New QCD Coherence for Multiple Soft Emissions using Glauber SCET

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Outline

1. Amplitude level coherence and the AMFS[1] result
2. Diagrammatic proof of the AMFS result at one-loop
3. Rederivation of the AMFS result in Glauber-SCET [Rothstein, Stewart 2016]
4. Towards two-loops

[1] [Angeles-Martinez, Forshaw, Seymour 2015]

The ordering problem

We are concerned with amplitude level results, and especially the regions of phase space where factorization breaks down. To illustrate the problem let us consider

- Resummation of non-global logarithms [Angeles Martinez, De Angelis, Forshaw, Plätzer, Seymour 2018]

$$\mathrm{d}\sigma_0 = \langle M^{(0)}|\mathbf{V}^\dagger_{Q_0,Q}\mathbf{V}_{Q_0,Q}|M^{(0)}\rangle \mathrm{d}\Pi_0$$

$$\mathrm{d}\sigma_1 = \langle M^{(0)}|(\ldots)^\dagger_\mu \left(\mathbf{V}_{Q_0,E_1}\mathbf{D}_1^\mu \mathbf{V}_{E_1,Q}\right)|M^{(0)}\rangle \mathrm{d}\Pi_1$$

$$\mathrm{d}\sigma_2 = \langle M^{(0)}|(\ldots)^\dagger_{\mu_1\mu_2} \left(\mathbf{V}_{Q_0,E_2}\mathbf{D}_2^{\mu_2} \mathbf{V}_{E_2,E_1}\mathbf{D}_1^{\mu_1}\mathbf{V}_{E_1,Q}\right)|M^{(0)}\rangle \mathrm{d}\Pi_1 \mathrm{d}\Pi_2$$

$$\mathbf{V}_{a,b} = \exp\left[\frac{-\alpha_s}{\pi}\int_a^b \frac{\mathrm{d}E_k}{E_k}\sum_{i<j}(-\mathbf{T}_i\cdot\mathbf{T}_j)\left\{\int \frac{\mathrm{d}y\,\mathrm{d}\phi}{2\pi}\frac{E_k^2}{k_\perp^{(ij)}} - \mathrm{i}\pi\tilde{\delta}_{ij}\right\}\right] \qquad \mathbf{D}_i^\mu = \sum_{j<i}\mathbf{T}_j E_i \frac{p_j^\mu}{p_j\cdot q_i}$$

- This algorithm satisfies an evolution equation, resums LL, subleading non-global logs and is equivalent to all the other approaches [Weigert 2004; Caron-Huot 2015; Larkoski et. al. 2015; Becher et. al. 2016]

The ordering problem

The algorithm involves ordering the subsequent emissions:

$$\mathrm{d}\sigma_0 = \langle M^{(0)}|\mathbf{V}^\dagger_{Q_0,Q}\mathbf{V}_{Q_0,Q}|M^{(0)}\rangle \mathrm{d}\Pi_0$$

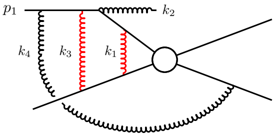
$$\mathrm{d}\sigma_1 = \langle M^{(0)}|(\ldots)^\dagger_\mu \left(\mathbf{V}_{Q_0,E_1}\mathbf{D}_1^\mu \mathbf{V}_{E_1,Q}\right)|M^{(0)}\rangle \mathrm{d}\Pi_1$$

$$\mathrm{d}\sigma_2 = \langle M^{(0)}|(\ldots)^\dagger_{\mu_1\mu_2} \left(\mathbf{V}_{Q_0,E_2}\mathbf{D}_2^{\mu_2} \mathbf{V}_{E_2,E_1}\mathbf{D}_1^{\mu_1}\mathbf{V}_{E_1,Q}\right)|M^{(0)}\rangle \mathrm{d}\Pi_1 \mathrm{d}\Pi_2$$

Ordering in energy only correct when insensitive to Glauber exchanges!

For the *leading* Superleading logs:

- k_T ordering gives $\frac{2C\alpha_s^4}{5!}\ln^5\frac{Q}{Q_0}$
- Energy ordering gives ∞
- Virtuality ordering gives $\frac{1}{2} \times \frac{2C\alpha_s^4}{5!}\ln^5\frac{Q}{Q_0}$
- Angular ordering gives 0



[Forshaw, Kyrieleisis, Seymour 2006; Banfi, Salam, Zanderighi 2010]

What is the correct ordering variable?

Soft gluon factorization

To begin answering this question start with the amplitude for N soft emissions ordered in their softness:

$$q_N \ll q_{N-1} \ll \ldots \ll q_1 \ll Q$$

and try to write an expression like the one before.

Ingredients for a one-loop calculation:

- Soft gluon factorization: [Catani, Grazzini 2000; Duhr, Gehrmann 2013; Li, Zhu 2013; Feige, Schwartz 2014]

$$|\mathcal{M}(q, p_1, \ldots p_n)\rangle \simeq g\mu^\epsilon \varepsilon^\mu(q)\mathbf{J}_\mu(q)|\mathcal{M}(p_1, \ldots, p_n)\rangle\,, \qquad \mathbf{J}(q) = \mathbf{J}^{(0)}(q) + \mathbf{J}^{(1)}(q) + \ldots$$

- Tree level soft current:

$$\mathbf{J}^{\mu(0)}(q) = \sum_{i=1}^{n} \mathbf{T}_i \frac{p_i^\mu}{p_i \cdot q} = \sum_{j=1}^{n} \mathbf{d}_{ij}^\mu(q)\,, \qquad \mathbf{d}_{ij}^\mu(q) = \mathbf{T}_j\left(\frac{p_j^\mu}{p_j \cdot q} - \frac{p_i^\mu}{p_i \cdot q}\right)$$

- One-loop soft current:

$$\mathbf{J}^{(1)}(q) = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \mathbf{d}_{jk}^{(1)}(q)\,, \qquad \mathbf{d}_{ij}^{(1)}(q) = \frac{\alpha_s}{2\pi}\frac{c_\Gamma}{\epsilon^2}\mathbf{T}_q \cdot \mathbf{T}_i\left(\frac{e^{-\mathrm{i}\pi\tilde{\delta}_{ij}}}{e^{-\mathrm{i}\pi\tilde{\delta}_{iq}}e^{-\mathrm{i}\pi\tilde{\delta}_{jq}}}\frac{4\pi\mu^2}{(q_\perp^{(ij)})^2}\right)^\epsilon \mathbf{d}_{ij}(q)$$

- One-loop hard virtual corrections:

$$|M_0^{(1)}\rangle = \sum_{i=2}^{n}\sum_{j=1}^{i-1} \mathbf{I}_{ij}(0, Q)|M_0^{(0)}\rangle\,, \qquad \mathbf{I}_{ij}(0, Q) = \frac{\alpha_s}{2\pi}\frac{\tilde{c}_\Gamma}{\epsilon^2}\mathbf{T}_i \cdot \mathbf{T}_j\left(e^{-\mathrm{i}\pi\tilde{\delta}_{ij}}\frac{4\pi\mu^2}{2p_i \cdot p_j}\right)^\epsilon$$

Ordering multiple soft emissions

And we arrive at the amplitude for ordered soft emissions:

$$\big|M_N\big\rangle = (g\mu^\epsilon)^N \mathbf{J}(q_N)\dots\mathbf{J}(q_1)\big|\mathcal{M}(p_1,\dots,p_n)\big\rangle\,,$$

Both soft gluon operator and the hard matrix element have loop expansions:

$$\mathbf{J}(q) = \mathbf{J}^{(0)}(q) + \mathbf{J}^{(1)}(q) + \dots\,, \qquad \big|M_0\big\rangle = \big|M_0^{(0)}\big\rangle + \big|M_0^{(1)}\big\rangle + \dots$$

One-loop pieces:

$$\mathbf{J}^{(1)}(q_{m+1}) = \frac{1}{2}\sum_{j=1}^{n+m}\sum_{k=1}^{n+m}\mathbf{d}^{(1)}_{jk}(q_{m+1})\,, \qquad \big|M_0^{(1)}\big\rangle = \sum_{i=2}^{n}\sum_{j=1}^{i-1}\mathbf{I}_{ij}(0,Q)\big|M_0^{(0)}\big\rangle$$

$$p_{n+m} = q_m\,, \qquad q_{j+1}\sim\kappa q_j \ll q_j$$

Note that both $\mathbf{I}_{ij}(0,Q)$ and $\mathbf{d}^{(1)}_{ij}$ are IR divergent whereas we are aiming at an expression where the intermediate virtual pieces are IR finite. We are almost there ...

Amplitude level coherence

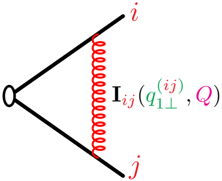
Remarkably, the same amplitude (at one-loop) can be expressed as [Angeles-Martinez, Forshaw, Seymour 2016]

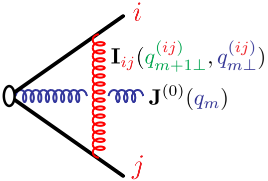
AMFS result

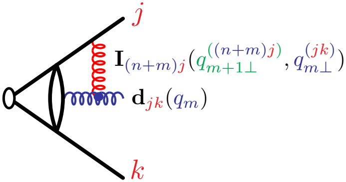
$$|M_N^{(1)}\rangle = (g\mu^\epsilon)^N \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_1)\left(\sum_{i=2}^{n}\sum_{j<i}\mathbf{I}_{ij}(q_{1\perp}^{(ij)}, Q)\right)|M_0^{(0)}\rangle$$

$$+ (g\mu^\epsilon)^N \sum_{m=1}^{N} \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_{m+1})\left(\sum_{i=2}^{n+m-1}\sum_{j<i}\mathbf{I}_{ij}(q_{m+1\perp}^{(ij)}, q_{m\perp}^{(ij)})\right)\mathbf{J}^{(0)}(q_m)\ldots\mathbf{J}^{(0)}(q_1)|M_0^{(0)}\rangle$$

$$+ (g\mu^\epsilon)^N \sum_{m=1}^{N} \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_{m+1})\left(\sum_{j,k=1}^{n+m-1}\mathbf{I}_{(n+m)j}(q_{m+1\perp}^{((n+m)j)}, q_{m\perp}^{(jk)})\mathbf{d}_{jk}(q_m)\right)\mathbf{J}^{(0)}(q_{m-1})\ldots\mathbf{J}^{(0)}(q_1)|M_0^{(0)}\rangle$$







Why is this result significant?

1. Unique ordering variable: QCD singles out dipole-k_T as the ordering parameter: dipole-k_T ordering is exact - no approximation of the limits of loop integrals!

$$\left(q_\perp^{(ij)}\right)^2 = \frac{2q \cdot p_i \, q \cdot p_j}{p_i \cdot p_j}$$

2. IR finite: The one-loop insertions $\mathbf{I}_{ij}$ are IR finite, no intermediate poles, everything in 4 dimensions (except for the very last emission).

$$\mathbf{I}_{ij}(a,b) = \frac{\alpha_s}{2\pi} \mathbf{T}_i \cdot \mathbf{T}_j \left[\left(\frac{4\pi\mu^2}{b^2}\right)^\epsilon \left(1 + \mathrm{i}\pi\tilde{\delta}_{ij} - \epsilon \ln \frac{2p_i \cdot p_j}{b^2}\right) - \left(\frac{4\pi\mu^2}{a^2}\right)^\epsilon \left(1 + \mathrm{i}\pi\tilde{\delta}_{ij} - \epsilon \ln \frac{2p_i \cdot p_j}{a^2}\right) \right]$$

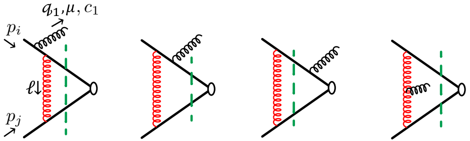
3. Markovian nature: Basis for future all-orders future amplitude level parton showers.
   [Forshaw, Holguin, Plätzer 2019]

4. No analog in SCET: Because of the weird third line that we saw, the expression simply doesn't exponentiate, and cannot be written as a solution of an evolution equation. Nonetheless, SCET will still prove to be very useful in understanding this result.

Diagrammatic proof of the AMFS result at one-loop

Switch mechanism for one real emission

We will focus on the imaginary part obtained by considering Eikonal cuts (cuts through the soft gluon and j vanish):



Contribution of graphs (a-c):

$$-\frac{\mathrm{i}\pi}{8\pi^2}\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\left[\mathbf{T}_i^{c_1}(\mathbf{T}_i\cdot\mathbf{T}_j)-(\mathbf{T}_i\cdot\mathbf{T}_j)\mathbf{T}_i^{c_1}+(\mathbf{T}_i\cdot\mathbf{T}_j)\mathbf{T}_i^{c_1}\right]\int_0^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}$$

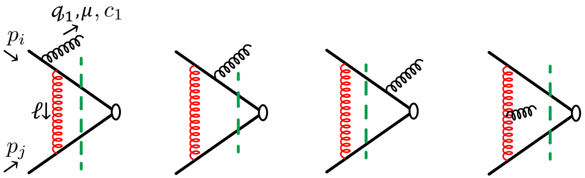
(d) involves

$$+\frac{\mathrm{i}\pi}{8\pi^2}\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\left[\mathbf{T}_i^{c_1}(\mathbf{T}_i\cdot\mathbf{T}_j)-(\mathbf{T}_i\cdot\mathbf{T}_j)\mathbf{T}_i^{c_1}\right]\frac{\vec{q}_{1\perp}^{\,2}}{2}\int_0^{Q^2}\frac{d^{d-2}\ell_\perp}{\vec{\ell}_\perp^{\,2}(\vec{\ell}_\perp+\vec{q}_{1\perp})^2}$$

Interestingly, this integral is an exact Θ-function!

$$\frac{\vec{q}_{1\perp}^{\,2}}{2}\int_0^{Q^2}\frac{d^2\ell_\perp}{\vec{\ell}_\perp^{\,2}(\vec{\ell}_\perp+\vec{q}_{1\perp})^2}=\int_0^{Q^2}\frac{d^2\ell_\perp}{\vec{\ell}_\perp^{\,2}}\frac{\vec{q}_\perp\cdot(\vec{\ell}_\perp+\vec{q}_\perp)}{(\vec{\ell}_\perp+\vec{q}_{1\perp})^2}=\int_0^{\vec{q}_{1\perp}^{\,2}}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}$$

Switch mechanism for one real emission



The loop integral of the triple gluon vertex graph acts as a switch:

$$-\frac{\mathrm{i}\pi}{8\pi^2}\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\left[\mathbf{T}_i^{c_1}(\mathbf{T}_i\cdot\mathbf{T}_j)\int_{\vec{q}_{1\perp}^{\,2}}^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}+(\mathbf{T}_i\cdot\mathbf{T}_j)\mathbf{T}_i^{c_1}\int_0^{\vec{q}_{1\perp}^{\,2}}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}\right]$$

The result can be generalized to final state partons as well. Thus we have:

$$\left[\mathbf{J}_1^{c_1}(q_1)\mathbf{C}_{q_{1\perp},Q}+\mathbf{C}_{0,q_{1\perp}}\mathbf{J}_1^{c_1}(q_1)\right]|n^{(0)}\rangle$$

$$\mathbf{J}_1^{c_1}(q_1)\equiv\sum_{i=1}^{n}\mathbf{T}_i^{c_1}\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\,,\qquad \mathbf{C}_{a,b}\equiv\frac{-\mathrm{i}\pi\mathbf{T}_i\cdot\mathbf{T}_j}{8\pi^2}\int_{a^2}^{b^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}$$

Switch mechanism: another useful way of looking at the result

Isolate the scenarios where the real emission comes about from the Glauber exchange long before, or as a part of the hard scattering.

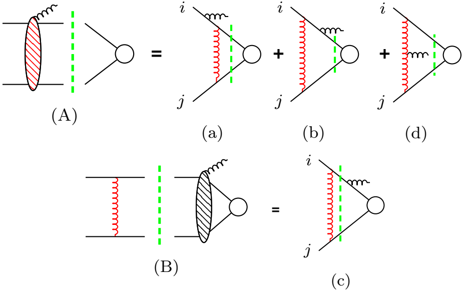
Cut A

$$-\frac{\mathrm{i}\pi}{8\pi^2}\mathbf{T}_j^b(\mathrm{i}f^{bc_1a})\mathbf{T}_i^a\left(\frac{p_j\cdot\varepsilon_1}{p_j\cdot q_1}-\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\right)\int_{\color{red}{\vec{q}_{1\perp}^{\,2}}}^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}\big|n^{(0)}\big\rangle$$

Cut B

$$\mathbf{C}_{0,Q}\mathbf{J}_1^{c_1}(q_1)\big|n^{(0)}\big\rangle$$

Both the contributions are gauge invariant.



Other *Wrongly time ordered cuts* that pass through the soft gluon vanish

Ordering soft real emissions: tree level

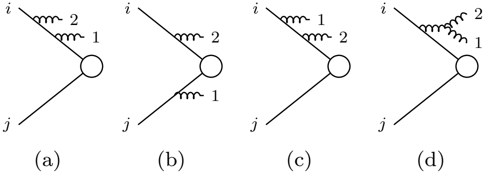
We are interested in the structure of the operator for two soft real emissions:

$$\left|n_{+2}^{(0)}\right\rangle = \mathbf{K}_2(q_1, q_2)\left|n^{(0)}\right\rangle$$

We assume that $\vec{q}_{2\perp}^{\,2} \ll \vec{q}_{1\perp}^{\,2}$ and consider kinematic limits for q_2 defined by the system $\{p_i, p_j, q_1\}$ to capture the leading contributions:

Limits on ordering of the two emissions

1. $q_1 \perp \{p_i, p_j\}$ and $q_2 \perp \{p_i, p_j, q_1\}$
2. $q_1 \perp \{p_i, p_j\}$ and $q_2 \,||\, \{q_1, p_i, p_j\}$
3. $q_1 \,||\, \{p_i, p_j\}$ and $q_2 \perp \{p_i, p_j\}$ Important for SLL!
4. $q_1 \,||\, \{p_i, p_j\}$ and $q_2 \,||\, \{p_i, p_j\}$



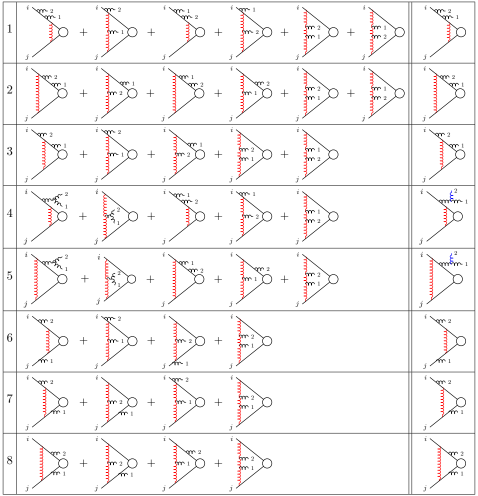
In all these limits we find

$$\mathbf{K}_2(q_1, q_2) = \mathbf{J}_2^{c_2 c_1 a}(q_2, q_1)\mathbf{J}_1^{a}(q_1)\,, \qquad \mathbf{J}_2^{c_2, c_1, a}(q_2, q_1) = \delta^{c_1 a}\mathbf{J}_1^{c_2}(q_2) + \frac{\mathrm{i} f^{c_1 c_2 a} q_1 \cdot \varepsilon_2}{q_1 \cdot q_2}$$

Two emissions at one-loop: Eikonal cuts

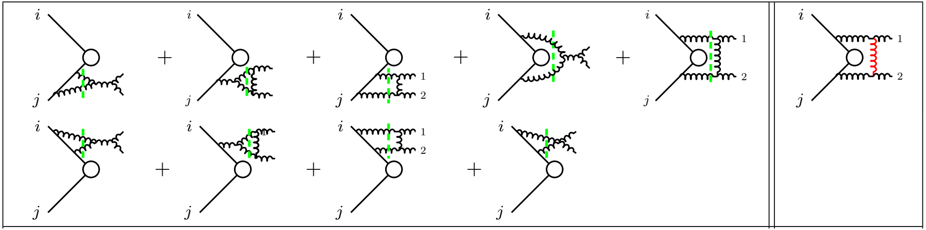
Consider cuts through the fast partons.
Careful grouping of graphs gives rise to
k_T-ordered expression:

$$\Big[\mathbf{J}_2^{c_2 c_1 a}(q_2, q_1)\mathbf{J}_1^a(q_1)\mathbf{C}_{q_{1\perp},Q}$$
$$+\,\mathbf{J}_2^{c_2 c_1 a}(q_2, q_1)\mathbf{C}_{q_{2\perp},q_{1\perp}}\mathbf{J}_1^a(q_1)$$
$$+\,\mathbf{C}_{0,q_{2\perp}}\mathbf{J}_2^{c_2 c_1 a}(q_2, q_1)\mathbf{J}_1^a(q_1)\Big]|M^{(0)}\rangle$$



Two emissions at one-loop: Soft gluon cuts

Another scenario where the cuts pass through two soft gluons. Leads to the nontrivial third line in the AMFS result



$$\mathbf{T}_i^a \mathbf{T}_j^b \left(\frac{p_j \cdot \varepsilon_1}{p_j \cdot q_1} - \frac{p_i \cdot \varepsilon_1}{p_i \cdot q_1} \right) \Bigg\{ \mathrm{i} f^{c_1 ae} \mathrm{i} f^{c_2 be} \left(\frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} - \frac{p_j \cdot \varepsilon_2}{p_j \cdot q_2} \right) \left[- \frac{\mathrm{i}\pi}{8\pi^2} \int_0^{\left(\vec{q}_{2\perp}^{\,(j1)}\right)^2} \frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}} \right]$$

$$- \mathrm{i} f^{c_2 ae} \mathrm{i} f^{c_1 be} \left(\frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} - \frac{p_i \cdot \varepsilon_2}{p_i \cdot q_2} \right) \left[- \frac{\mathrm{i}\pi}{8\pi^2} \int_0^{\left(\vec{q}_{2\perp}^{\,(i1)}\right)^2} \frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}} \right] \Bigg\} \left| M^{(0)} \right\rangle$$

The insertions are bounded by the k_T in the dipole rest frame of the q_1 and the *parent* (i or j) of q_2:

$$\left(\vec{q}_{2\perp}^{\,(i1)}\right)^2 = \frac{2 p_i \cdot q_2 \, q_1 \cdot q_2}{p_i \cdot q_1}$$

Arbitrary number of emissions at one-loop

1. Can prove similar ordering in emissions from outgoing partons.
2. Calculate the real part in the Eikonal approximation.
3. Do some color algebra.

AMFS result

$$\big|M_N^{(1)}\big\rangle = (g\mu^\epsilon)^N \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_1)\left(\sum_{i=2}^{n}\sum_{j<i}\mathbf{I}_{ij}(q_{1\perp}^{(ij)}, Q)\right)\big|M_0^{(0)}\big\rangle$$

$$+ (g\mu^\epsilon)^N \sum_{m=1}^{N} \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_{m+1})\left(\sum_{i=2}^{n+m-1}\sum_{j<i}\mathbf{I}_{ij}(q_{m+1\perp}^{(ij)}, q_{m\perp}^{(ij)})\right)\mathbf{J}^{(0)}(q_m)\ldots\mathbf{J}^{(0)}(q_1)\big|M_0^{(0)}\big\rangle$$

$$+ (g\mu^\epsilon)^N \sum_{m=1}^{N} \mathbf{J}^{(0)}(q_N)\ldots\mathbf{J}^{(0)}(q_{m+1})\left(\sum_{j,k=1}^{n+m-1}\mathbf{I}_{(n+m)j}(q_{m+1\perp}^{((n+m)j)}, q_{m\perp}^{(jk)})\mathbf{d}_{jk}(q_m)\right)\mathbf{J}^{(0)}(q_{m-1})\ldots\mathbf{J}^{(0)}(q_1)\big|M_0^{(0)}\big\rangle$$

This result is equivalent to Catani-Grazzini result because

$$\mathbf{d}_{ij}^{(1)}(q_m) \approx -\mathbf{J}^{(0)}(q_m)\mathbf{I}_{ij}(0, q_{m\perp}^{(ij)}) + \mathbf{I}_{ij}(0, q_{m\perp}^{(ij)})\mathbf{J}^{(0)}(q_m) + \mathbf{I}_{(n+m)i}(0, q_{m\perp}^{(ij)})\mathbf{d}_{ij}(q_m) + \mathbf{I}_{(n+m)j}(0, q_{m\perp}^{(ij)})\mathbf{d}_{ji}(q_m)$$

Re-derivation of the AMFS result in SCET

Starting point

We now consider the same problem in $\mathrm{SCET_{II}}$ with hard scattering and Glauber operators:

$$\mathcal{L}_{\mathrm{SCET_{II}}}^{\text{hard scattering}} = \bar{\xi}_n W_n S_n^\dagger \Gamma S_{\bar{n}} W_{\bar{n}}^\dagger \xi_{\bar{n}}$$

$$\mathcal{L}_{\mathrm{SCET_{II}}}^{\text{dynamical}} = \mathcal{L}_S^{(0)}(\psi_S, A_S) + \sum_{n,\bar{n}} \mathcal{L}_{n_i}^{(0)}(\xi_{n_i}, A_{n_i}) + \mathcal{L}_G^{\mathrm{II}(0)}(\{\xi_{n_i}, A_{n_i}\}, \psi_S, A_S)\,,$$

and make use of

a) The Glauber operators

$$\mathcal{L}_G^{\mathrm{II}(0)} = e^{-\mathrm{i}x\cdot\mathcal{P}} \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + e^{-\mathrm{i}x\cdot\mathcal{P}} \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

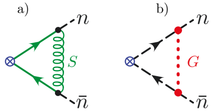
$$\begin{aligned}
\mathcal{O}_n^{qB} &= \overline{\chi}_n T^B \tfrac{\bar{\not{n}}}{2}\, \chi_n & \mathcal{O}_n^{gB} &= \tfrac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^{C}\, \tfrac{\bar{n}}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{n\perp}^{D\mu} \\
\mathcal{O}_{\bar{n}}^{qB} &= \overline{\chi}_{\bar{n}} T^B \tfrac{\not{n}}{2}\, \chi_{\bar{n}} & \mathcal{O}_{\bar{n}}^{gB} &= \tfrac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^{C}\, \tfrac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{\bar{n}\perp}^{D\mu}
\end{aligned}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \tfrac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}$$

$$\begin{aligned}
\mathcal{O}_s^{q_n B} &= 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \tfrac{\not{n}}{2} \psi_S^n \right) & \mathcal{O}_s^{g_n B} &= 8\pi\alpha_s \left(\tfrac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC}\, \tfrac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{S\perp}^{nD\mu} \right) \\
\mathcal{O}_s^{q_{\bar{n}} B} &= 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} T^B \tfrac{\bar{\not{n}}}{2} \psi_S^{\bar{n}} \right) & \mathcal{O}_s^{g_{\bar{n}} B} &= 8\pi\alpha_s \left(\tfrac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C}\, \tfrac{\bar{n}}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)
\end{aligned}$$

b) *Soft-Glauber correspondence* (for the cases where it works):

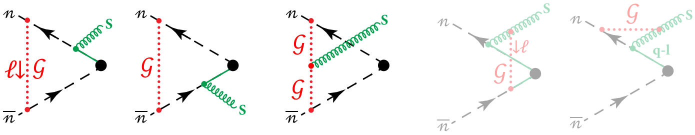
$$S^{(G)} = G$$



[Rothstein, Stewart 2016]:
"It is then interesting to note that the simplest method of computing these (iπ) terms is by making use of the G_i Glauber diagrams."

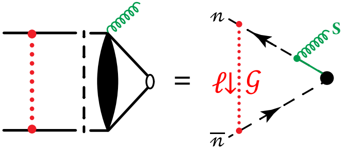
The “G_i Glauber graphs”

For the case of imaginary part of the one-loop one-real emission, we have 3 diagrams[2]. The fourth in the row equals its zero bin. Fifth vanishes trivially.

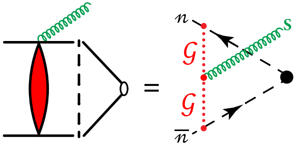


Hard vertex correction

Lipatov vertex



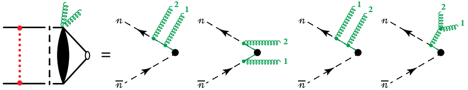
$g^3 \mathbf{C}_{0,Q} \mathbf{J}_1^{c_1}(q_1) \big| M^{(0)} \big\rangle$



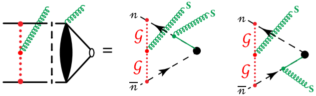
$g^3 \big[\mathbf{J}_1^{c_1}(q) \mathbf{C}_{q,Q} + \mathbf{C}_{0,q}\, \mathbf{J}_1^{c_1}(q) - \mathbf{C}_{0,Q}\, \mathbf{J}_1^{c_1}(q) \big] \big| M_0 \big\rangle$

[2] had 7 in full theory

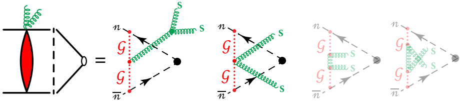
Eikonal cuts for one loop 2 soft emissions in SCET



$$\left[-\frac{i\pi}{8\pi^2}\int_0^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}\right]\mathbf{K}^{c_1,c_2}(q_1,q_2)\big|M^{(0)}\big\rangle$$



$$\mathbf{T}_j^b(if^{bc_2a})\mathbf{T}_i^a\left(\frac{p_j\cdot\varepsilon_1}{p_j\cdot q_2}-\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_2}\right)\left[-\frac{i\pi}{8\pi^2}\int_{\vec{q}_{2\perp}^{\,2}}^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}\right]\mathbf{J}^{c_1}(q_1)|M^{(0)}\rangle$$



$$\mathbf{T}_i^a\mathbf{T}_j^b\left(\frac{p_j\cdot\varepsilon_1}{p_j\cdot q_1}-\frac{p_i\cdot\varepsilon_1}{p_i\cdot q_1}\right)\Bigg[if^{c_1ae}if^{c_2be}\left(\frac{q_1\cdot\varepsilon_2}{q_1\cdot q_2}-\frac{p_j\cdot\varepsilon_2}{p_j\cdot q_2}\right)$$
$$-\,if^{c_2ae}if^{c_1be}\left(\frac{q_1\cdot\varepsilon_2}{q_1\cdot q_2}-\frac{p_i\cdot\varepsilon_2}{p_i\cdot q_2}\right)\Bigg]\left[-\frac{i\pi}{8\pi^2}\int_{\vec{q}_{1\perp}^{\,2}}^{Q^2}\frac{d\vec{\ell}_\perp^{\,2}}{\vec{\ell}_\perp^{\,2}}\right]|M^{(0)}\rangle$$

- Individual contributions are gauge invariant and add up to yield the previous result for Eikonal cuts
- Two gluon emission in forward scattering regime involves only two graphs[3]! The ones with soft propagator are subleading in strong ordering.

[3] had 36 total in full theory

Soft gluon cut graph(s) in SCET

To consider the diagrams corresponding to "Soft gluon cuts" we distinguish between the rapidities of the two soft emissions, and express the momentum q_2 in the $(i1)$ dipole frame:

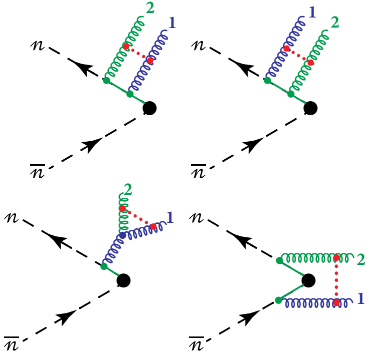
$$k^\mu = \frac{p_i^\mu}{p_i \cdot q_1} q_1 \cdot k + \frac{q_1^\mu}{p_i \cdot q_1} p_i \cdot k + k^\mu_{\perp(i1)}\,, \qquad \vec{k}^{\,2}_{\perp(i1)} = \frac{2p_i \cdot k\, q_1 \cdot k}{p_i \cdot q_1}$$

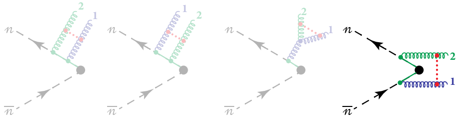
Consider the forward scattering: $g(q_1') + g(q_2') \to g(q_1) + g(q_2)$

Scaling in the $(i1)$ dipole frame:

$$\begin{aligned} q_1 &\sim q_1^{+(i1)}(1,0,0)_{i1} \\ p_i &\sim p_i^{-(i1)}(0,1,0)_{i1} \\ q_1' &\sim q_1^{+(i1)}(1,\lambda^2,\lambda)_{i1} \\ q_2 \sim q_2' &\sim q_1^{+(i1)}(\lambda,\lambda,\lambda)_{i1} \\ \ell &\sim q_1^{+(i1)}(\lambda,\lambda^2,\lambda)_{i1}\,, \end{aligned}$$

$$q_1^{+(i1)} = p_i^{-(i1)} = \sqrt{2p_i \cdot q_1}\,, \qquad \lambda = \frac{q_{2\perp(i1)}}{q_1^{+(i1)}} = \frac{\sqrt{p_i \cdot q_2\, q_1 \cdot q_2}}{p_i \cdot q_1}$$





$$\mathcal{S}^{(2,1)}_{(i1)j} = g^2 \int đ^d \ell \, \frac{(-\mathrm{i})^2 \mathbf{K}^{ab}_{\mu\lambda}(q_1+\ell, q_2-\ell)}{[(q_1+\ell)^2+\mathrm{i}0]\,[(q_2-\ell)^2+\mathrm{i}0]} \varepsilon_{1\nu}\varepsilon_{2\tau} \Big[\frac{2\mathrm{i}g^2}{\ell^2_{\perp(i1)}}\Big] \frac{\mathrm{i}f^{ac_1e}\mathrm{i}f^{bc_2e}}{\kappa_{i1}}$$

$$\times \Big[p_i\cdot q_1\, g^{\mu\nu}_{\perp(i1)} - p_i^\mu (q_{1\perp(i1)}+\ell_{\perp(i1)})^\nu - p_i^\nu q^\mu_{1\perp(i1)} + \frac{q_{1\perp(i1)}\cdot(q_{1\perp(i1)}+\ell_{\perp(i1)})p_i^\mu p_i^\nu}{p_i\cdot q_1}\Big]$$

$$\times \Big[q_1\cdot q_2\, g^{\lambda\tau}_{\perp(i1)} - q_1^\lambda (q_{2\perp(i1)}-\ell_{\perp(i1)})^\tau - q_1^\tau q^\lambda_{2\perp(i1)} + \frac{(q_{2\perp(i1)}-\ell_{\perp(i1)})\cdot q_{2\perp(i1)} q_1^\lambda q_1^\tau}{q_1\cdot q_2}\Big].$$

$$K^{c_2c_1a}_{\mu\lambda}(q_1,q_2) = \mathbf{J}^{c_2c_1a}_{\lambda}(q_2,q_1)\mathbf{J}^a_\mu(q_1)\,, \qquad \mathbf{J}^{c_2c_1a}_{\lambda} = \delta_{c_1a}\sum_{i=1}^{n}\mathbf{T}_i^{c_2}\Big(\frac{p_{i\lambda}}{p_i\cdot q_2} - \frac{q_{1\lambda}}{q_1\cdot q_2}\Big)$$

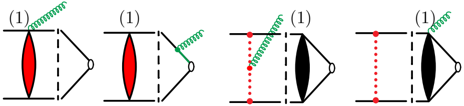
1. For $(i1)$ coordinates, graphs with q_1 attachments to i fast parton, and q_2 to q_1 vanish.
2. Gauge invariance allows us to pick p_i as the auxiliary momentum for each term in $\mathbf{J}^{c_2c_1a}_\lambda$.
3. The only diagram for each ij combination reproduces the previous result.[4]

[4] had 9 in full theory

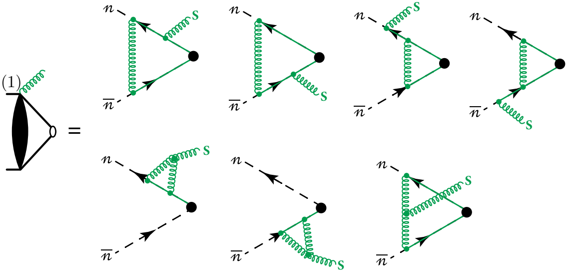
Towards two-loops

Extending the results to two-loops

The imaginary part of the two loop result can be obtained by considering one-loop corrections to the Glauber exchange part or the hard vertex (Only displaying graphs with two hard partons):

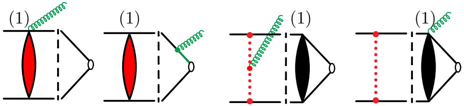


The hard vertex corrections are precisely the real part of the one-loop graphs (after Glauber bin subtractions)

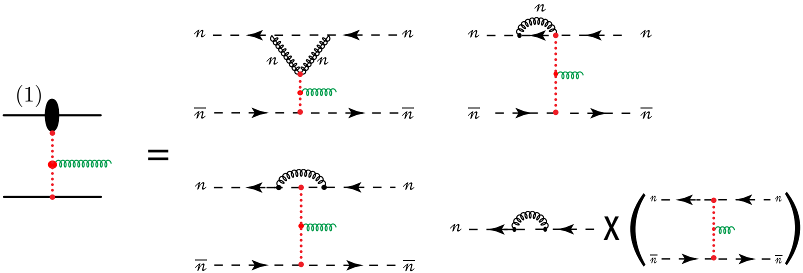


Collinear loops

The imaginary part of the two loop result can be obtained by considering one-loop corrections to the Glauber exchange part or the hard vertex:

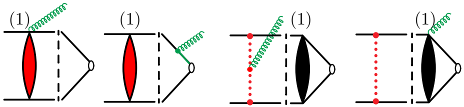


These graphs can be obtained from one-loop matching of forward scattering cross section to the full theory [Rothstein, Stewart 2016]:

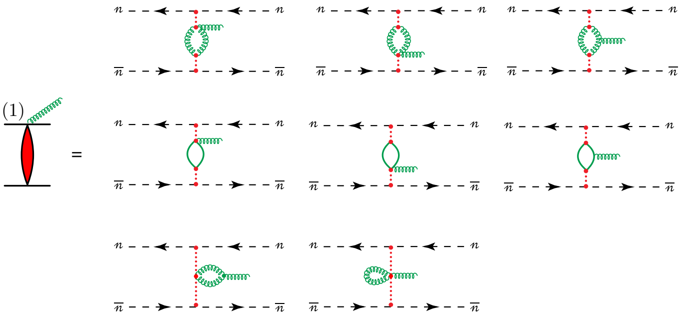


Soft loops

The imaginary part of the two loop result can be obtained by considering one-loop corrections to the Glauber exchange part or the hard vertex:

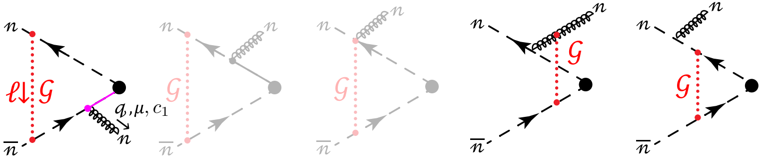


An additional soft emission tied to the loop momentum is crucial to introduce the Θ-function:



Outlook

- Do the two-loop calculation in SCET in ordered limits and test the one-loop conjecture. Also see [Plätzer, Ruffa 2020] for efforts towards a complete two-loop calculation in the full theory.
- Include collinear emissions in the chain (important for superleading logs) [Schwartz, Yan, Zhu 2017]



- Can we get around the IR divergence in the Regge trajectory at the next order?
- What is the connection of the AMFS result with virtual RG renormalization in SCET?

Conclusion

- Presented an interesting formulation of the Catani-Grazzini result that involves IR finite virtual insertions, exact Θ-functions governing the loop integral limits, emergence of unique dipole-k_T ordering variable, and an expression that won't let you exponentiate it.
- SCET can significantly simplify the analysis of ordered amplitudes even if a deeper connection is obscure.
- Graphs in SCET were automatically grouped and led immediately to the final expressions. Interestingly only contributions from the gauge invariant O_S^{AB} operator survived in the soft gluon ordering!
- Due to a lot fewer diagrams a two-loop extension of this result is tractable.

Thank you

Ordering soft real emissions: tree level

1. Both emissions at wide angles: $q_2^\mu \sim \kappa q_1^\mu$

$$\mathbf{K}_2^{c_1,c_2}(q_1,q_2)\Big|_{\text{limit 1}} = \frac{\mathbf{T}_i^{c_2}\, n\cdot\varepsilon_2}{n\cdot q_2}\left[\frac{\mathbf{T}_i^{c_1}\, n\cdot\varepsilon_1}{n\cdot q_1} + \frac{\mathbf{T}_j^{c_1}\, \bar{n}\cdot\varepsilon_1}{\bar{n}\cdot q_1}\right] + \frac{\mathrm{i} f^{c_1 c_2 a}\varepsilon_2\cdot q_1}{q_2\cdot q_1}\frac{\mathbf{T}_i^{a}\, n\cdot\varepsilon_1}{n\cdot q_1} + (i\leftrightarrow j)$$

2. Higher k_T gluon at wide angles, other collinear: $q_1 \sim q_{1\perp}(1,1,1)$ and $q_2 \sim q_{1\perp}(\kappa^2,1,\kappa)$, $q_1^- \sim q_2^-$

$$\mathbf{K}_2^{c_1,c_2}(q_1,q_2)\Big|_{\text{limit 2}} = \frac{\mathbf{T}_i^{c_2}\, n\cdot\varepsilon_2}{n\cdot q_2}\left[\frac{\mathbf{T}_i^{c_1}\, n\cdot\varepsilon_1}{n\cdot q_1} + \frac{\mathbf{T}_j^{c_1}\, \bar{n}\cdot\varepsilon_1}{\bar{n}\cdot q_1}\right]$$

3. Higher k_T gluon collinear, other at wide angles:

$$\mathbf{K}_2^{c_1,c_2}(q_1,q_2)\Big|_{\text{limit 3}} = \frac{\mathbf{T}_i^{c_2}\varepsilon_2^+}{q_2^+}\frac{\mathbf{T}_i^{c_1}\varepsilon_1^+}{q_1^+ + q_2^+} + \frac{\mathbf{T}_j^{c_2}\varepsilon_2^-}{q_2^-}\frac{\mathbf{T}_i^{c_1}\varepsilon_1^+}{q_1^+} + \frac{\mathbf{T}_i^{c_1}\varepsilon_1^+}{q_1^+}\frac{\mathbf{T}_i^{c_2}\varepsilon_2^+}{q_1^+ + q_2^+} + \frac{\mathrm{i} f^{c_1 c_2 a}\varepsilon_2^+\mathbf{T}_i^{a}\varepsilon_1^+}{q_2^+(q_1^+ + q_2^+)}$$
$$= \mathbf{J}_2^{c_2,c_1,a}(q_2,q_1)\frac{\mathbf{T}_i^{c_1}\varepsilon_1^+}{q_1^+}$$

The *wrongly ordered* graph always combines in these limits to give *the ordered result*:

$$\mathbf{K}_2(q_1,q_2) = \mathbf{J}_2^{c_2 c_1 a}(q_2,q_1)\mathbf{J}_1^{a}(q_1)$$