NLP soft functions for the Drell-Yan process at threshold

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Outline

- Introduction, summary of the bare factorization formula at NLP

  [Beneke, AB, Garny, Jaskiewicz, Szafron, Vernazza, Wang `18] [Beneke, AB, Jaskiewicz, Vernazza `19]

- Generalized soft functions at NLP accuracy

- Calculation: diagrams, reduction to MIs, evaluation of MIs using DE method

- Integrated results and checks at cross-section level

- Conclusions & Outlook

Work in progress with Sebastian Jaskiewicz and Leonardo Vernazza

Motivation

- Provide insight on higher order perturbative functions in NLP factorization theorems

- Complete the check of our bare NLP factorization formula to NNLO by computing all the perturbative ingredients that are needed

Threshold Kinematics

- Threshold resummation and fixed-order expansions have been applied to many different processes at LP: Drell-Yan [Becher, Neubert, Xu `07], Higgs production [Ahrens, Becher, Neubert, Yang `09], ttbar [Ahrens, Ferroglia, Neubert, Pecjak, Yang `10, `11], ttbar+V [AB, Ferroglia, Pecjak, Ossola, Yang, Signer `15, `16, `17]…DY at NLP: Expansion by regions and LBKD [Bonocore, Laenen, Magnea, Vernazza, White `14, `16] [Bonocore, Laenen, Magnea, Melville, Vernazza, White `15], [Bahjat-Abbas, Sinninghe Damsté, Vernazza, White `18]. LL resummation in SCET [Beneke, AB, Garny, Jaskiewicz, Szafron, Vernazza, Wang `18], Factorization theorem in SCET [Beneke, AB, Jaskiewicz, Vernazza `19], Diagrammatic resummation of threshold effects at NLP [Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, Vernazza, White `19], Generalized threshold kinematics [Lustermans, Michel, Tackmann `19].

$$\frac{d\sigma}{dM^2} \sim \sum_{ab} \int_\tau^1 \frac{dz}{z} ff_{ab}(\tau/z)\hat{\sigma}_{ab}(z)$$

When real radiation is present in the final state $\longrightarrow$ $\hat{s} \neq Q^2$, $z = Q^2/\hat{s} \to 1$

$$\hat{\sigma}_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[\underbrace{c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z}\right]_+}_{\text{LP}} + \underbrace{d_{nm} \ln^m(1-z) \right)}_{\text{NLP}} + \ldots \right]$$

$(1-z)$ expansion

LP factorization & Soft function

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\rm DY}(Q(1-z))$$

[Becher, Neubert, Xu `07]

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi} e^{i\Omega x^0/2} \frac{1}{N_c} \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

NLP Factorization formula & Soft functions

$q\bar{q}$ channel [Beneke,AB,Jaskiewicz,Vernazza `19]

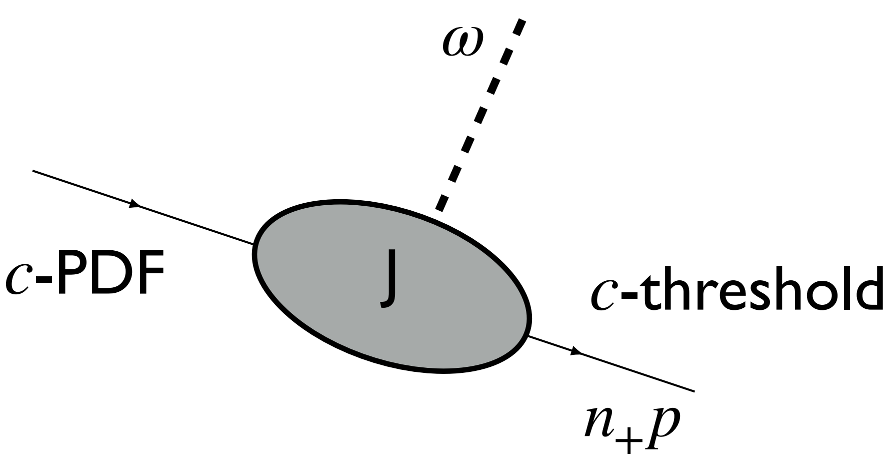
$$\Delta(z) = \frac{1}{(1-\epsilon)}\frac{\hat{\sigma}(z)}{z} \qquad \Delta_{\rm NLP} = \Delta^{kin}_{\rm NLP}(z) + \boxed{\Delta^{dyn}_{\rm NLP}(z)}$$

Kinematic contributions are related to "phase space" corrections to LP factorization formula (LP soft function needed with $(x_0, \vec{x})$ dependence)

$$\Delta^{dyn}_{\rm NLP}(z) = -\frac{2}{(1-\epsilon)}\, Q \left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^{\rho}_{\perp}\right]_{\beta\gamma}$$

$$\times \int d(n_+p)\, C^{A0,A0}(n_+p, x_b n_- p_B)\, C^{*A0A0}(x_a n_+ p_A,\, x_b n_- p_B)$$

$$\times \sum_{i=1}^{5}\int \{d\omega_j\}\, \boxed{J_{i,\gamma\beta}(n_+p, x_a n_+ p_A; \{\omega_j\})}\, \boxed{S_i(\Omega; \{\omega_j\})} + \text{h.c.}\,,$$



Generalized Soft Functions

Collinear functions (contain derivative contribution), calculated up to $\mathcal{O}(\alpha_s)$ in [Beneke,AB,Jaskiewicz,Vernazza `19]

$$S_i(\Omega; \{\omega_j\}) = \int \frac{dx^0}{4\pi}\, e^{i\Omega\, x^0/2} \int \left\{\frac{dz_{j-}}{2\pi}\right\} e^{-i\omega_j z_{j-}}\, S_i(x_0; \{z_{j-}\})$$

At NLP the power suppression is entirely coming from Lagrangian insertions in time ordered products operators

Generalized Soft Functions

- List of the soft functions with a non-zero contribution to the NLP cross section

$$S_1(x^0;z_-) = \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\nu_\perp}(z_-)\right)|0\rangle$$

$$S_3(x^0;z_-) = \frac{1}{N_c}\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]$$
$$\times\,\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]\frac{1}{(in_-\partial)^2}\left[\mathcal{B}^{+\,\mu_\perp}(z_-),\left[in_-\partial\mathcal{B}^+_{\mu_\perp}(z_-)\right]\right]\right)|0\rangle$$

generates all the LL contributions

$$S^{AB}_{4;\mu\nu,bf}(x^0;z_{1-},z_{2-}) = \frac{1}{N_c}\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]_{ba}$$
$$\times\,\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\mathcal{B}^{+A}_{\mu_\perp}(z_{1-})\mathcal{B}^{+B}_{\nu_\perp}(z_{2-})\right)|0\rangle$$

$$S_{5;bfgh,\sigma\lambda}(x^0;z_{1-},z_{2-}) = \frac{1}{N_c}\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]_{ba}$$
$$\times\,\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\frac{g_s^2}{(in_-\partial_{z_1})(in_-\partial_{z_2})}q_{+\sigma g}(z_{1-})\bar{q}_{+\lambda h}(z_{2-})\right)|0\rangle$$

From $\mathscr{L}^{(1)}\ \mathscr{L}^{(1)}$ insertions on the same (collinear) leg

$$\mathcal{B}^\mu_\pm = Y_\pm^\dagger\left[i\,D_s^\mu\,Y_\pm\right]$$
$$q^\pm = Y_\pm^\dagger\,q_s$$

Generalized Soft Functions

- Extract the soft operators matrix elements up to two emissions. At NLO only S_1 contributes. Virtual-real was already computed in [Beneke,AB,Jaskiewicz,Vernazza `19].
- I will mainly focus on S_1 since it is the soft function with the most interesting structure
- S_1 matrix elements with one and two soft emissions

$$\langle g^K(k)|\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\nu_\perp}(z_-)|0\rangle = \mathbf{T}^K\frac{g_s}{(n_-k)}\left[k_\perp^\eta - \frac{k_\perp^2}{(n_-k)}n_-^\eta\right]\epsilon^*_\eta(k)\boxed{e^{iz_-k}}$$

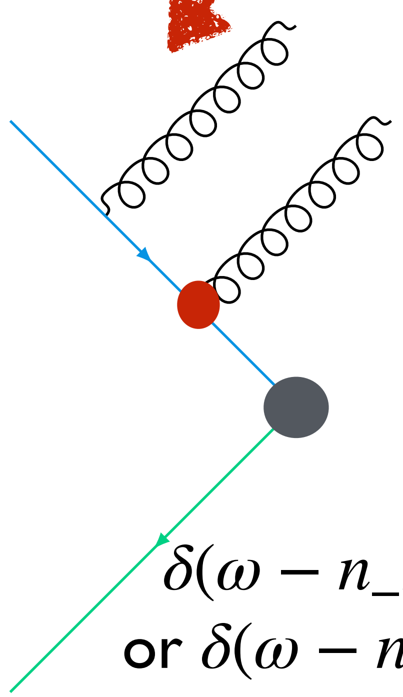
in momentum space this corresponds to $\delta(\omega - n\_k)$

$$\langle g^{K_1}(k_1)g^{K_2}(k_2)|\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_-)\right]|0\rangle =$$

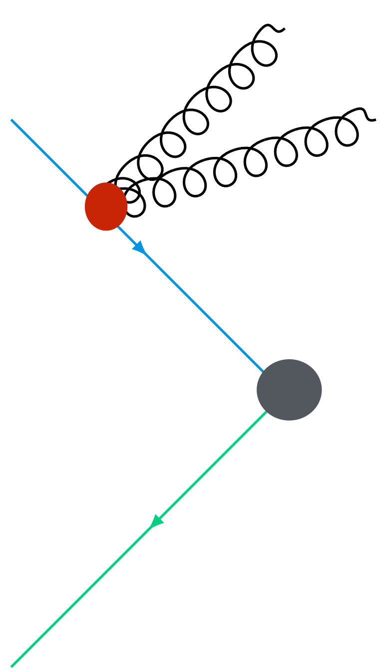
$$g_s^2\mathbf{T}^{K_2}\,\mathbf{T}^{K_1}\frac{1}{(n_-k_1)}\frac{n_-^{\eta_2}}{(n_-k_2)}\left[k_{1\perp}^{\eta_1} - \frac{k_{1\perp}^2}{(n_-k_1)}n_-^{\eta_1}\right]\epsilon^*_{\eta_1}(k_1)\epsilon^*_{\eta_2}(k_2)\boxed{e^{iz_-k_1}}$$

$+$............

One emission from the soft building block and one emission from the Wilson line

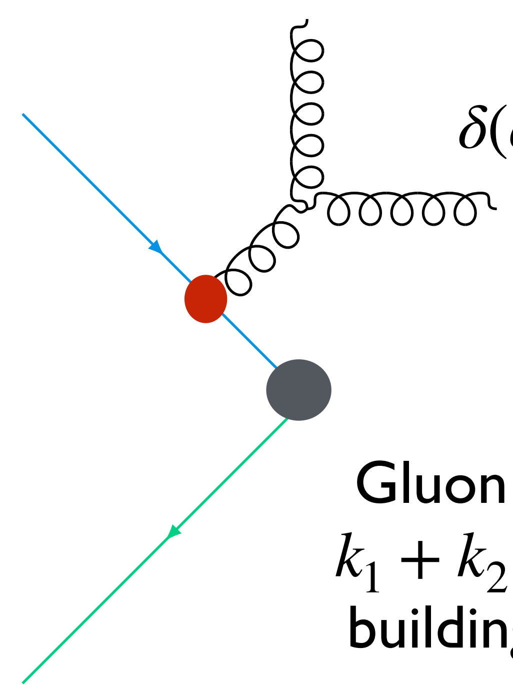


$\delta(\omega - n\_k_1)$
or $\delta(\omega - n\_k_2)$



Both soft gluons generated from soft building block

$\delta(\omega - n\_k_1 - n\_k_2)$



$\delta(\omega - n\_k_1 - n\_k_2)$

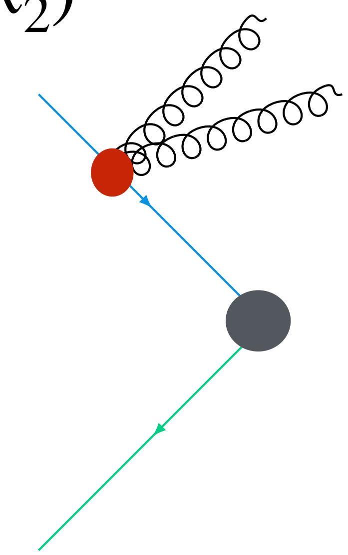
$+\ldots$

Gluon with momentum $k_1 + k_2$ generated by soft building block that splits

Generalized Soft Functions

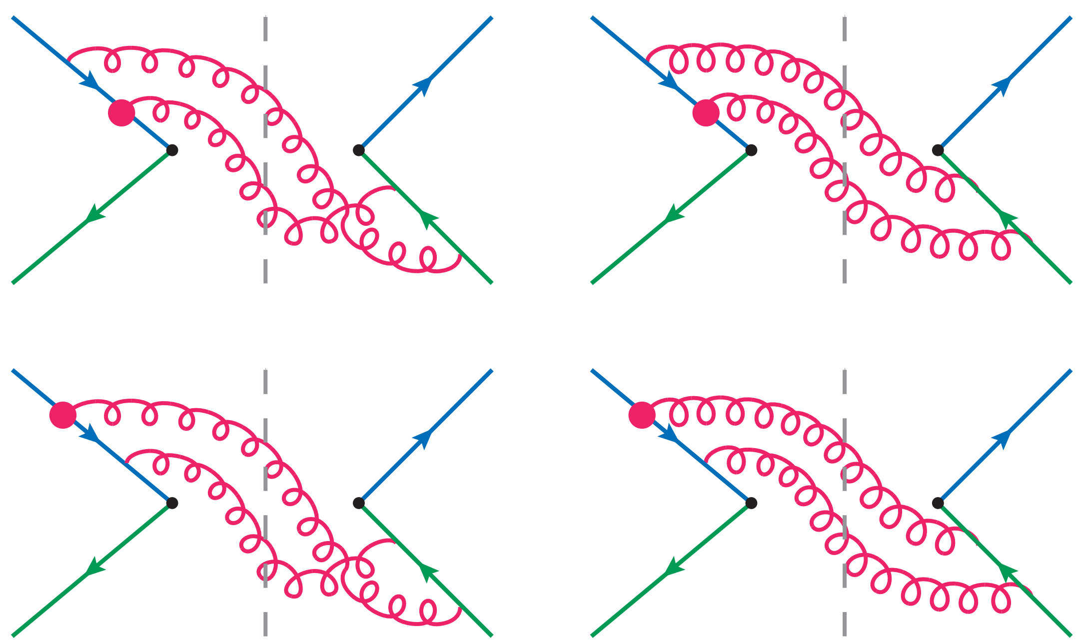
- S_4 and S_5 contribute only to the structure $\delta(\omega_1 - n\_k_1)\,\delta(\omega_2 - n\_k_2)$
- S_3 contributes only $\delta(\omega - n\_k_1 - n\_k_2)$ terms

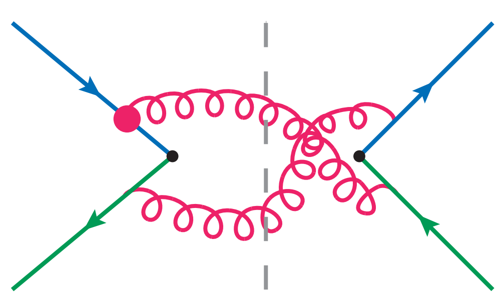
$$\langle g^{K_1}(k_1) g^{K_2}(k_2)| \frac{1}{(in\_\partial)^2} \left[\mathcal{B}^{+\,\mu_\perp}(z_-), \left[in\_\partial \mathcal{B}^{+}_{\mu_\perp}(z_-)\right]\right] |0\rangle \quad \sim$$



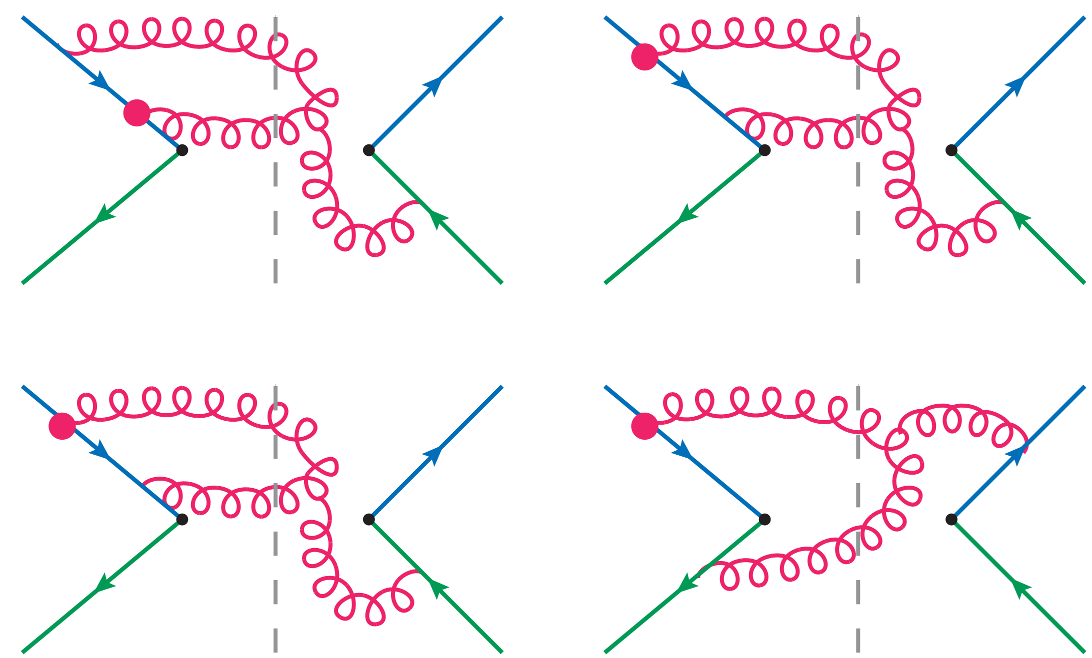
$\times\, e^{i\,z_-(k_1+k_2)}$

- We need to interfere these matrix elements with the LP* amplitude



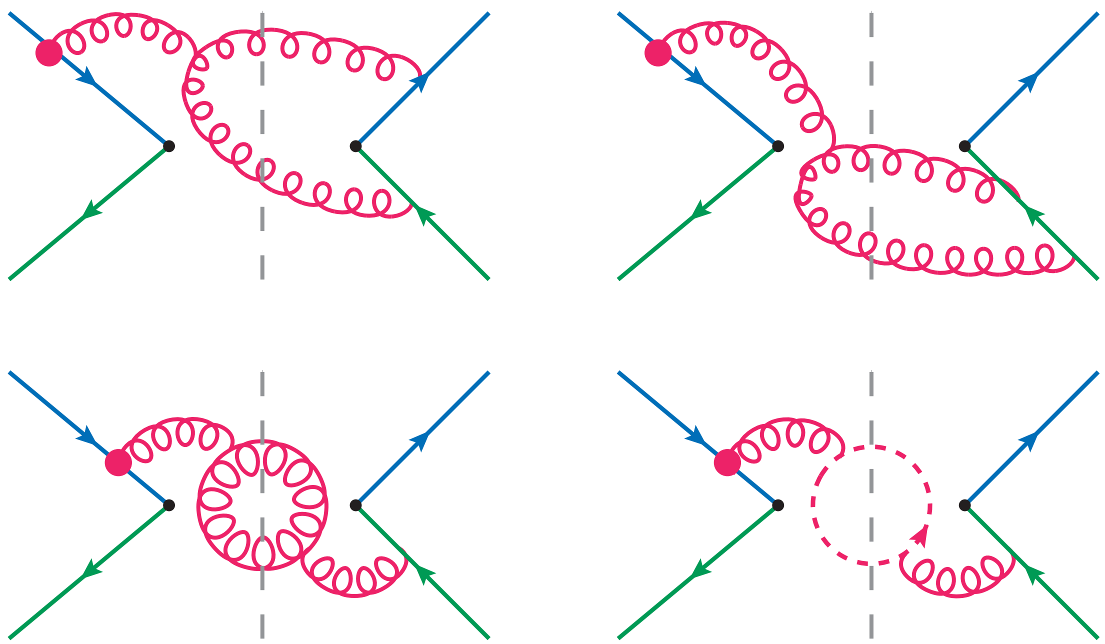


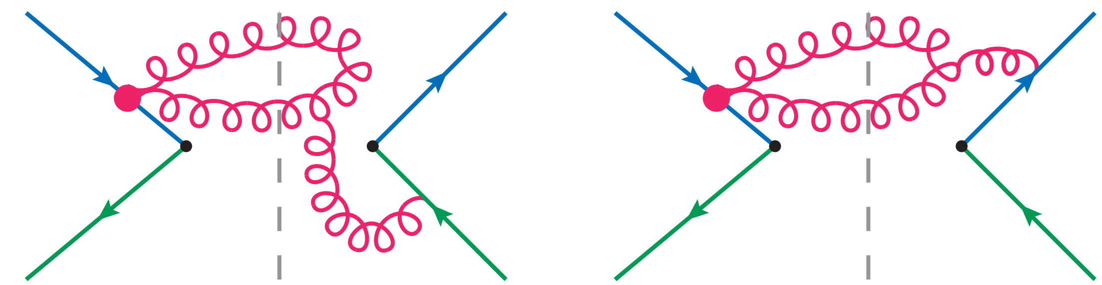
contribute only
to S_1 both to C_F^2
and $C_A C_F$ with $\delta(\omega - n\_k_1)$
or $\delta(\omega - n\_k_2)$ structures



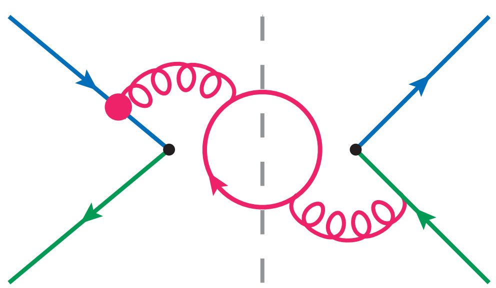
contribute only
to S_1 ($C_A C_F$) with $\delta(\omega - n\_k_1)$
or $\delta(\omega - n\_k_2)$ structures

Generalized Soft Functions

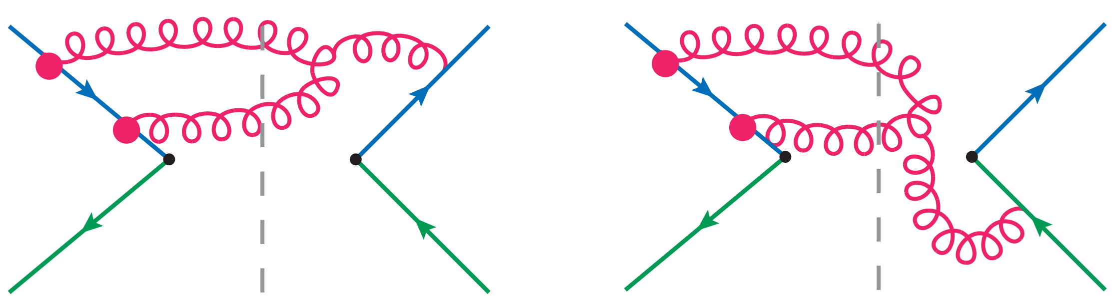




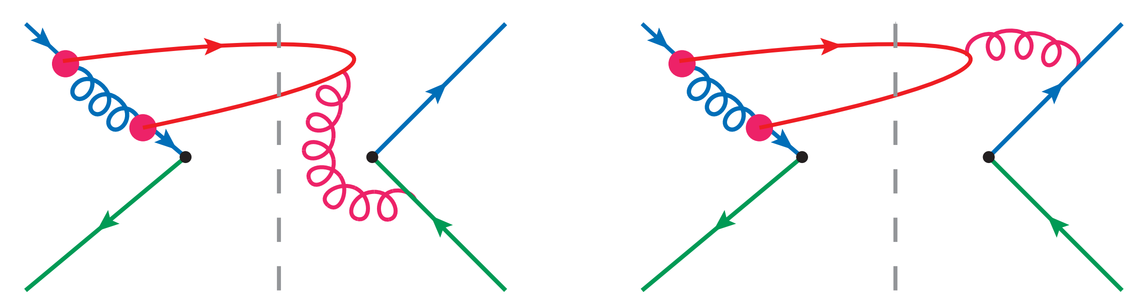
contribute both
to S_1 and S_3 with
$\delta(\omega - n\_k_1 - n\_k_2)$



contribute only
to S_1 with
$\delta(\omega - n\_k_1 - n\_k_2)$



contribute to S_4 with
$\delta(\omega_1 - n\_k_1)\,\delta(\omega_2 - n\_k_2)$



contribute to S_5 with
$\delta(\omega_1 - n\_k_1)\,\delta(\omega_2 - n\_k_2)$

Reduction to Master Integrals

- Soft functions at LP [Li, Mantry, Petriello `11],[Becher, Bell, Marti `12], [Ferroglia, Pecjak,Yang `12]
- 9 auxiliary topologies needed to reduce the soft functions (we used Litered)
- In total 8 MIs are found, 5 of them with the constraint $\delta(\omega - n\_k_1)$, 2 with$\delta(\omega - n_k_1 - n_k_2)$ and one with $\delta(\omega_1 - n\_k_1)\,\delta(\omega_2 - n\_k_2)$
- 3 MIs can be computed by direct integration
- 5 MIs computed with the differential equation method. They all belong to the same topology with the $\delta(\omega - n\_k_1)$ constraint

$$
\begin{aligned}
S_1^{(2)2r0v}(\Omega,\omega)^{\delta(\omega-n\_k_1)} \;&=\; C_F^2\,\frac{8\,(2-9\epsilon+9\epsilon^2)}{\epsilon^2\,\omega\,(\Omega-\omega)^2}\,\hat{\mathrm{I}}_1 \\
&\quad +C_F C_A\Bigg[\frac{(2-3\epsilon)\,(-4\Omega+\epsilon\,(\omega+19\Omega)+4\epsilon^2\,(\omega-7\Omega)-16\epsilon^3(\omega-\Omega))}{\epsilon^2(1-2\epsilon)\,\omega\,\Omega\,(\Omega-\omega)^2}\,\hat{\mathrm{I}}_1 \\
&\qquad -\frac{(1-4\epsilon^2)}{\epsilon\,\omega\,\Omega}\hat{\mathrm{I}}_2+\frac{(3\Omega-10\epsilon\,\Omega+16\epsilon^2(\omega+\Omega))}{2(1-2\epsilon)\,\omega\,\Omega}\hat{\mathrm{I}}_3 \\
&\qquad +\frac{(\Omega-3\omega)}{2\omega}\,\hat{\mathrm{I}}_4\;+\;\Omega\,\hat{\mathrm{I}}_5\Bigg] \\
S_1^{(2)2r0v}(\Omega,\omega)^{\delta(\omega-n\_k_1-n\_k_2)} \;&=\; C_F C_A\left[\frac{9-20\epsilon+12\epsilon^2-2\epsilon^3}{\epsilon^2\,(3-2\epsilon)\omega^2(\Omega-\omega)}\hat{\mathrm{I}}_6+(\Omega-\omega)\hat{\mathrm{I}}_7\right] \\
&\quad -C_F\,n_f\,\frac{4(1-\epsilon)^2}{\epsilon\,(3-2\epsilon)\omega^2(\Omega-\omega)}\,\hat{\mathrm{I}}_6
\end{aligned}
$$

Master Integrals

Let's focus on this particular auxiliary topology

$$P_1 = (k_1+k_2)^2, \quad P_2 = n_+ k_2, \quad P_3 = n_-(k_1+k_2)$$

$$\boxed{P_4 = k_1^2, \quad P_5 = k_2^2, \quad P_6 = \left(\Omega - n_- k_1 - n_- k_2 - n_+ k_1 - n_+ k_2\right), \quad P_7 = \left(\omega - n_- k_1\right)}$$

Cut prop.

[Anastasiou, Melnikov `02] $\delta(k_1^2) = \frac{1}{2\pi i}\left[\frac{1}{k_1^2 + i0^+} - \frac{1}{k_1^2 - i0^+}\right]$

Implement the constraints

$$\hat{I}(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7) = (4\pi)^4 \left(\frac{e^{\gamma_E}\mu^2}{4\pi}\right)^{2\epsilon} \int \frac{d^dk_1}{(2\pi)^{d-1}} \frac{d^dk_2}{(2\pi)^{d-1}} \prod_{i=1}^{7} \frac{1}{P_i^{\alpha_i}}$$

5 MIs found, it is convenient to make the variable change $\omega \to r\,\Omega$ and redefine the MIs

$$I_1'(r) = \frac{1}{\Omega^2}\left(\frac{\Omega}{\mu}\right)^{4\epsilon} \hat{I}_1(\Omega, r) \qquad I_2'(r) = \frac{1}{\Omega}\left(\frac{\Omega}{\mu}\right)^{4\epsilon} \hat{I}_2(\Omega, r)$$

$$I_3'(r) = \left(\frac{\Omega}{\mu}\right)^{4\epsilon} \hat{I}_3(\Omega, r) \qquad I_4'(r) = \Omega\left(\frac{\Omega}{\mu}\right)^{4\epsilon} \hat{I}_4(\Omega, r)$$

$$I_5'(r) = \Omega^2\left(\frac{\Omega}{\mu}\right)^{4\epsilon} \hat{I}_5(\Omega, r)$$

Ω dependence is trivial, non-trivial dependence only on variable r

Master Integrals

The system of DEs can be put in canonical form (exclude $I_2'(r)$ part of a subsystem with $I_1'(r)$)

$$\frac{d\vec{I}(r)}{dr} = \epsilon\, A(r) \cdot \vec{I}(r)$$

$$\begin{aligned}
I_1'(r) &= \frac{2(1-r)^2}{2-9\epsilon+9\epsilon^2} I_1(r) \\
I_3'(r) &= \frac{1}{\epsilon^2} I_3(r) \\
I_4'(r) &= -\frac{1}{\epsilon^2(1-r)} I_4(r) \\
I_5'(r) &= -\frac{1+r}{2\epsilon^2(1-r)r} I_4(r) + \frac{1}{\epsilon^2 r} I_5(r)
\end{aligned}$$

$$A(r) = \begin{bmatrix} -\frac{1}{r}+\frac{3}{1-r} & 0 & 0 & 0 \\ \frac{2}{r} & -\frac{2}{r} & 0 & 0 \\ \frac{2}{r} & \frac{2}{r} & \frac{4}{1-r} & 0 \\ \frac{1}{r} & \frac{1}{r} & \frac{1}{r} & -\frac{2}{r} \end{bmatrix}$$

- Integral $I_1(r)$ has to be computed by direct integration
- $I_3(r)$ is obtained in d-dimensions via DE
- $I_4(r)$ is more complicated but it can still be computed in exact d-dimensions using an integral representation for ${}_2F_1$
- $I_5(r)$ is too complicated (integral over ${}_3F_2$) we need to expand it in $\epsilon = (4-d)/2 \to 0$
- Fixing of the integration constants done by looking at specific limits in r or by comparing to the integrated version of these integrals (easy to obtain even in d dimensions)

Master Integrals

Preliminary

$$\frac{dI_5(r)}{dr} = \epsilon\left(\frac{1}{r}I_1(r) + \frac{1}{r}I_3(r) + \frac{1}{r}I_4(r) - \frac{2}{r}I_5(r)\right)$$

$$I_5(r) = r^{-2\epsilon}\left(C_5(\epsilon) + \int_1^r dr'\, f_{I_5}(r',\epsilon)\right)$$

structure of the solution for $I_5(r)$

Non-canonical $I_5'(r)$

$$\begin{aligned}
I_5'(r) = &-\frac{\delta(1-r)+\delta(r)}{2\epsilon^3} + \frac{1}{\epsilon^2}\left(2\left[\frac{1}{1-r}\right]_+ + \left[\frac{1}{r}\right]_+\right) + \frac{1}{12\epsilon}\Bigg(5\pi^2\delta(1-r) - \pi^2\delta(r) \\
&- 96\left[\frac{\ln(1-r)}{1-r}\right]_+ - 24\left[\frac{\ln r}{r}\right]_+ - \frac{48\ln(1-r)}{r} - \frac{12\ln r}{1-r}\Bigg) \\
&+ \frac{\zeta_3}{3}\big(28\delta(1-r) - 5\delta(r)\big) - \frac{5\pi^2}{3}\left[\frac{1}{1-r}\right]_+ + \frac{\pi^2}{6}\left[\frac{1}{r}\right]_+ + 32\left[\frac{\ln^2(1-r)}{1-r}\right]_+ + 4\left[\frac{\ln^2 r}{r}\right]_+ \\
&+ 8\frac{\ln^2(1-r)}{r} + \frac{2(1+r)}{r(1-r)}\ln(1-r)\ln(r) + \frac{\ln^2 r}{2(r-1)} - \frac{7\pi^2}{6} \\
&+ \frac{(6-7r)}{(r-1)r}\left(\mathrm{Li}_2(r) - \frac{\pi^2 r}{6}\right) + \mathcal{O}(\epsilon)
\end{aligned}$$

We can directly compute the integrated version of this integrals (without $\delta(\omega - n\_k_1)$ constraint) and compare to the integral of $I_5'(r)$ over r in $[0,1]$ and we find agreement

Cross section at NNLO

By combining these result with the LO collinear functions and integrating over ωs (equivalently r) we get the contributions to the cross section. We set $\Omega = Q(1-z)$ and $\mu = Q$

$$\Delta^{dyn\,(2)}_{\mathrm{NLP}-soft,S_i} \sim H^0 \int d\omega\, J_i^{(0)}\big(x_a(n_+p_A);\omega\big)\, S_i^{(2)}(\Omega,\omega)$$

$$\Delta^{dyn\,(2)1r1v}_{\mathrm{NLP-soft},S_{1,C_FC_A}}(z) = \frac{\alpha_s^2}{(4\pi)^2} C_F C_A \left(\boxed{-\frac{8}{\epsilon^3}} + \frac{32\ln(1-z)}{\epsilon^2} - \frac{64\ln^2(1-z)}{\epsilon} + \frac{28\pi^2}{3\epsilon} \right.$$
$$\left. \boxed{+\frac{256}{3}\ln^3(1-z)} - \frac{112}{3}\pi^2\ln(1-z) + \frac{448\zeta(3)}{3} + \mathcal{O}(\epsilon)\right)$$

$$\Delta^{dyn\,(2)2r0v}_{\mathrm{NLP-soft},S_{1,C_FC_A}}(z) = \frac{\alpha_s^2}{(4\pi)^2} C_F C_A \left(\boxed{\frac{8}{\epsilon^3}} - \frac{4}{3\epsilon^2}(24\ln(1-z) - 11) \right.$$
$$-\frac{16}{9\epsilon}\left(-36\ln^2(1-z) + 33\ln(1-z) + 6\pi^2 - 16\right)$$
$$\boxed{-\frac{256}{3}\ln^3(1-z)} + \frac{352}{3}\ln^2(1-z) + \frac{128}{3}\pi^2\ln(1-z)$$
$$\left. -\frac{1024}{9}\ln(1-z) - \frac{616\zeta(3)}{3} - \frac{154\pi^2}{9} + \frac{1484}{27} + \mathcal{O}(\epsilon)\right)$$

Cancellation of LL between virtual-real and double real in the $C_F C_A$ term. LL C_F^2 only

Cross section at NNLO

$$\Delta^{dyn\,(2)}_{\text{NLP}-soft,\,S_i} \sim H^0 \int d\omega\, J_i^{(0)}\big(x_a(n_+p_A);\omega\big)\, S_i^{(2)}(\Omega,\omega)$$

$$\Delta^{dyn\,(2)2r0v}_{\text{NLP}-\text{soft},S_{1,C_F^2}}(z) = \frac{\alpha_s^2}{(4\pi)^2} C_F^2 \bigg(\frac{32}{\epsilon^3} - \frac{128}{\epsilon^2}\ln(1-z) + \frac{256}{\epsilon}\ln^2(1-z) - \frac{112\pi^2}{3\epsilon} + \frac{32}{3}\left(-32\ln^3(1-z) + 14\pi^2\ln(1-z) - 62\zeta(3)\right) + \mathcal{O}(\epsilon) \bigg)$$

There is also a contribution to S_1 with $C_F n_f$ structure

S_3 and S_4 cancel each other exactly at NNLO

$$\Delta^{dyn\,(2)2r0v}_{\text{NLP}-\text{soft},S_3}(z) = 4\frac{\alpha_s^2}{(4\pi)^2} C_F C_A \left(\frac{\Omega^4}{\mu^4}\right)^{-\epsilon} \frac{1}{\epsilon} \frac{(1-\epsilon)}{(1-2\epsilon)^2(3-2\epsilon)} \frac{e^{2\epsilon\gamma_E}\Gamma[1-\epsilon]^2}{\Gamma[1-4\epsilon]}$$

$$\Delta^{dyn\,(2)2r0v}_{\text{NLP}-\text{soft},S_4}(z) = \boxed{-4\frac{\alpha_s^2}{(4\pi)^2} C_F C_A \left(\frac{\Omega^4}{\mu^4}\right)^{-\epsilon} \frac{1}{\epsilon} \frac{(1-\epsilon)}{(1-2\epsilon)^2(3-2\epsilon)} \frac{e^{2\epsilon\gamma_E}\Gamma[1-\epsilon]^2}{\Gamma[1-4\epsilon]}}$$

S_5 contribution (soft-quarks)

$$\Delta^{dyn\,(2)2r0v}_{\text{NLP}-\text{soft},S_5}(z) = \frac{\alpha_s^2}{(4\pi)^2}\left(C_F^2 - \frac{1}{2}C_F C_A\right)\left(\frac{8}{\epsilon} - 32\ln(1-z) + 24 + \mathcal{O}(\epsilon)\right)$$

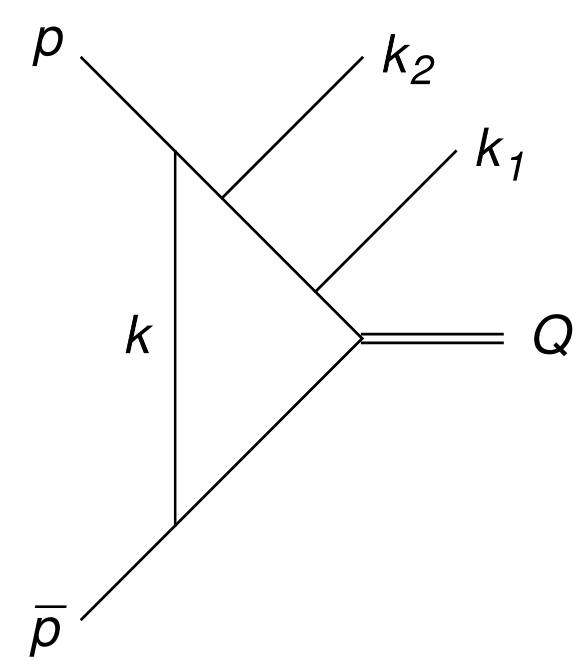
In the end only S_1 and S_5 contribute to the NNLO cross section

Checks

- We calculated the contribution to the cross section directly starting from SCET Feynman rules at subleading power (before decoupling transformation) and we obtained the same result for the cross section

- Our results reproduce the expansion by region calculation

- We can convolute with the $\mathcal{O}(\alpha_s)$ collinear functions and compare with the C_F^3 term at N^{3}LO from expansion by regions [Bahjat-Abbas, Sinninghe Damsté, Vernazza, White `18] with a collinear loop and two real soft emission. We find agreement



Conclusions & Outlook

- We calculated the soft functions appearing in the NLP factorization formula for the Drell-Yan process at threshold, they depend on Ω and on convolution variables ω or ω_1, ω_2

- After integration over the ω's we find agreement at NNLO with the expansion by region method at the cross section level and with the calculation carried out starting directly from the SCET Feynman rules at subleading power

- After convolution with the $\mathcal{O}(\alpha_s)$ collinear functions we find agreement with the C_F^3 term at N^{3}LO from expansion by regions [Bahjat-Abbas, Sinninghe Damsté, Vernazza, White `18]

- Provide useful information for the divergent convolution problem in our case

- Still to do: compare at NNLO to the [Hamberg, van Neerven, Matsuura `90] result for the cross section

Thank you!