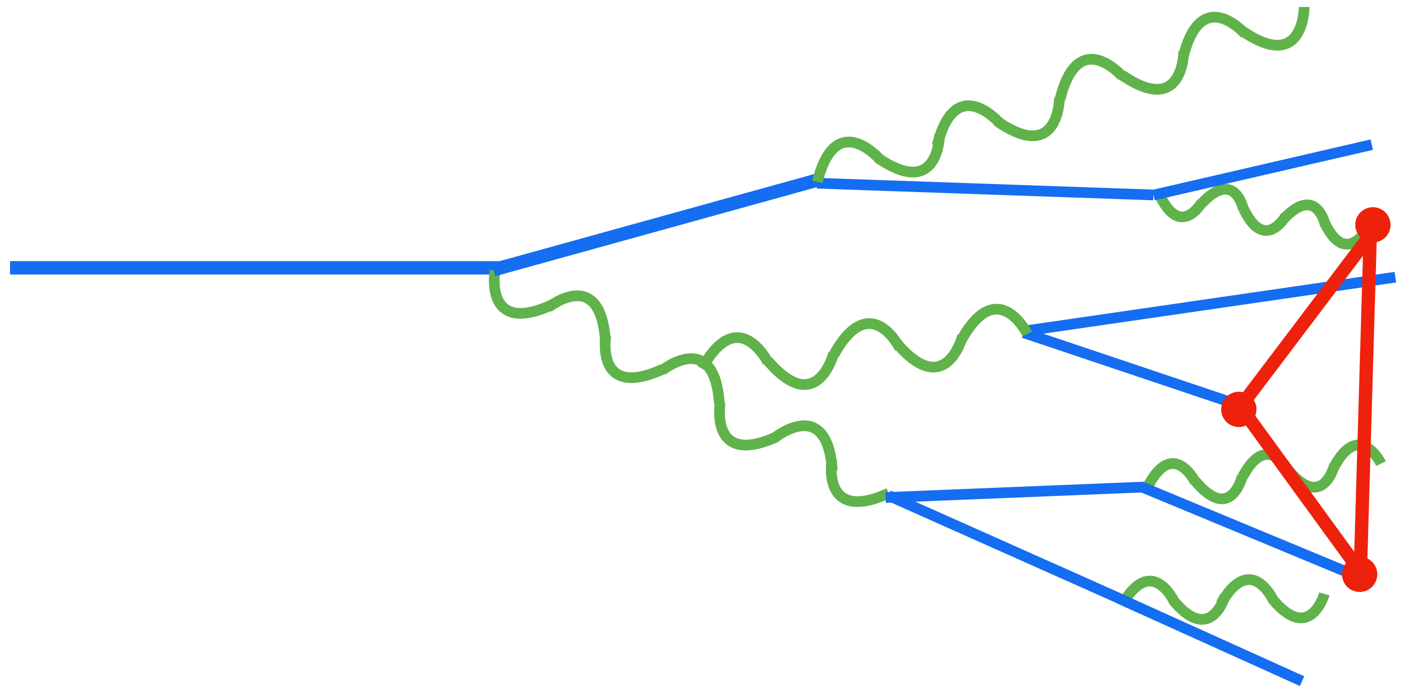
Re-organizing Power Corrections with Lorentz/Conformal Symmetry



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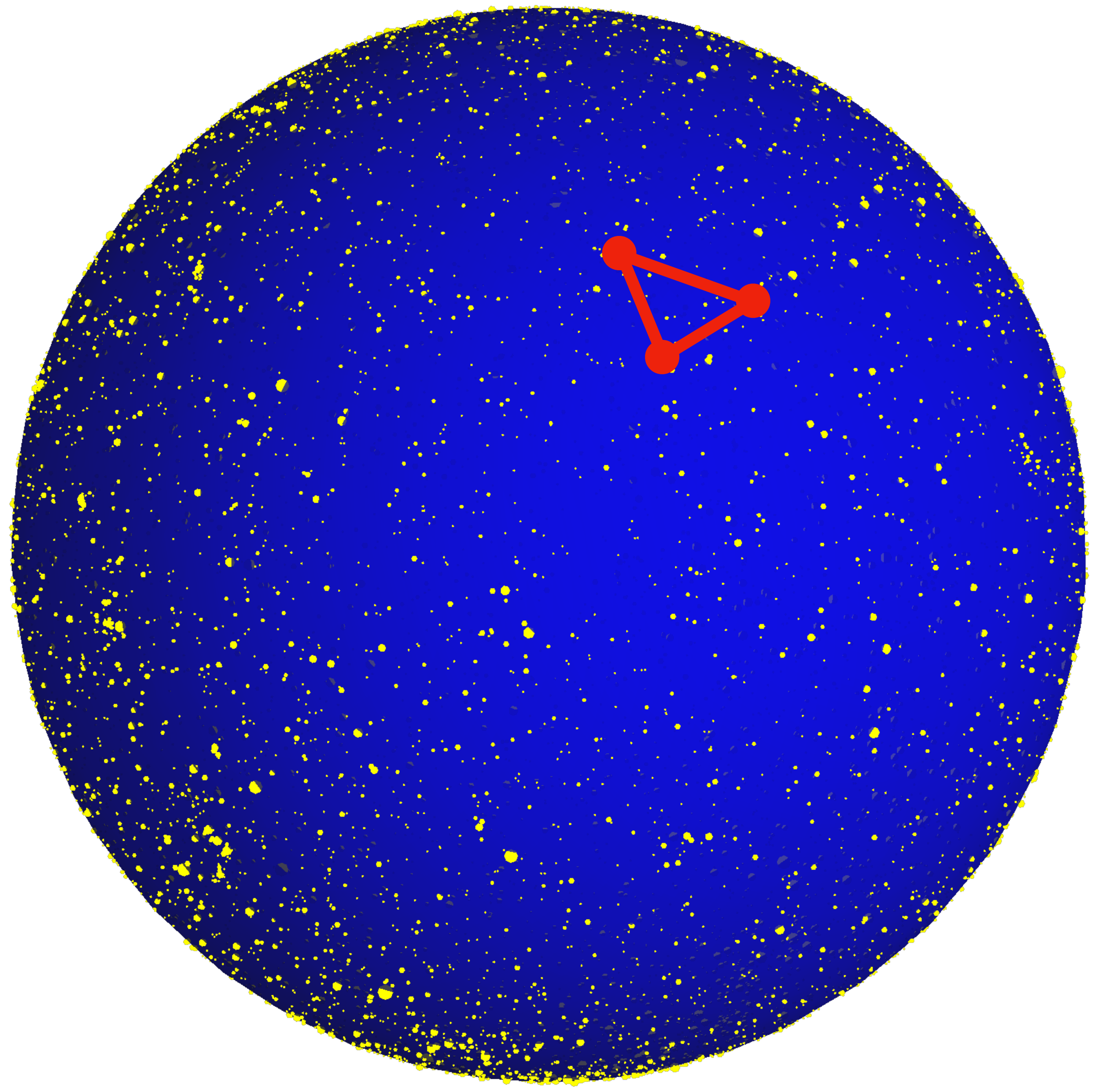


based on a work in preparation

with Ian Moult, Joshua Sandor, HuaXing Zhu

World SCET 2021 April 23

Power Corrections

Motivation

- Important for precision test of Standard Model
- Interesting for understanding the field theory structure in kinematic limits, e.g. the RG evolution, anomalous dimensions…

Recent progress in NLP study

- Event shapes (thrust) $\tau \ll 1$

  [Moult, Stewart, Vita, Zhu, 1804.04665, 1910.14038]
- Transverse momentum distribution

  [Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 1812.08189] [Moult, Vita, Yan, 1912.02188]
- Threshold limit

  [Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 1809.10631]
  [Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, Vernazza, White, 1905.13710]
  [Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 1910.12685]
- Higgs decay $(m_b/M_h \ll 1)$

  [Liu, Neubert, 1912.08818; J. Wang, 1912.0992] [Liu, Mecaj, Neubert, Wang, 2009.06779]

Goals

The goal of this talk is to introduce a number of new techniques for understanding the structure of power expansion in the collinear limit, motivated by CFT.

① Celestial blocks:

$$g(z,\bar{z}) = \sum_{\delta,j} \underbrace{c(\delta,j)}_{\text{dynamics}} \underbrace{g_{\delta,j}(z,\bar{z})}_{\text{symmetries}}$$

Allows the resummation of infinite series of power corrections within the same representation of Lorentz group.

② Lorentzian inversion:

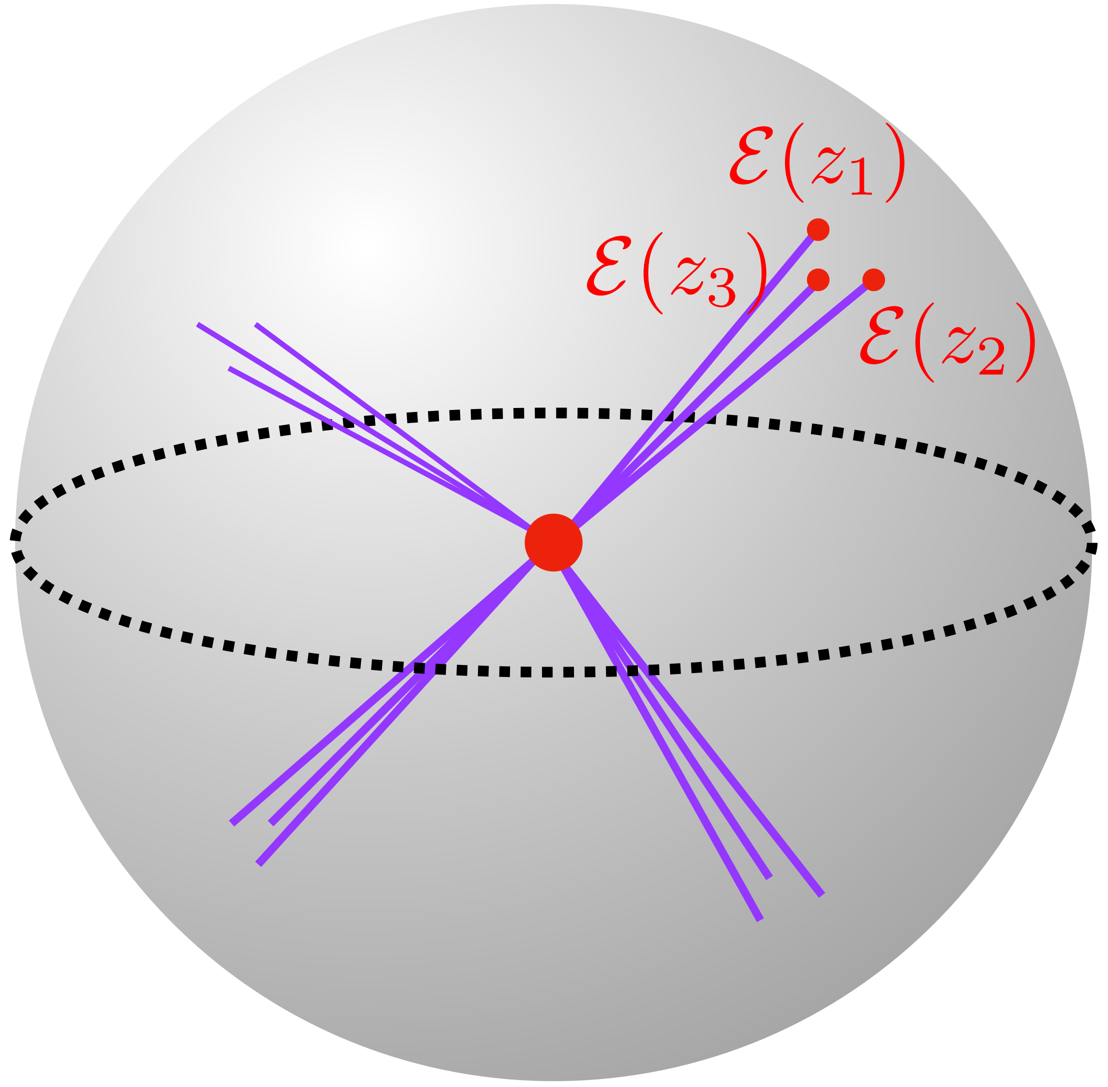
$$c^t(\delta,j) = \frac{\kappa_{\delta+j}}{4}\int_0^1 dz d\bar{z} \mu(z,\bar{z}) g_{j+d-1,\delta+1-d}(z,\bar{z}) \mathrm{dDisc}\left[g(z,\bar{z})\right]$$

Coefficients in power expansion are **analytic** function of j, and lie on Regge trajectories.

Outline

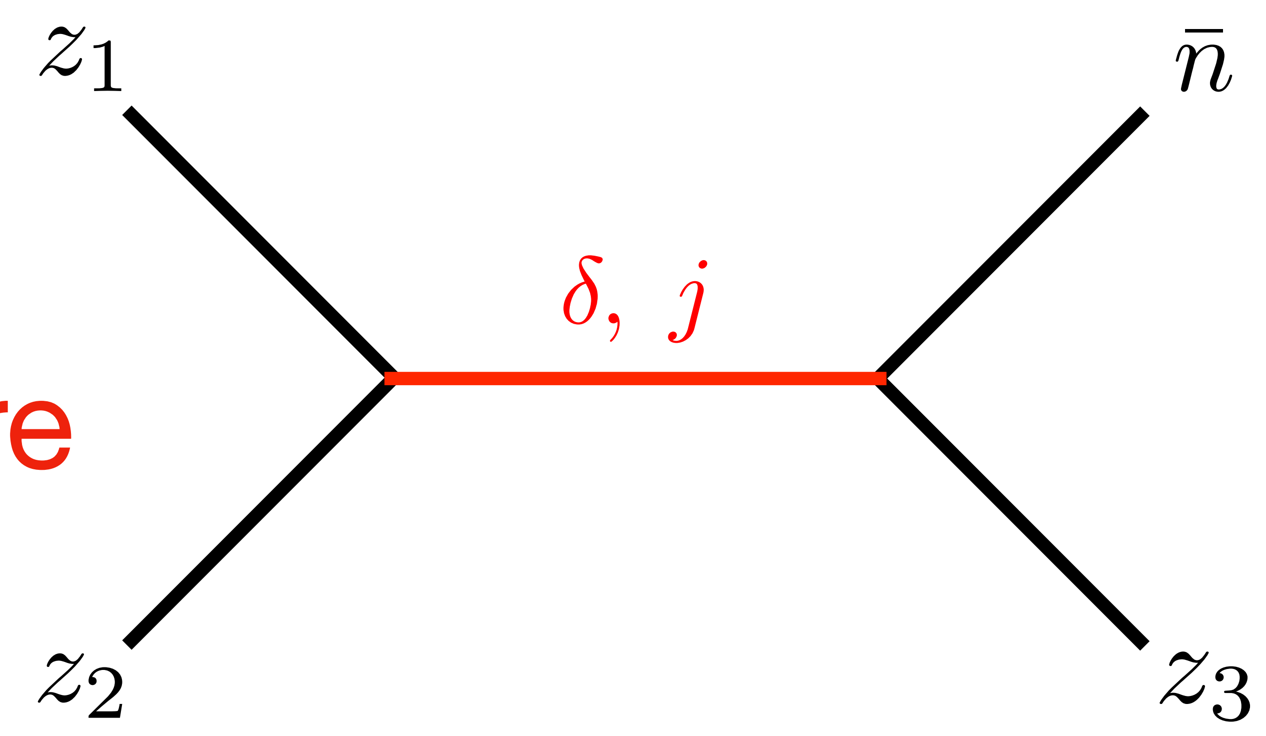
- Introduction
  - EEC-like event shapes
  - Light-ray operators and OPE

Advocate their theoretical and experimental usefulness



- Collinear Triple Energy Correlation
  - Properties
  - Conformal block decomposition on the celestial sphere
  - Lorentzian inversion formula

Lorentz symmetry



New way to organize power corrections for 3-point energy correlator

State of the art for Energy Correlators

EEC calculations (full angle):

QCD: **LO** [Basham, Brown, Ellis, Love, 1978]

NLO [Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]

$\mathcal{N}=4$ SYM: **NLO** [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2014]

NNLO [Henn, Sokatchev, Yan, Zhiboedov, 2019]

Collinear limit:

QCD: [Dixon, Moult, Zhu, 2019]

$\mathcal{N}=4$ SYM: [Korchemsky, 2019; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

[Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020]

3-point (LO): [HC, Luo, Moult, Yang, Zhang, Zhu, 2019]

Back-to-back limit:

QCD: **N^3LL** [Moult, Zhu, 2018] [Ebert, Mistlberger, Vita, 2020]

NLP [Moult, Vita, Yan, 2019]

$\mathcal{N}=4$ SYM: [Korchemsky, 2019]

Other colliders:

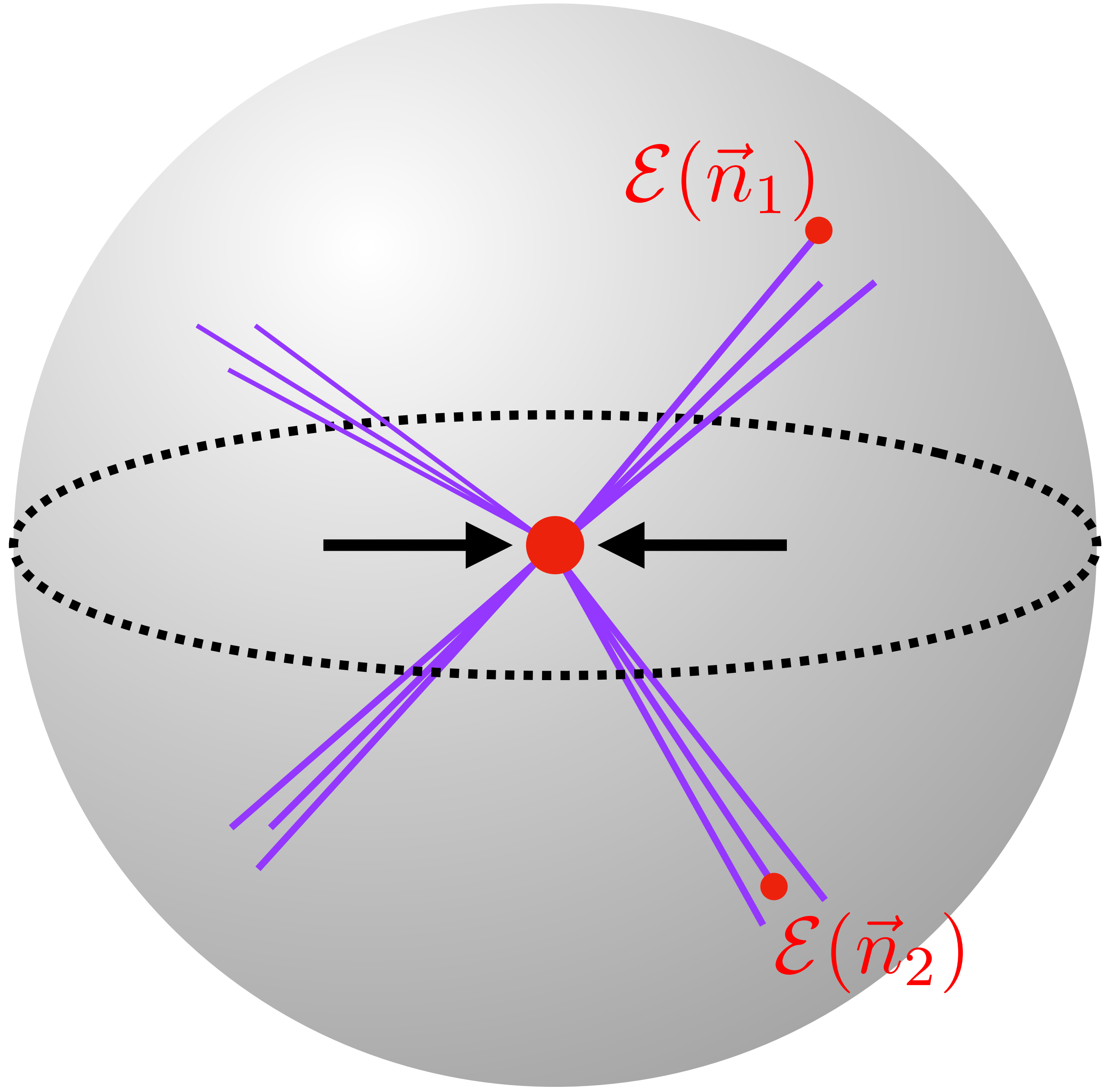
pp [Gao, Li, Moult, Zhu, 2019] ep [Li, Vitev, Zhu, 2020; Li, Makris, Vitev, 2021]

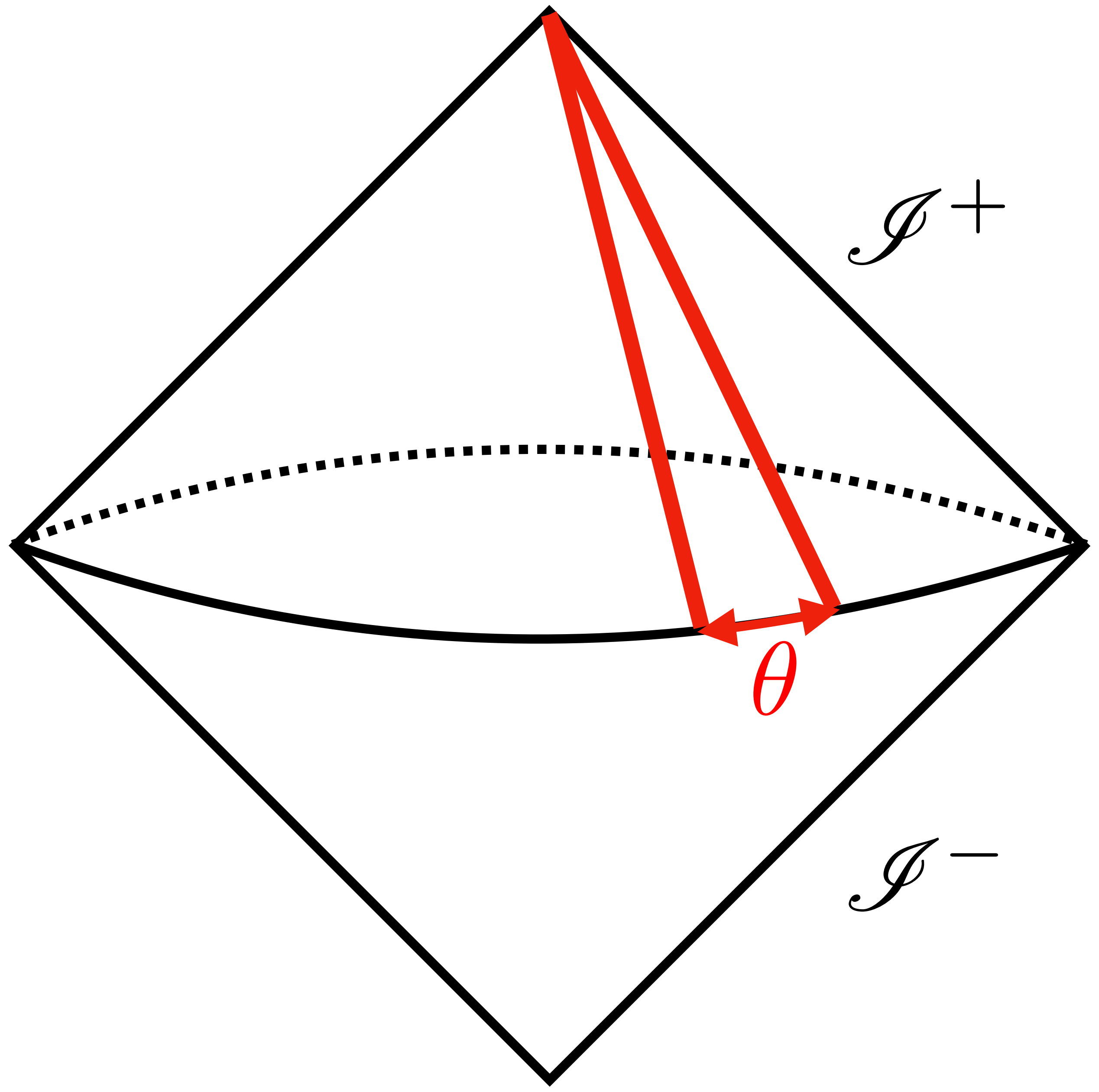
Energy Flow Operators and EEC

[Basham, Brown, Ellis and Love, 1978]
introduced energy-energy correlation

$$\frac{d\Sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$

which characterizes the correlation of two energy detectors (calorimeters) at spatial infinity (celestial sphere).





The energy detector has a nice operator definition:

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\vec{n})$$

[Korchemsky, Sterman, 1999; Hofman, Maldacena, 2008; Bauer, Fleming, Lee, Sterman, 2008; ...]

allowing alternative definition of EEC and its multi-point generalization as correlation function of multiple insertion of **energy flow** operators

$$\langle O'(-q)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots|O(q)\rangle$$

Source with total momentum $q = (Q,0,0,0)$

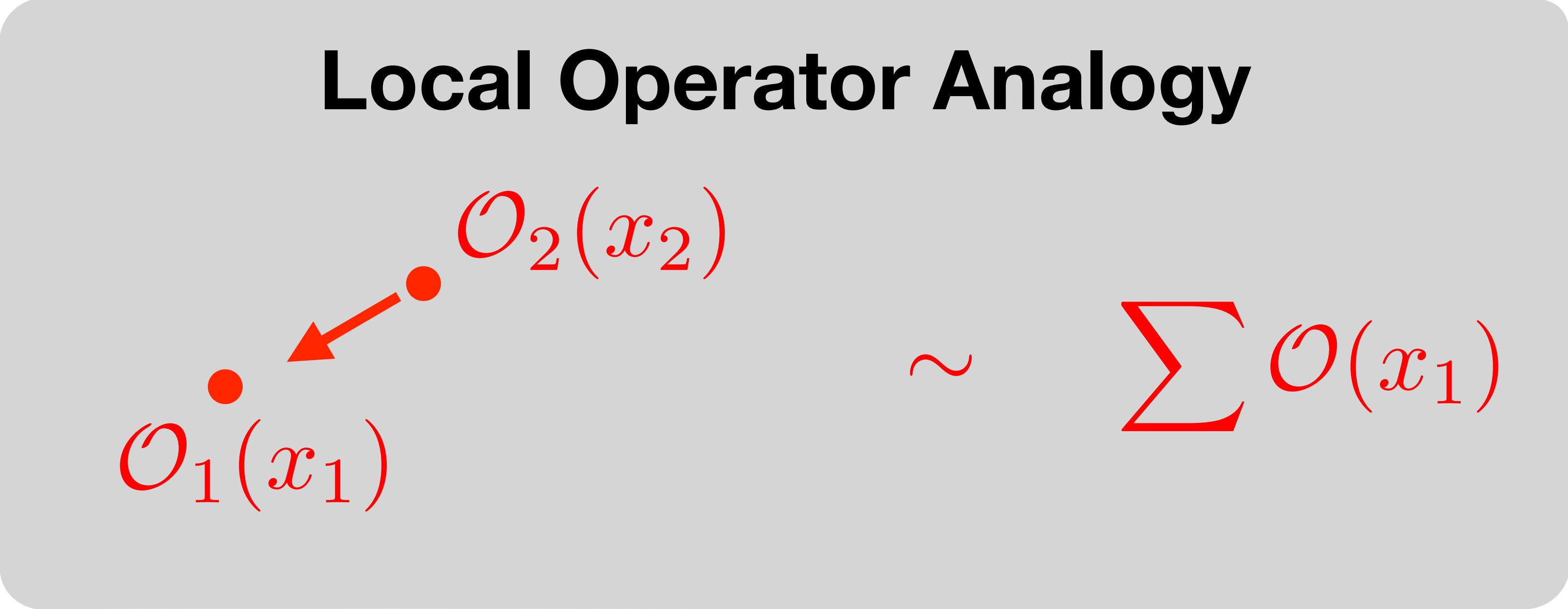
Light-ray Operators and OPE

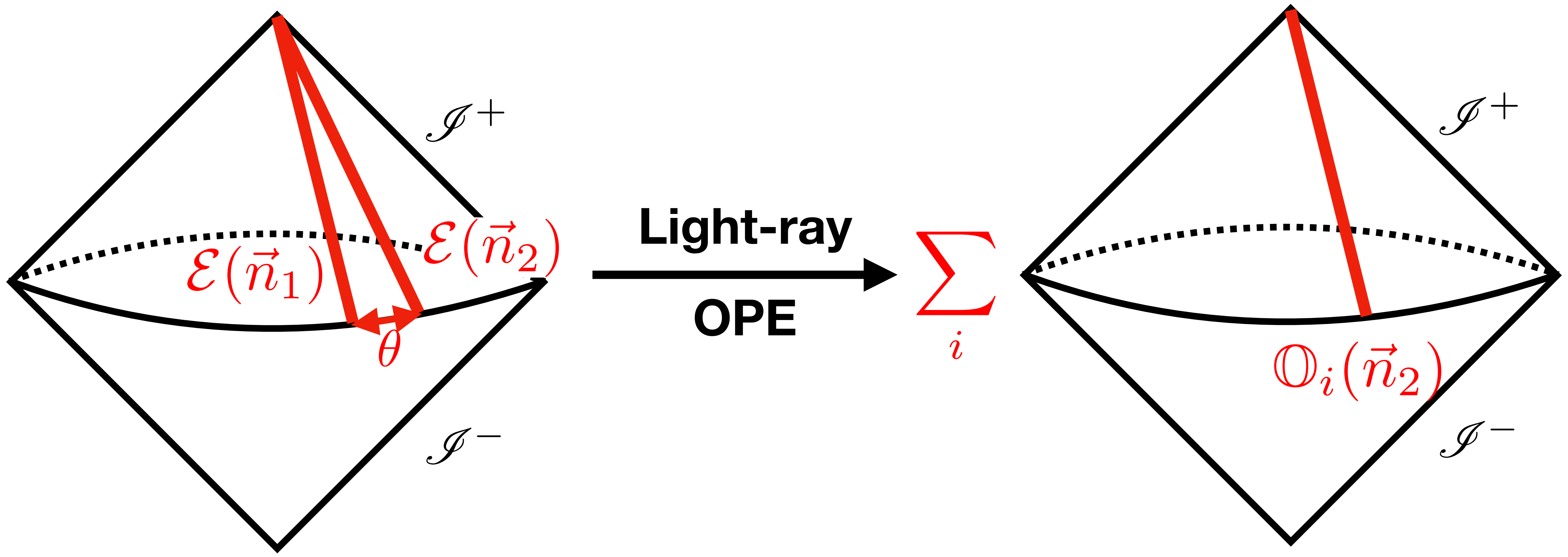
Generalization of $\mathcal{E}(\vec{n})$:

[Kravchuk, Simmons-Duffin, 2018]

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta - J} \int_0^\infty dt\, O^{\mu_1 \cdots \mu_J}(t, r\vec{n}) \bar{n}_{\mu_1} \ldots \bar{n}_{\mu_J}$$

twist ($\Delta - J$)





Small angle behavior is controlled by the OPE of these light-ray operators.

Light-ray OPE $\mathcal{E}(\vec{n}_1)\, \mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i - 4} \mathbb{O}_i(\vec{n}_2)$ [Hofman, Maldacena, 2008]

dominated by leading twist

Light-ray OPE in CFT is rigorous and convergent. [Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019] [Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020]

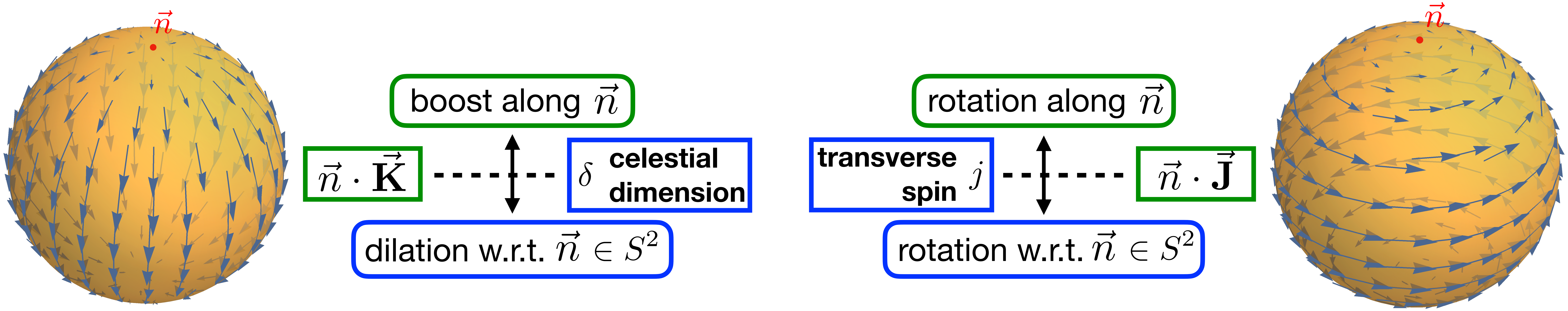
In QCD, things are less understood, but the leading power contribution is. [HC, Moult, Zhu, 2020]

Celestial Sphere and Celestial Block

Light-ray operators are local on the celestial sphere.

It has long been realized that the **Lorentz group** is equivalent to the **conformal group** on the **celestial sphere**.

Can we use CFT techniques to study energy correlators? power corrections?



For 2-point EEC, **[Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]** had used this fact to give rigorous light-ray OPE and organize it into "**celestial blocks**".

sum all descendants like conformal blocks

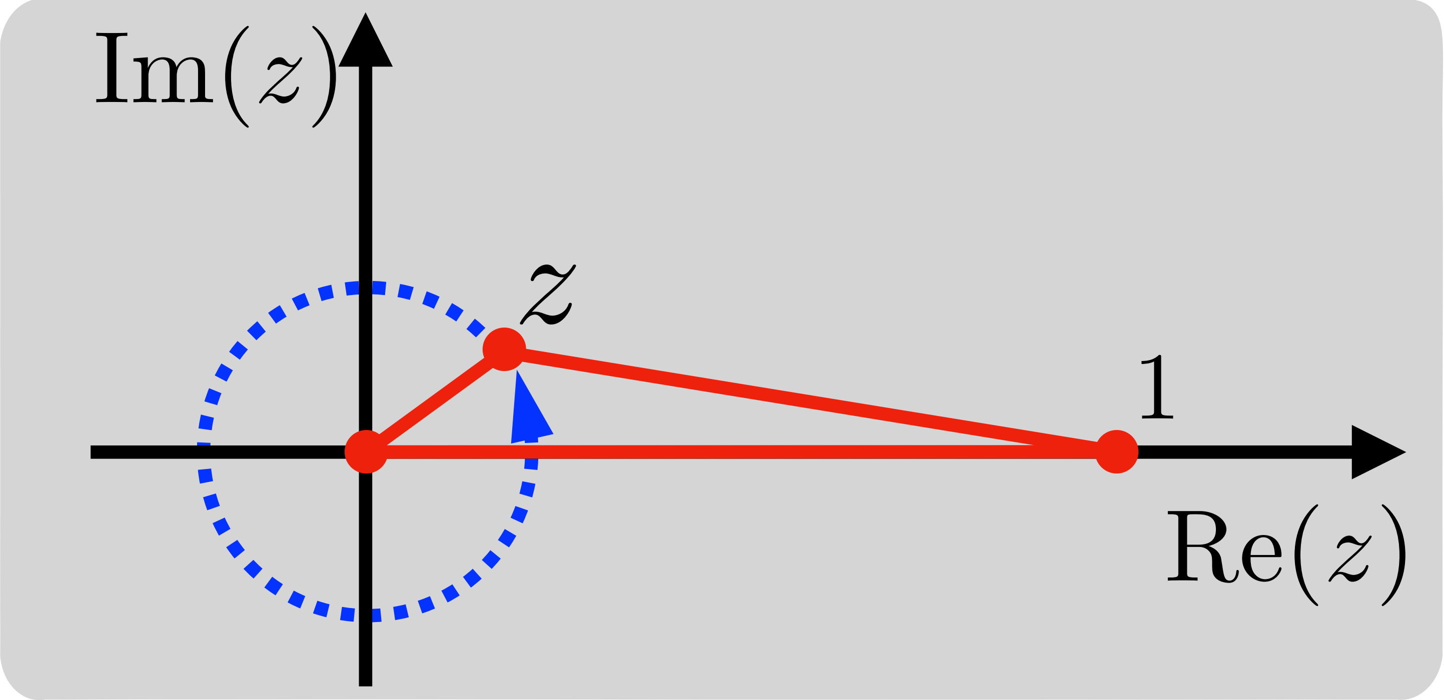
But this does not realize conformal symmetry on the celestial sphere because the sources live inside Minkowski space.

$$\langle O'(-q)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots|O(q)\rangle$$

Collinear EEEC Kinematics

In the collinear limit, **[HC, Luo, Moult, Yang, Zhang, Zhu, 2019]**

EEEC configuration can be approximated by a triangle.

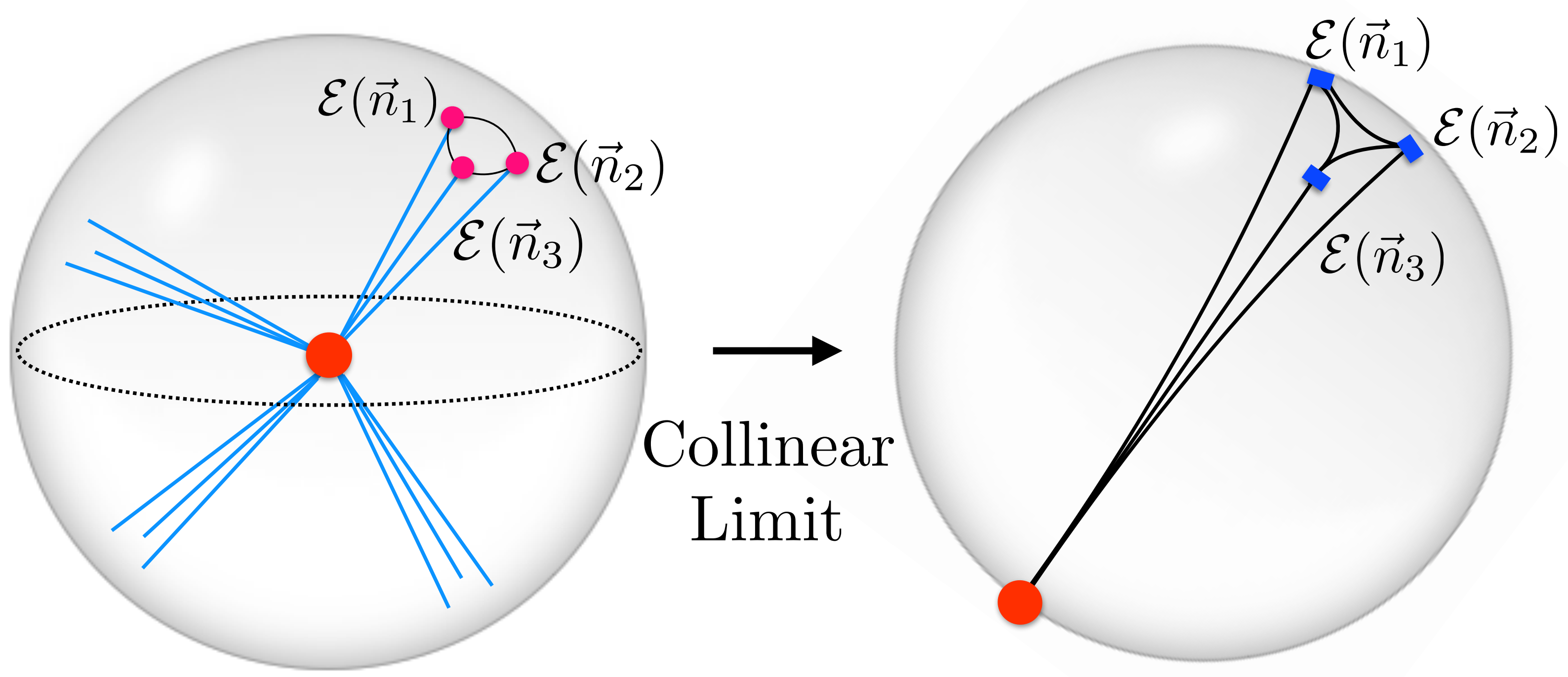


Parameterized in terms of

(1) the longest side x_L **[Size]** ← **LP**

(2) a complex number z **[Shape]**

study power correction in the squeezed limit $z \to 0$

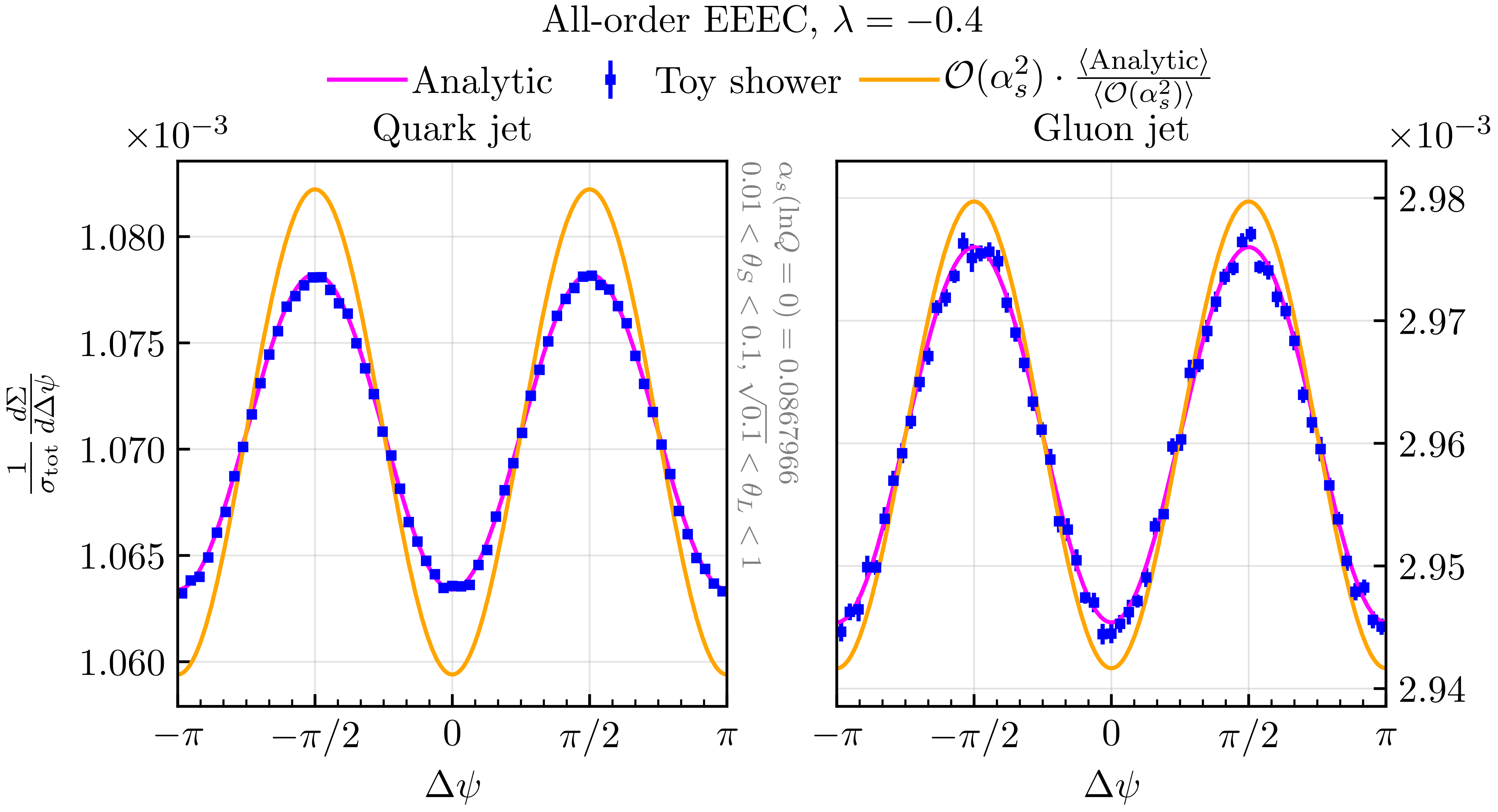


Squeezed limit encodes spin correlation information and the resummation of LP is done with light-ray OPE framework.

[HC, Moult, Zhu, 2020]

Recently, when collinear spin correlation is included in the **PanScales** family of parton showers, our resummed result provides validation of shower results.

[Karlberg, Salam, Scyboz, Verheyen, 2021]



3-Point Energy Correlator

with collinear quark source

Operator Definition

$$\int dt\, e^{it\bar{n}\cdot P} \langle\Omega|\bar{\chi}(t\bar{n})\bar{\not{n}}\mathcal{E}(z_1)\mathcal{E}(z_2)\mathcal{E}(z_3)\chi(0)|\Omega\rangle$$

Properties

- depends on scalar products $\bar{n}\cdot P,$ dimensionless $\bar{n}\cdot z_i, \quad z_i\cdot z_j$
- dimension = 5
- homogeneous in $\bar{n},\quad z_1,\ z_2,\ z_3$ RPI 0, -3; celestial dimension $\mathcal{E}(\lambda z_i) = \lambda^{-3}\mathcal{E}(z_i)$

Functional Form

$$(\bar{n}\cdot P)^5 \frac{1}{(z_1\cdot z_2)^3}\frac{1}{(z_3\cdot\bar{n})^4}\left(\frac{z_1\cdot z_3}{z_1\cdot\bar{n}}\right) g(u,v)$$

4 point conformal correlator on the celestial sphere

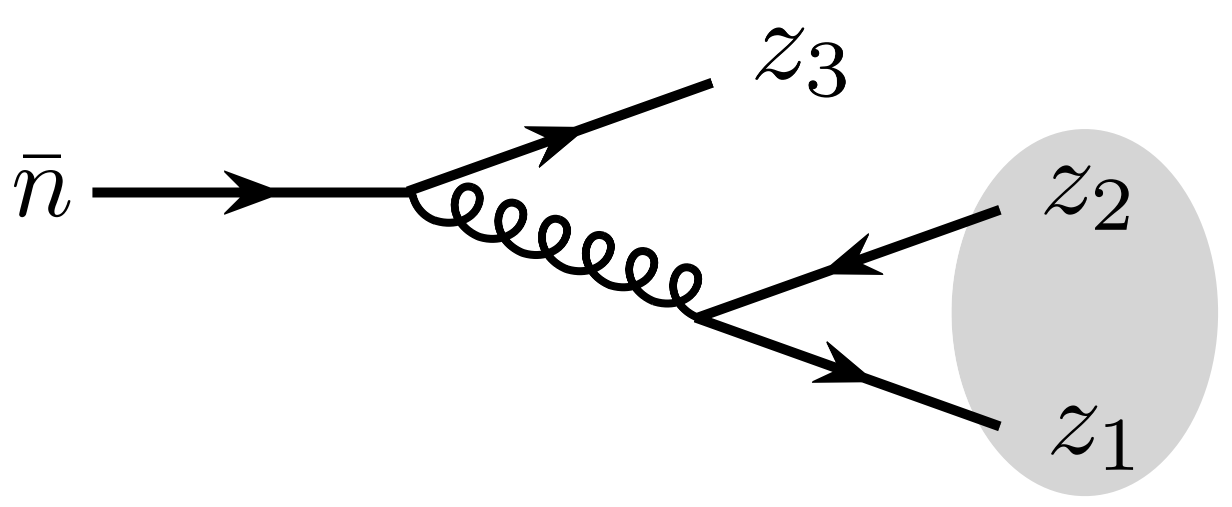
cross-ratios

$$u = \frac{(z_1\cdot z_2)(z_3\cdot\bar{n})}{(z_1\cdot z_3)(z_2\cdot\bar{n})}$$

$$v = \frac{(z_1\cdot\bar{n})(z_2\cdot z_3)}{(z_1\cdot z_3)(z_2\cdot\bar{n})}$$

Example

For simplicity, tagging final state quarks



Squeezed limit: $z_1 \cdot z_2 \to 0$

$$u = z\bar{z}, \; v = (1-z)(1-\bar{z})$$

Expanding the full result:

highest transverse spin series

$$g(u,v) \equiv g(z,\bar{z}) \propto \; -z^3\bar{z} \;(\cos 2\phi) \; + \frac{39}{10} z^2\bar{z}^2 \; - z\bar{z}^3 \qquad \textbf{LP}$$

$$-z^4\bar{z} \;(\cos 3\phi) \; + \frac{39}{20} z^3\bar{z}^2 \; + \frac{39}{20} z^2\bar{z}^3 \; - z\bar{z}^4 \qquad \textbf{NLP}$$

$$-\frac{6}{7} z^5\bar{z} \;(\cos 4\phi) \; + \frac{229}{140} z^4\bar{z}^2 \; - \frac{211}{140} z^3\bar{z}^3 \; + \frac{229}{140} z^2\bar{z}^4 \; - \frac{6}{7} z\bar{z}^5 \qquad \textbf{NNLP}$$

$$-\frac{5}{7} z^6\bar{z} \;(\cos 5\phi) \; + \frac{207}{140} z^5\bar{z}^2 \; - \frac{233}{140} z^4\bar{z}^3 \; - \frac{233}{140} z^3\bar{z}^4 \; + \frac{207}{140} z^2\bar{z}^5 \; - \frac{5}{7} z\bar{z}^6 \qquad \textbf{NNNLP}$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$-z^3\bar{z}\; {}_2F_1\left(3,2,6,z\right)$$

How to understand?

Casimir Equation

on the celestial sphere

Finding a good **basis** that respects **symmetry**.

$$G(z_1, z_2, z_3, \bar{n}) = \frac{1}{(z_1 \cdot z_2)^3} \frac{1}{(z_3 \cdot \bar{n})^4} \left(\frac{z_1 \cdot z_3}{z_1 \cdot \bar{n}} \right) g(z, \bar{z}) \longrightarrow \boxed{???}$$

Symmetry: Lorentz Group

Representation labels:
- δ **celestial dimension** ------ $\vec{n} \cdot \vec{\mathbf{K}}$
- j **transverse spin** ------ $\vec{n} \cdot \vec{\mathbf{J}}$

Quadratic Casimir: $\frac{1}{2} M_{\mu\nu} M^{\mu\nu} \xrightarrow{\text{eigenvalue}} -\left(\delta(\delta - 2) + j^2\right)$

Casimir Equation: acting Casimir operator on $z_1,\ z_2$

$$\mathcal{L}^{\mu\nu}(z_1, z_2)\mathcal{L}_{\mu\nu}(z_1, z_2) G_{\delta,j} = -(\delta(\delta - 2) + j^2) G_{\delta,j}$$

$$\mathcal{L}^{\mu\nu}(z_1, z_2) \equiv \sum_{i=1,2} \left(z_i^\mu \frac{\partial}{\partial z_{i\nu}} - z_i^\nu \frac{\partial}{\partial z_{i\mu}} \right)$$

[Dolan, Osborn, 2003]

Rotation Group $\mathrm{SO}(3)$

$$f(\theta, \phi) = \sum_{\ell,m} f_{\ell,m} \boxed{Y_{\ell,m}(\theta, \phi)}$$

Origin of Spherical Harmonics

Cartan subalgebra basis: $\mathbf{L}_3$

Casimir operator: $\mathbf{L}_1^2 + \mathbf{L}_2^2 + \mathbf{L}_3^2$

Differential operator form:

$$\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2}$$

Solutions: $Y_{\ell,m}(\theta, \phi)$

Eigenvalue: $-\ell(\ell + 1)$

Label ℓ **is the eigenvalue of** $\mathbf{L}_3$ **when the solution is annihilated by** $\mathbf{L}_1 + i\mathbf{L}_2$

Conformal Blocks

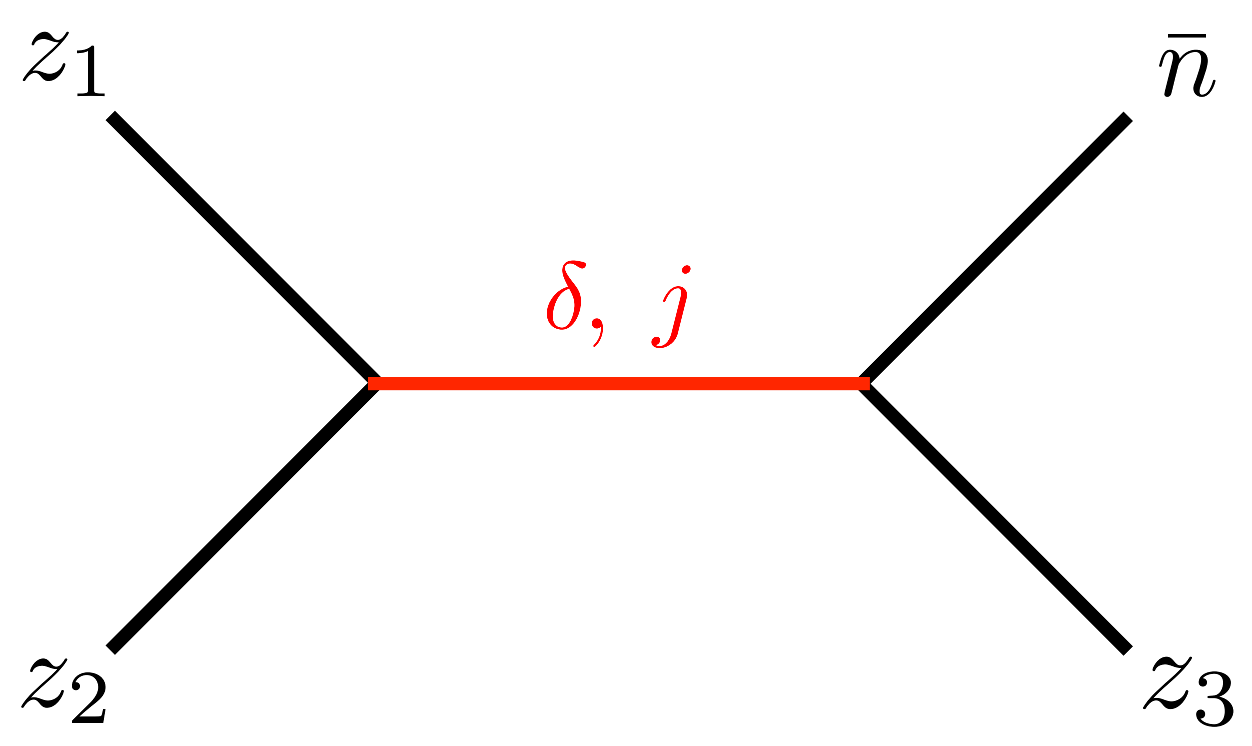
on the celestial sphere

Solutions:

$$g_{\delta,j}(z,\bar{z}) = \frac{1}{1+\delta_{j,0}}\left(k_{\delta-j}(z)k_{\delta+j}(\bar{z}) + k_{\delta+j}(z)k_{\delta-j}(\bar{z})\right)$$

[Notations] In our case, $a = 0,\ b = -1$

$$k_\beta(x) \equiv x^{\beta/2}\ {}_2F_1\left(\frac{\beta}{2}+a, \frac{\beta}{2}+b, \beta, x\right)$$



We find **celestial blocks** for **collinear EEEC** turn out to be **2D conformal blocks**.

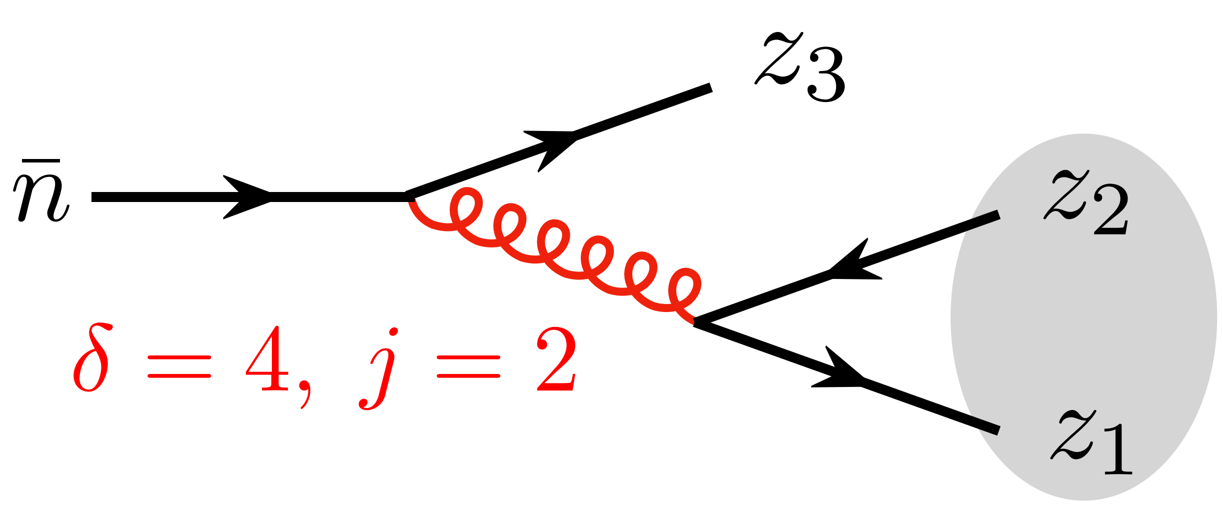
Decomposition:

$$g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$$

Previous example: $-z^3\bar{z}\ {}_2F_1(3,2,6,z)$

set $\delta = 4,\ j = 2$

$$k_6(z) = z^3\ {}_2F_1(3,2,6,z) \qquad k_2(\bar{z}) = \bar{z}$$



Contributing Operator

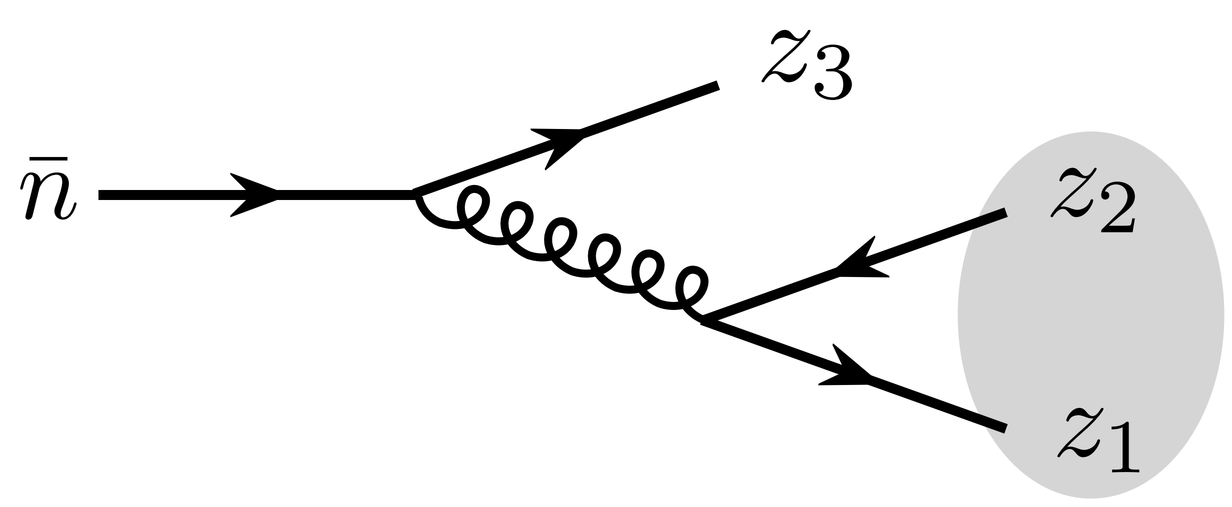
$F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$

twist-2, transverse spin-2 gluonic operator

Conformal Block Decomposition

on the celestial sphere

Example:



For simplicity, tagging final state quarks

Squeezed limit: $z_1 \cdot z_2 \to 0$

j

$$g(z,\bar{z}) = -\frac{1}{720} g_{4,2}(z,\bar{z}) + \frac{163}{252000} g_{6,2}(z,\bar{z}) - \frac{2057}{4233600} g_{8,2}(z,\bar{z}) - \frac{82667}{768398400} g_{10,2}(z,\bar{z})$$

$$+\frac{13}{2400} g_{4,0}(z,\bar{z}) - \frac{139}{40320} g_{6,0}(z,\bar{z}) - \frac{10211}{5880000} g_{8,0}(z,\bar{z}) + \cdots$$

δ

$$-\frac{1}{168} \partial_\delta g_{8,0}(z,\bar{z}) - \frac{1}{1386} \partial_\delta g_{10,2}(z,\bar{z}) + \cdots$$

[derivative of blocks, contain $\log|z|$]

Conformal blocks nicely re-organize the power correction of this small angle expansion.

In particular, in this example, only $j = 0, 2$ blocks exist.

Lorentzian Inversion Formula [Caron-Huot, 2017]

Extracting block coefficients from **double discontinuity** of CFT 4-point correlator

Gribov-Froissart Formula

Partial wave expansion

$$A(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell(z), \quad z = \cos\theta$$

Gribov-Froissart formula

$$a_\ell(s) = \frac{1}{2\pi} \int_1^\infty dz\, Q_\ell(z) [\mathrm{Disc}_t A(s,z) + (-1)^\ell \mathrm{Disc}_u A(s,-z)]$$

partial wave ← **Discontinuity of amplitude**

Conformal block expansion $\quad g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$

Lorentzian inversion $\quad c(\delta,j) = c^t(\delta,j) + (-1)^j c^u(\delta,j)$

$$c^t(\delta,j) = \frac{\kappa_{\delta+j}}{4} \int_0^1 dz d\bar{z} \mu(z,\bar{z}) g_{j+d-1,\delta+1-d}(z,\bar{z}) \mathrm{dDisc}\left[g(z,\bar{z})\right]$$

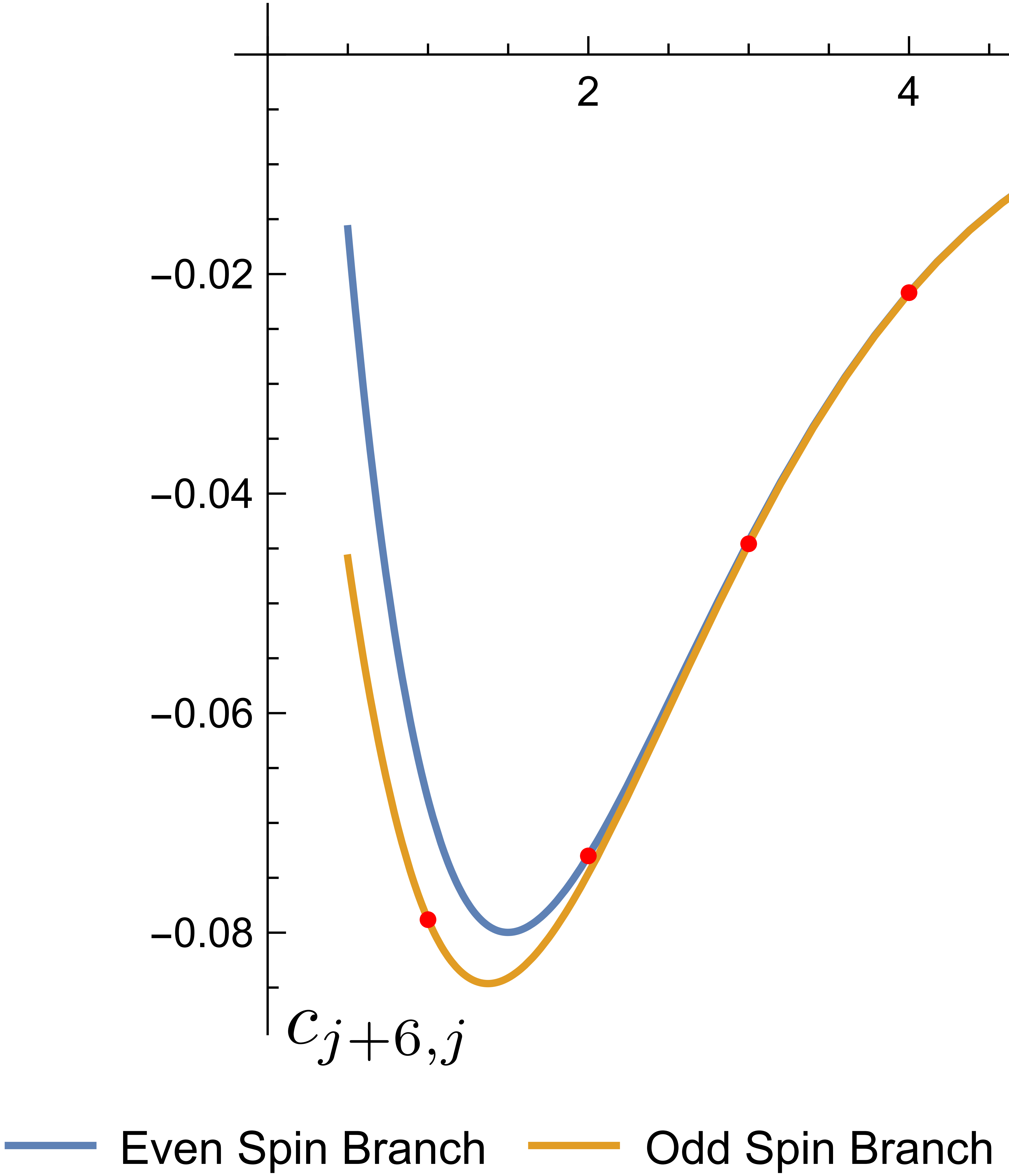
$A(s,t)$	$g(z,\bar{z})$
$a_\ell(s)$	$c_{\delta,j}$
$\mathrm{Disc} A$	$\mathrm{dDisc}\, g$

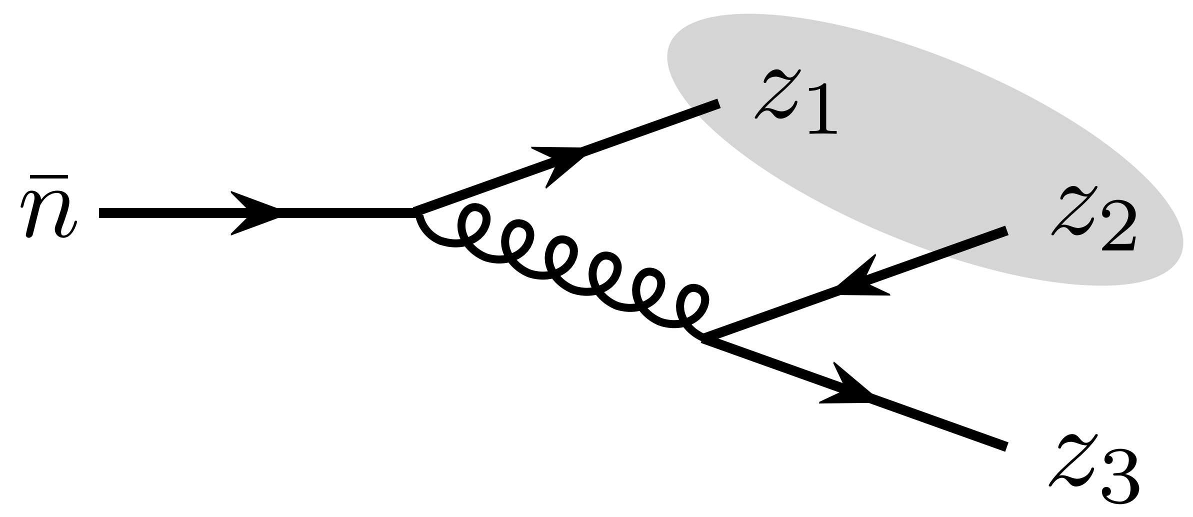
$P_\ell(z)$	→	$Q_\ell(z)$
$g_{\delta,j}$	→	$g_{j+d-1,\delta+1-d}$

Double Discontinuity

$$\mathrm{dDisc}\, g(z,\bar{z}) = \cos(\pi(a+b)) g(z,\bar{z}) - \frac{1}{2} e^{i\pi(a+b)} g^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2} e^{-i\pi(a+b)} g^{\circlearrowright}(z,\bar{z})$$

Lorentzian Inversion Formula





Squeezed limit

$$z_1 \cdot z_2 \to 0$$

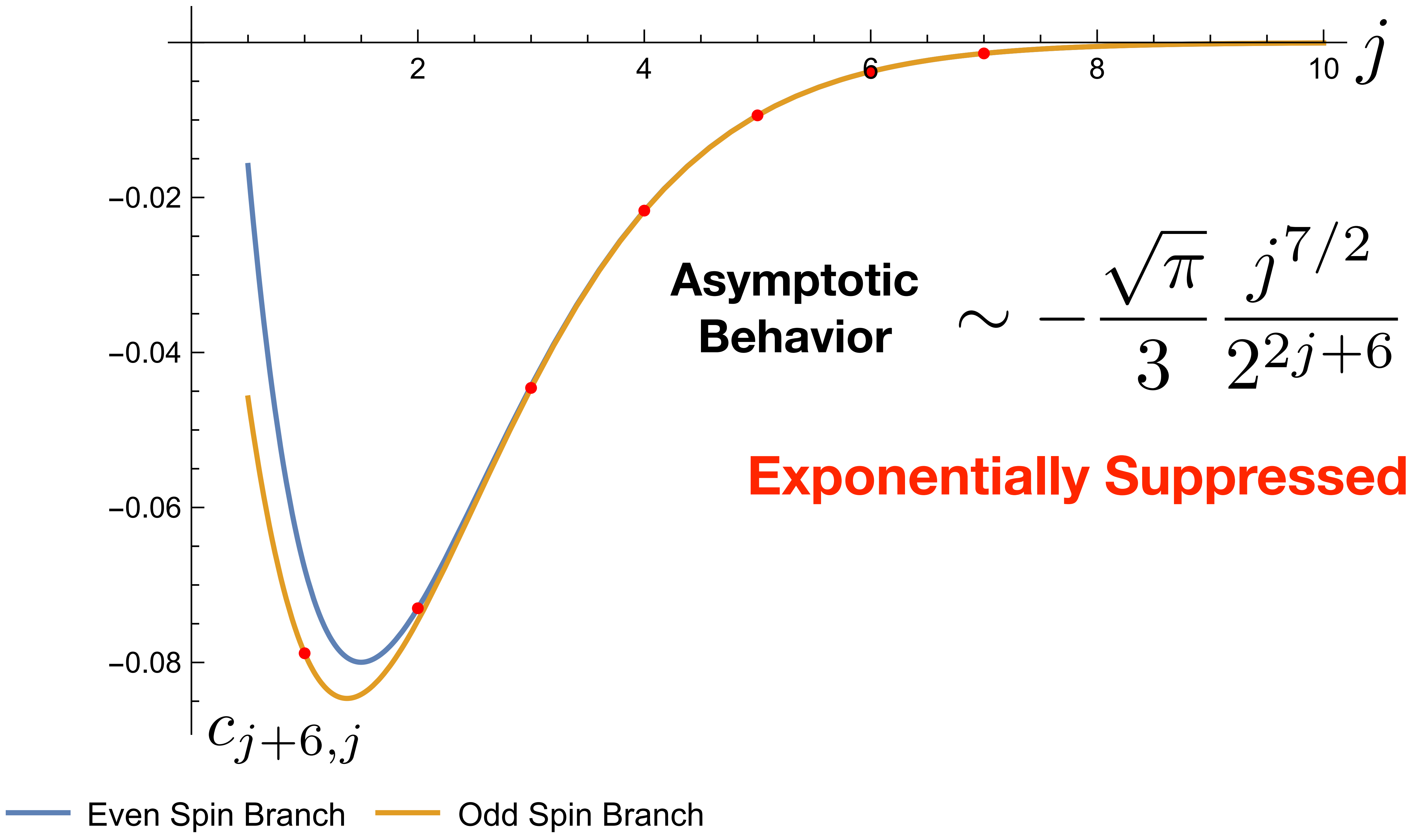
Analytic in transverse spin j (odd and even, resp.)

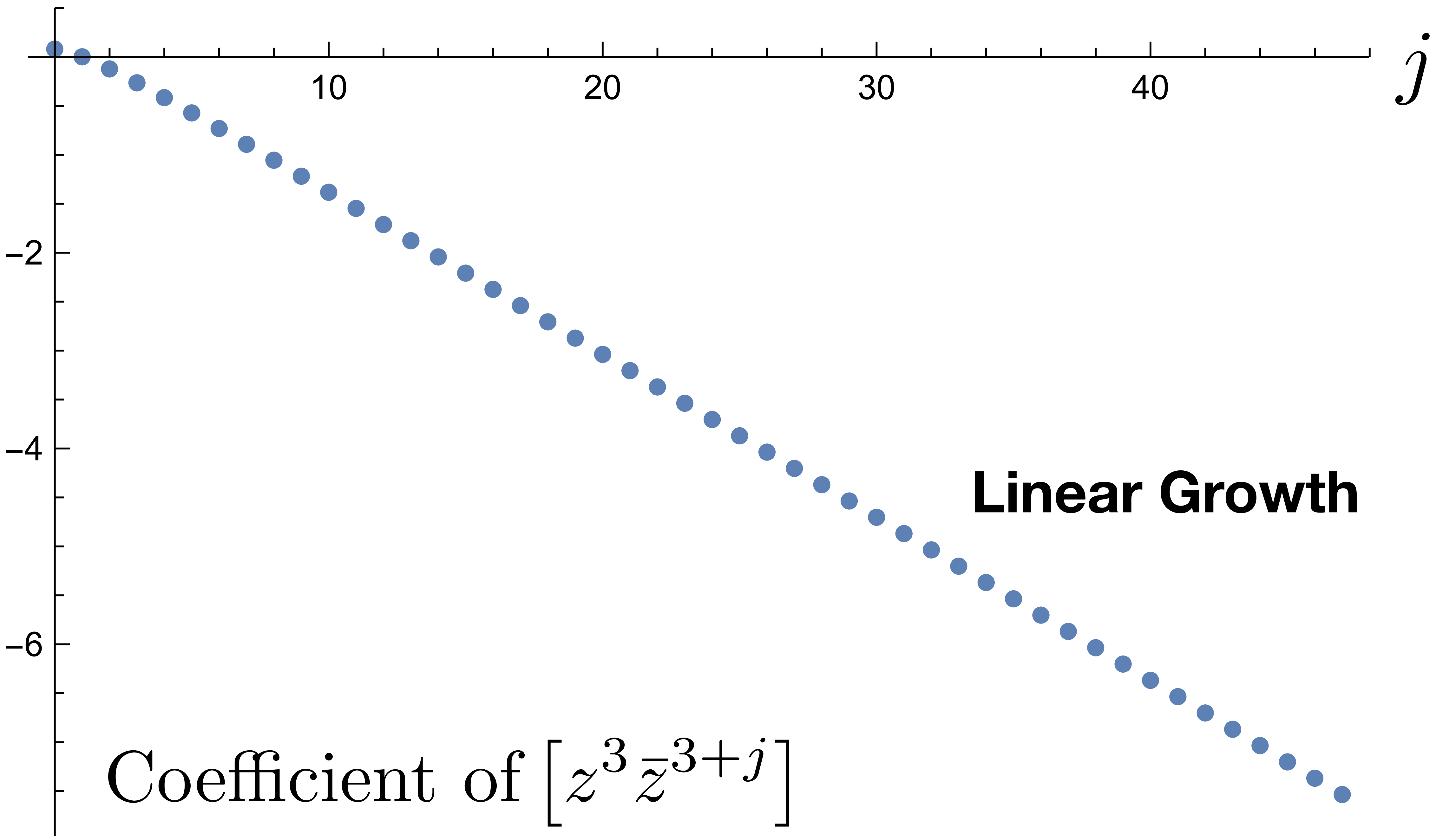
Compared with block coefficients **(red dots)**, it's valid for $j \geq 1$

$$\begin{aligned} g(z,\bar{z}) \supset & \left(\frac{11939}{1200} - \pi^2\right) g_{6,0}(z,\bar{z}) + \left(\frac{2953}{100} - 3\pi^2\right) g_{7,1}(z,\bar{z}) \\ & + \left(\frac{233603}{4200} - \frac{79}{14}\pi^2\right) g_{8,2}(z,\bar{z}) + \left(\frac{662863}{8400} - 8\pi^2\right) g_{9,3}(z,\bar{z}) \\ & + \left(\frac{9863251}{110880} - \frac{595}{66}\pi^2\right) g_{10,4}(z,\bar{z}) + \left(\frac{99805933}{1201200} - \frac{1204}{143}\pi^2\right) g_{11,5}(z,\bar{z}) \\ & + \cdots \end{aligned}$$

Block Expansion vs Taylor Expansion

In addition to theory insights, block expansion also helps numerical approximations.

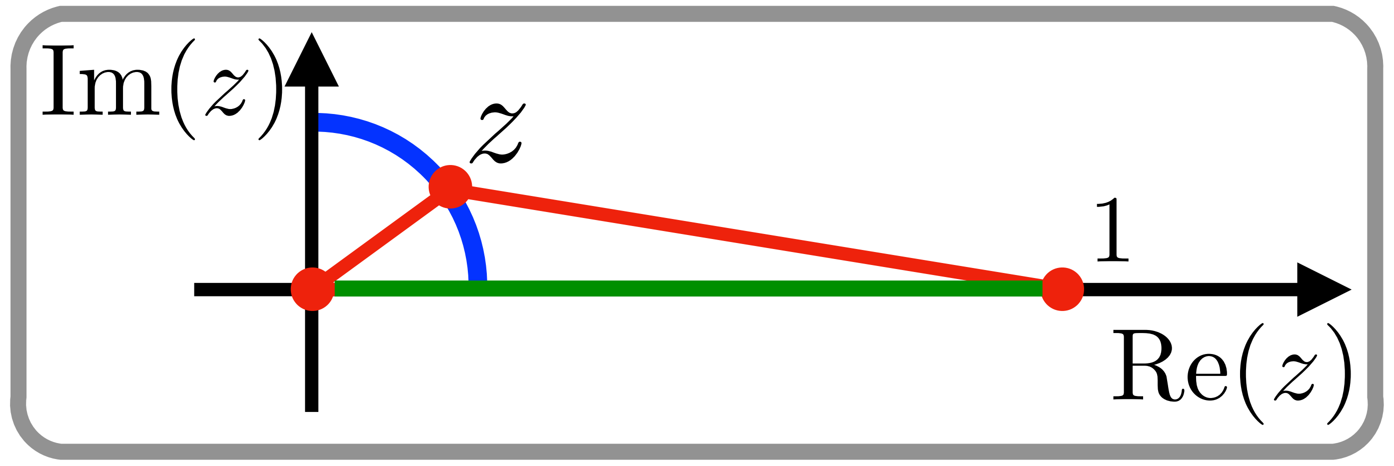


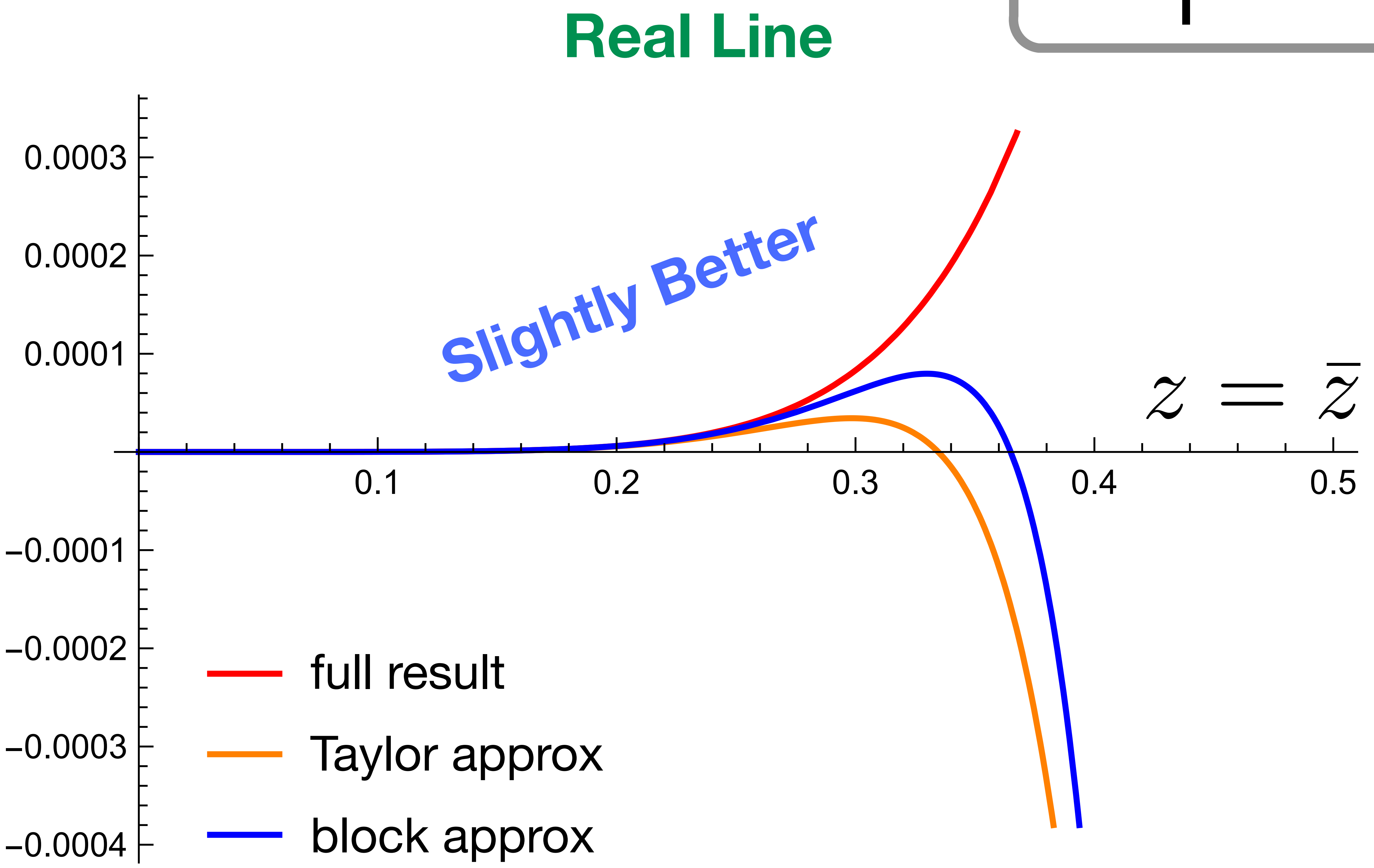


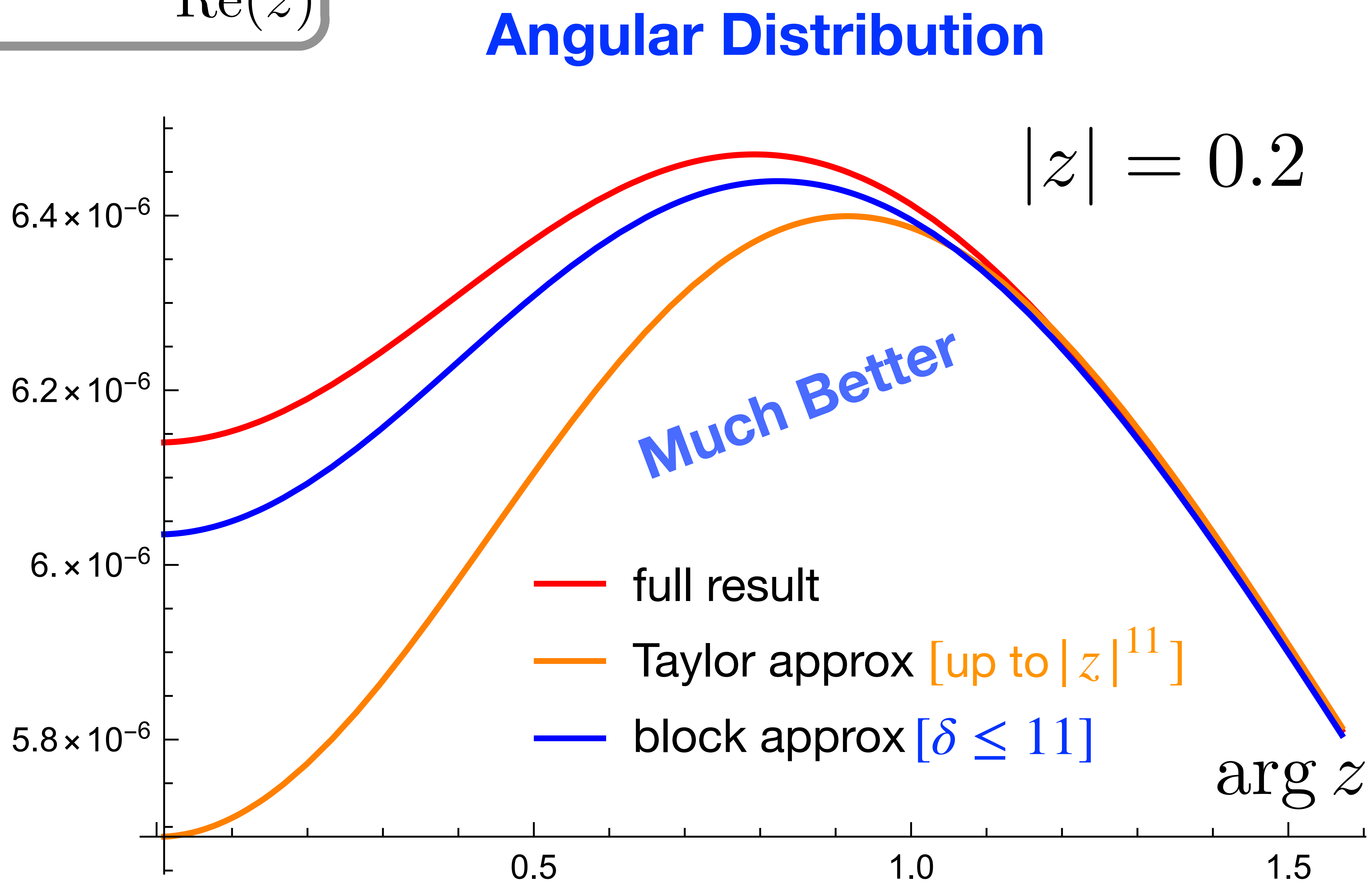
Better convergence in coefficients if organized with conformal blocks.

⟶ Approximation with blocks makes the coefficient of leading term in the remainder much smaller.

Block Expansion vs Taylor Expansion







For angular distribution, blue curve is uniformly close to red curve.

Conformal block decomposition captures symmetry structure.

Summary

- For celestial observables, Lorentz/conformal symmetry provides a natural and clean way to organize power corrections.

- To exploit these symmetries, we introduced several new techniques to study the EEEC
    - Celestial blocks
    - Lorentzian inversion formula

- Outlook:
    - Interplay of blocks and RG
    - Effective CFT on the celestial sphere in the collinear limit? Bootstrap?


Thanks

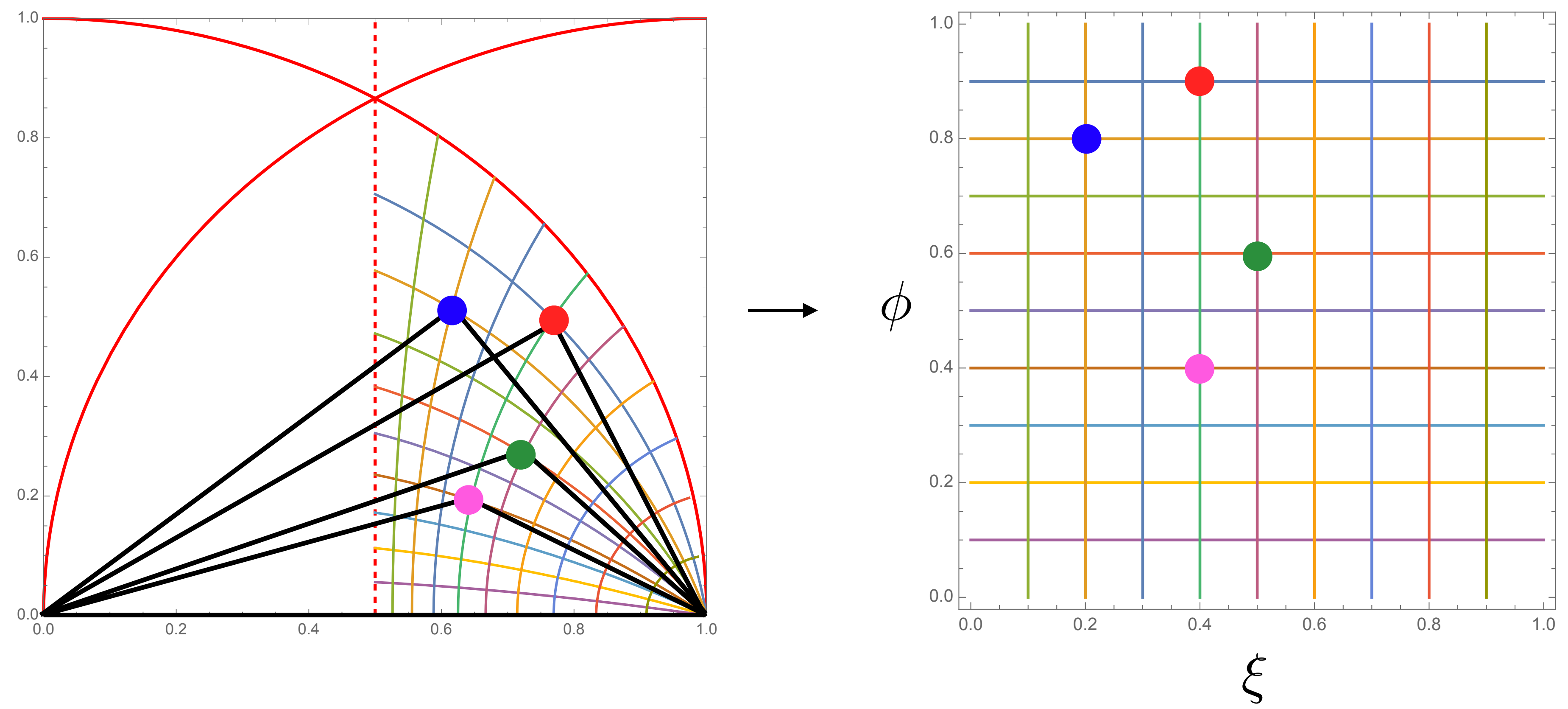
Backup

Shape Dependence

The shape dependence $g(z, \bar{z})$ in collinear EEEC can be directly measured at LHC!

[Komiske, Moult, Thaler, Zhu, Forthcoming]

This work also measures the scaling behavior with “MIT Open Data”.



Moduli space and parametrization

Imaging of 3-point energy correlator $g(z, \bar{z})/(z\bar{z})^3$