Factorization-compatible hadronization model for Monte Carlo event generators

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Doktoratskolleg Particles and Interactions

Der Wissenschaftsfonds.

Motivation

- Hadronization effects are often a bottle neck for precision in QCD calculations

- For some observables hadronization can be modeled with a simple shape function, but in general rely on MC event generators (cluster model, string model,…)

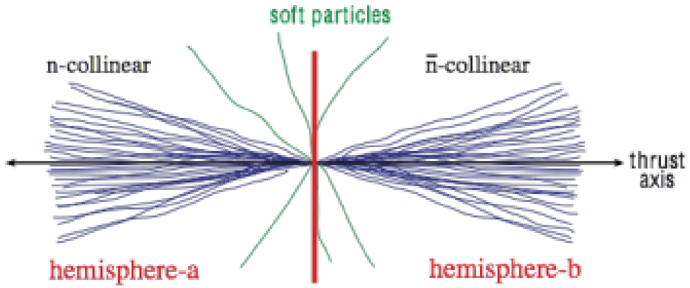
- Perturbative resummation in MC (= parton shower) has some cutoff to separate pert. and non-pert. physics

- Often we try to make connection between MC results and analytic calculations , e.g.
  - ➔ top mass scheme from measurements based on direct reconstruction
  - ➔ extraction of had. corrections from MC in α_s fits
  - ➔ …

- Usually no cutoff in the analytic calculation
  e.g. not clear how calculation in e.g. $\overline{\text{MS}}$ relates to parton shower with hard cutoff ~1 GeV

Motivation

- To have a consistent connection we need
  - to understand impact of the cutoff on perturbative calculations
  - to have a hadronization model that is consistent with the factorization properties of the observable $\leftarrow$ this talk
- We will study a simple observable that we understand well enough
  - Thrust in e^+e^--collisions $\frac{1}{Q}\min_{\hat{t}}\sum_i (p_i^0 - |\hat{t}\cdot\vec{p}_i|)$
  - only peak region $\tau << 1$
  - strictly massless



- Factorization theorem for thrust ($\tau_Q = Q\tau$)

Becher, Schwartz (2008); Abbate, Fickinger, Hoang, Mateu, Stewart (2011)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_Q}(\tau_Q) \sim H \times \int \mathrm{d}\tau'\, J\big(Q(\tau_Q - \tau'_Q)\big) S(\tau'_Q) \qquad S(k) = \int \mathrm{d}k'\, S_{\mathrm{pert}}(k') S_{\mathrm{mod}}(k - k')$$

Parton Shower and Cutoff

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- On the MC side we use Herwig 7 with angular ordered parton shower based on coherent branching algorihtm (CB)

  Catani, Marchesini, Webber (1991); Gieseke, Stephens, Webber (2003)

- Without cutoff CB equivalent to SCET results for τ_Q up to NLL precision

- Leading (linear) effect of cutoff $p_T > Q_0$ comes from soft radiation

- Equivalent to calculation of the SCET soft function with p_T cutoff

$$S_{\rm pert}(k, Q_0) = S_{\rm pert}(k) - \Delta(Q_0) \times S'_{\rm pert}(k) + \ldots \qquad \Delta(Q_0) = 16 Q_0 \frac{\alpha_s(Q_0) C_F}{4\pi} + \mathcal{O}(Q_0 \alpha_s^2)$$

- Massive case: cutoff dependent coherent branching mass scheme

$$m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \tfrac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(Q_0 \alpha_s^2)$$

- Physical prediction is unchanged if change of Q_0 is absorbed in a gap in the model function

$$S(k) = \int \mathrm{d}k'\, S_{\rm part}(k', Q_0) S_{\rm mod}(k - k' - \Delta(Q_0))$$

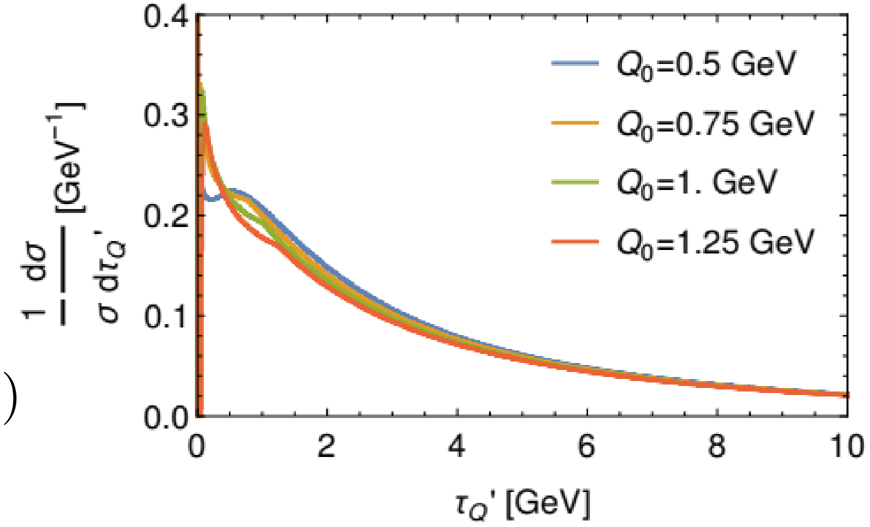
- R-evolution of gap parameter $\Delta(Q_0') = \Delta(Q_0) + 16 \int_{Q_0}^{Q_0'} \mathrm{d}R \frac{\alpha_s(R) C_F}{4\pi}$

Parton Shower and Cutoff

- The same (i.e. NOT shifted) model function applied to the partonic results of Herwig for different cutoffs leads to a shift of the peak in the hadronic distribution

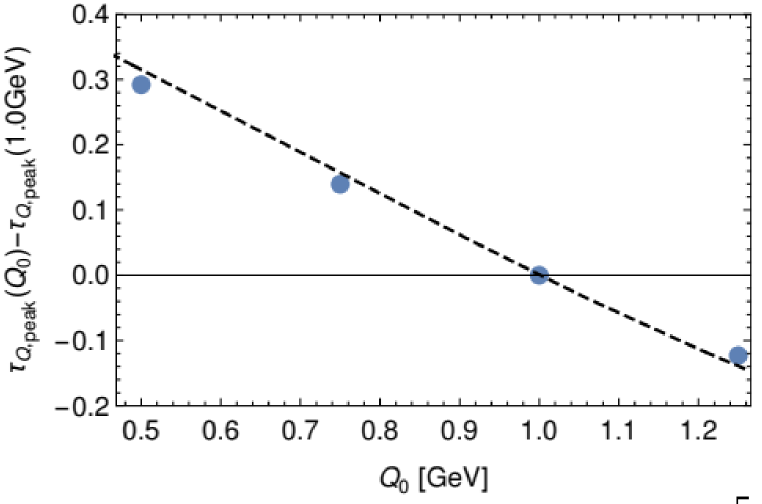
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_Q}(\tau_Q, Q_0) = \int \mathrm{d}\tau'_Q \frac{\mathrm{d}\sigma^{\mathrm{MC}}_{\mathrm{part}}}{\mathrm{d}\tau_Q}(\tau'_Q, Q_0) S_{\mathrm{mod}}(\tau_Q - \tau'_Q)$$

$$S_{\mathrm{mod}}(k) = \frac{128k^3}{3\Lambda^4}\mathrm{e}^{\frac{-4k}{\Lambda}}$$



- The peak shift follows the cutoff dependence of the gap

$$\frac{\mathrm{d}\tau_Q^{(\mathrm{peak})}(Q_0)}{\mathrm{d}Q_0} = \frac{\mathrm{d}\Delta(Q_0)}{\mathrm{d}Q_0} = 16\frac{\alpha_s(Q_0)C_F}{4\pi}$$



- This only works because of the „$\tau_Q - \tau'_Q$ form" of the convolution with the non-pert. model function

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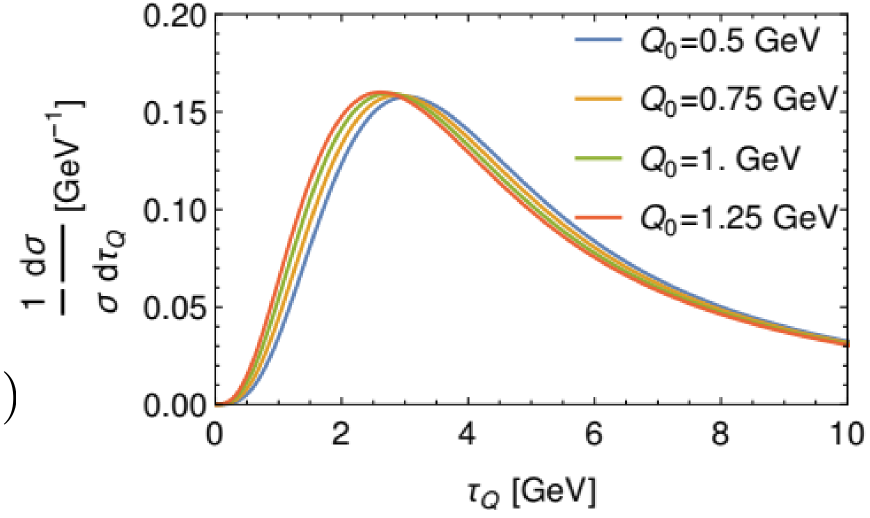

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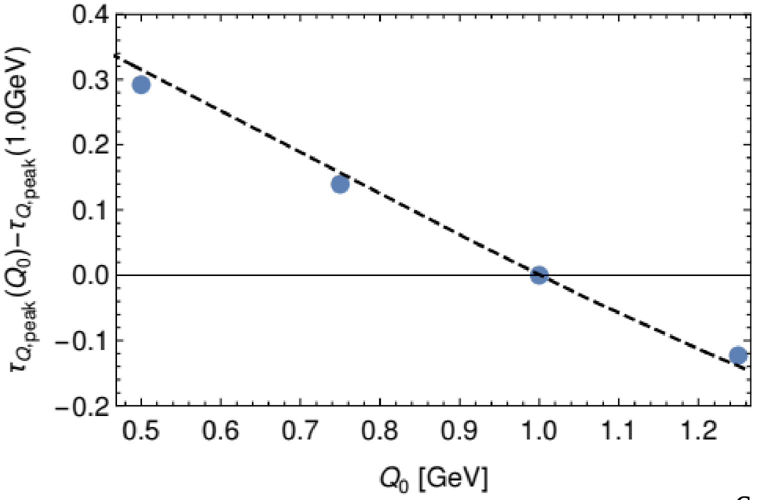

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MC hadronization function

- Define *hadronization function* $\tilde{S}_{\rm had}(y, y', Q, Q_0)$ of a given had. model for observable y
- $\int \mathrm{d}y\, \tilde{S}_{\rm had}(y, y', Q, Q_0) = 1$
- Most general form of transforming a partonic distribution to a hadronic distribution

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y, Q) = \int \mathrm{d}y' \, \frac{\mathrm{d}\sigma^{\rm MC}_{\rm part}}{\mathrm{d}y}(y', Q, Q_0)\tilde{S}_{\rm had}(y, y', Q, Q_0)$$

- $\tilde{S}_{\rm had}(y, y', Q, Q_0)$ is the probability distribution that an event with partonic value y' before hadronization ends up with hadronic value y after haronization
- Extrac had. function from MC had. model and see if it is consistent with fact. theorem
- Use Herwig 7 (angular ordered PS): can make connection to analytic calculations for thrust

NLL precise for τ_Q (also in the massive case)

understand cutoff dependence (at least in the peak): $\Delta(Q_0)$

Expectations from the Factorization Theorem

- Compare with factorzation theorem for (rescaled) thrust

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_Q}(\tau_Q, Q) = \int \mathrm{d}\tau'_Q \, \frac{\mathrm{d}\sigma_{\mathrm{pert}}}{\mathrm{d}\tau_Q}(\tau'_Q, Q, Q_0) S_{\mathrm{mod}}(\tau_Q - \tau'_Q - \Delta(Q_0)) \qquad \frac{\mathrm{d}\sigma_{\mathrm{pert}}}{\mathrm{d}\tau_Q} \sim H \times J \otimes S_{\mathrm{pert}}$$

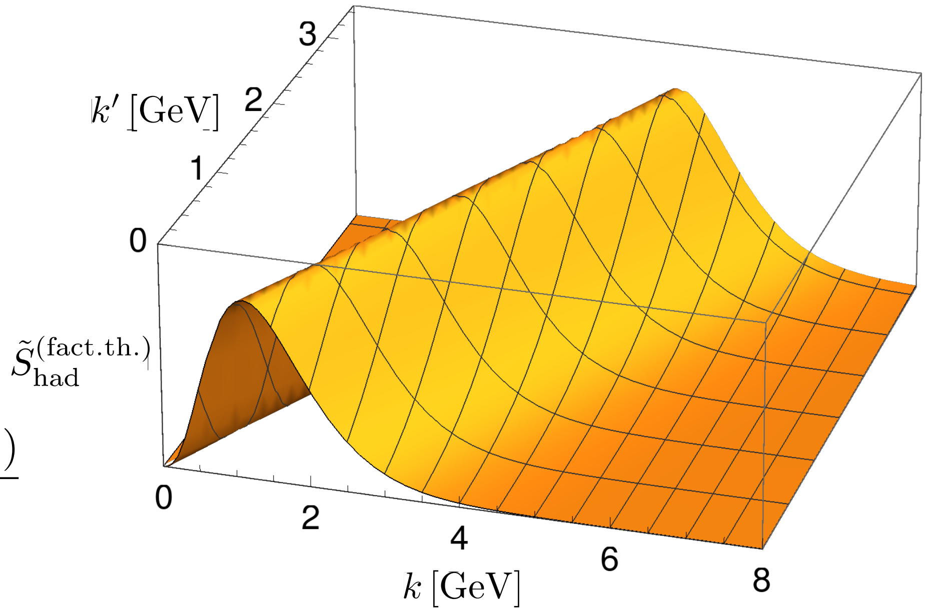
- Had. fct. $\tilde{S}_{\mathrm{had}}(k, k', Q, Q_0)$ in the fact. theorem of the form:

$$\tilde{S}_{\mathrm{had}}^{(\mathrm{fact.th.})}(k, k', Q, Q_0) = S_{\mathrm{mod}}(k - k' - \Delta(Q_0))$$

- Define shifted hadronization function:

$$S_{\mathrm{had}}(k - k', k', Q, Q_0) = \tilde{S}_{\mathrm{had}}(k, k', Q, Q_0)$$

$$\Rightarrow S_{\mathrm{had}}^{(\mathrm{fact.th.})}(k, k', Q, Q_0) = S_{\mathrm{mod}}(k - \Delta(Q_0))$$



- Cutoff dependence of the first moment known from R-evolution of the gap parameter

$$\Omega_1(Q_0) = \int \mathrm{d}k \, k \, S_{\mathrm{mod}}(k - \Delta(Q_0)) \;\Rightarrow\; \frac{\mathrm{d}\Omega_1(Q_0)}{\mathrm{d}Q_0} = \frac{\mathrm{d}\Delta(Q_0)}{\mathrm{d}Q_0}$$

e.g. for thrust: $\displaystyle \frac{\mathrm{d}\Delta(Q_0)}{\mathrm{d}Q_0} = 16 \frac{\alpha_s(Q_0) C_F}{4\pi}$

- Relations of the moment of the non-pert. soft function for various event shapes are known, e.g. C-parameter, thrust and angularities

$$\Omega_1^{(c)} = \frac{3\pi}{2} \Omega_1^{(\tau)} = \frac{3\pi(1-a)}{2} \Omega_1^{(\tau_a)}$$

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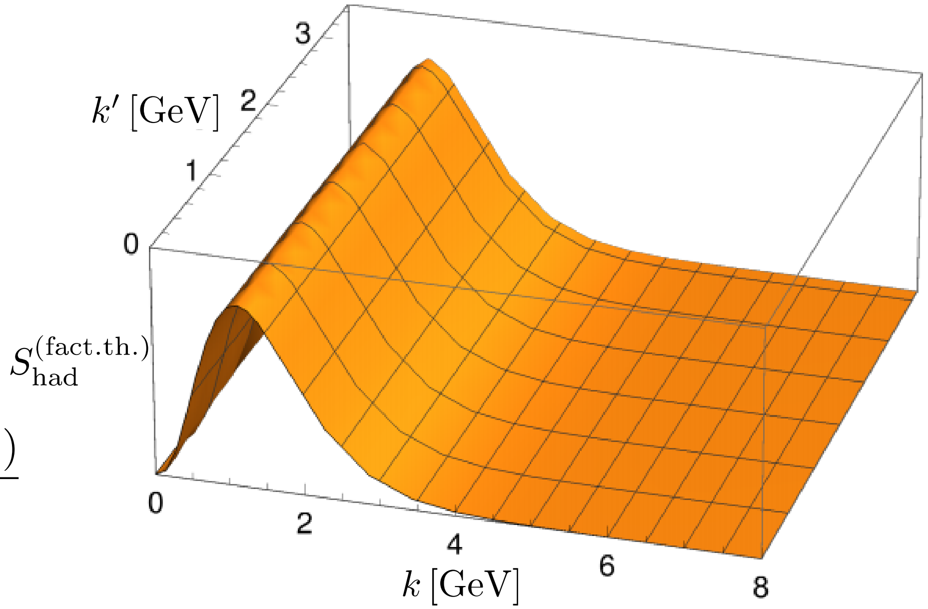

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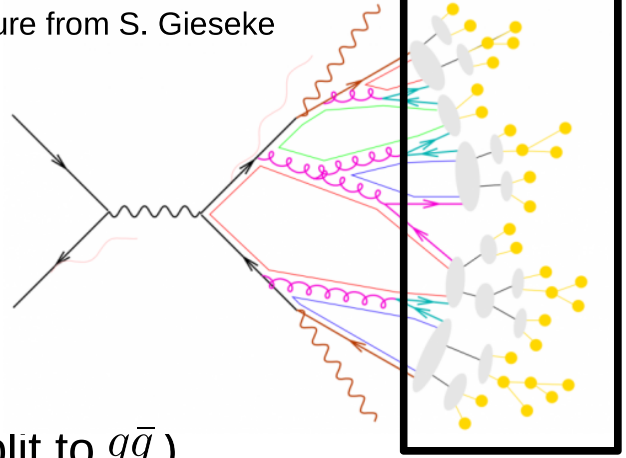
Herwig's cluster model

Figure from S. Gieseke



- Standard hadronization model of Herwig: cluster hadronizaiton model

  Webber (1984)

- Color-connected quarks combined into preconfined clusters (all partons get constituent masses, final state gluons forced to split to $q\bar{q}$)

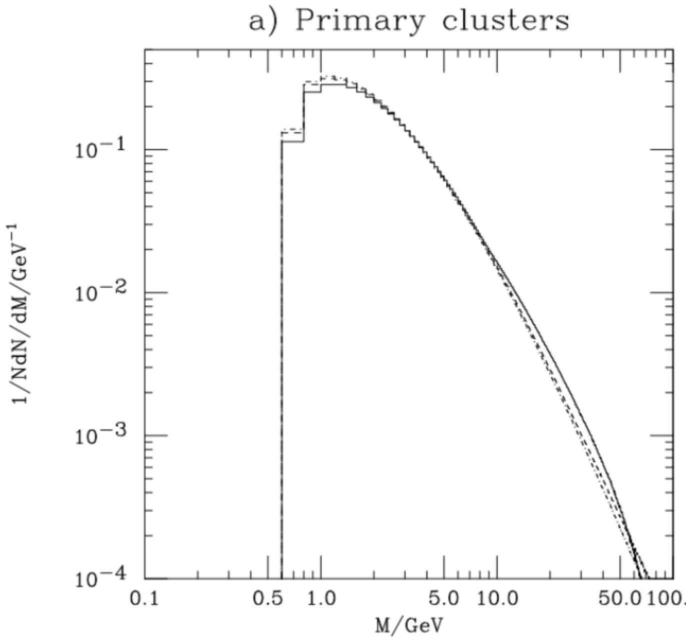


Figure from Herwig ++ Manual

- Cluster mass spectrum universal over a very large energy range

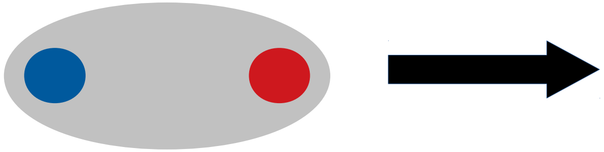
- Peaked at low cluster masses

- Clusters can be seen as highly excited hadrons that subsequently decay (isotropically) into actual hadrons

Herwig’s cluster model

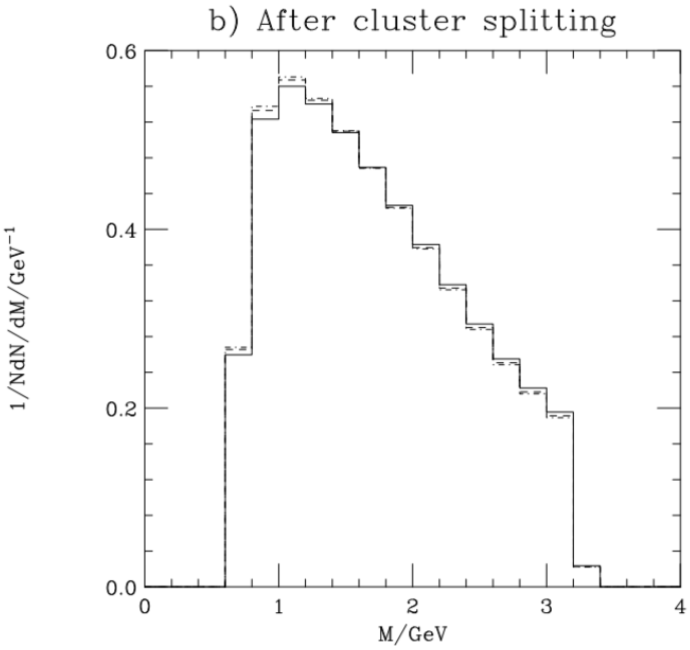
- Some cluster can be very heavy: picture of excited hadrons not applicable any more

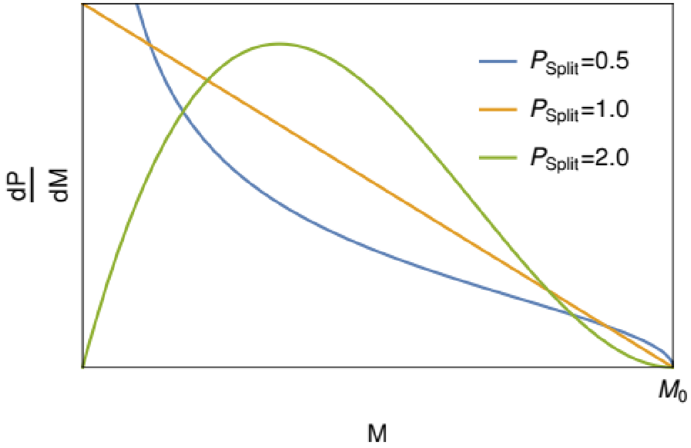
- Undergo 1 → 2 fission process along axis of constituent quarks until they are light enough



- Various tuning parameters: e.g.
  - mass spectrum of daughter cluster in fission
  - cutoff criterion for fission
  - constituent masses

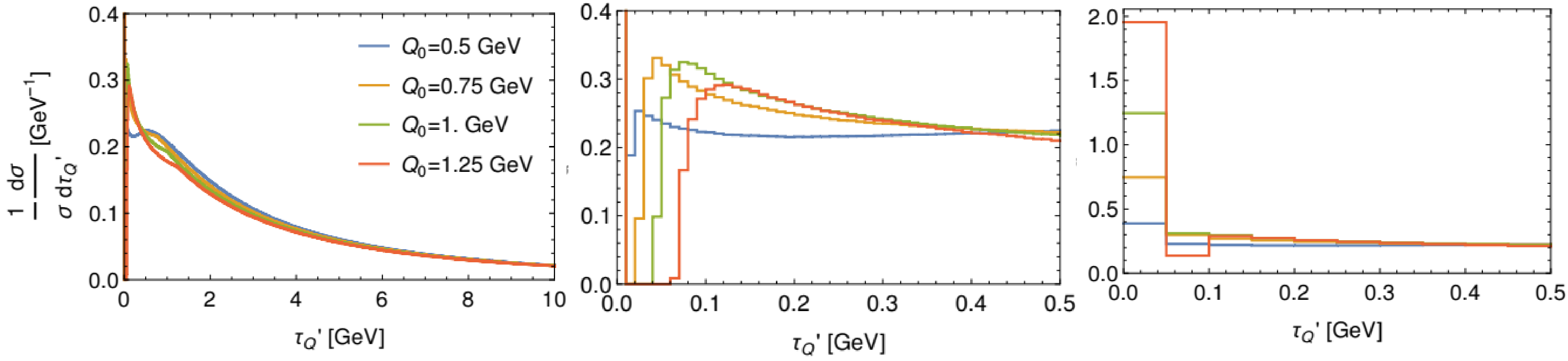
Figure from Herwig ++ Manual



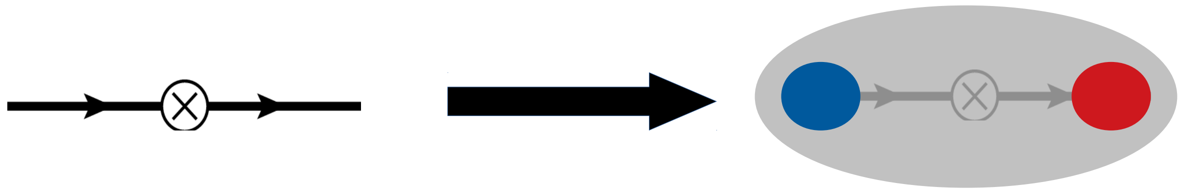


Cluster fission and shower cutoff dependence

- Parton shower cutoff affects partonic distribution mostly at low p_T, i.e. relevant for the peak



- Events with low p_T can produce heavy clusters (→ cluster fission):
  extreme case: no perturbative radiation at all



- Cutoff dependence of the model will be senstive to the cluster fission process

Hadronization in the event generator

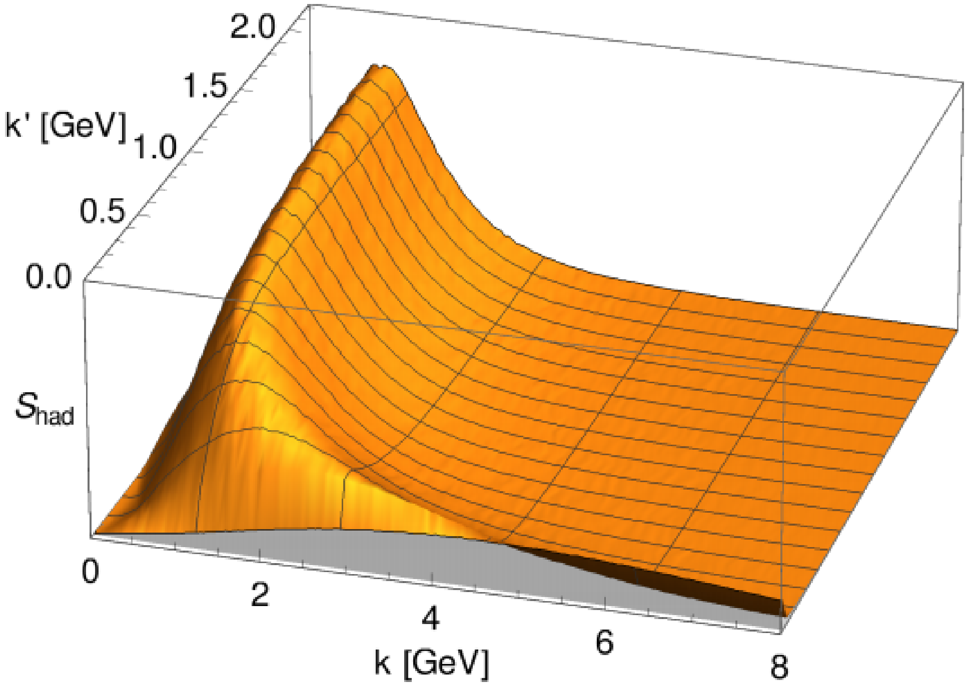
- We can now directly check if the hadronization functions of Herwig for different eventshapes (here thrust and C-Parameter) are consistent with he factorization properties

- Procedure:
  - Set the parton shower cutoff to different values
  - Tune the model to some (fake) data
  - Run the MC and calculate the eventshape before and after hadronization for each event and fill it in a 2D-histogram → hadronization function
  - Check the behavior of the hadronizaton function against the fact. theorem

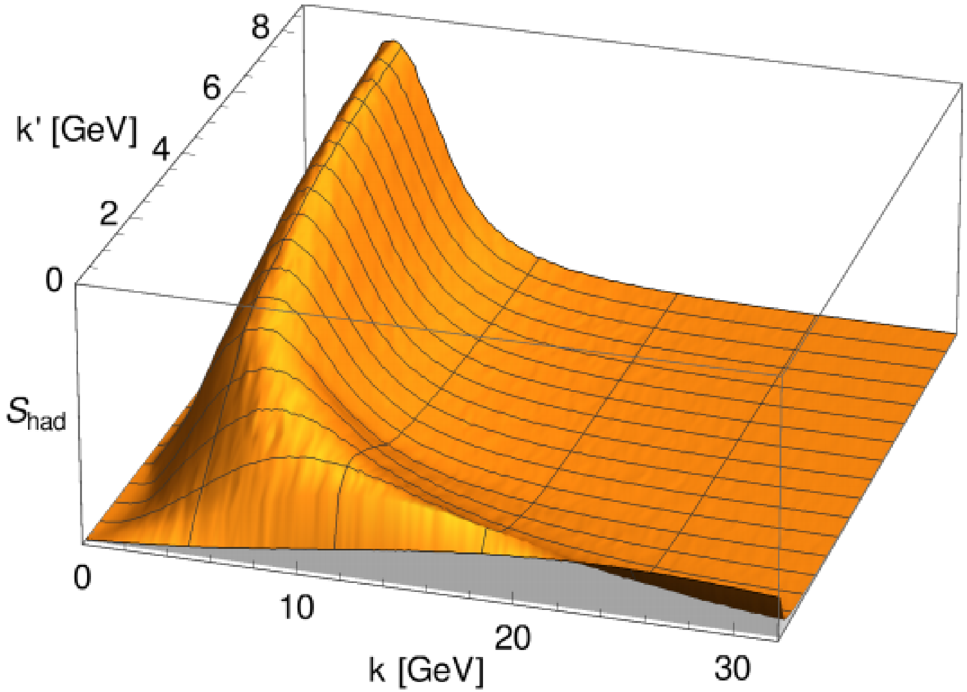
Hadronization function of Herwig's default model

$Q = m_Z$
$Q_0 = 1\,\mathrm{GeV}$

Hadronization function $S_{\mathrm{had}}(k, k', Q, Q_0)$ of Herwig:



Thrust



C-Parameter

Hadronization function of Herwig's default model

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- Check behavior of first moment

$$\Omega_1(k', Q, Q_0) = \int \mathrm{d}k\, k S_{\mathrm{had}}(k, k', Q, Q_0)$$

- Prediction from evolution of the gap parameter:

$$\frac{\mathrm{d}\Omega_1^{(\tau)}(Q_0)}{\mathrm{d}Q_0} = 16\frac{\alpha_s(Q_0)C_F}{4\pi}$$

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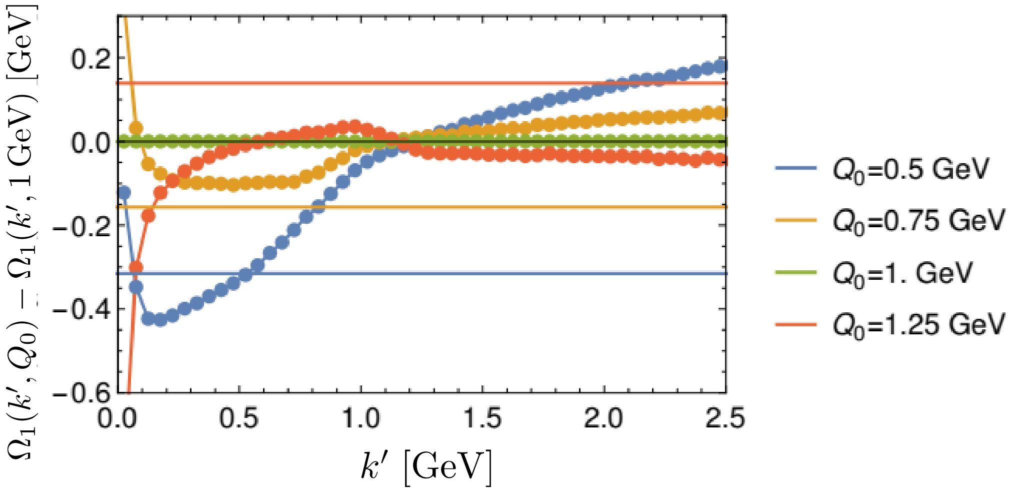
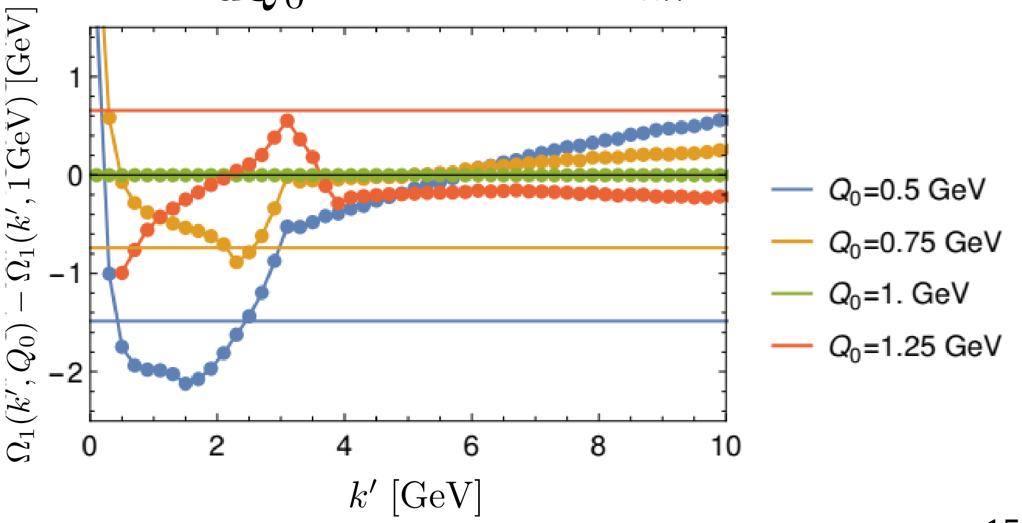




Dynamic Model

- Current implementation of the cluster model is not able to produce hadronization effects in a way consistent with factorization properties of the observable

- Try to modify it to improve this behavior

- We understand and can control the cutoff dependence in the parton shower

  => basic guideline:

  try to make a smoother transition at the cutoff from the parton shower to the hadronization model

- Cluster (and gluon) masses generated dynamically from splitting: → „dynamic model”

Forced gluon splitting: current implementation

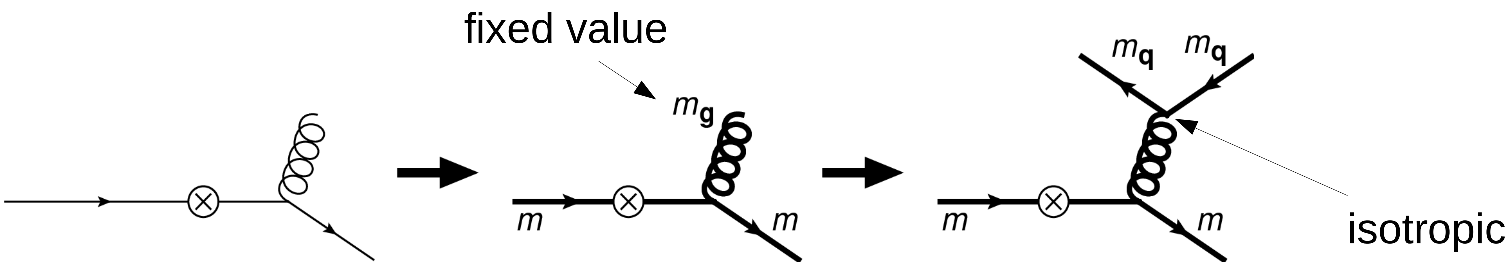
after parton shower stops:

- set all final state particles' masses to their constituent values

  m_g: fixed value (tuning parameter, default ca. 1 GeV)

- do kinematic reconstruction

- split gluons into $q\bar{q}$ pairs : isotropic decay in gluon's rest frame



Dynamic model: forced gluon splitting

- If the splitting had taken place in the parton shower it would have been generated from the splitting function

$$\mathrm{d}P(g \to q\bar{q}) \sim \frac{\mathrm{d}q^2}{q^2}\alpha_s(q^2)\Big(1 - 2z(1-z) + \frac{2m_q^2}{q^2}\Big)\Theta\big(q^2 z(1-z) - m_q^2\big)$$

  giving the gluon a virtuality q^2

- Use the probability distribution for this dynamically generated virtuality as „gluon constituent mass" m_g

- Set a highest possible scale for the non-pert. gluon splitting $\tilde{Q}_g$

  (new tuning parameter instead of fixed m_g)

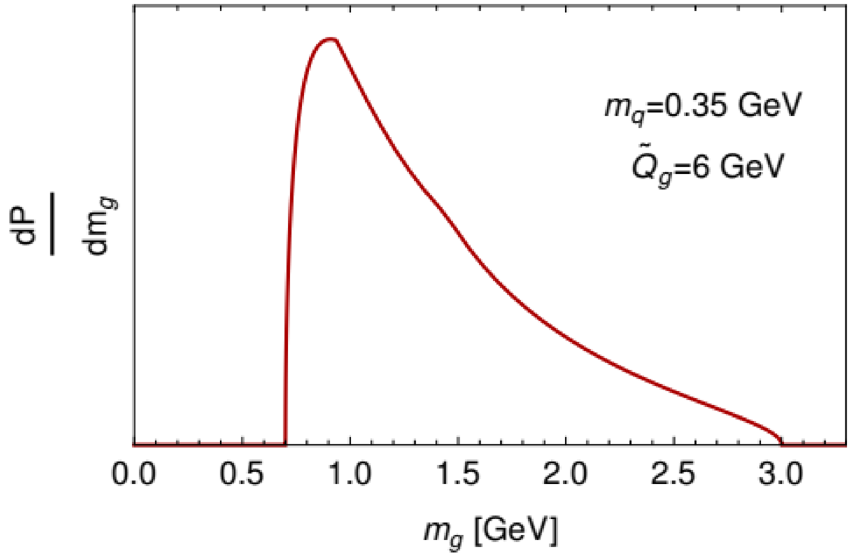
- Need to IR regularize the splitting function (because evolve below cutoff): $\mathrm{d}P(g \to q\bar{q}) \to \mathrm{d}\tilde{P}(g \to q\bar{q})$

  freeze out strong coupling at some low scale to avoid Landau pole

  use constituent masses for quarks

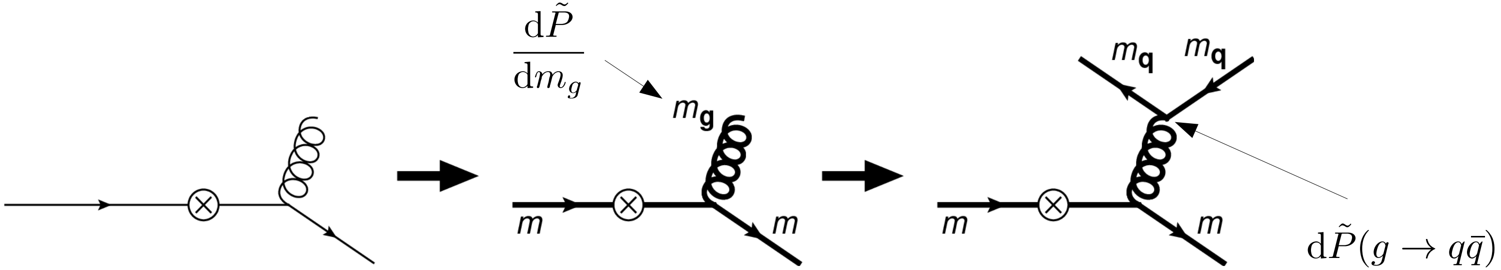
Dynamic model: forced gluon splitting

1) Generate random mass for each gluon from resulting probability distribution



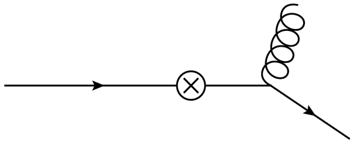
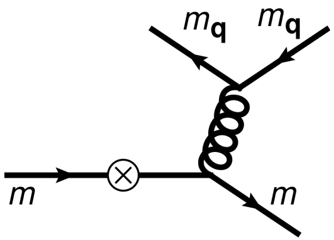
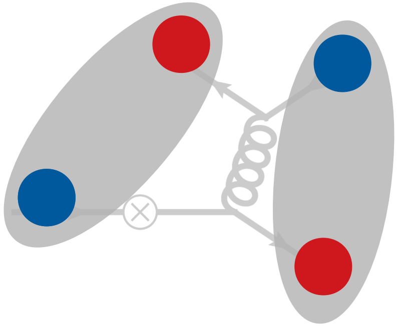
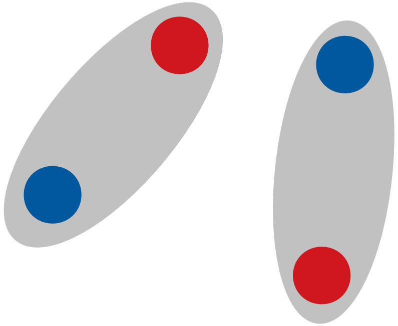
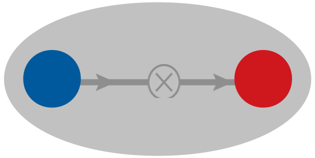
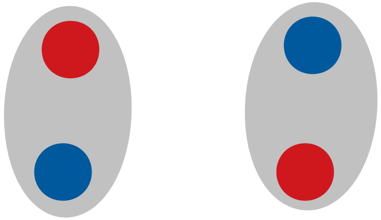
2) Do the kinematic reconstruction with all partons on their consituent mass as usual

3) Split the gluons to $q\bar{q}$ pairs according to the (IR regularized) splitting function $\mathrm{d}\tilde{P}(g \to q\bar{q})$



Dynamic model: cluster fission

- Use the same philosophy also for the cluster fission:

parton shower	forced splitting and constituent masses	cluster formation	cluster fission
			
			

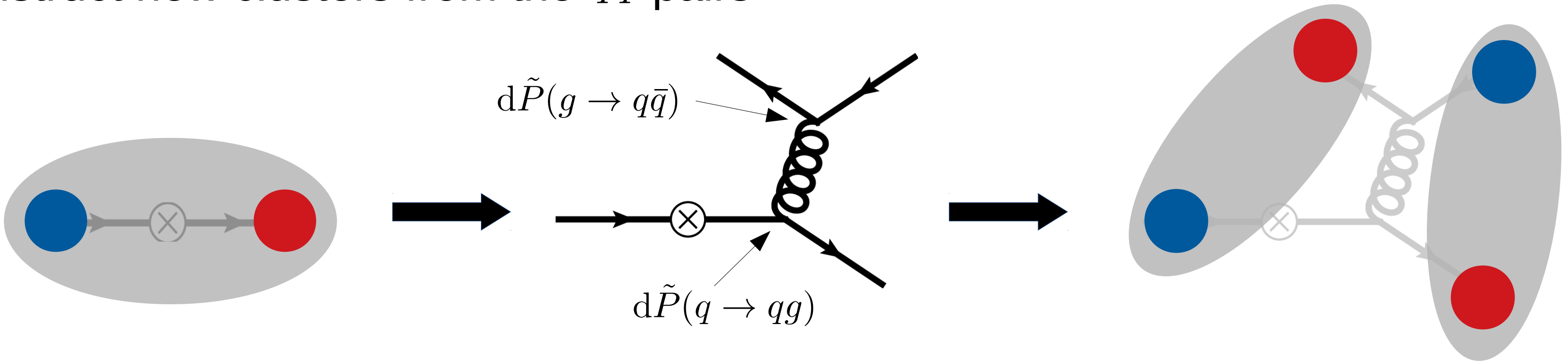
Dynamic model: cluster fission

- Want a mass distribution of the daughter clusters that resembles as much as possible the mass distribution dynamically generated by low scale emissions of the parton shower

- Radiate a gluon from one of the cluster's constituents according to $\mathrm{d}\tilde{P}(q \to qg)$ set a maximum scale $\tilde{Q}_q$ of the splitting (new tuning parameter for fission instead of P_{Split})

- Split the gluon according to $\mathrm{d}\tilde{P}(g \to q\bar{q})$

- Construct new clusters from the $q\bar{q}$ pairs



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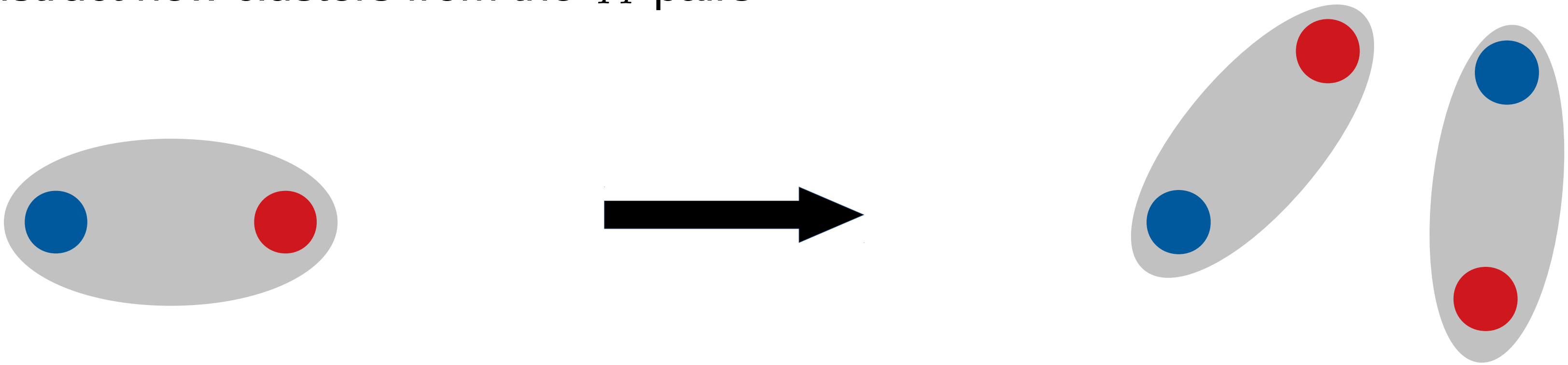

Hadronization function of the dynamic model

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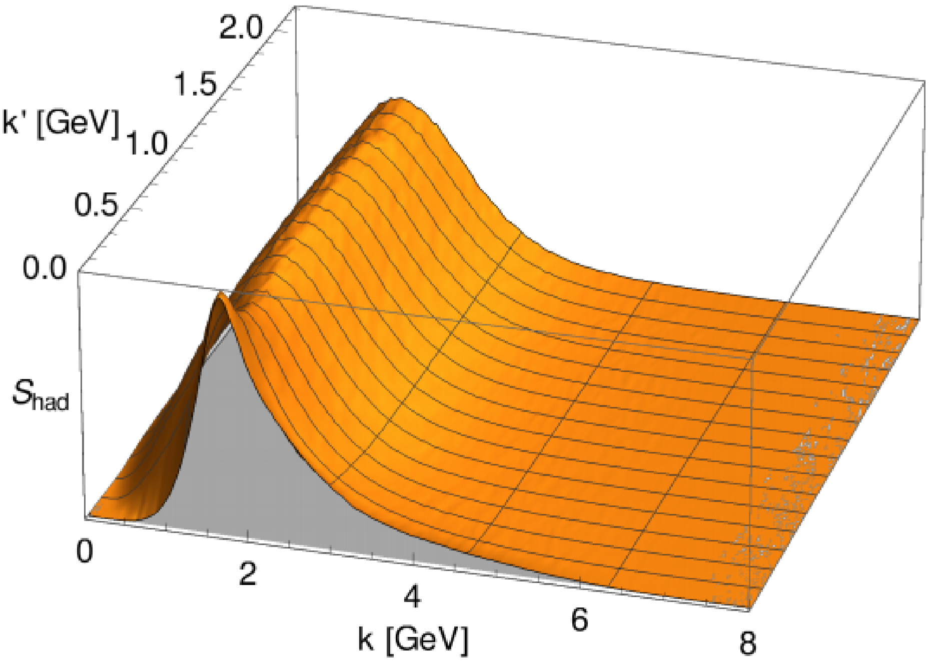
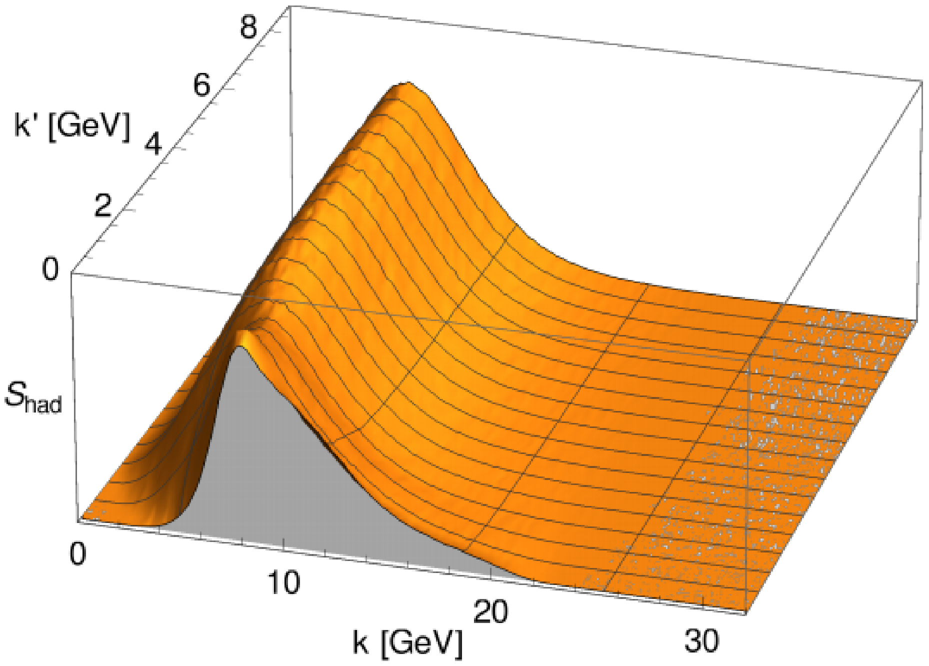
Hadronization function $S_{\mathrm{had}}(k, k', Q, Q_0)$ of Herwig for the new model:



Thrust



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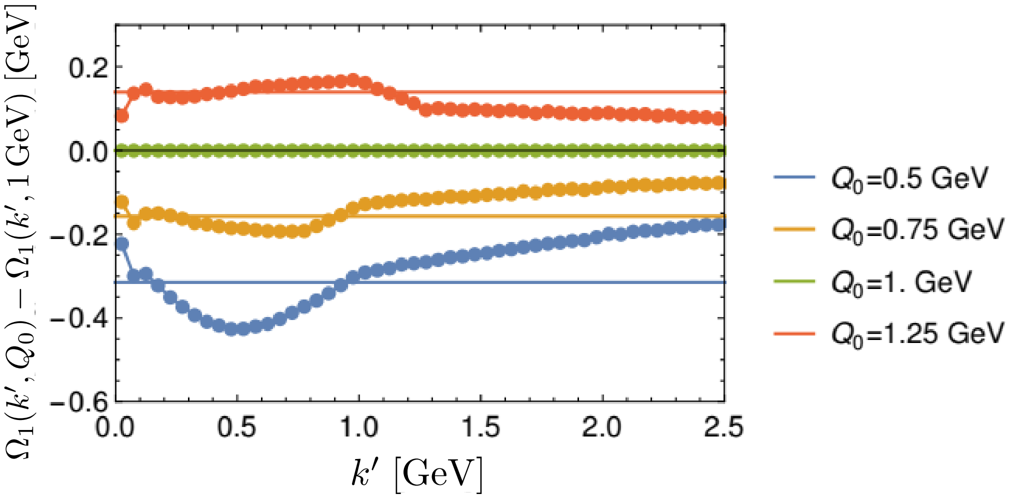
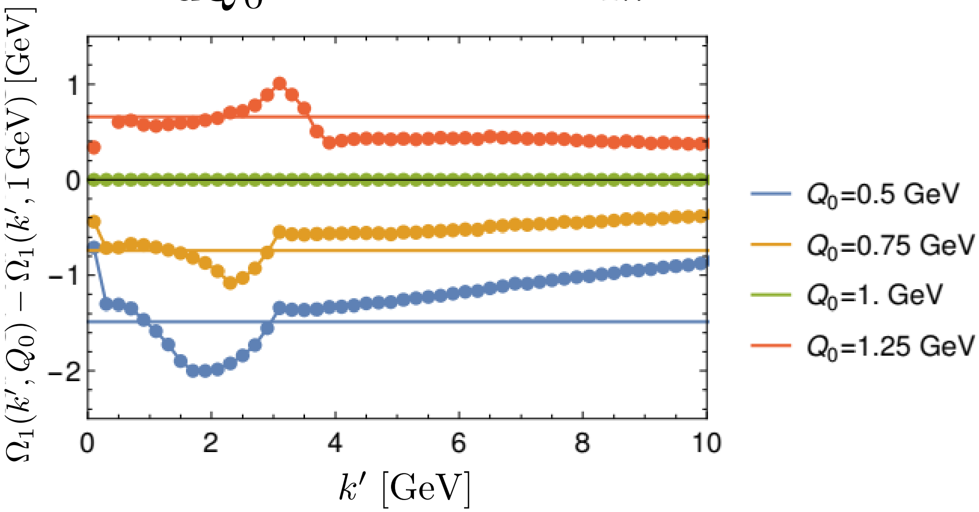
$$\frac{\mathrm{d}\Omega_1^{(c)}(Q_0)}{\mathrm{d}Q_0} = 24\pi\frac{\alpha_s(Q_0)C_F}{4\pi}$$





Hadronization function of the dynamic model

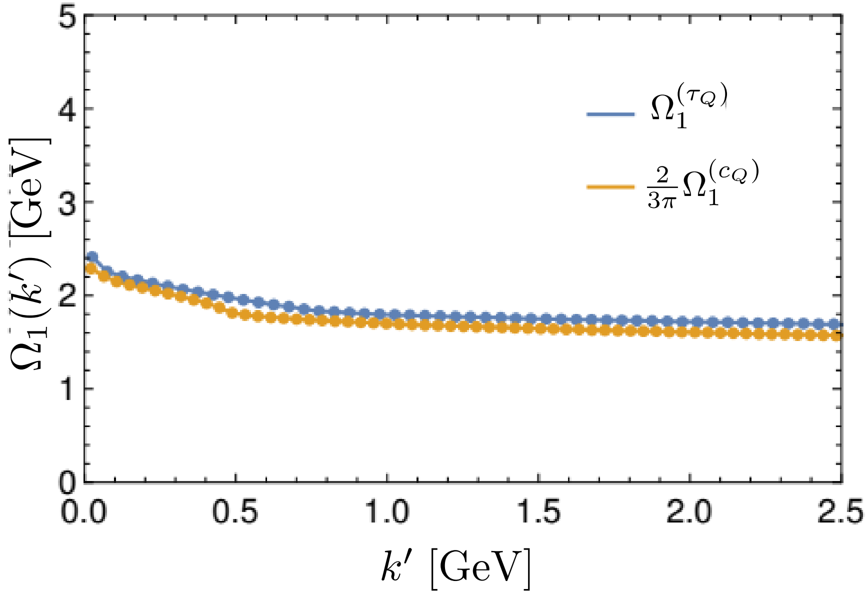
- Check behavior of first moment

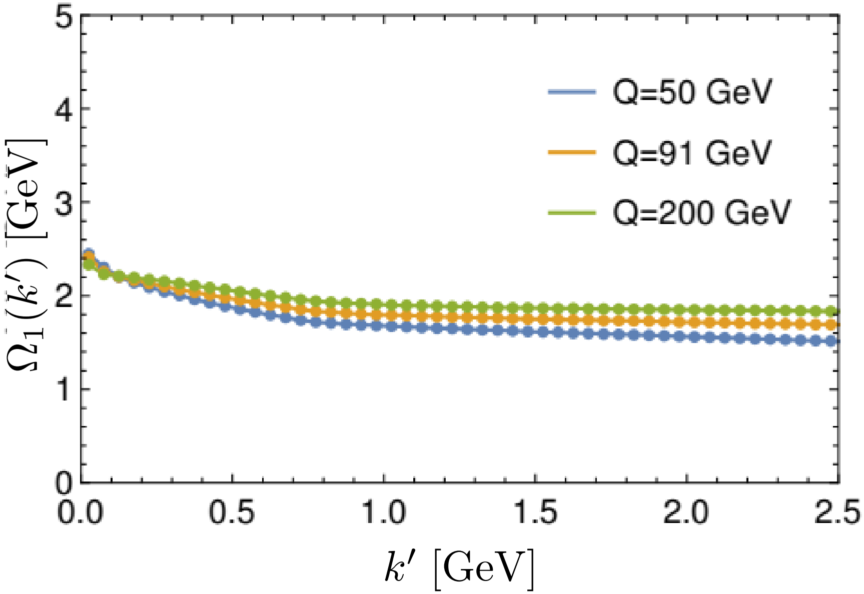
$$\Omega_1(k', Q, Q_0) = \int \mathrm{d}k\, k S_{\rm had}(k, k', Q, Q_0)$$

- Predictions of factorization theorem:

$$\Omega_1^{(\tau)} = \frac{2}{3\pi}\Omega_1^{(c)}$$

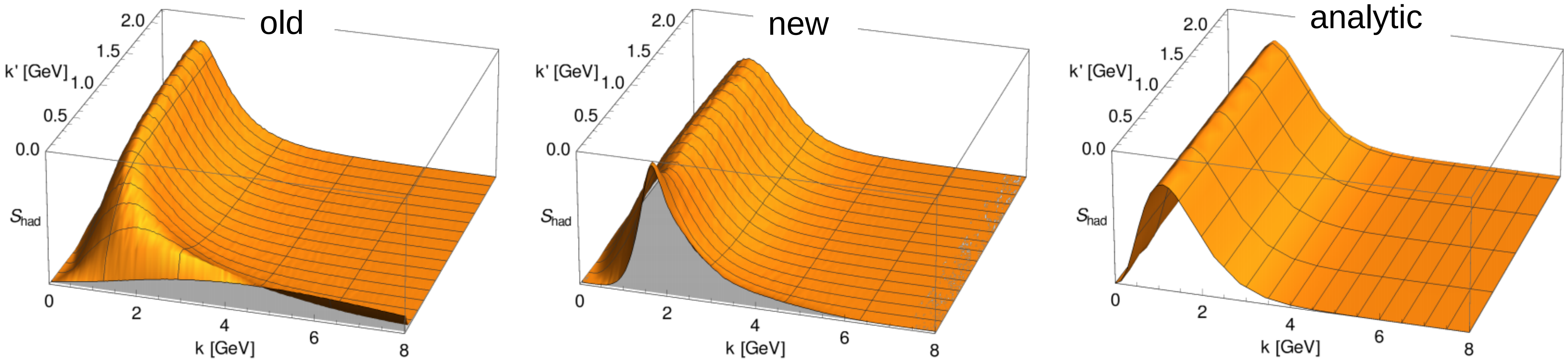
Independent of Q





Outlook

- New model looks promising for hadronization model with correct factorization properties



- Still needs a lot of work, e.g. :
  - → tuning to other observables
  - → observable dependence of the tunes
  - → cutoff dependence of the tunes
  - → different tuning parameters?

- Go to massive case ($t\bar{t}$ -production) to check possible hadronization effects on MC top mass scheme $m_t^{\mathrm{MC}}(Q_0)$

Outlook

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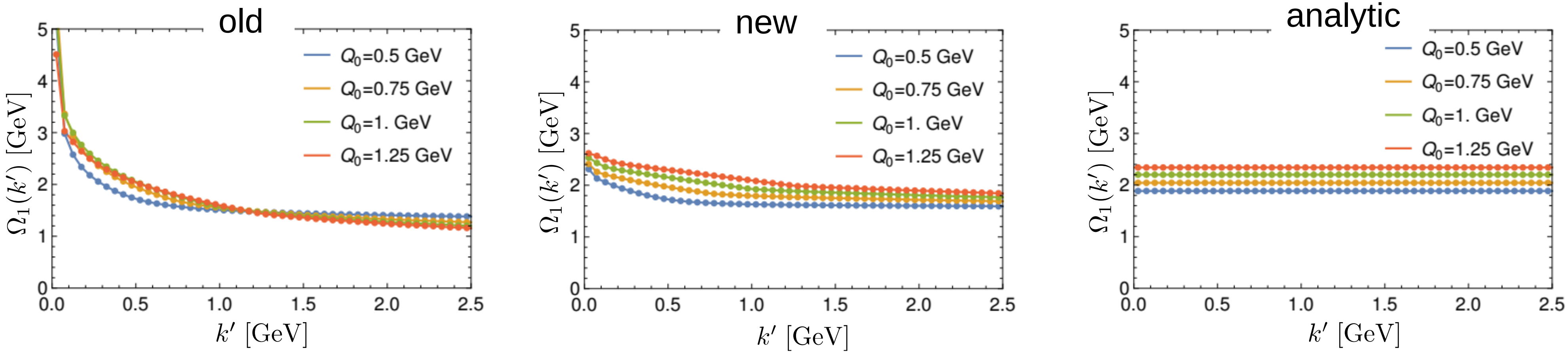

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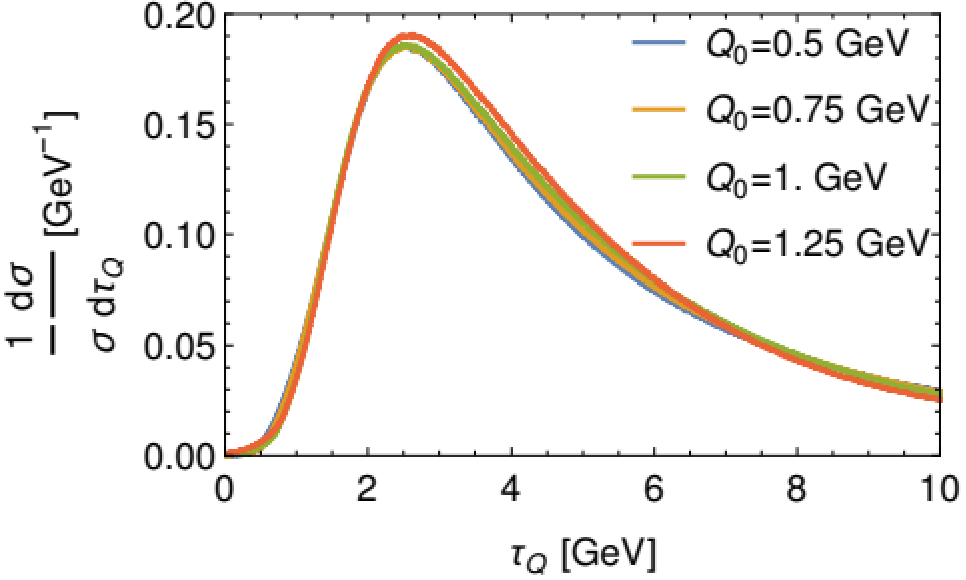
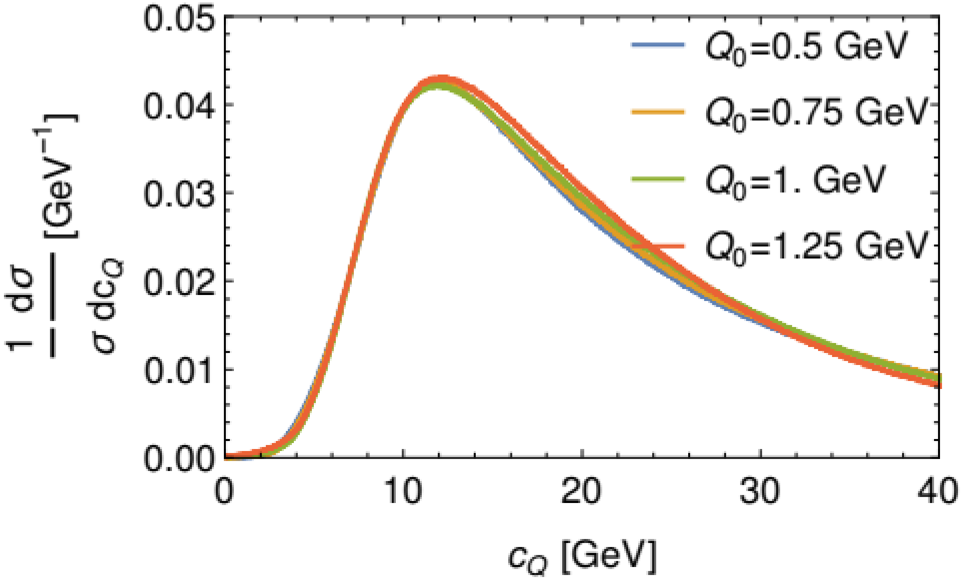
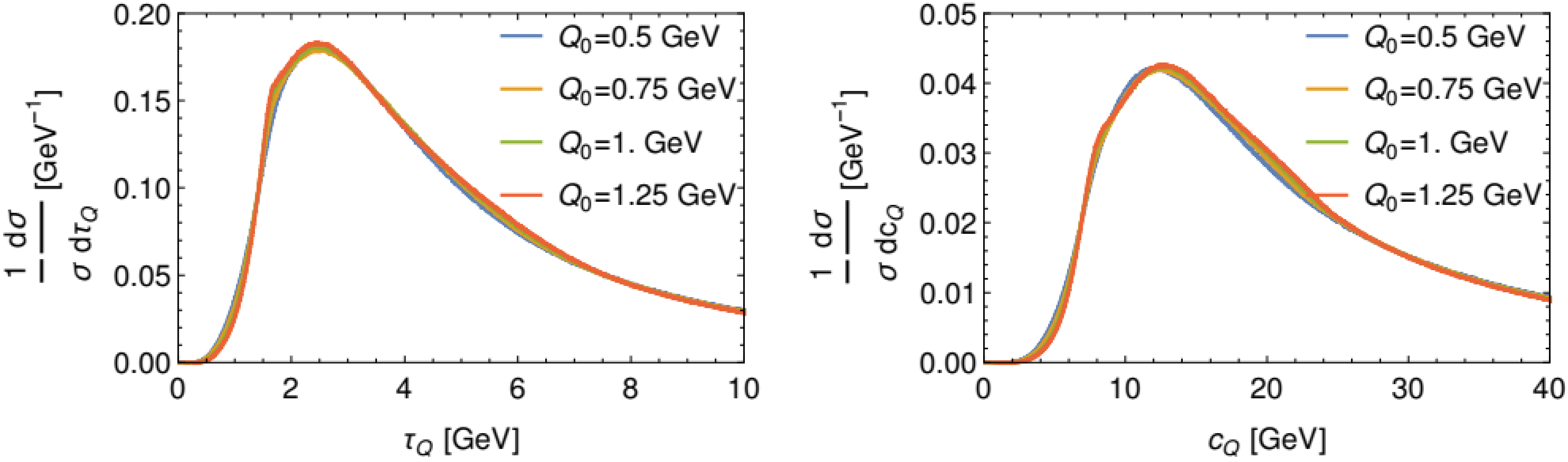
Backup

Hadronic distribution after tuning: default model





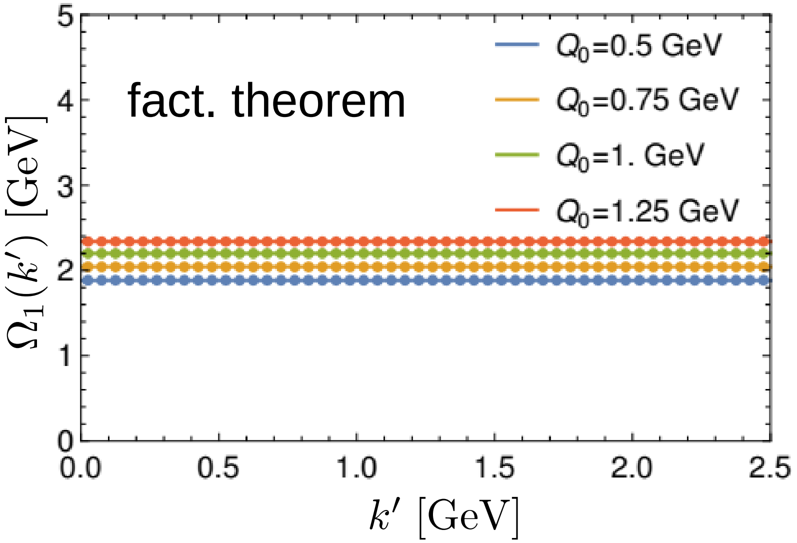
Hadronic distribution after tuning: dynamic model

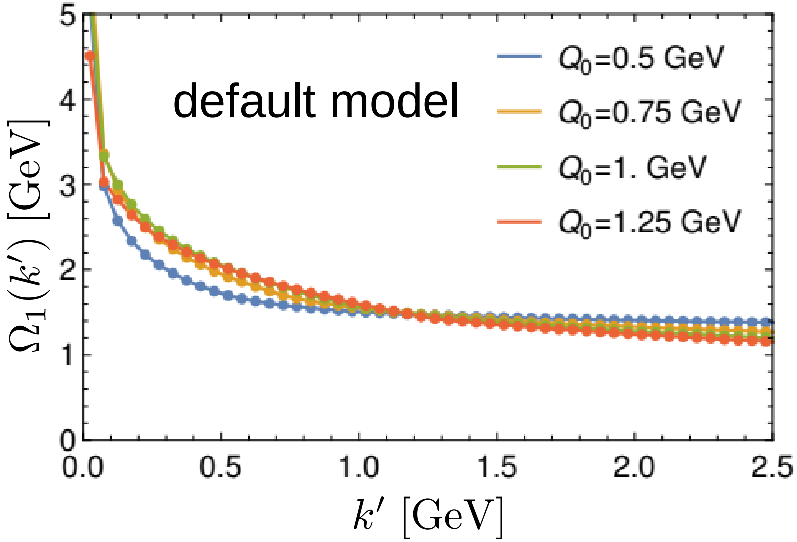


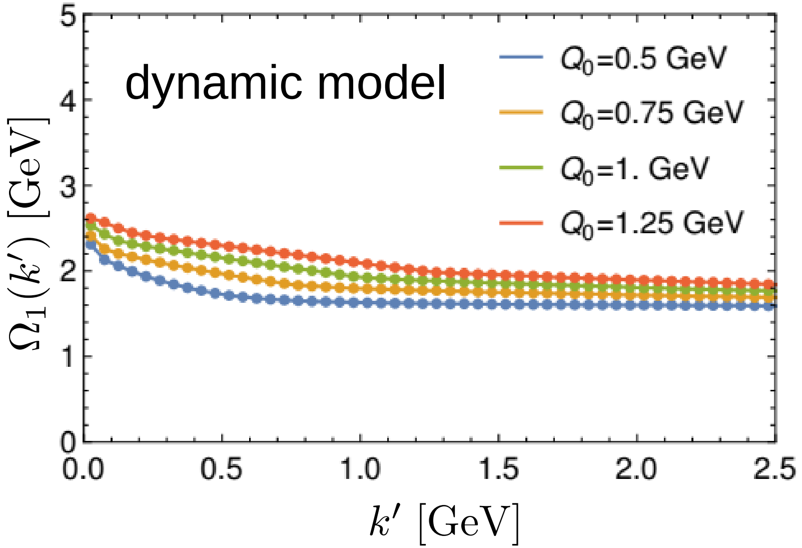
First moments of the hadronization function:

$$\Omega_1(k', Q, Q_0) = \int \mathrm{d}k\, k S_{\mathrm{had}}(k, k', Q, Q_0)$$

$$\frac{\mathrm{d}\Omega_1^{(\tau)}(Q_0)}{\mathrm{d}Q_0} = 16 \frac{\alpha_s(Q_0) C_F}{4\pi}$$







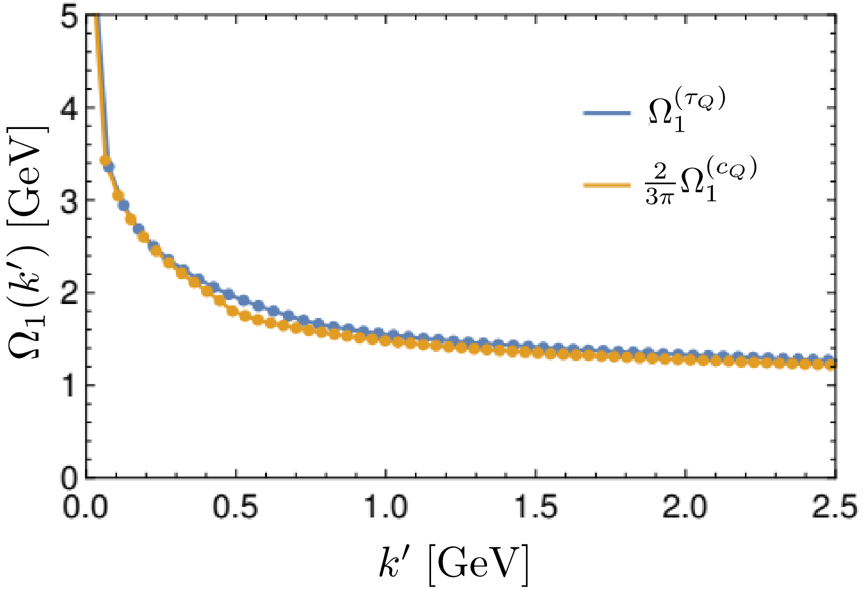
Hadronization function of default model

- Check behavior of first moment

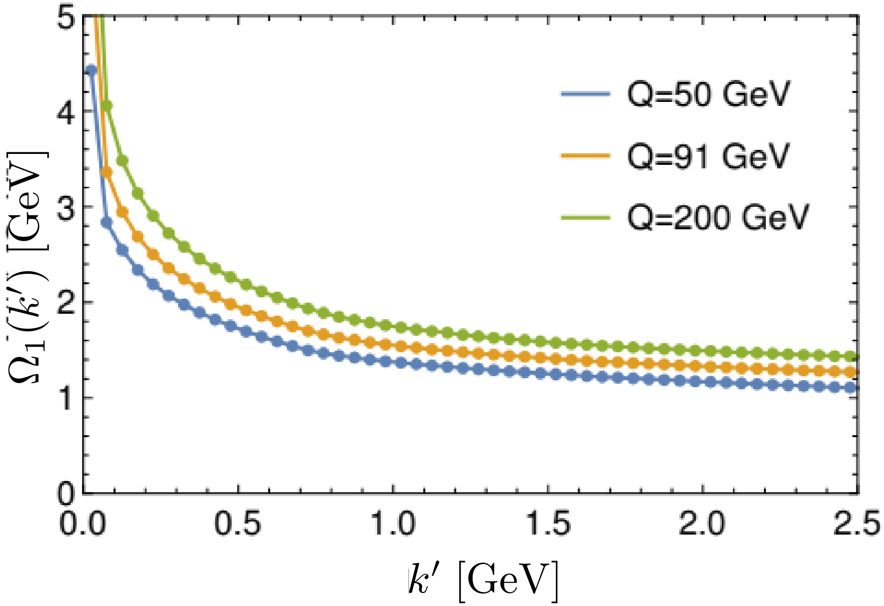
$$\Omega_1(k', Q, Q_0) = \int \mathrm{d}k\, k S_{\mathrm{had}}(k, k', Q, Q_0)$$

- Predictions of factorization theorem:

$$\Omega_1^{(\tau)} = \frac{2}{3\pi}\Omega_1^{(c)}$$

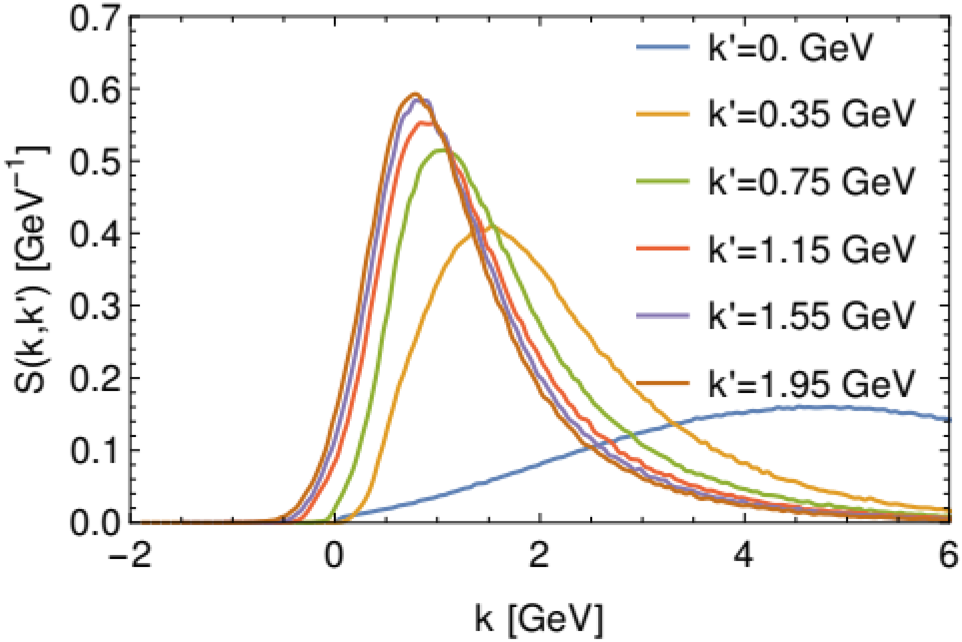


Independent of Q

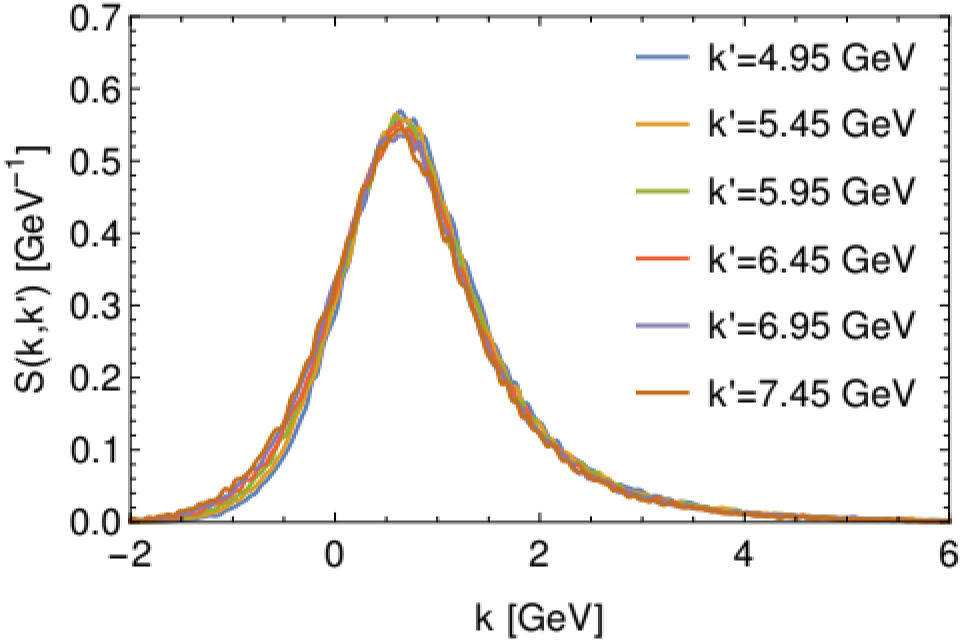


Default Model:

S_{had}(k,k') for fixed k' in the peak

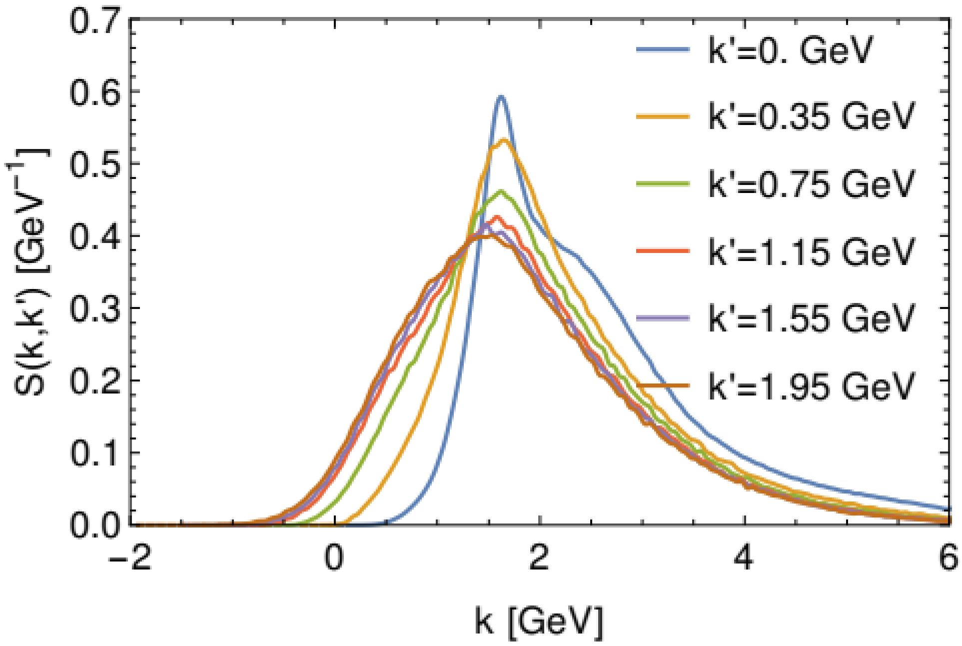


S_{had}(k,k') for fixed k' in the tail

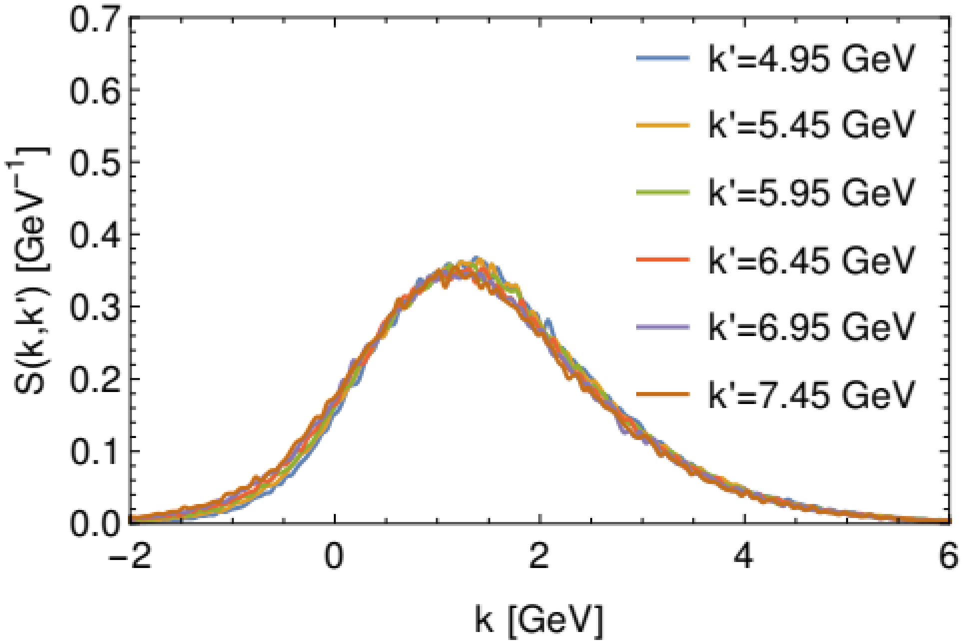


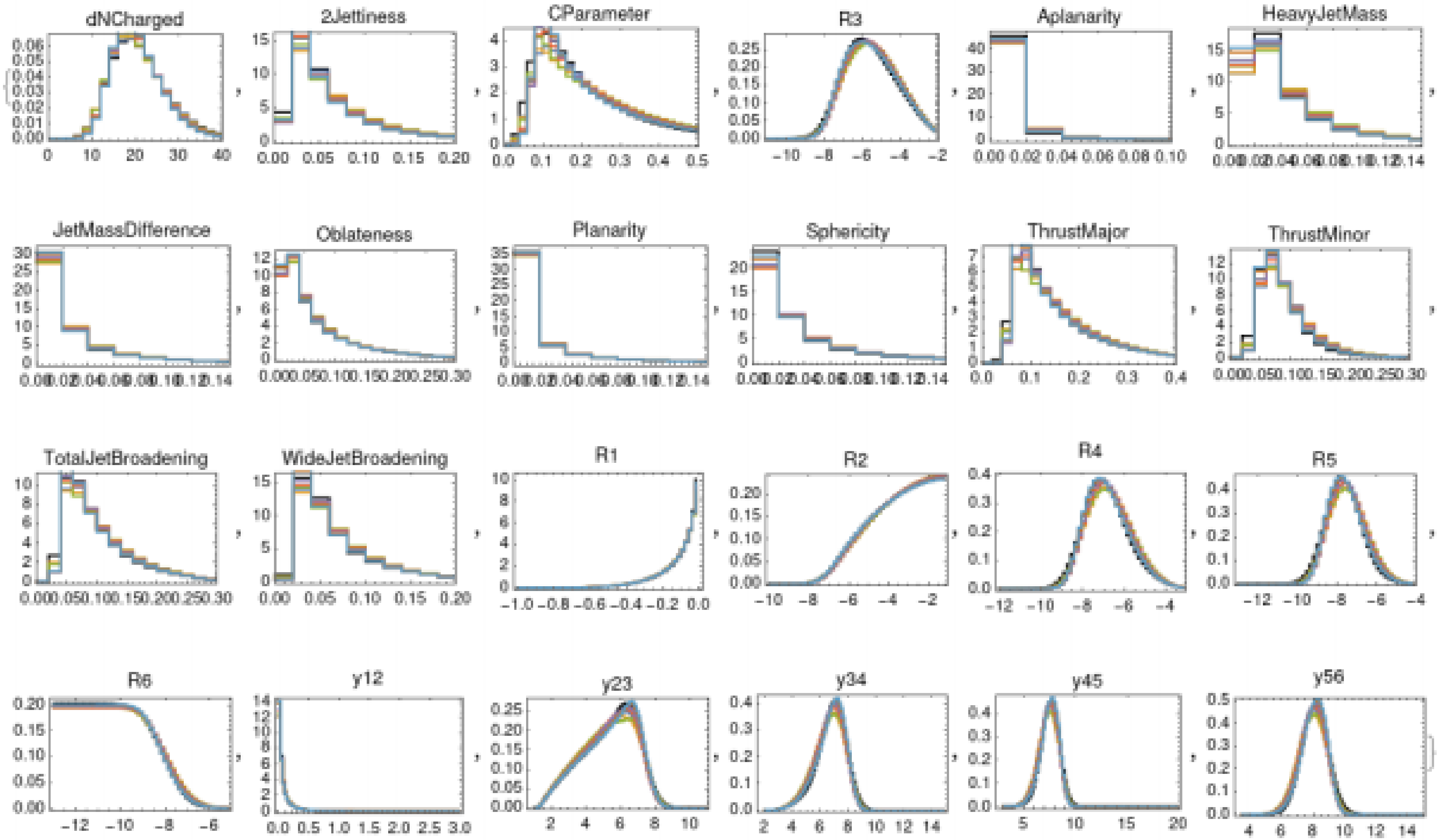
Dynamic Model:

$S_{had}(k,k')$ for fixed k' in the peak

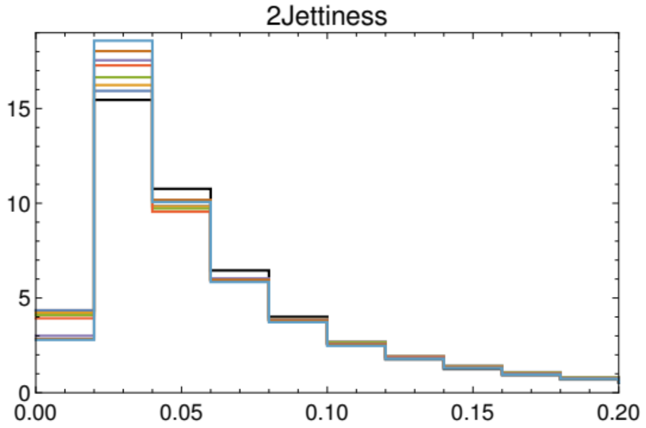


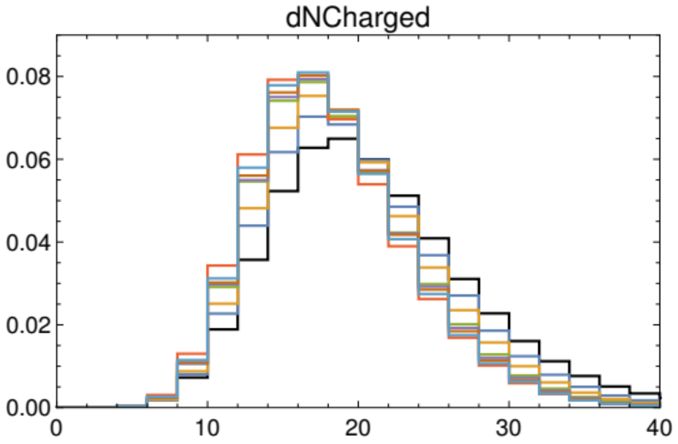
$S_{had}(k,k')$ for fixed k' in the tail



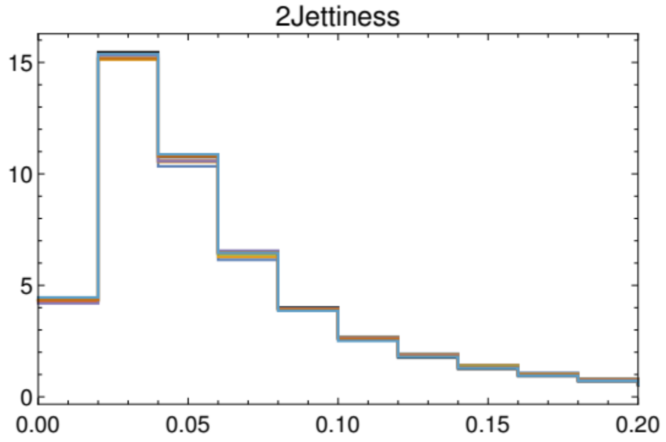

dNCharged
2Jettiness
CParameter
R3
Aplanarity
HeavyJetMass
JetMassDifference
Oblateness
Planarity
Sphericity
ThrustMajor
ThrustMinor
TotalJetBroadening
WideJetBroadening
R1
R2
R4
R5
R6
y12
y23
y34
y45
y56

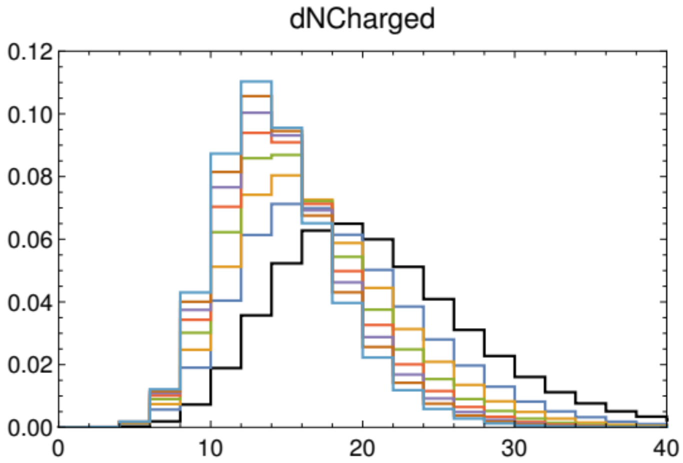
Tuning to all eventshapes (peak):



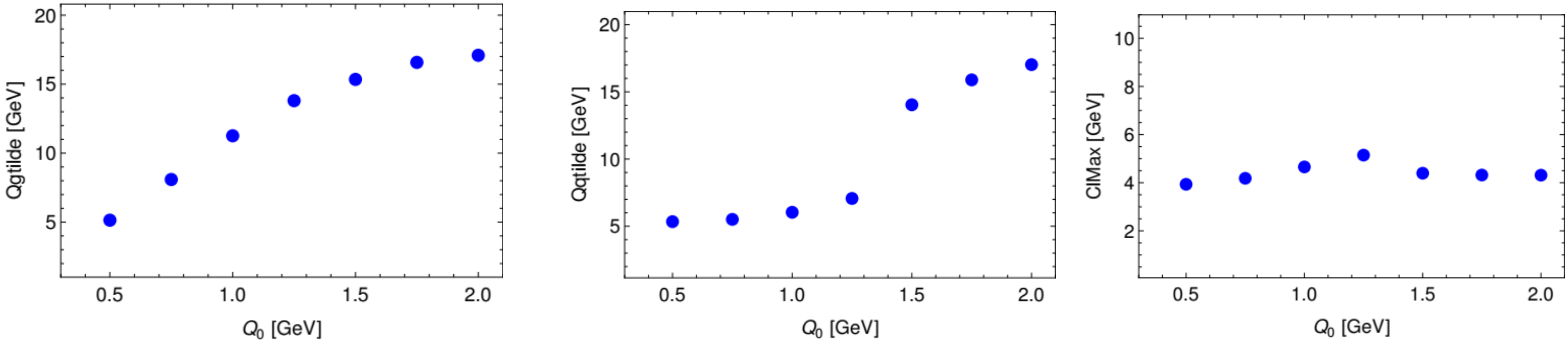


Tuning to thrust, C-parameter and angularities (peak):





Tuning to all eventshapes (peak):



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