EW (width Γ_t in peak region) [Fleming et al, 07,08]
  - 2-loop bHQET jet fct. $J_{B,t}$ [Stewart et al, 08]
  - 2-loop SCET - bHQET hard matching H_t [Pathak et al, 15]
  - 2-loop SCET massive jet fct. J_t [Lepenik et al, 18]
  - Study at N^{3}LL (peak region) [Hoang et al, 20] $\rightarrow$ poster
- Goal: systematic inclusion of NLO EW effects
  - Weak Sudakov logarithms $\sim \alpha \ln Q/M$, where $M \sim M_W, M_Z$ (normalization)
  - EW logs (essentially QED) $\sim \alpha \ln \tau$ (shape)
  - EW interference effects beyond Γ_t (shape)

$e^+e^- \to t\bar{t}$ with electroweak corrections

- Observable: 2-jettiness τ [Stewart et al, 10]
- Factorization for $m_t \hat{s}_t = M_t^2 - m_t^2 \ll m_t^2$ in QCD: [Fleming et al, 07]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H_Q * H_t * J_{B,t}^{(\Gamma_t)} \otimes J_{B,\bar{t}}^{(\Gamma_t)} \otimes S$$

- Hierarchy: $Q^2 \gg M^2 \sim m_t^2 \gg M_t^2 - m_t^2 \gtrsim m_t \Gamma_t \gg \Lambda_{\mathrm{QCD}}^2$
- Leptonic objects also charged under $SU(2) \times U(1)$
- In this hierarchy:
  - Weak (massive) corrections: virtual effects only $\to$ amplitude level calculation/factorization suffices in this hieararchy
  - QED corrections to all factorization quantities
- Initial-state collinear objects (QED) can be treated with beam functions B_{e^-}, B_{e^+} [Stewart et al, 10]



Operator and channel space notation for $\ell\bar{\ell} \to q\bar{q}$

- Isospin/operator structure, e.g. for LL chiralities (QCD and Dirac indices implicit):

$$O_I = (\bar{\ell}\,\Gamma_I \ell)(\bar{q}\,\Gamma_I q) \equiv \Gamma_I \otimes \Gamma_I$$

$$\langle O_I \rangle_i = \langle i_3, i_4 | O_I | i_1, i_2 \rangle \equiv \mathcal{M}_{iI}$$

- 6 "physical" combinations for $\Gamma_I \otimes \Gamma_I$ and i out of 16 in total (with net $\mathrm{I}_3 = 0$)

$$i \sim \begin{pmatrix} (\bar{\nu}\nu)(\bar{u}u) \\ (\bar{\nu}\nu)(\bar{d}d) \\ (\bar{e}e)(\bar{u}u) \\ (\bar{e}e)(\bar{d}d) \\ (\bar{e}\nu)(\bar{u}d) \\ (\bar{\nu}e)(\bar{d}u) \end{pmatrix} \qquad \Gamma_I \otimes \Gamma_I \sim \begin{pmatrix} 1 \otimes 1 \\ t^a \otimes t^a \\ t^3 \otimes t^3 - \frac{1}{6} t^a \otimes t^a \\ t^3 \otimes 1 \\ 1 \otimes t^3 \\ t^+ \otimes t^- - t^- \otimes t^+ \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{symmetric} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{broken} \end{matrix}$$

"channel space" "operator space"

- The two bases are related by an *orthogonal* transformation B_{iI}
- Operator space: easy to keep track of symmetry-breaking (SB) effects
- Amplitude $\mathcal{M}$ is some vector in either space, and (factorizable) radiative corrections A act as matrices:

$$\mathcal{M}_i = \sum_j A_{ij} \mathcal{M}_j^{\text{tree}} = \sum_I A_{iI} \mathcal{M}_I^{\text{tree}}$$

SCET_{EW}

[Chiu et al, 07, 08, 09]

- Hard SM tree-level (expand in $M/Q \ll 1$):

$$\mathcal{M}_I^{\text{tree}} \sim \begin{pmatrix} \star & \star & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

- O_I are formulated as SCET_{EW} operators with corresponding Wilson coefficients C_I

$$O_I \to \mathcal{O}_I^{\text{SCET}_{\text{EW}}} \sim \sum_{ij} \{\tilde{\chi}_n\}_i^{\text{EW}} S_{ij}^{\text{EW}} B_{jI}, \qquad \mathcal{L}_{\text{eff}} \sim \sum_I \mathcal{O}_I^{\text{SCET}_{\text{EW}}} C_I$$

- Collinear and soft operators:

$$\{\chi_n\}_i = \chi_{i_1} \overline{\chi}_{i_2} \chi_{i_3} \overline{\chi}_{i_4} \qquad S_{ij} = (Y_1)_{j_1 i_1} (\overline{Y}_2)_{i_2 j_2} (Y_3)_{i_3 j_3} (\overline{Y}_4)_{j_4 i_4}$$

- Matching at high scale $\mu_Q \sim Q$ + evolution from $\mu_Q \sim Q$ to $\mu_M \sim M$

$$\langle q\overline{q} | \ell\overline{\ell} \rangle_i = \sum_I \langle \mathcal{O}_I^{\text{SCET}_{\text{EW}}} \rangle_i \, C_I$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} C_I = \sum_J \Gamma_{IJ}^{\text{EW}} C_J \quad \to \quad C_I(\mu_M) = \sum_J U_{IJ}^{\text{EW}}(\mu_M \leftarrow \mu_Q) \, C_J(\mu_Q)$$

- C_J and evolution Γ_{IJ}^{EW} are free of SB effects:

$$\Gamma_{IJ}^{\text{EW}} \sim \begin{pmatrix} \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & 0 & 0 & 0 & 0 \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \end{pmatrix}$$

Matching at M: virtuality

[Chiu et al, 07, 08, 09]

- Matching at EW scale $\mu_M \sim M$ onto QED + QCD (SCET$_\gamma$)

$$\mathcal{O}_I^{\mathrm{SCET_{EW}}} = \sum_J \mathcal{O}_J^{\mathrm{SCET}_\gamma}\, \mathcal{M}_{M,JI} \qquad \mathcal{O}_i^{\mathrm{SCET}_\gamma} \sim \{\chi_n\}_i^\gamma\, S_i^\gamma$$

- Contains SB effects (most easily seen in operator space):

$$\mathcal{M}_{M,IJ} \quad \sim \quad \begin{pmatrix} \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \\ \star & \star & \star & \star & \star & \star \end{pmatrix}$$

- Mass-mode matching $\mathcal{M}_M$ and its evolution in virtuality μ well-defined:
  - Anomalous dimensions are free of SB effects above and below M respectively

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathcal{M}_M = \mathcal{M}_M \Gamma^{\mathrm{EW}} - \Gamma^\gamma \mathcal{M}_M$$

  - SB effects (boson masses etc.) are contained in the matching itself
- Rapidity log remains

$$\ln \mathcal{M}_M \sim \alpha \ln M/Q \times \ln M/\mu_M$$

Matching at M: rapidity

- "Soft-collinear factorization" (= regulator-dependent split) of $\mathcal{M}_M$ at $\mu \sim M$

$$\mathcal{M}_M = \mathcal{M}_c \, \mathcal{M}_s ,$$

$$\langle q\bar{q} | \{\tilde{\chi}_n\}_j^{\mathrm{EW}} | \ell\bar{\ell} \rangle_i = \langle q\bar{q} | \{\chi_n\}_i^{\gamma} | \ell\bar{\ell} \rangle_i \, \mathcal{M}_{c,ij}$$

$$\langle 0 | S_{ij}^{\mathrm{EW}} | 0 \rangle = \langle 0 | S_i^{\gamma} | 0 \rangle_i \, \mathcal{M}_{s,ij}$$

- Using the symmetric rapidity regulator with evolution in ν [Rothstein et al, 11]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} \mathcal{M}_c = \mathcal{M}_c \, \gamma_M^{(\nu)} \qquad \frac{\mathrm{d}}{\mathrm{d}\ln\nu} \mathcal{M}_s = -\gamma_M^{(\nu)} \, \mathcal{M}_s$$

- Summary:
  - Approach correctly treats: FO EW one-loop + LL EW resummation
  - Structure and soft-collinear factorization at M at higher loops so far conjectured

$$O_I \to \mathcal{O}_I^{\mathrm{SCET_{EW}}} \sim \sum_{ij} \{\tilde{\chi}_n\}_i^{\mathrm{EW}} \, S_{ij}^{\mathrm{EW}} B_{jI} , \qquad \mathcal{L}_{\mathrm{eff}} \sim \sum_I \mathcal{O}_I^{\mathrm{SCET_{EW}}} \, C_I$$

$\Rightarrow$ checks against two-loop FO SM calculation [Denner et al, 06, 08]
$\Rightarrow$ checks against two-loop Sudakov FF [Assi, Kniehl, 20] $\to$ poster
(*work in progress*)

Factorization theorem for top production

- In the regime $m_t^2 \sim M_t^2 \gg M_t^2 - m_t^2 \sim m_t \Gamma_t$ the factorization theorem takes the form

$$\frac{\mathrm{d}\sigma_{e^+e^-t\bar{t}}}{\mathrm{d}\tau} \sim \mathrm{Tr}\left[\rho_{e^+e^-t\bar{t}} \mathcal{M}_c \mathcal{M}_s C C^\dagger \mathcal{M}_s^\dagger \mathcal{M}_c^\dagger\right] * J_{B,t}^{(\Gamma_t)} \otimes \bar{J}_{B,t}^{(\Gamma_t)} \otimes B_{e^-} \otimes B_{e^+} \otimes S_{e^+e^-t\bar{t}}$$

- ρ_i is the density matrix for the measurement of channel i, e.g. for $e^+e^-t\bar{t}$

$$\rho_{e^+e^-t\bar{t}} = \mathrm{diag}(0,0,1,0,0,0) \qquad \text{in channel space.}$$

- Tr denotes the trace in operator or channel space
- QED + weak Sudakov logs can be resumed by evolution ✓
- Above formula describes the double-resonant cross section only, with bHQET jet functions

$$J_{B,t}^{(\Gamma_t)} \sim \mathrm{Im}\left[i\,\langle 0|\mathrm{T}\, h_t \bar{h}_t|0\rangle^{(\Gamma_t)}\right]$$

- In top channel(s), e.g. $i = e^+e^-t\bar{t}$:

$$\mathrm{Im}\left[\mathcal{M}_{c,i}\right] \neq 0 \quad \text{due to} \quad m_t > M_W + m_b$$

$\rightarrow$ these imaginary parts cancel in $\mathcal{M}_c^\dagger \mathcal{M}_c$.

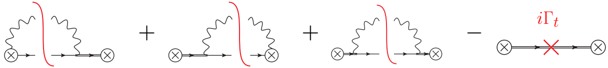
- Resonant/non-resonant interference contributions are obtained by [Hoang, Reisser, 04]

$$\mathrm{Im}\left[\mathrm{Tr}\left[\tilde{\mathcal{M}}_c^\dagger \rho_{e^+e^-t\bar{t}} \mathcal{M}_c \ldots\right] * i\,\langle 0|\mathrm{T}\, h_t \bar{h}_t|0\rangle^{(\Gamma_t)}\right]$$

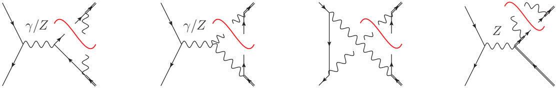
$\rightarrow$ $bW-$cuts in $\tilde{\mathcal{M}}_c^\dagger$ have same sign as in $\mathcal{M}_c$!

Resonant/non-resonant interference effects

- $\mathrm{Im}\Big[\,\mathrm{Tr}\big[\tilde{\mathcal{M}}_c^\dagger \rho_{e^+e^-t\bar{t}} \mathcal{M}_c \ldots\big]\Big]$ diagrammatically at $\mathcal{O}(\alpha_w/\hat{s}_t)$:

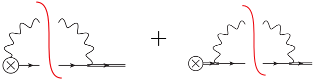


- Crucial aspects:
  - Full-theory cuts (no label $= W^\pm$)

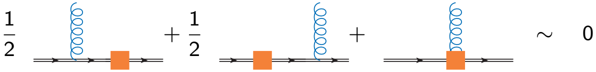


reproduced by the EFT imaginary parts ✓

$\mathrm{Im}\big[\mathcal{M}_{c,t}\big] \quad \sim$



  - Factorization of ultracollinear (bHQET) and mass-mode effects
    $\rightarrow$ checked diagramatically for the jet vacuum correlator at $\mathcal{O}(\alpha_w \alpha_s)$ ✓
    e.g. cancellations at $\mathcal{O}(1/\hat{s}_t)$ such as

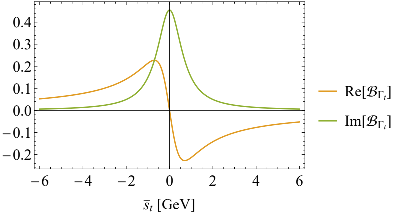


Numerical example

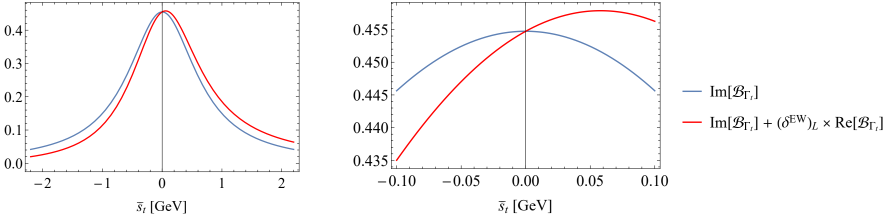
- Interference with tree-level bHQET jet function

$$i \langle 0|\mathrm{T}\, h_t \bar{h}_t|0\rangle^{(\Gamma_t)\mathrm{tree}} \sim -\frac{1}{\pi}\frac{1}{\bar{s}_t + i\Gamma_t/2} = \mathcal{B}_{\Gamma_t}(\hat{s}_t), \qquad \bar{s}_t = \hat{s}_t/2$$

$$\mathrm{Im}\big[(A+iB)\mathcal{B}_{\Gamma_t}\big] =$$
$$= A\,\mathrm{Im}\big[\mathcal{B}_{\Gamma_t}\big] + B\,\mathrm{Re}\big[\mathcal{B}_{\Gamma_t}\big]$$



- Induces a shift of peak:



Summary and outlook

- Summary:
  - EW corrections in the peak region for boosted top pair production at lepton colliders
  - Summation of Sudakov logarithms (virtuality + rapidity)
  - Resonant/non-resonant interference effects for inclusive boosted top jets

- Outlook:
  - Connect with QCD calculations
  - Refine treatment of QED for initial-state
  - Test of EW soft-collinear factorization against SM FO (two-loop)
  - NLL resummation
  - Extension of formalism to LHC observables