RPI-Assisted Renormalization of Subleading SCET Operators

Julian Strohm
TU München

World SCET 2021
23. April 2021

Work with Martin Beneke and Mathias Garny; in Preparation.

Introduction

SCET beyond Leading Power

Unlike most other EFTs, including higher-power corrections in SCET is challenging — even from a conceptual standpoint.

$\longrightarrow$ Breakdown of Factorization, Endpoint-Divergent Convolutions, ...

[Beneke et al., Moult et al., Neubert et al., Stewart et al.]

Even standard renormalization is not completely understood!

Naive calculations yield a subleading anomalous dimension that is incompatible with QCD.

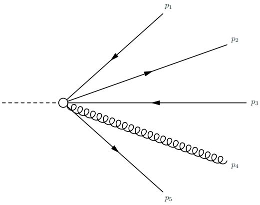
Use **Reparameterization Invariance (RPI)** to reconstruct the proper operator mixing.

$\longrightarrow$ Throughout the talk: Position-Space Formalism, SCET I

N-Jet Operators

N-Jet operators describe hard scattering processes involving N well-separated jets.

$$J(x) = \int \mathrm{d}\mathbf{t}\, C(\mathbf{t}) \prod_{i=1}^{N} J_i(t)$$



Built out of **collinear quark fields** χ_i and **collinear gauge fields** $\mathcal{A}_{\perp}^{\mu}$.

Operator Basis at Subleading Power

A0 Operator $\quad J^{(A0)} = \prod_{j \neq i} \psi_j(t_j n_{j+})\, \chi_i(t_i n_{i+})$

A1 Operator $\quad J^{(A1)\mu} = \prod_{j \neq i} \psi_j(t_j n_{j+})\, i\partial_{\perp_i}^{\mu} \chi_i(t_i n_{i+})$

B1 Operator $\quad J^{(B1)\mu} = \prod_{j \neq i} \psi_j(t_j n_{j+})\, \mathcal{A}_{\perp_i}^{\mu}(t_{i_2} n_{i+})\, \chi_i(t_{i_1} n_{i+})$

T1 Operator $\quad J^{(T1)} = i \int \mathrm{d}^4x\, T\left\{ J^{(A0)}, \mathcal{L}^{(1)}(x) \right\}$

[Beneke, Garny, Szafron, Wang, 1712.04416]

Renormalization of SCET beyond Leading Power

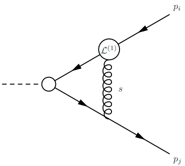
Standard Problem: Renormalize subleading N-jet operators.
$\longrightarrow$ Regulate IR divergences with small off-shellness p_i^2.
$\longrightarrow$ Apparently no major problems.

[Beneke, Garny, Szafron, Wang, 1712.04416,1808.04742]

But: UV poles do not match the IR divergences of QCD!

Only a **specific set of fields** $\hat{\xi}$, $\hat{A}_c$ correctly reproduces QCD amplitudes **off-shell**.

$$\psi \xrightarrow[\text{Collinear Fields}]{\text{Split into Soft \&}} \hat{\xi} + q \xrightarrow{\text{Field Redefinition}} \xi + q$$



[Beneke, Garny, Szafron, Wang, 1907.05463]

Treat SCET as a full-fledged, stand-alone EFT:
Renormalize subleading N-Jet operators without referencing QCD or preferred fields.

Reparameterization Transformations

Reparameterization Transformations

$$\text{I} \quad \begin{aligned} n_{i+} &\longrightarrow n_{i+} \\ n_{i-} &\longrightarrow n_{i-} + \Delta_{\perp_i} \end{aligned} \qquad \text{II} \quad \begin{aligned} n_{i+} &\longrightarrow n_{i+} + \epsilon_{\perp_i} \\ n_{i-} &\longrightarrow n_{i-} \end{aligned} \qquad \text{III} \quad \begin{aligned} n_{i+} &\longrightarrow n_{i+} + \alpha\, n_{i+} \\ n_{i-} &\longrightarrow n_{i-} - \alpha\, n_{i-} \end{aligned}$$

[Manohar et al., 0204229; Marcantonini, Stewart, 0809.1093]

Transformations of the SCET Fields?

Derive the transformations directly in SCET by demanding:

A **The SCET Lagrangian is reparameterization-invariant**.

B **Reparameterizations preserve the projection** $\not{n}_{i-}\chi_i = 0$**.**

$$\begin{aligned} \delta_{\text{I}}\, \chi_i =& \frac{\not{\Delta}_{\perp_i} \not{n}_{i+}}{4} \chi_i + \mathcal{O}\left(\lambda^2 \chi_i\right), \\ \delta_{\text{II}}\, \chi_i =& \frac{1}{i n_{i+}\partial} \frac{\not{\epsilon}_{\perp}}{2} \left(i\not{\partial}_{\perp_i} + \not{\mathcal{A}}_{\perp_i}\right) \chi_i + \left[\frac{1}{i n_{i+}\partial} \epsilon_{\perp_i} \cdot \mathcal{A}_{\perp_i}\right] \chi_i + \mathcal{O}\left(\lambda^2 \chi_i\right), \\ \delta_{\text{III}}\, \chi_i =& 0 \end{aligned}$$

Reparameterization Constraints on the Wilson Coefficients

Ansatz:

$$J(x) = \int \mathrm{d}\mathbf{t}\, C^{(A0)}(t)\, J^{(A0)}(t) + \int \mathrm{d}\mathbf{t}\, C^{(A0)}(t)\, J^{(T1)}(t)$$
$$+ \int \mathrm{d}\mathbf{t}\, C^{(A1)}(t)\, J^{(A1)}(t) + \int \mathrm{d}\mathbf{t}\, C^{(B1)}(t)\, J^{(B1)}(t) + \mathcal{O}\left(\lambda^2 J^{(A0)}\right)$$

Reparameterization Constraints on the Wilson Coefficients of Subleading Operators:

$$C^{(A1)\mu}\left(Q_{ij},\mu^2\right) = \frac{1}{n_{i+}p_i} C^{(A0)}\left(Q_{ij},\mu^2\right) \frac{\not{n}_{i+}}{2}\gamma^{\mu}_{\perp_i}$$
$$- \frac{1}{n_{i+}p_i} \sum_{j\neq i} \left[Q^2_{ij} \frac{\partial}{\partial Q^2_{ij}} C^{(A0)}\left(Q_{ij},\mu^2\right) \right] \frac{2n^{\mu}_{j-}}{n_{i-}n_{j-}}$$

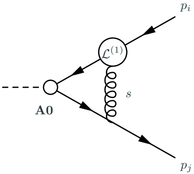
$$C^{(B1)A\mu}\left(Q_{ij},x,\mu^2\right) = \frac{1}{n_{i+}p_i} C^{(A0)}\left(Q_{ij},\mu^2\right) \frac{\not{n}_{i+}}{2}\gamma^{\mu}_{\perp_i} T^A$$
$$+ \frac{1}{n_{i+}p_i} \sum_{j\neq i} \frac{1}{x} C^{(A0)}\left(Q_{ij},\mu^2\right) \frac{2n^{\mu}_{j-}}{n_{i-}n_{j-}} T^A,$$
$$+ \text{(unconstrained)},$$

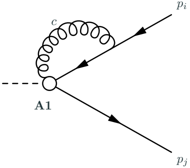
where x denotes the fraction of momentum carried by the additional gluon.

Reconstruction of the Missing Mixing Terms

Subleading Anomalous Dimension

	$J^{(T1)}$	$J^{(A1)}$	$J^{(B1)}$
$J^{(T1)}$	$\mathbf{\Gamma}_{A0A0}$	$\mathbf{\Gamma}_{T1A1}$	$\mathbf{\Gamma}_{T1B1}$
$J^{(A1)}$	$\mathbf{\Gamma}_{A1T1}$	$\mathbf{\Gamma}_{A1A1}$	$\mathbf{\Gamma}_{A1B1}$
$J^{(B1)}$	$\mathbf{\Gamma}_{B1T1}$	$\mathbf{\Gamma}_{B1A1}$	$\mathbf{\Gamma}_{B1B1}$





The field representations differ only at higher power,

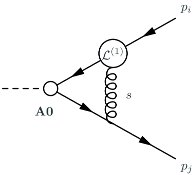
$$\hat{\chi}_i = \chi_i + \lambda F^{(1)} + \lambda^2 F^{(2)} + \ldots$$

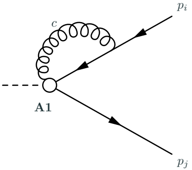
Only the mixing of **time-ordered products** is affected!

Reconstruction of the Missing Mixing Terms

Subleading Anomalous Dimension

	$J^{(T1)}$	$J^{(A1)}$	$J^{(B1)}$
$J^{(T1)}$	$\mathbf{\Gamma}_{A0A0}$	$\mathbf{\Gamma}_{T1A1}$	$\mathbf{\Gamma}_{T1B1}$
$J^{(A1)}$	$\mathbf{\Gamma}_{A1T1}$	$\mathbf{\Gamma}_{A1A1}$	$\mathbf{\Gamma}_{A1B1}$
$J^{(B1)}$	$\mathbf{\Gamma}_{B1T1}$	$\mathbf{\Gamma}_{B1A1}$	$\mathbf{\Gamma}_{B1B1}$



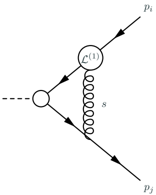


The field representations differ only at higher power,

$$\hat{\chi}_i = \chi_i + \lambda F^{(1)} + \lambda^2 F^{(2)} + \dots$$

Only the mixing of **time-ordered products** is affected!

Reconstruction of the Mixing into $A1$ Operators



The $A1$ Wilson coefficient is **fully constrained** by RPI,

$$C^{(A1)\mu}\left(Q_{ij},\mu^2\right) = \frac{1}{n_{i+}p_i} C^{(A0)}\left(Q_{ij},\mu^2\right) \frac{\not{n}_{i+}}{2}\gamma^\mu_{\perp_i} - \frac{1}{n_{i+}p_i}\sum_{j\neq i}\left[Q_{ij}^2 \frac{\partial}{\partial Q_{ij}^2} C^{(A0)}\left(Q_{ij},\mu^2\right)\right]\frac{2n^\mu_{j-}}{n_{i-}n_{j-}}.$$

RPI constraints hold **independently of the factorization scale** μ,

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} C^{(A1)\mu} = \frac{1}{n_{i+}p_i} \mathbf{\Gamma}_{A0A0} C^{(A0)} \frac{\not{n}_{i+}}{2}\gamma^\mu_{\perp_i} - \frac{1}{n_{i+}p_i}\sum_{j\neq i}\left[Q_{ij}^2 \frac{\partial}{\partial Q_{ij}^2} \mathbf{\Gamma}_{A0A0} C^{(A0)}\right]\frac{2\,n^\mu_{j-}}{n_{i-}n_{j-}}$$

$$= \mathbf{\Gamma}_{T1A1} C^{(T1)} + \mathbf{\Gamma}_{A1A1} C^{(A1)}.$$

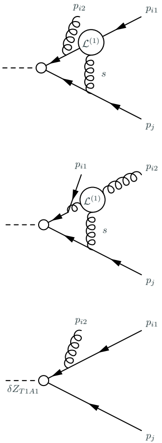
Solve for the missing T1 $\to$ A1 mixing: [Beneke et al., 1907.05463]

$$\mathbf{\Gamma}^\mu_{T1A1} = -\frac{1}{n_{i+}p_i}\gamma_{\text{cusp}}\left(\alpha_s\right)\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j \frac{2\,n^\mu_{j-}}{n_{i-}n_{j-}}$$

Reconstruction of the Missing Mixing Terms

Subleading Anomalous Dimension

	$J^{(T1)}$	$J^{(A1)}$	$J^{(B1)}$
$J^{(T1)}$	$\mathbf{\Gamma}_{A0A0}$	$\mathbf{\Gamma}_{T1A1}$	$\mathbf{\Gamma}_{T1B1}$
$J^{(A1)}$	$\mathbf{\Gamma}_{A1T1}$	$\mathbf{\Gamma}_{A1A1}$	$\mathbf{\Gamma}_{A1B1}$
$J^{(B1)}$	$\mathbf{\Gamma}_{B1T1}$	$\mathbf{\Gamma}_{B1A1}$	$\mathbf{\Gamma}_{B1B1}$



Remaining: Reconstruct T1 $\rightarrow$ B1 Mixing.

But first: Ensure Convolutions and Anomalous Dimension are well-defined!

Mixing into $\mathrm{B1}$ Operators: Endpoint Divergences

Complication 1: Endpoint-Divergent Convolution

$$\int_0^1 \mathrm{d}x \underbrace{C^{(B1)}(x)}_{\sim\frac{1}{x}+\ldots} \left\langle J^{(B1)}(x) \right\rangle$$

But: The singular part of the convolution is **constrained by RPI**,

$$C^{(B1)A\mu}(x) = \frac{1}{n_{i+}p_i} C^{(A0)} \frac{\not{n}_{i+}}{2} \gamma^\mu_{\perp_i} T^A + \frac{1}{n_{i+}p_i} \sum_{j\neq i} \boxed{\frac{1}{x}} C^{(A0)} \frac{2n^\mu_{j-}}{n_{i-}n_{j-}} T^A,$$

$$+ \text{(unconstrained)}.$$

The unconstrained part of the $B1$ Wilson coefficient is regular,

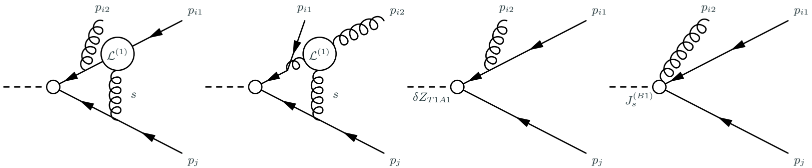
$$C^{(B1)}_{\text{unconstrained}} = 1 + \frac{\alpha_s}{4\pi} C_F \left[\frac{\pi^2}{6} - 3 + \frac{4}{x} \ln \bar{x} + \ln \frac{Q^2}{\mu^2} - \frac{1}{x} \ln^2 \bar{x} - \frac{2}{x} \ln \bar{x} \ln \frac{Q^2}{\mu^2} - \ln^2 \frac{Q^2}{\mu^2} \right]$$
$$+ \frac{\alpha_s}{4\pi} C_A \left[-\frac{2}{x} \ln \bar{x} - \frac{1}{\bar{x}} \ln x + \frac{1}{2x} \ln^2 \bar{x} + \frac{1}{x} \ln \bar{x} \ln \frac{Q^2}{\mu^2} \right].$$

Absorb the endpoint divergence into a **Singular $\mathrm{B1}$ Operator**: [1907.05463]

$$\boxed{J^{(B1)\mu}_s = \prod_{j\neq i} \psi_j(t_j n_{j+}) \left[\boxed{\frac{1}{i n_{i+}\partial}} \mathcal{A}^\mu_{\perp_i}(t_i n_{i+}) \right] \chi_i(t_i n_{i+}).}$$

Mixing into $\mathrm{B1}$ Operators: Non-Local Poles

Complication 2: Non-Local Poles



Non-Local Poles proportional to $\dfrac{1}{\epsilon} \times \dfrac{1}{(p_{i,1}+p_{i,2})^2}$!

Solution: Combine the $T1$ operator and the singular $B1$ operator into $\check{J}$ such that the non-local poles cancel,

$$\check{J} = J^{(T1)} + \sum_{j\neq i} \frac{2n^\mu_{j-}}{n_{i-}n_{j-}} J^{(B1)}_{s\mu}.$$

[Beneke, Garny, Szafron, Wang, 1907.05463]

Reconstruction of the Missing Mixing Terms

Subleading Anomalous Dimension

$$
\begin{array}{c|ccc}
 & \check{J} & J^{(A1)} & J^{(B1)}_{\text{reg}} \\
\hline
\check{J} & \mathbf{\Gamma}_{A0A0} & \mathbf{\Gamma}_{\check{J}\,J^{(A1)}} & \mathbf{\Gamma}_{\check{J}\,J^{(B1)}_{\text{reg}}} \\
J^{(A1)} & \mathbf{\Gamma}_{J^{(A1)}\,\check{J}} & \mathbf{\Gamma}_{J^{(A1)}\,J^{(A1)}} & \mathbf{\Gamma}_{J^{(A1)}\,J^{(B1)}_{\text{reg}}} \\
J^{(B1)}_{\text{reg}} & \mathbf{\Gamma}_{J^{(B1)}_{\text{reg}}\,\check{J}} & \mathbf{\Gamma}_{J^{(B1)}_{\text{reg}}\,J^{(A1)}} & \mathbf{\Gamma}_{J^{(B1)}_{\text{reg}}\,J^{(B1)}_{\text{reg}}}
\end{array}
$$

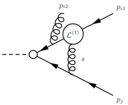
Mixing of $\check{\mathrm{J}}$ into the regular $\mathrm{B1}$ Operator

The $B1$ Wilson coefficient is **only partially constrained** by RPI,

$$C^{(B1)A\mu}_{\text{reg}}(x) = \frac{1}{n_{i+}p_i} C^{(A0)} \frac{\not{n}_{i+}}{2}\gamma^{\mu}_{\perp_i} T^A + (\text{unconstrained}).$$

The $\check{J} \to J^{(B1)}_{\text{reg}}$ mixing thus **cannot be reconstructed** using RPI constraints.

But: $T1$ diagrams with extra collinear emission produce **only non-local** poles!



$$\propto \frac{1}{(p_{i,1}+p_{i,2})^2} \left\langle q\bar{q}g \middle| J^{(B1)}_{\text{reg}} \middle| 0 \right\rangle$$

Only the singular $B1$ operator contributes to the $\check{J} \to J^{(B1)}_{\text{reg}}$ mixing,

$$\Gamma_{\check{J} J^{(B1)\mu}_{\text{reg}}} = \underbrace{\Gamma_{J^{(T1)} J^{(B1)\mu}_{\text{reg}}}}_{=0} + \Gamma_{J^{(B1)}_{s} J^{(B1)\mu}_{\text{reg}}} = \Gamma_{J^{(B1)}_{s} J^{(B1)\mu}_{\text{reg}}}$$

$$= \frac{\alpha_s}{2\pi} \frac{1}{n_{i+}p_i} \left[\left(C_F - \frac{C_A}{2} \right) \frac{\ln \bar{x}}{x} + C_F \right] \sum_{j\neq i} \frac{2 n^{\nu}_{j-}}{n_{i-}n_{j-}} \gamma^{\nu}_{\perp_i} \gamma^{\mu}_{\perp_i}.$$

[Beneke, Garny, Szafron, Wang, 1907.05463]

Summary

Renormalization of Subleading N-Jet Operators

Treat SCET as a full-fledged, stand-alone EFT.

Subleading renormalization can be completely calculated without referencing QCD or a preferred field representation.

I The **RPI transformations** of SCET fields can be obtained from the SCET Lagrangian and their projection properties.

II RPI constraints reconstruct the $\check{J} \to J^{(A1)}$ mixing.

III **Non-Local Poles** and **End-Point Divergences** require a rearrangement of the operator basis.

$$\check{J} = J^{(T1)} + J_s^{(B1)}$$

IV Only the singular $B1$ Operator contributes to the $\check{J} \to J^{(A1)}$ mixing.

Thanks for Your Attention!