Soft drop double differential cross section

Based on Aditya Pathak, Iain Stewart, Varun Vaidya, LZ, arXiv:2012.15568

Lorenzo Zoppi, 21 Apr – World SCET 2021

Soft drop

[Larkoski, Marzani, Soyez, Thaler '14]
(modified Mass Drop Tagger [Dasgupta, Fregoso, Marzani, Salam '13])

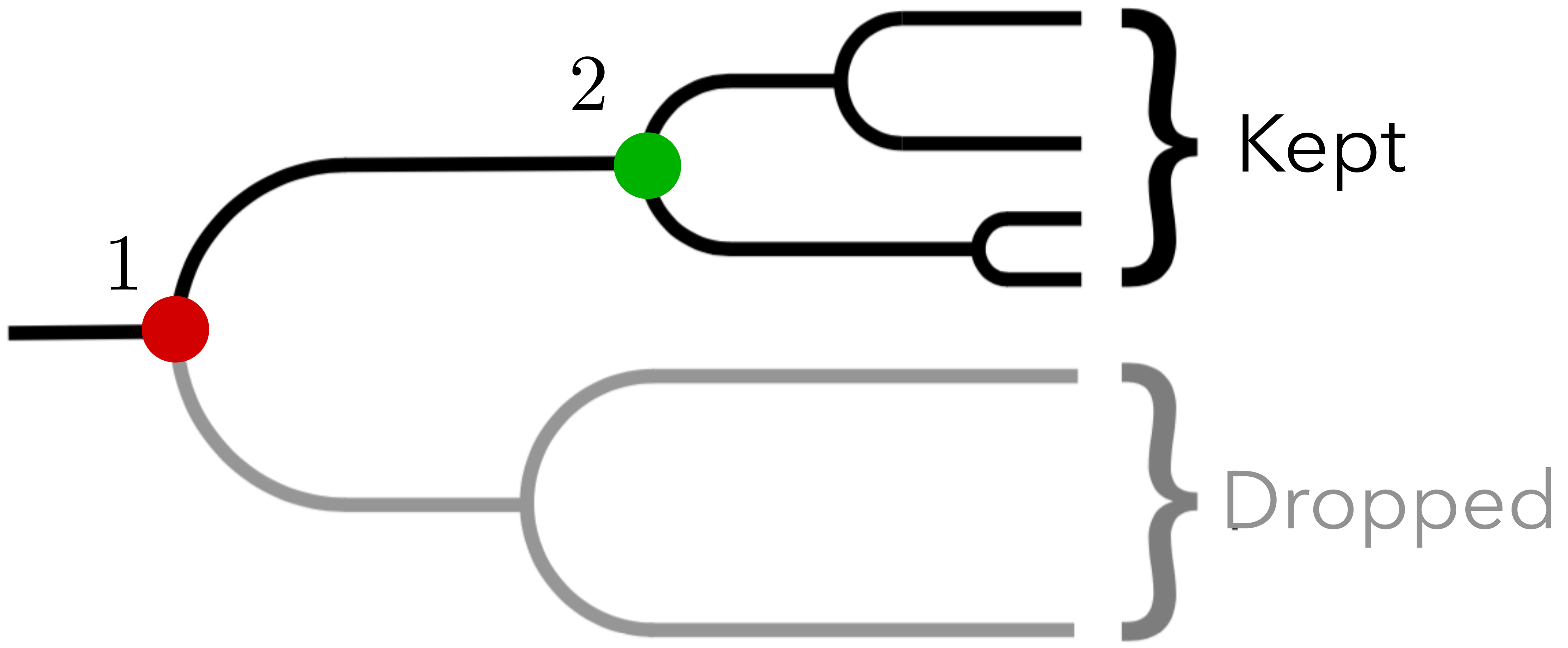
- Re-cluster jet based on angular separation

- Starting from last clustering node, test*:

$$\left(\frac{\min(E_i, E_j)}{E_i + E_j} > z_{\rm cut}\left[\sin(\theta_{ij}/2)\right]^{\beta}\right)?$$

Pass → Keep both branches. Stop.

Fail → Drop the branch with lower energy. Move to next node



*Throughout the presentation, e^+e^- case with $R_0^{ee} = \pi/2$
arXiv:2012.15568 generalizes to pp collisions

Effect: jet is cleaned up from contaminating low-energy radiation
(cuts down impact of pileup, hadronization, multiparton interactions…)

Soft drop groomed jet mass

Benchmark substructure observable, measures jet energy spread

$$m_J^2 = \Big| \sum_{i \in (\mathrm{SDjet})} p_i^\mu \Big|^2 \simeq \sum_{i \in (\mathrm{SDjet})} z_i \, \theta_i^2$$

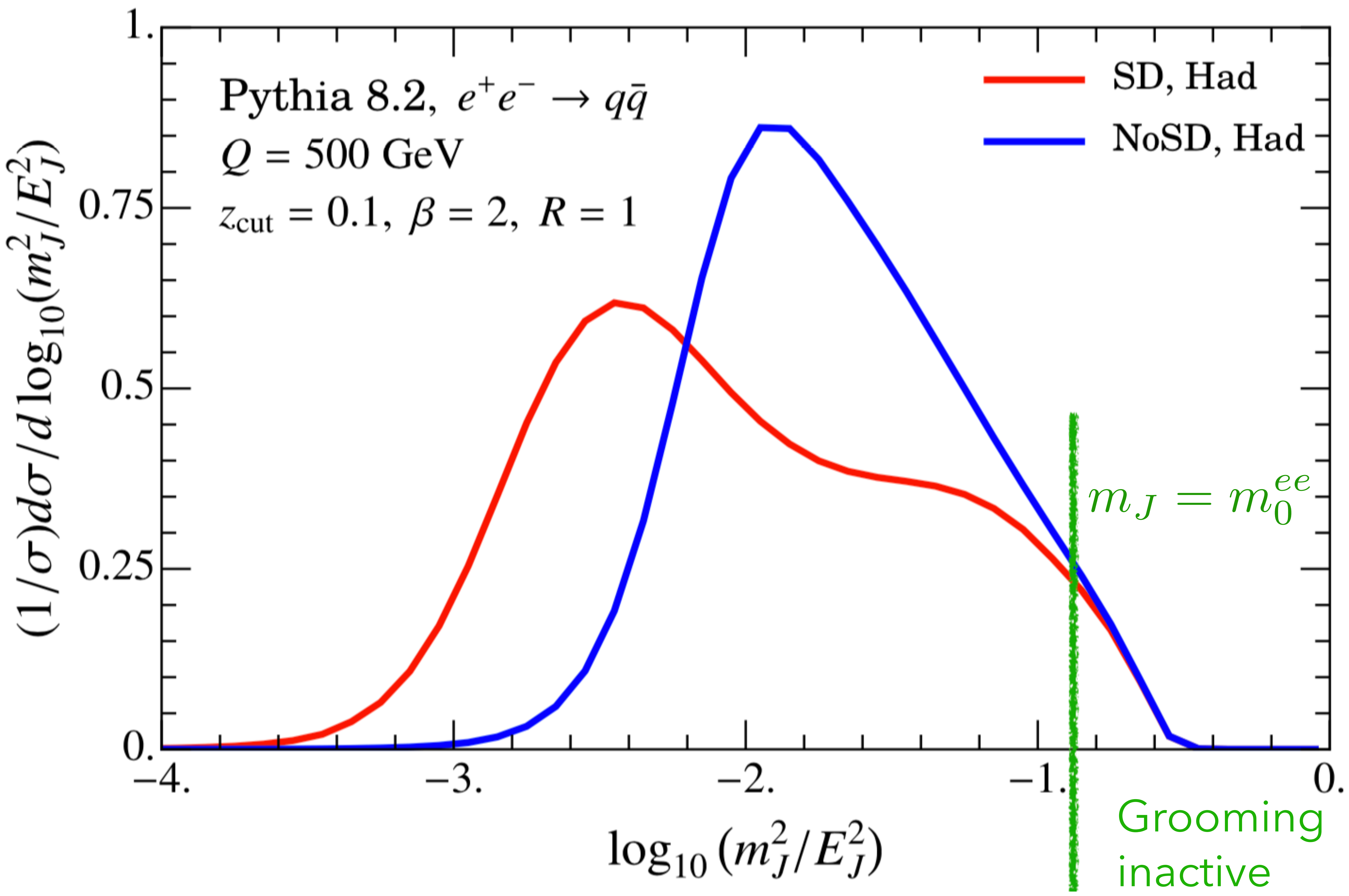
Fixed order spectrum known at NNLO
[Kardos, Somogyi, Trócsányi '18]

Resummation framework available
[Frye, Larkoski, Schwartz, Yan '16]

Resummed predictions for the LHC
[Marzani, Schunk, Soyez '17]

Joint resummation of $m_J/Q, R, z_{\mathrm{cut}}$ logarithms
[Kang, Lee, Liu, Ringer '18]

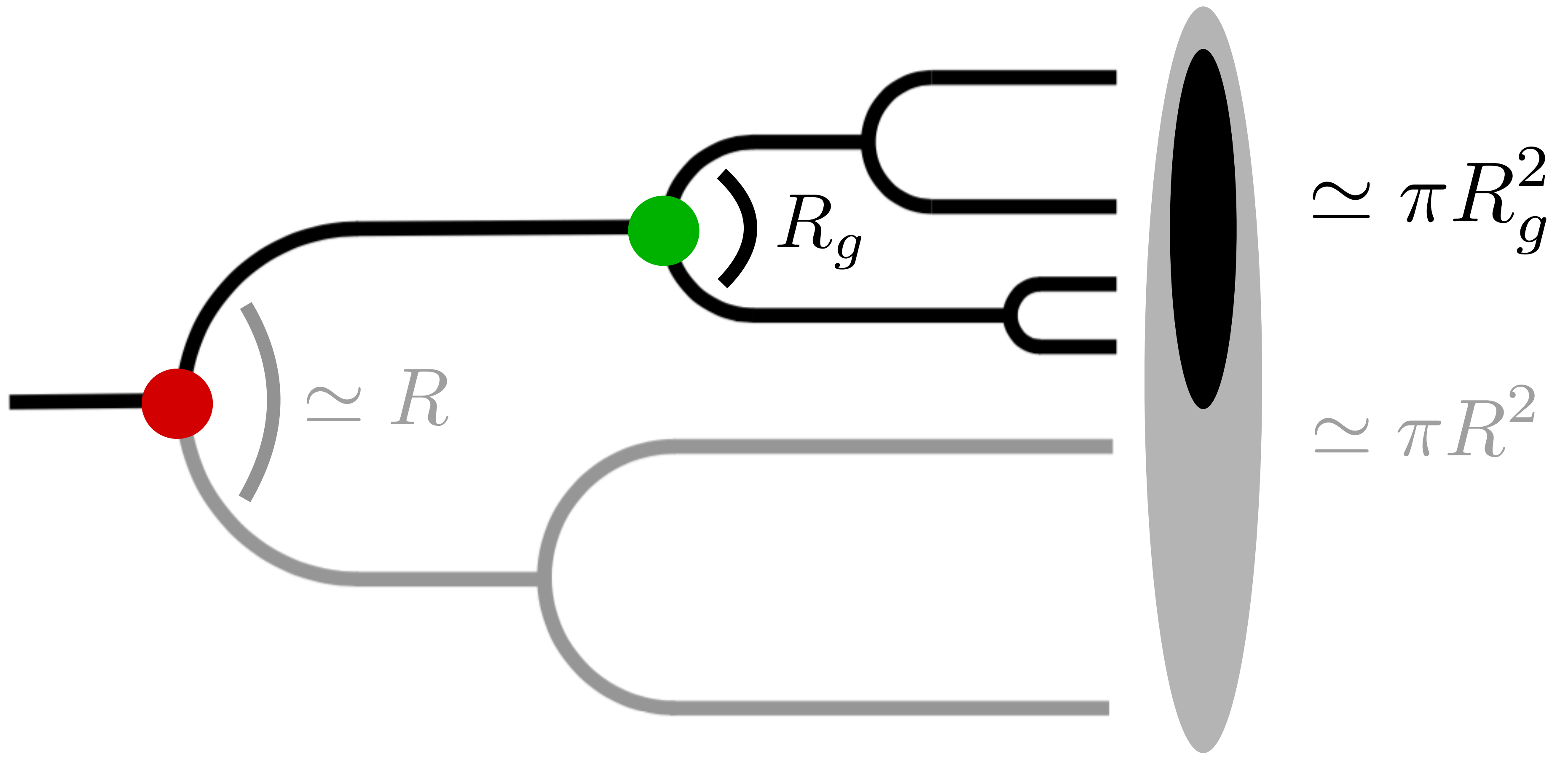
Cusps, fixed-order z_{cut} corrections under control
[Larkoski '20]



Soft drop groomed jet radius

R_g: angle of the branching that stops soft drop

Different from the ungroomed jet radius R, R_g is set dynamically [Larkoski, Marzani, Soyez, Thaler '14]



SCET framework set up in [Kang, Lee, Liu, Neill, Ringer '19]

Cross section differential in R_g at NLL accuracy

- Resummation of logarithms of $R, z_{\rm cut}, R_g/R$
- Non Global Logarithms (NGL) + C/A clustering effects [Dasgupta, Salam '01] [Banfi, Marchesini, Smye '02]
- Abelian clustering logarithms [Delenda, Appleby, Dasgupta, Banfi '06]

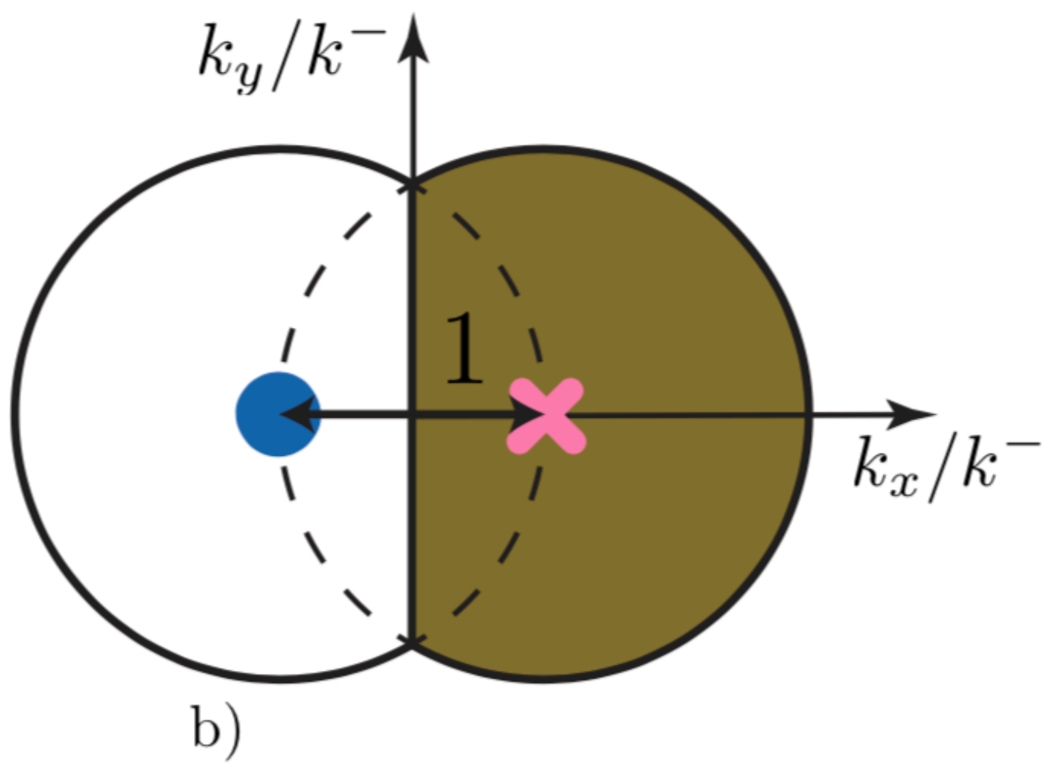
} MC LL resummation in the large N_c limit [Kang, Lee, Liu, Neill, Ringer '19]

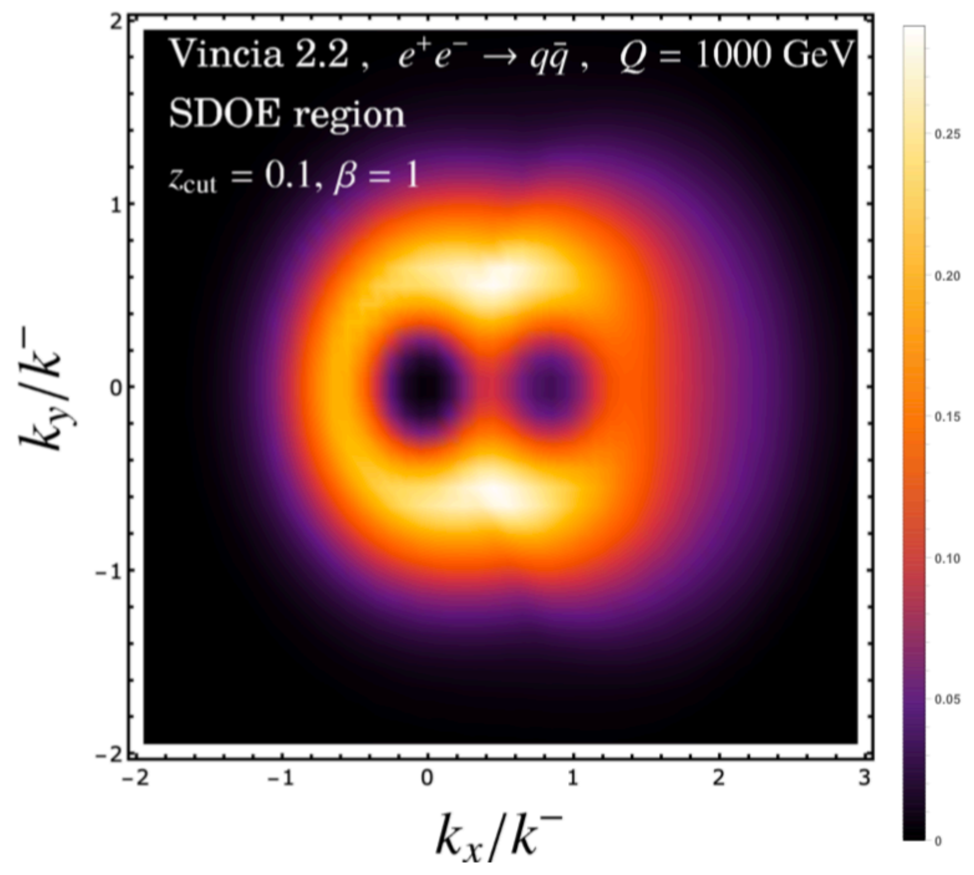
Outline

- SCET framework for the double differential cross section

$$\frac{d^2\sigma}{dm_J^2 dR_g} \longrightarrow \sum_j \widehat{N}_q^j(Q,R,\mu) \otimes_\Omega \widehat{S}_G^{q,j}(Q_{\rm cut},R,\beta,\mu) \times 2E_J \int d\ell^+ J_q(m_J^2 - Q\ell^+,\mu) \sum_k \widehat{S}_{c_m}^{q,k}\Big(\frac{\ell^+}{R_g/2},\mu\Big) \otimes_\Omega \widehat{S}_{c_g}^{q,k}\Big(\frac{R_g}{2} Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\Big)$$

- Application: nonperturbative (NP) corrections to groomed jet mass spectrum





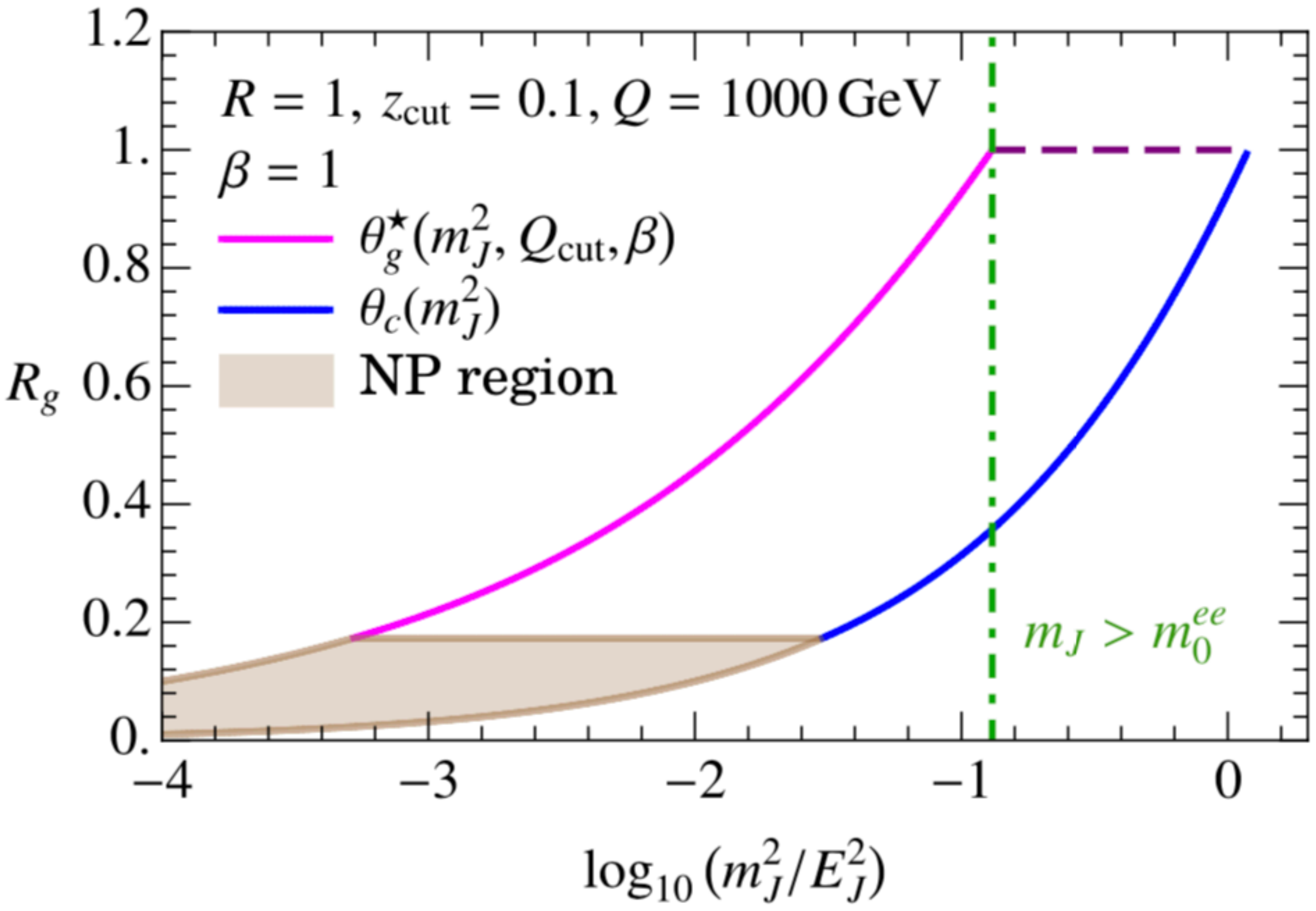
- Outlook

Framework

Kinematical regions

At fixed jet mass, the range in groomed jet radius is constrained (NLL values)

$$\frac{m_J}{E_J} \equiv \theta_c \leq R_g \leq \theta_g^\star \equiv \left(\frac{m_J^2}{E_J^2 z_{\rm cut}}\right)^{\frac{1}{2+\beta}}$$



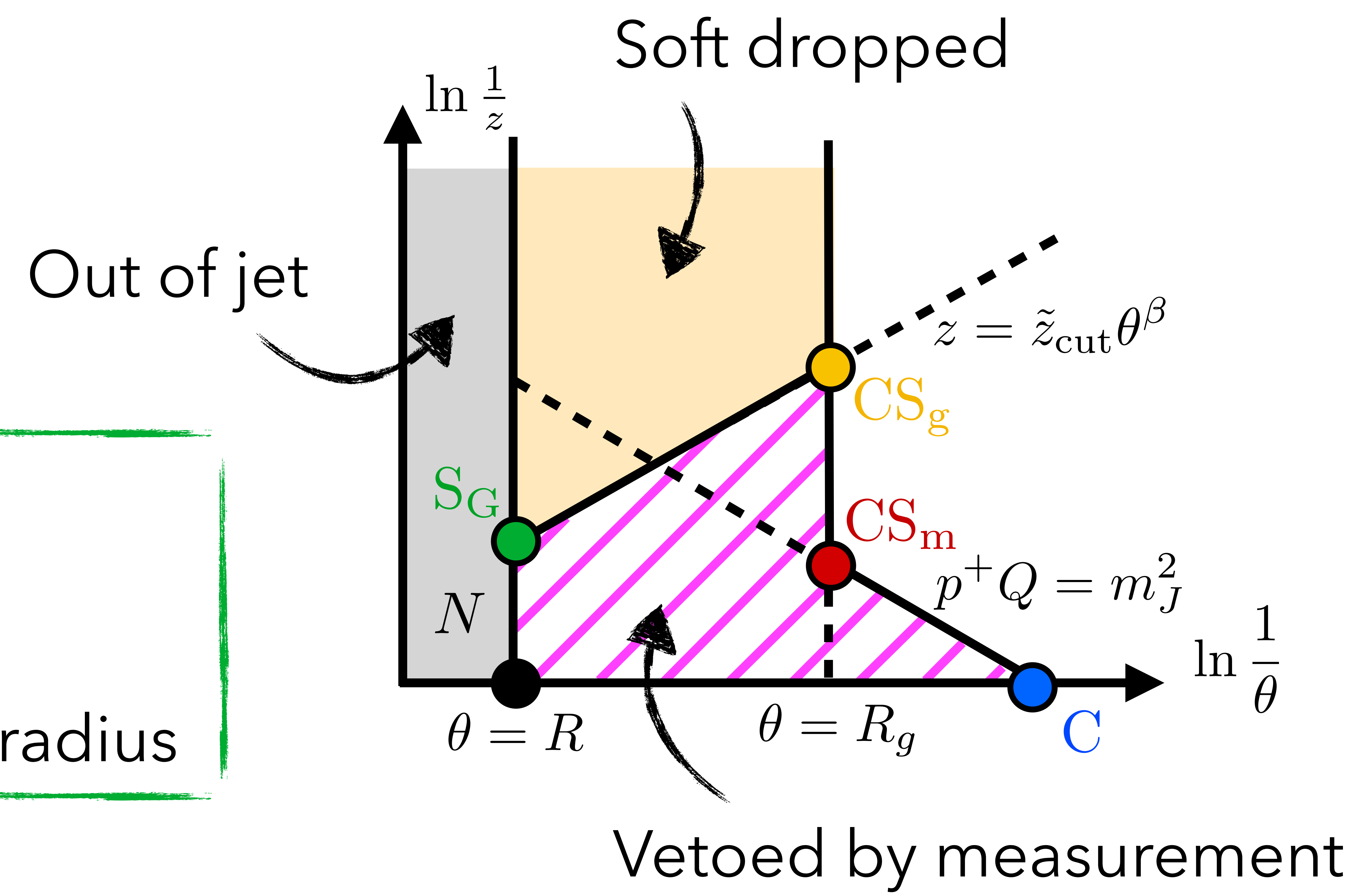
At large jet mass, R_g is limited by R instead

$$m_J^2 \geq (m_0^{ee})^2 \equiv E_J^2 z_{\rm cut} (2 \sin \tfrac{R}{2})^{2+\beta}$$

At small jet radii, nonperturbative effects are large

$$R_g \lesssim (R_g)_{\rm NP} \equiv \left(\frac{\Lambda_{\rm QCD}}{E_J}\right)^{\frac{1}{2+\beta}}$$

Mode analysis



Collinear-Soft$_{\text{grooming}}$
Stops SD,
setting groomed jet radius

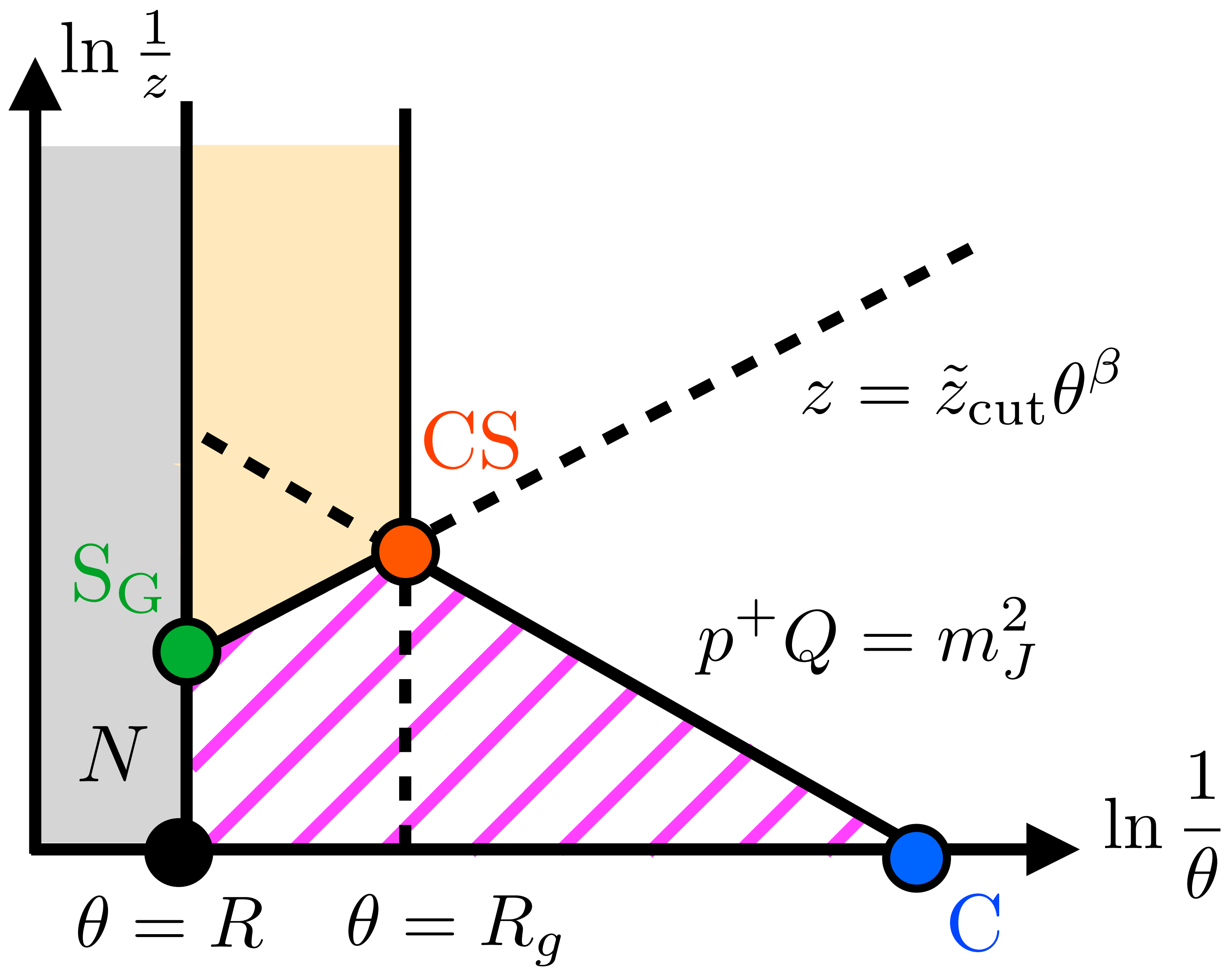
Soft$_{\text{Global}}$
Set by SD
Knows about
ungroomed jet radius

Collinear-Soft$_{\text{mass}}$
Knows about both measurements
Always passes SD

Collinear
Knows about jet mass
Set by smallest angle

Large Rg

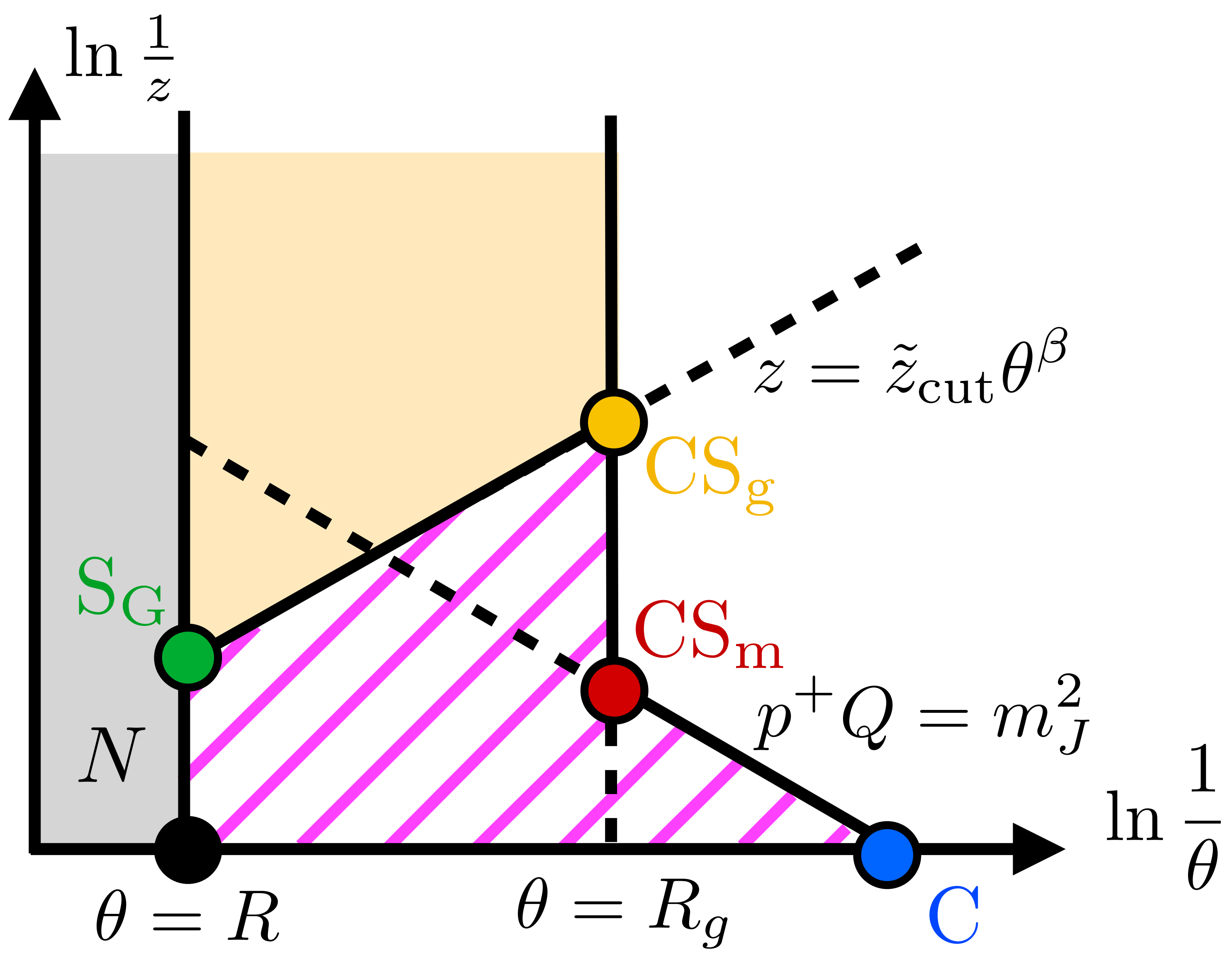
$\theta_g^\star \gtrsim R_g \gg \theta_c$



Collinear-Soft
Stops SD
Set by largest angle

Intermediate Rg

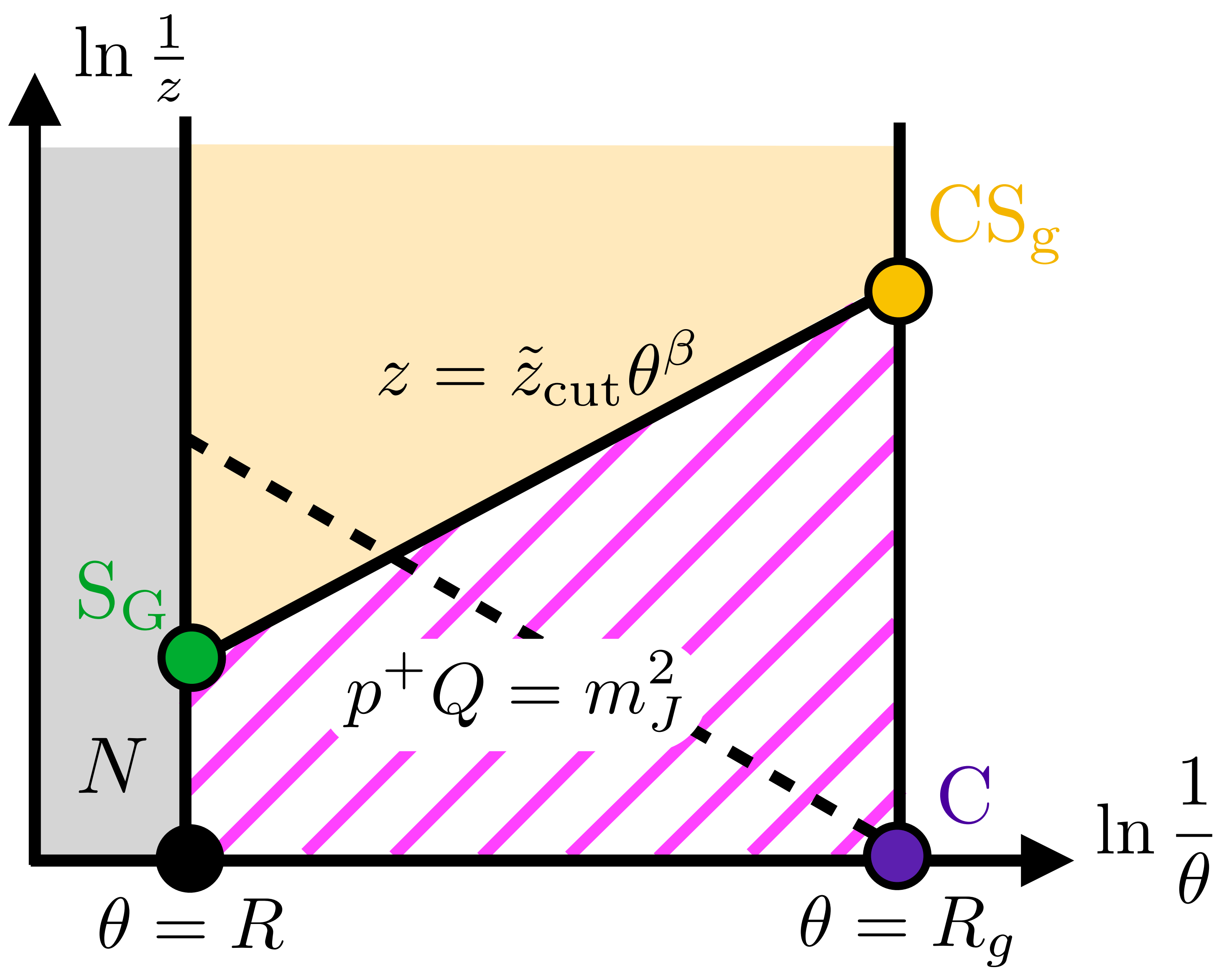
$\theta_g^\star \gg R_g \gg \theta_c$



(the two measurements factorize here)

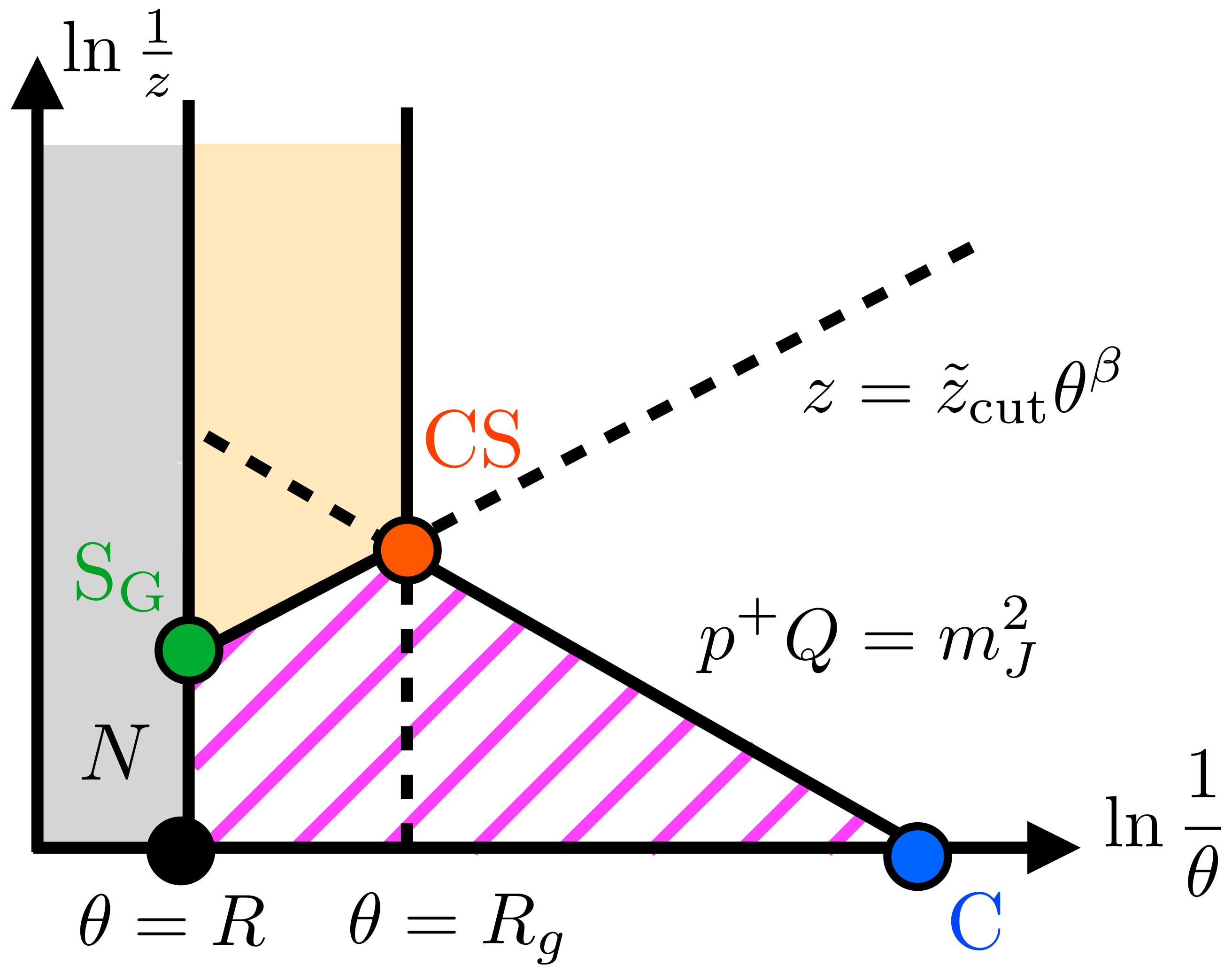
Small Rg

$\theta_g^\star \gg R_g \gtrsim \theta_c$



Collinear
Set by smallest angle

Factorization – large Rg



NGLs formally require summing over arbitrary number of emissions

[Dasgupta, Salam '01]
[Larkoski, Moult, Neill '15]
[Becher, Neubert, Rothen, Shao '15]

Soft drop global soft function

[Frye, Larkoski, Schwartz, Yan '16]
[Bell, Rahn, Talbert '18]

$$\frac{d\Sigma(R_g)}{dm_J^2} = \sum_j \widehat{N}_q^j(Q,R,\mu) \otimes_\Omega \widehat{S}_G^{q,j}(Q_{\rm cut}, R, \beta, \mu)$$

$$\times\, Q_{\rm cut}^{\frac{1}{1+\beta}} \int d\ell^+ J_q(m_J^2 - Q\ell^+, \mu)\, S_c^q\left(\ell^+ Q_{\rm cut}^{\frac{1}{1+\beta}}, \tfrac{1}{2} R_g Q_{\rm cut}^{\frac{1}{1+\beta}}, \beta, \mu\right)$$

$$\frac{d\Sigma(R_g)}{dm_J^2} = \int_0^{R_g} d\theta_g \frac{d^2\sigma(R_g)}{dm_J^2 d\theta_g}$$

$$Q_{\rm cut} = 2^{\beta+1} E_J z_{\rm cut}$$

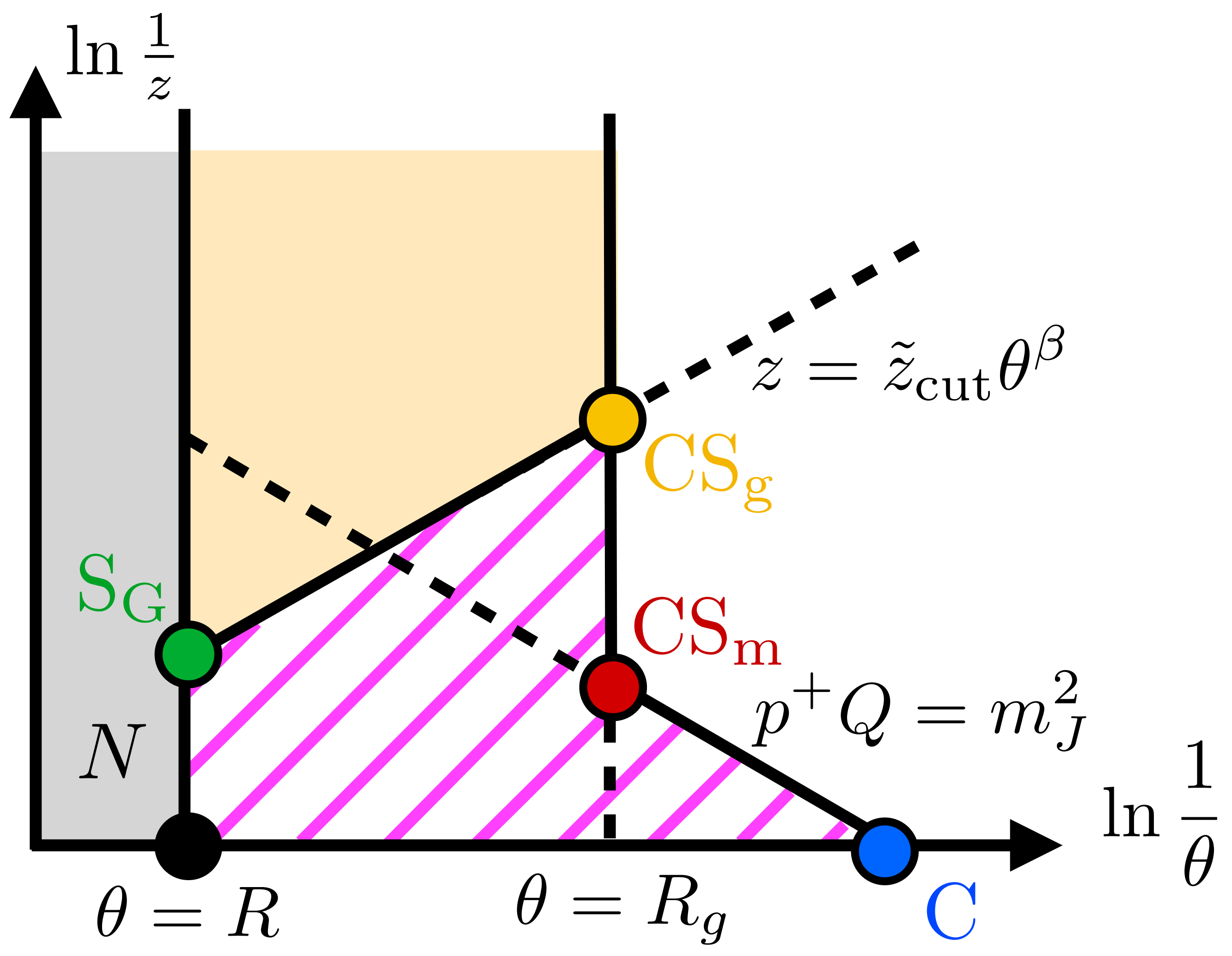
Universal jet function

[Bauer, Manohar '04]
[S. Bosch, B. Lange, M. Neubert and G. Paz '04]
[Becher, Neubert '06]
[Brüser, Liu, Stahlhofen '18]

New collinear soft function we computed at one loop

Note: no large logarithms of R_g

Factorization – intermediate Rg



Most factorized scenario
Large logarithms of both R_g and $m_J^2/(E_J R_g)^2$

More NGLs, related to groomed jet radius
These also affect the shape!
[Kang, Lee, Liu, Neill, Ringer '19]

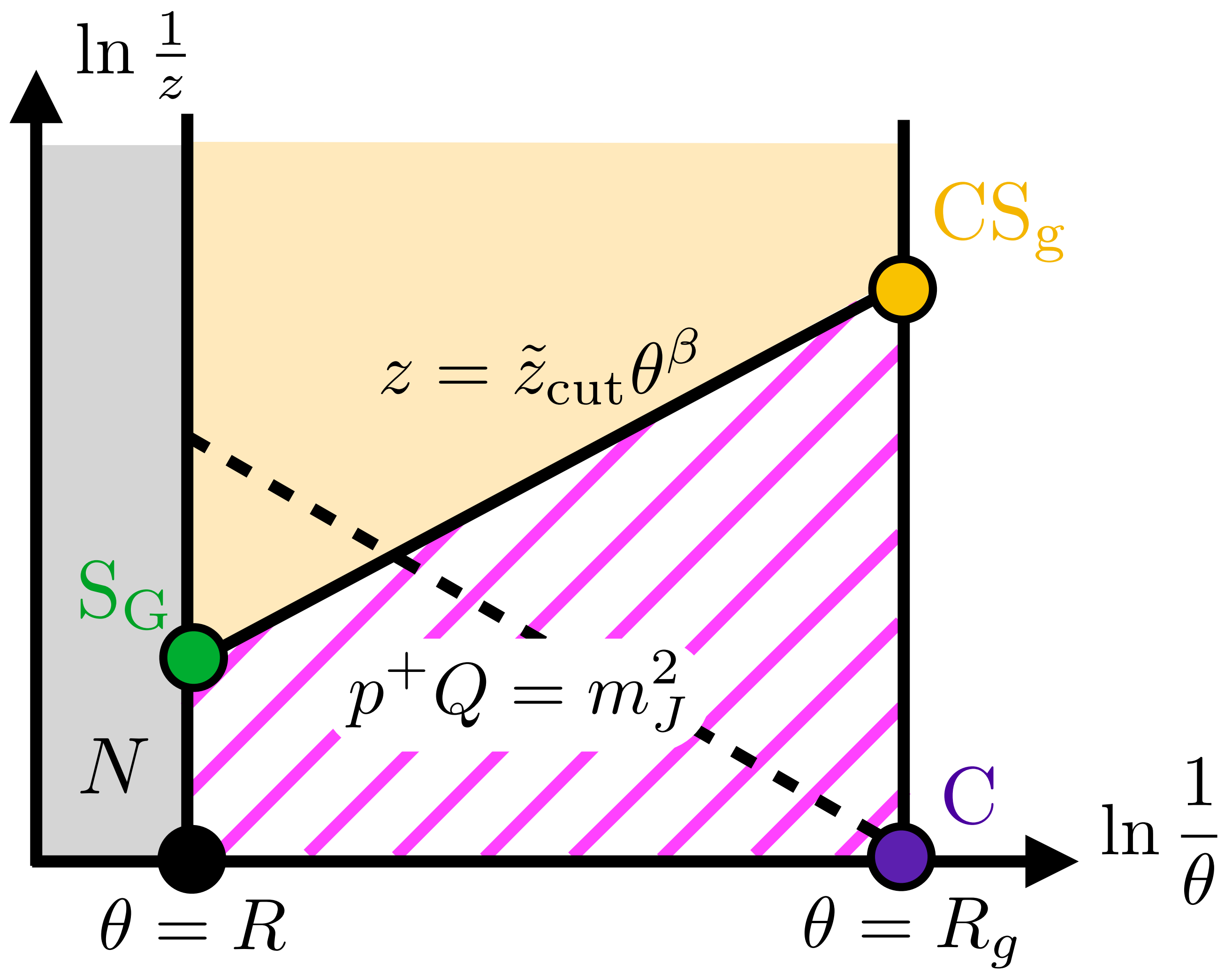
$$\frac{d\sigma(R_g)}{dm_J^2} = \sum_j \widehat{N}_q^j(Q,R,\mu) \otimes_\Omega \widehat{S}_G^{q,j}(Q_{\rm cut},R,\beta,\mu)$$

$$\times\, 2E_J \int d\ell^+ J_q(m_J^2 - Q\ell^+,\mu) \sum_k \widehat{S}_{c_m}^{q,k}\left(\frac{\ell^+}{R_g/2},\mu\right) \otimes_\Omega \widehat{S}_{c_g}^{q,k}\left(\frac{R_g}{2} Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\right)$$

Collinear soft function
[Kang, Lee, Liu, Neill, Ringer '19]

Collinear soft function
[Ellis, Vermillion, Walsh, Hornig, Lee, '10]

Factorization – small Rg



Groomed jet mass measurements treated at fixed order; no large logarithms of $m_J^2/(E_J R_g)^2$

Similar modes as for single differential in groomed jet radius [Kang, Lee, Liu, Neill, Ringer '19]

$$\frac{d\sigma(R_g)}{dm_J^2} = \sum_j \widehat{N}_q^j(Q,R,\mu) \otimes_\Omega \widehat{S}_G^{q,j}(Q_{\rm cut},R,\beta,\mu)$$

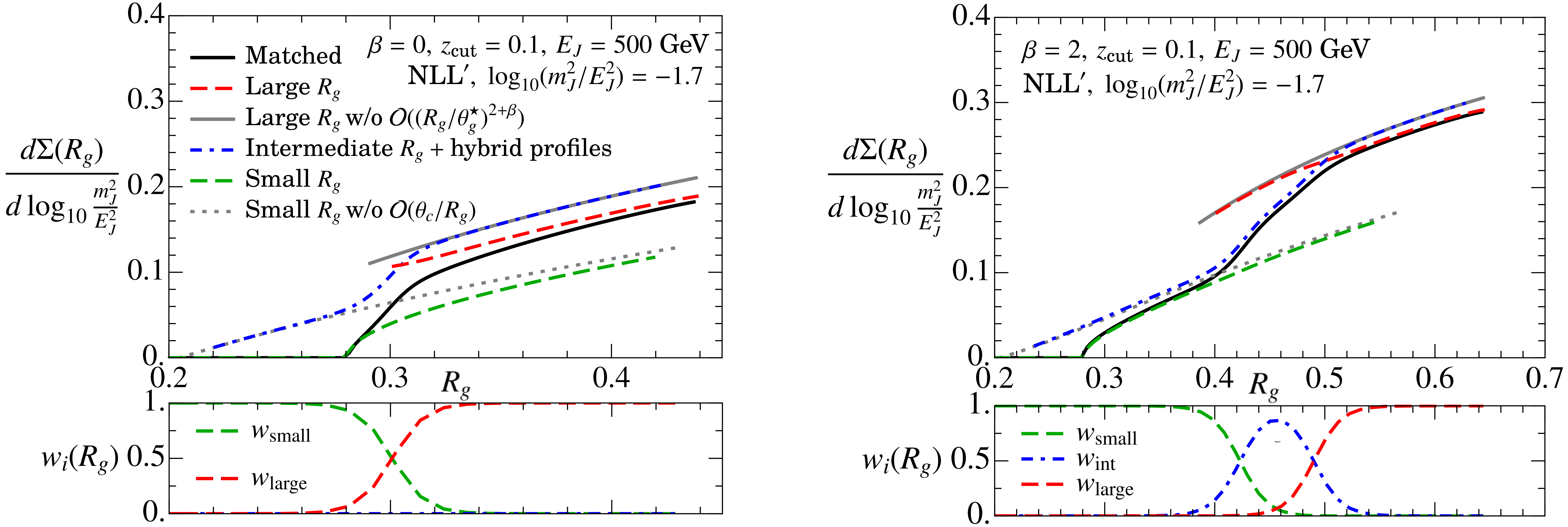
$$\times \frac{1}{(E_J R_g)^2} \sum_k \widehat{\mathcal{C}}_k^q\Big(\frac{m_J^2}{E_J^2 R_g^2}, E_J R_g, \mu\Big) \otimes_\Omega \widehat{S}_{c_g}^{q,k}\Big(\frac{R_g}{2} Q_{\rm cut}^{\frac{1}{1+\beta}}, \beta, \mu\Big)$$

New collinear function
we computed at one loop

Matching the three regimes

- We consider the cumulant Rg cross section at fixed jet mass, aiming at NLL′ accuracy
- We include those leading NGLs that affect the shape (rather than the normalization)

Matching uses weight functions $w_i(R_g)$ and 2D profile scales



Depending on $\theta_c(m_J^2)$, $\theta_g^\star(m_J^2, z_{\rm cut}, \beta)$, there may or may not be room for intermediate EFT

Application

Nonperturbative corrections to groomed jet mass distribution

NP corrections to soft drop groomed jet mass

[Hoang, Mantry, Pathak, Stewart '19]

Single differential

$$\frac{d\sigma_\kappa^{\rm had}}{dm_J^2} = \frac{d\sigma_\kappa}{dm_J^2} - 2E_J\Omega_{1\kappa}\frac{d}{dm_J^2}\left[C_1^\kappa(m_J^2, 2E_J, z_{\rm cut}, \beta, R)\frac{d\sigma_\kappa}{dm_J^2}\right]$$
$$+\frac{2E_J}{m_J^2}(\Upsilon_{1,0}^\kappa + \beta\Upsilon_{1,1}^\kappa)C_2^\kappa(m_J^2, 2E_J, z_{\rm cut}, \beta, R)\frac{d\sigma_\kappa}{dm_J^2} + \dots$$

Subleading corrections

NP parameters   Perturbative coefficients

At LL,

$$C_1^\kappa(m_J^2) = \frac{1}{\langle 1\rangle(m_J^2)}\left\langle\frac{\theta_g}{2}\right\rangle$$

"Shift coefficient"

$$C_2^\kappa(m_J^2) = \frac{m_J^2/(4E_J^2)}{\langle 1\rangle(m_J^2)}\left\langle\frac{2}{\theta_g}\delta(z_g - z_{\rm cut}\theta_g^\beta)\right\rangle$$

"Boundary coefficient"

General strategy

Compute the matching coefficients as moments of the double-differential distribution

$$C_1^q(m_J^2) \simeq M_1^q(m_J^2, z_{\rm cut}, \beta, R) \equiv \left(\frac{d\sigma}{dm_J^2}\right)^{-1} \int d\theta_g \frac{\theta_g}{2} \frac{d^2\sigma}{dm_J^2 d\theta_g}$$

$$C_2^q(m_J^2) \simeq M_{-1}^{q\circledcirc}(m_J^2, z_{\rm cut}, \beta, R) \equiv \left(\frac{d\sigma}{dm_J^2}\right)^{-1} \int d\theta_g \frac{2}{\theta_g} \left[\frac{d}{d\varepsilon} \frac{d^2\sigma(\varepsilon)}{dm_J^2 d\theta_g}\Big|_{\theta_g \sim \theta_g^\star}\right]\Big|_{\varepsilon \to 0}$$

Boundary distribution: $\overline{\Theta}_{\rm sd} = \Theta(z - z_{\rm cut}\theta_g^\beta) \to \overline{\Theta}_{\rm sd}(\varepsilon) = \Theta(z - z_{\rm cut}\theta_g^\beta + \varepsilon)$

Caveat

- Geometry of the catchment area requires evaluating the boundary distribution at $\theta_g \sim \theta_g^\star$
- The $d^2\sigma$ distribution itself receives NP corrections, but these should not enter $C_i^\kappa(m_J^2)$

Reaching NLL' accuracy – intermediate Rg

In the most factorized scenario, (Laplace space) resummation is immediate

$$\frac{d\Sigma(R_g)}{dm_J^2} = N_q^{\mathrm{evol}}(Q, Q_{\mathrm{cut}}, \beta, R, \mu) \frac{d\Sigma_{\mathrm{int}}^q(R_g, \partial_\Omega)}{dm_J^2} \left[\frac{e^{\gamma_E \Omega}}{\Gamma(-\Omega)} \right]$$

$$\frac{d\Sigma_{\mathrm{int}}^q(R_g, \partial_\Omega)}{dm_J^2} \equiv \frac{1}{m_J^2}\, e^{\left[K_{cs_g}(\mu,\mu_{cs_g}) + K_{cs_m}(\mu,\mu_{cs_m}) + K_J(\mu,\mu_J)\right]}$$

$$\times \left(\frac{\mu_{cs_g}}{Q_{\mathrm{cut}}(R_g/2)^{1+\beta}} \right)^{\omega_{cs_g}(\mu,\mu_{cs_g})} \left(\frac{\mu_J^2}{m_J^2} \right)^{\omega_J(\mu,\mu_J)} \left(\frac{Q R_g \mu_{cs_m}}{2m_J^2} \right)^{\omega_{cs_m}(\mu,\mu_{cs_m})}$$

$$\times\; S_{c_g}^q\left[\frac{R_g}{2} Q_{\mathrm{cut}}^{\frac{1}{1+\beta}}, \mu_{cs_g} \right] \tilde{J}_q\left[\partial_\Omega + \log\left(\frac{\mu_J^2}{m_J^2} \right), \alpha_s(\mu_J) \right] \tilde{S}_{c_m}^q\left[\partial_\Omega + \log\left(\frac{Q R_g \mu_{cs_m}}{2m_J^2} \right), \alpha_s(\mu_{cs_m}) \right]$$

Purely logarithmic dependence: trade logs for derivatives

[Korchemsky, Marchesini '93], [Balzereit, Mannel, Kilian '98]

[…]

Reaching NLL' accuracy – small, large Rg

$$C^q\left[\frac{4m_J^2}{Q^2R_g^2},\frac{QR_g}{2},\mu\right] = \delta\left(\frac{4m_J^2}{R_g^2Q^2}\right) + \frac{\alpha_s C_F}{2\pi}\left\{\delta\left(\frac{4m_J^2}{Q^2R_g^2}\right)\left(\frac{1}{2}\ln^2\frac{4\mu^2}{Q^2R_g^2}+\frac{3}{2}\ln\frac{4\mu^2}{Q^2R_g^2}+\frac{7}{2}-\frac{5\pi^2}{12}\right)\right.$$

$$+\,\theta(QR_g-4m_J)\mathcal{L}_0\left(\frac{4m_J^2}{Q^2R_g^2}\right)\left[4\ln\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{16m_J^2}{Q^2R_g^2}}\right)-\frac{3}{2}\sqrt{1-\frac{16m_J^2}{Q^2R_g^2}}\,\right]$$

$$\left.-\,2\theta(QR_g-4m_J)\mathcal{L}_1\left(\frac{4m_J^2}{Q^2R_g^2}\right)\right\}$$

Small Rg regime:
Evolution independent of groomed jet mass

$$S_c^\kappa\left[\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}},\frac{R_g}{2}Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\right] = S_c^\kappa\left[\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\right]$$

$$+\,\frac{\alpha_s C_\kappa}{\pi}\left[-\frac{2}{2+\beta}\Theta\left(\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}}-\left(\frac{R_g}{2}Q_{\rm cut}^{\frac{1}{1+\beta}}\right)^{2+\beta}\right)\frac{1}{\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}}}\log\left(\frac{\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}}}{\left(Q_{\rm cut}^{\frac{1}{1+\beta}}\frac{R_g}{2}\right)^{2+\beta}}\right)\right]$$

Large Rg regime:
Evolution independent of groomed jet radius

Complication: in both cases, the double differential distribution effectively starts at $\mathcal{O}(\alpha_s)$

Solution: rearrange the perturbative series in a multiplicative form

$$\frac{m_J^2}{(Q\frac{R_g}{2})^2}\,\mathcal{C}^q\left[\frac{4m_J^2}{Q^2R_g^2},\frac{QR_g}{2},\mu\right] \equiv \frac{\alpha_s C_F}{\pi}a_{10}^{\mathcal{C}}\left(\frac{\theta_c^2}{R_g^2}\right)\left[1+\frac{\alpha_s C_F}{\pi}\left(\frac{1}{4}\ln^2\frac{4\mu^2}{Q^2R_g^2}+\frac{3}{4}\ln\frac{4\mu^2}{Q^2R_g^2}\right)\right.$$

$$\left.+\frac{\alpha_s\beta_0}{4\pi}\ln\frac{4\mu^2}{Q^2R_g^2}\right]+\left(\frac{\alpha_s}{\pi}a_{20}^{\mathcal{C}}\right)\frac{\alpha_s C_F}{\pi}a_{10}^{\mathcal{C}}\left(\frac{64}{25}\frac{(\theta_c/2)^2}{R_g^2}\right)$$

Log terms from RG evolution

One-loop, non-log terms

Currently unknown two-loop, non-log terms variations included in uncertainty bands

Two-loop shift of the endpoint [Marzani, Schunk, Soyez '17]

(Similar treatment for the Large Rg case, with multiplication ⟶ convolution)

$$S_c^\kappa\left[\ell^+Q_{\rm cut}^{\frac{1}{1+\beta}},\frac{R_g}{2}Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\right]=\int d\ell'^+Q_{\rm cut}^{\frac{1}{1+\beta}}\,S_c^\kappa\left[(\ell^+-\ell'^+)Q_{\rm cut}^{\frac{1}{1+\beta}},\beta,\mu\right]\Delta S_c^\kappa\left[\ell'^+Q_{\rm cut}^{\frac{1}{1+\beta}},\frac{R_g}{2}Q_{\rm cut}^{\frac{1}{1+\beta}},\beta\right]$$

Reaching NLL′ accuracy – recap

$$\mathcal{F}(x,\mu) = \sum_{m=0}\sum_{n=1}^{2m} a^{\mathcal{F}}_{mn}\left(\frac{\alpha_s}{4\pi}\right)^m L^n_{\mathcal{F}}$$

Generic factorization ingredient

	Γ^{cusp}	γ	$\beta(\alpha_s)$	$\alpha_s^m L_{\mathcal{F}}^{n\geq 0}$	Large R_g additions	Small R_g additions
LL	α_s	-	α_s	-	$+\alpha_s\, a_{10}^{S_c}\left(\frac{R_g}{\theta_g^\star}\right)$	$+\alpha_s\, a_{10}^{\mathcal{C}}\left(\frac{\theta_c^2}{R_g^2}\right)$
NLL	α_s^2	α_s	α_s^2	-	$+\alpha_s\, a_{10}^{S_c}\left(\frac{R_g}{\theta_g^\star}\right)$	$+\alpha_s\, a_{10}^{\mathcal{C}}\left(\frac{\theta_c^2}{R_g^2}\right)$
NLL′	α_s^2	α_s	α_s^2	$\alpha_s \sum_{j=0}^{2} a^{\mathcal{F}}_{1j} L^j_{\mathcal{F}}$	$+\alpha_s^2\, a_{10}^{S_c}\left(\frac{R_g}{\theta_g^\star}\right)\sum_{j=0}^{2} a^{\mathcal{F}}_{1j} L^j_{\mathcal{F}}$	$+\alpha_s^2\, a_{10}^{\mathcal{C}}\left(\frac{\theta_c^2}{R_g^2}\right)\sum_{j=0}^{2} a^{\mathcal{F}}_{1j} L^j_{\mathcal{F}}$

Standard picture

Additions

Reaching NLL' accuracy for the boundary cross section

$$\left[\frac{d}{d\varepsilon}\frac{d^2\sigma(\varepsilon)}{dm_J^2 d\theta_g}\bigg|_{\theta\sim\theta_g^\star}\right]_{\varepsilon\to 0} \qquad \overline{\Theta}_{\rm sd} = \Theta(z - z_{\rm cut}\theta_g^\beta) \to \overline{\Theta}_{\rm sd}(\varepsilon) = \Theta(z - z_{\rm cut}\theta_g^\beta + \varepsilon)$$

Shifted soft drop condition induces two classes of modifications

$$\frac{d\Sigma(R_g, \overline{\Theta}_{\rm sd}(\varepsilon))}{dm_J^2} = \underline{\frac{d\Sigma\big(R_g, \delta_{\beta,0}\gamma_0(\varepsilon, z_{\rm cut})\big)}{dm_J^2}} + \underline{\frac{Q\varepsilon}{Q_{\rm cut}}\frac{d\Delta\Sigma_\varepsilon(R_g)}{dm_J^2}} + \mathcal{O}(\varepsilon^2)$$

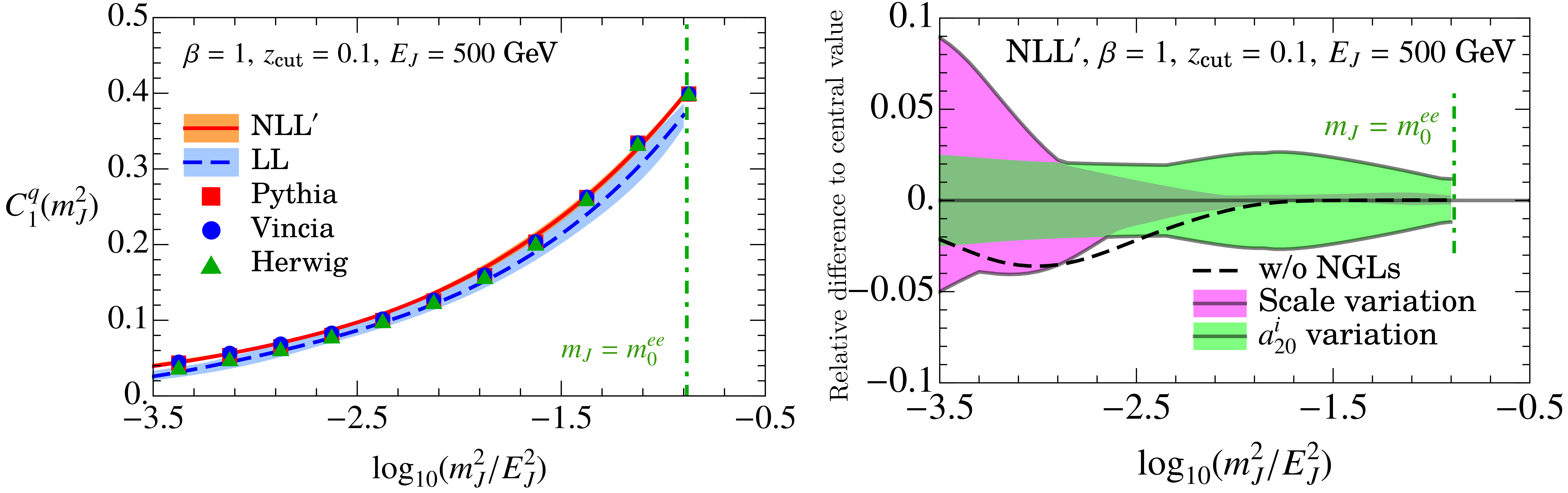
Corrections to the NLL evolution, due to new anomalous dimension $\gamma_0^{S_c^\kappa}(\varepsilon, z_{\rm cut}) = -8C_\kappa\frac{Q\varepsilon}{Q_{\rm cut}}$

Corrections to the Rg dependence, require recomputing NLO ingredients with shifted soft drop

$$Q_{\rm cut}^{\frac{1}{1+\beta}} S_c^{\kappa[1],\rm bare}\left[\ell^+, R_g, Q_{\rm cut}, \overline{\Theta}_{\rm sd}(\varepsilon), \beta, \mu\right]$$

$$\equiv \frac{\alpha_s C_\kappa}{\pi}\frac{(\mu^2 e^{\gamma_E})^\epsilon}{\Gamma(1-\epsilon)}\int\frac{dp^+dp^-}{(p^+p^-)^{1+\epsilon}}\overline{\Theta}_{\rm sd}(\varepsilon)\left[\overline{\Theta}_{R_g}\delta\big(\ell^+ - p^+\big) - \delta(\ell^+)\right]$$

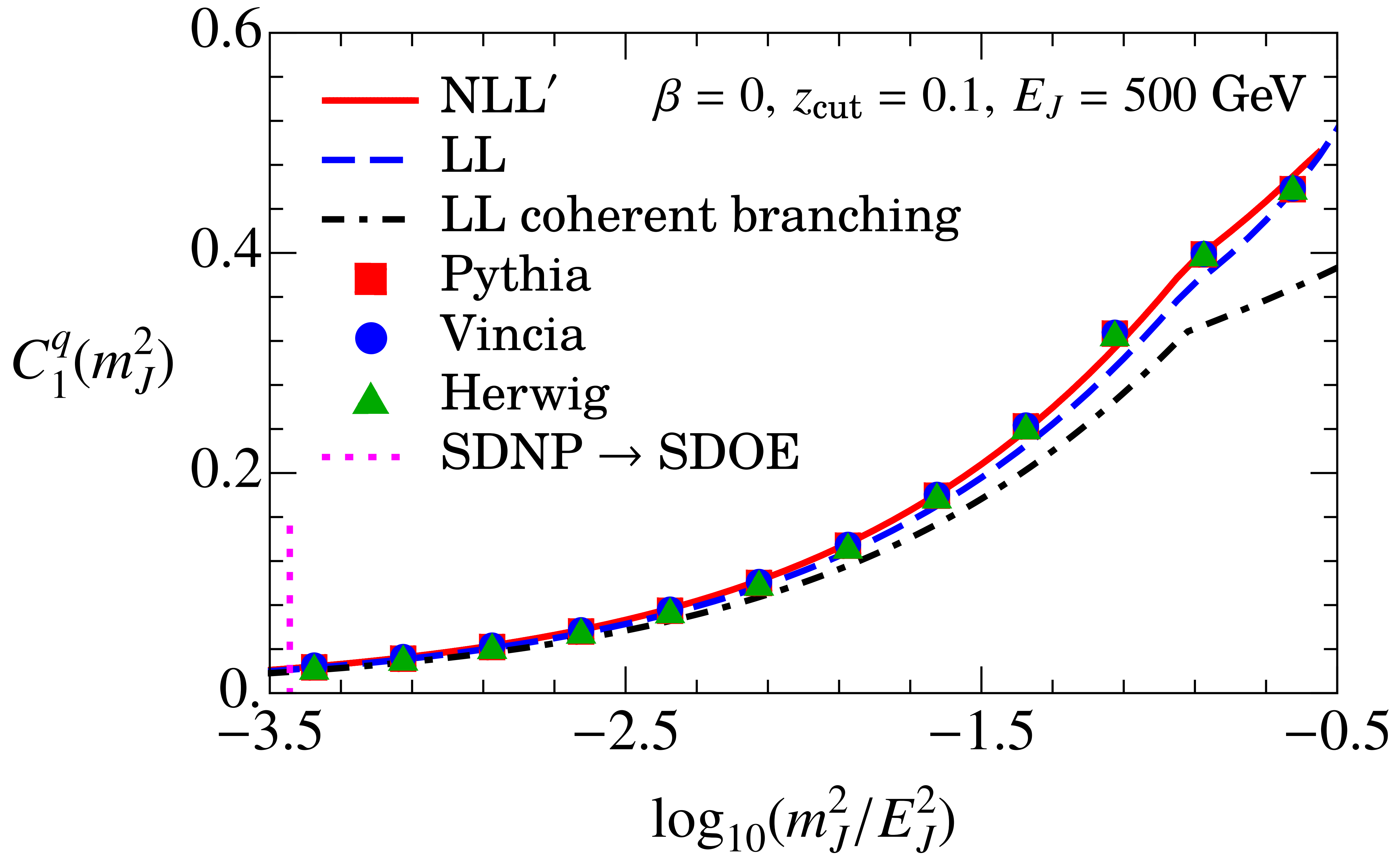
Results - C1

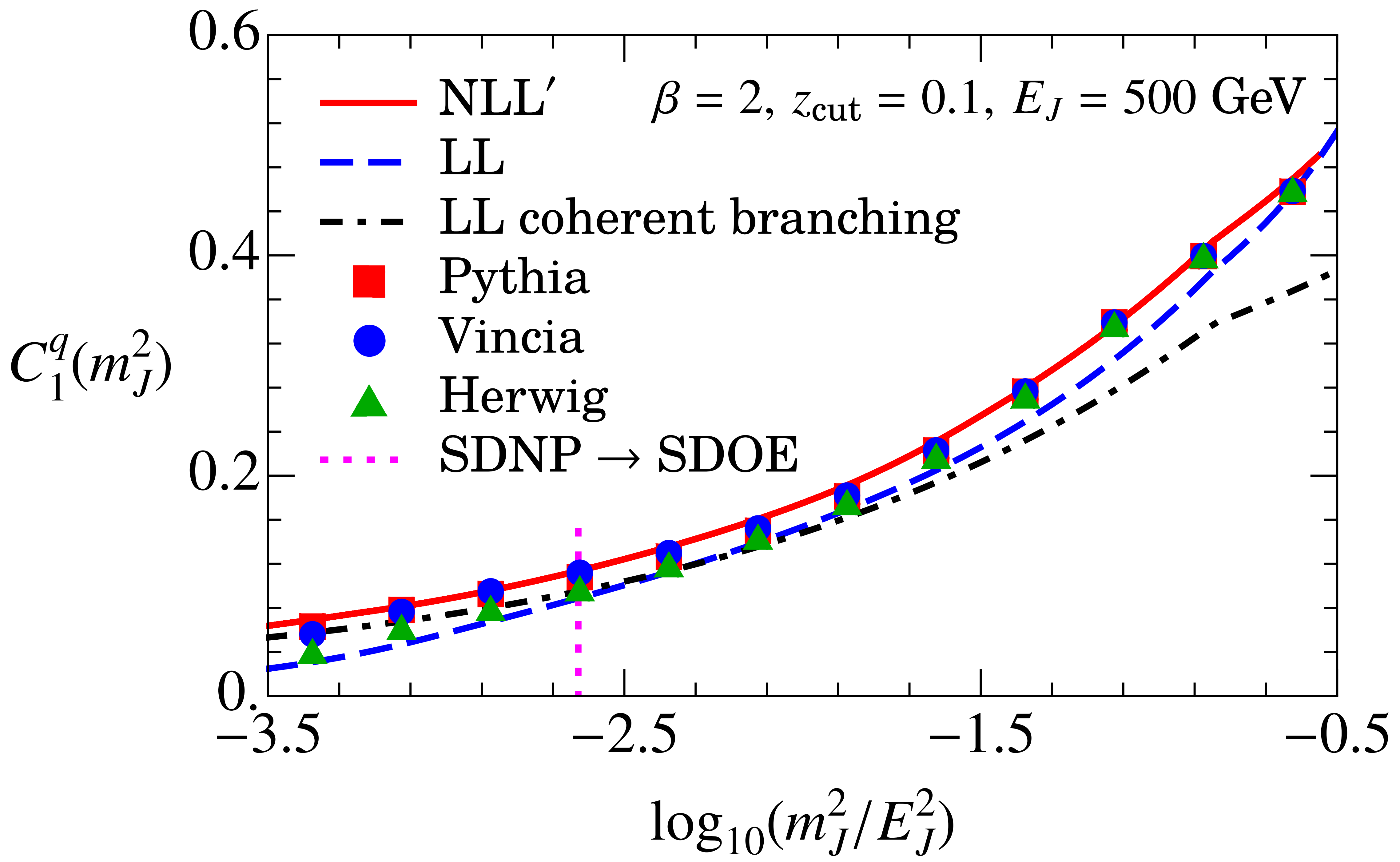


Note: double logarithms, hard/soft prefactors cancel out in the ratio $C_1^\kappa(m_J^2) = \frac{1}{\langle 1\rangle(m_J^2)}\left\langle \frac{\theta_g}{2} \right\rangle$

Including leading NGLs has small, but sizable effect

Results - C1 (different β values)

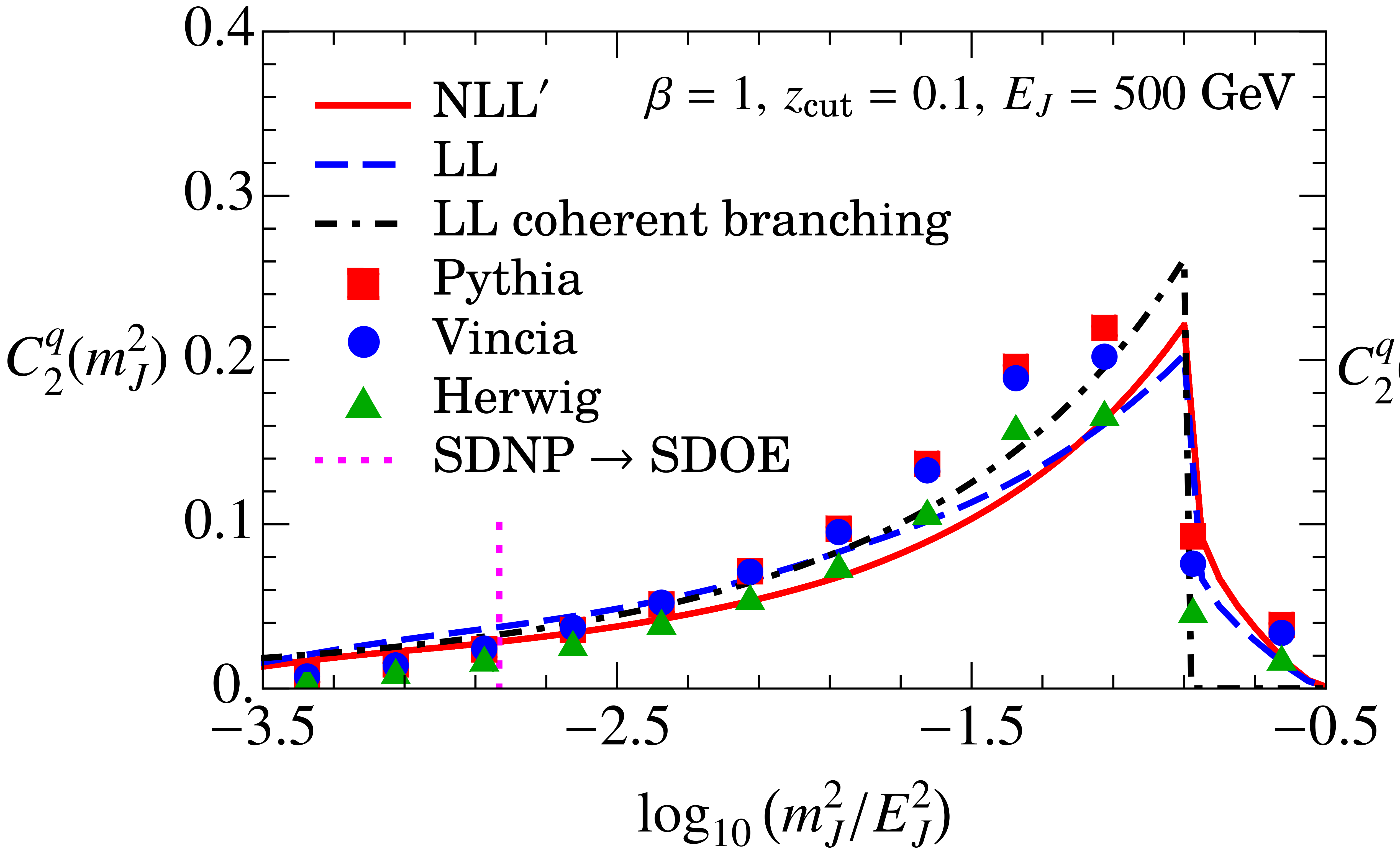


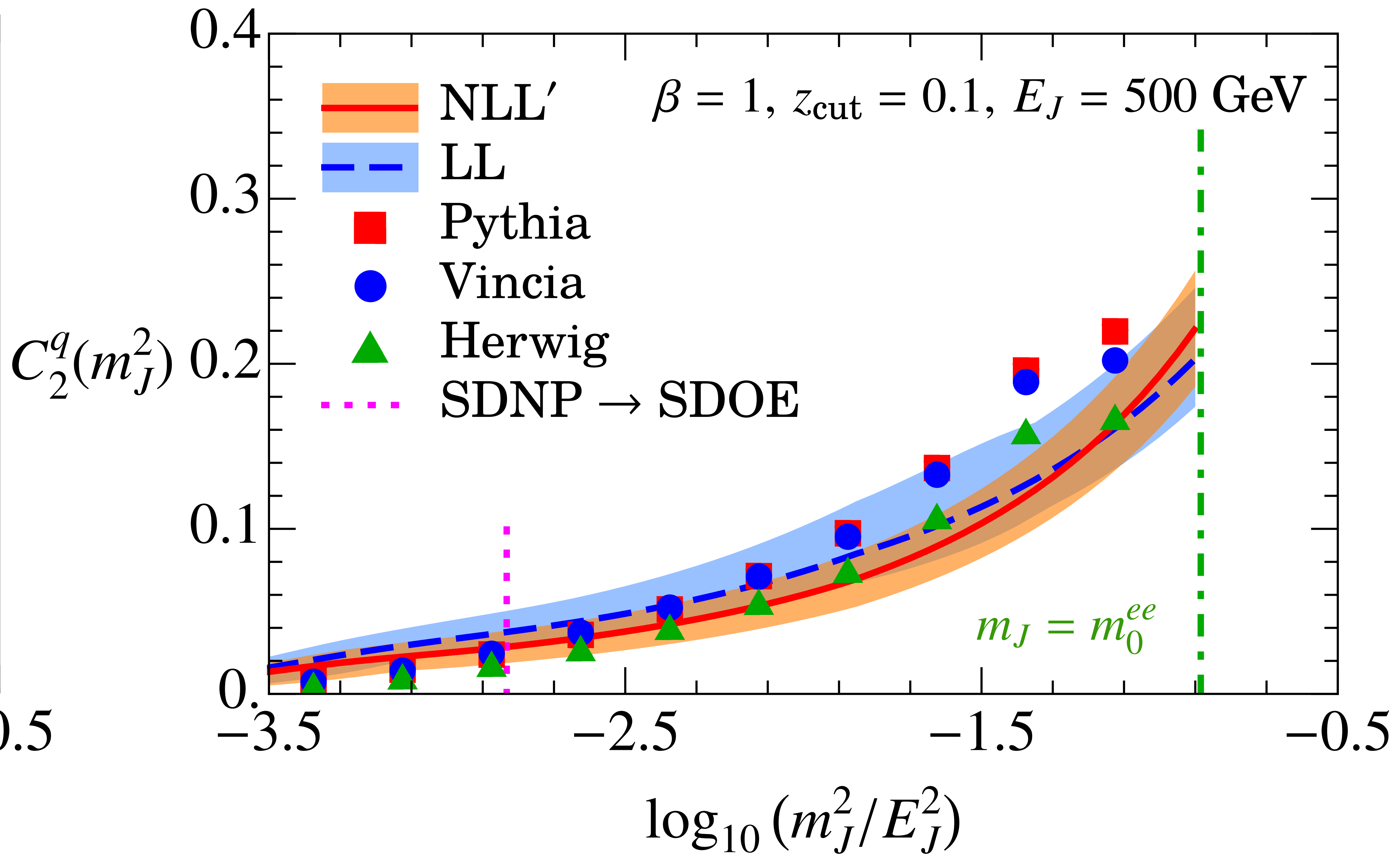


LL coherent branching from [Hoang, Mantry, Pathak, Stewart '19]

NP effects kick in earlier at larger Soft Drop exponent β

Results - C2

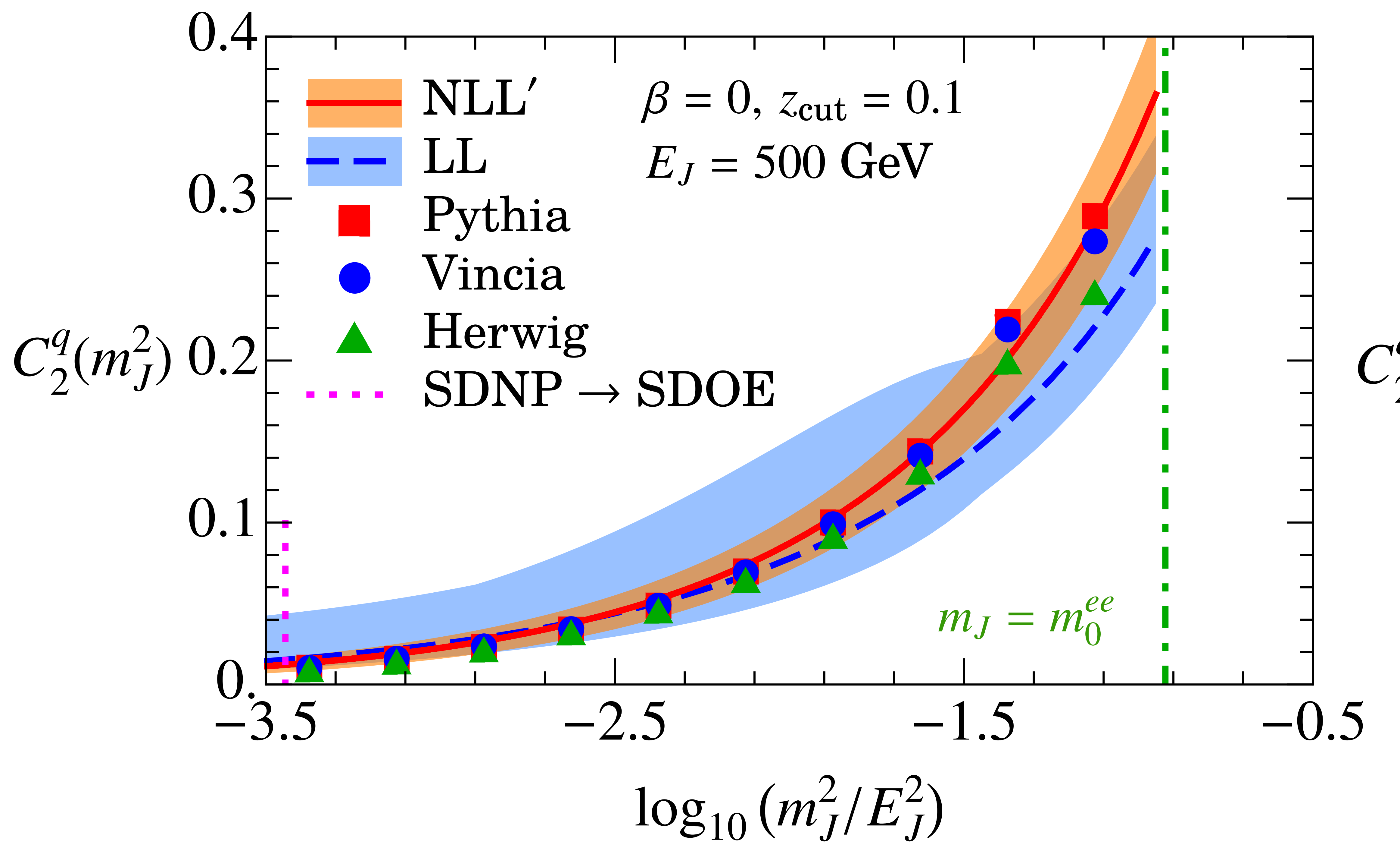


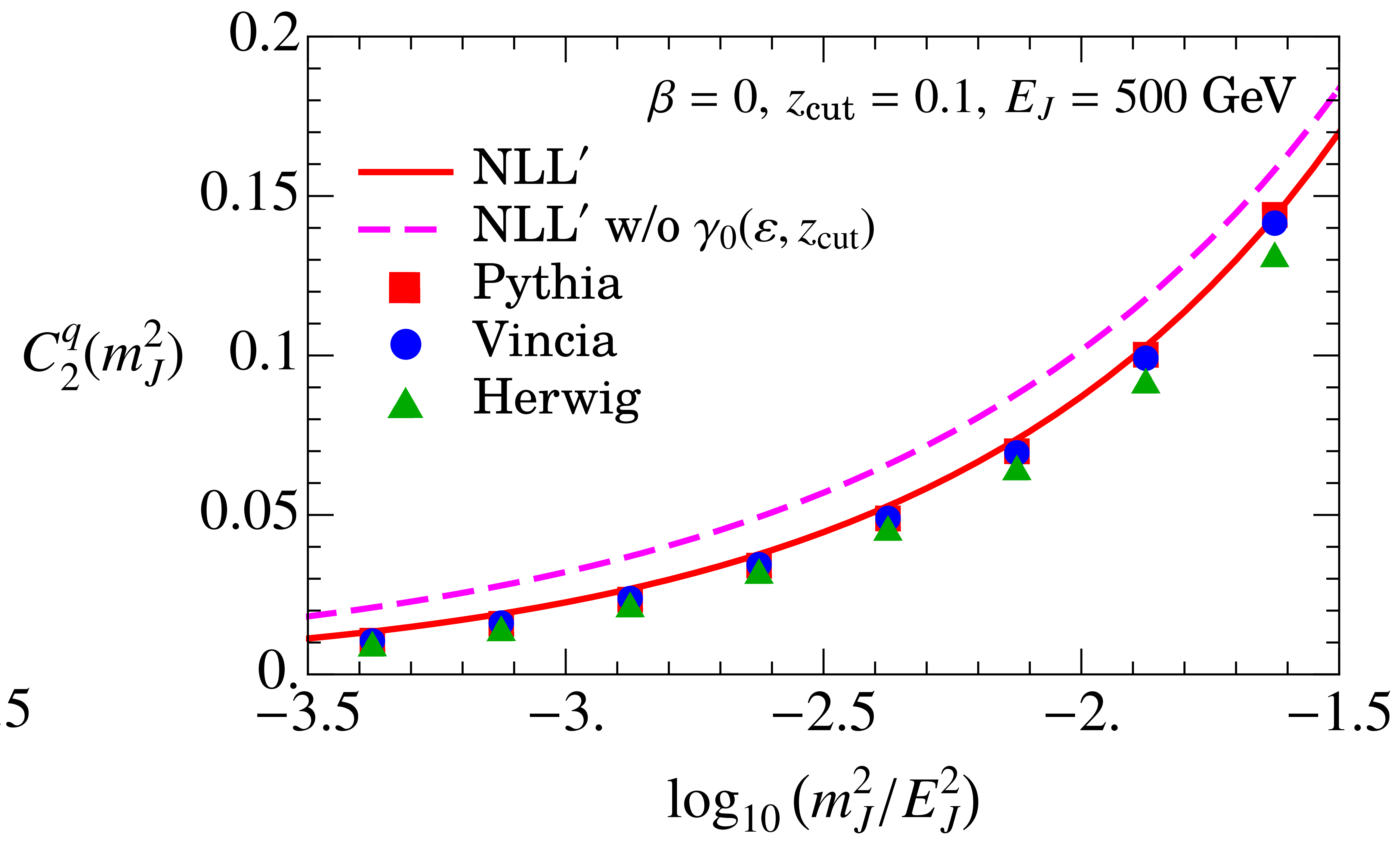


Larger spread between MC

Larger uncertainty bands at NLL′, mainly due to unknown two-loop non-logarithmic terms

Results - C2 ($\beta = 0$)





For $\beta = 0$, boundary corrections to the RG evolution are fundamental

A few more technical points (in brief)

- $C_1(m_J^2)$ uses the same matching as slide 13. $C_2(m_J^2)$ uses only large Rg by definition
- Profile functions (and their variations) needed to switch off resummation smoothly
- Scales must be frozen when $R_g \lesssim (R_g)_{\rm NP} \equiv \left(\frac{\Lambda_{\rm QCD}}{E_J}\right)^{\frac{1}{2+\beta}}$

Outlook

Further ideas

- Our approach systematically improves $C_i^\kappa(m_J^2)$ beyond LL.
  Beyond LL, one may also consider subleading NP effects

- A full description of the double differential requires treating also the ungroomed region

- The moments $M_i^\kappa(m_J^2)$ govern $C_i^\kappa(m_J^2)$, but are interesting observables in their own right

- Part of the complexity of the framework derives from $d^2\sigma$ starting at $\mathcal{O}(\alpha_s)$
  Can we turn it into an advantage? (e.g. α_s measurements)

- In the pp case, jet grooming allows for cleaner access to the proton structure
  Applications to TMD physics?

Conclusions

- We developed a SCET framework for the double differential distribution in groomed jet mass and groomed jet radius.
- First application: we improved the calculation of leading NP corrections to the soft drop groomed jet mass distribution
- Awaits further applications!