Factorization of the Transverse Momentum Distribution in Semi-Inclusive DIS at Subleading Power

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work in progress

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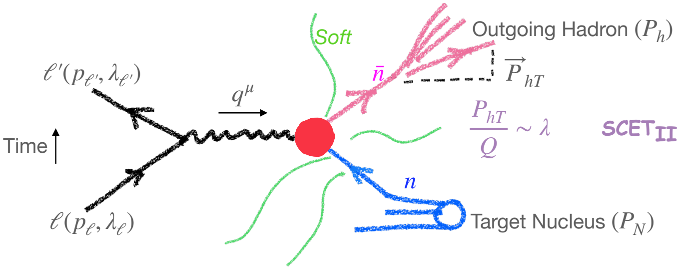
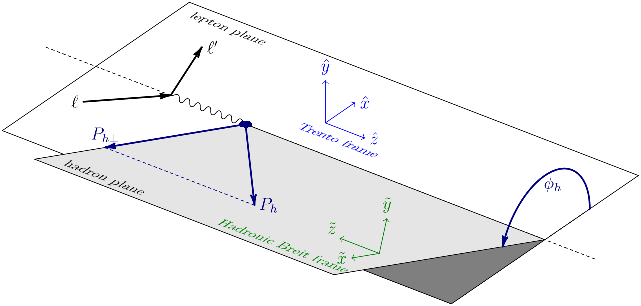
Basics



- Only consider one photon exchange and unpolarized proton in this talk
- Lorentz invariants $Q = \sqrt{-q^2}\,, \quad x = \frac{Q^2}{2P_N \cdot q}\,, \quad y = \frac{P_N \cdot q}{P_N \cdot p_\ell}\,, \quad z = \frac{P_N \cdot P_h}{P_N \cdot q}$
- $$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4}\frac{y}{z} L_{\mu\nu}(p_\ell, p_{\ell'}) W^{\mu\nu}(q, P_N, P_h)$$
- $$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q} \, \langle N | J^{\dagger\,\mu}(b) | h, X\rangle \, \langle h, X | J^\nu(0) | N\rangle$$
$$\begin{aligned} L^{\mu\nu}(p_\ell, p_{\ell'}) &= \langle \ell | J_e^{\dagger\mu} | \ell' \rangle \, \langle \ell' | J_e^\nu | \ell \rangle \\ &= 2\delta_{\lambda_\ell \lambda_{\ell'}} \left[\left(p_\ell^\mu p_{\ell'}^\nu + p_\ell^\nu p_{\ell'}^\mu - p_\ell \cdot p_{\ell'}\, g^{\mu\nu} \right) + \mathrm{i}\lambda_\ell\, \epsilon^{\mu\nu\rho\sigma} p_{\ell\rho} p_{\ell'\sigma} \right] \end{aligned}$$

Kinematics and Reference Frames

- Trento frame: used for experimental analysis [Bacchetta et al '04], target at rest, $\hat{x}$ on the lepton plane
- Hadronic Breit frame: $q^\mu = (0, 0, 0, -Q)_B$, $\vec{P}_{hT} = (P_{hT}, 0)_B$
- Factorization frame (not in the figure):
  $$P_N^\mu = P_N^- \frac{n^\mu}{2}\,, \quad P_h^\mu = P_h^+ \frac{\bar{n}^\mu}{2}\,, \quad \vec{q}_T = (-q_T, 0)_F$$
- $g_{F\perp}^{\mu\nu} = g_{B\perp}^{\mu\nu} + \mathcal{O}(\lambda)$



Tensor Decomposition and Motivation

- Standard structure function decomposition [Bacchetta et al '06]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\delta_{\lambda_\ell\lambda_{\ell'}}}{1-\epsilon}\Big[\big(W_{-1}+\epsilon W_0\big)+\epsilon\cos(2\phi_h)W_3 + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h W_1 + \lambda\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h W_2\Big]\,.$$

- $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$
- $W_i = P_i^{\mu\nu}W_{\mu\nu}$ with projectors $P_i^{\mu\nu}$
- $P_{-1}^{\mu\nu} = (\tilde{x}^\mu\tilde{x}^\nu + \tilde{y}^\mu\tilde{y}^\nu)\,,\quad P_3^{\mu\nu} = \tilde{x}^\mu\tilde{x}^\nu - \tilde{y}^\mu\tilde{y}^\nu\,,$

  $P_1^{\mu\nu} = -(\tilde{t}^\mu\tilde{x}^\nu + \tilde{x}^\mu\tilde{t}^\nu)\,,\quad P_2^{\mu\nu} = \mathrm{i}\big(\tilde{t}^\mu\tilde{x}^\nu - \tilde{x}^\mu\tilde{t}^\nu\big)\,,\quad P_0^{\mu\nu} = \tilde{t}^\mu\tilde{t}^\nu\,,$
- $W_{-1}\,,\,W_3 \sim \mathcal{O}(\lambda^0)$, standard factorization theorems (CSS, SCET)
- $W_1\,,\,W_2 \sim \mathcal{O}(\lambda)$
  - First treated in parton model (tree level matching) [Mulders, Tangerman '95]
  - Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - Conjecture: Resolved by adding a soft function [Bacchetta et al '19]

⇒ *Use SCET to derive all-order factorization at subleading power*

Factorization: the General Procedure

$$W^{\mu\nu}(q, P_N, P_h) = \sum\!\!\!\!\!\!\int_X \int \frac{d^4 b}{(2\pi)^4} e^{ib\cdot q} \langle N| J^{\dagger\,\mu}(b) |h, X\rangle \langle h, X| J^\nu(0) |N\rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at subleading power

- Match SCET currents onto QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \dots$
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $\quad W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand projectors in the factorization frame $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \dots$

Categories of power corrections

1) Subleading operator contributions, $P_i^{(0)\mu\nu} W_{\mu\nu}^{(1)}$
2) Kinematic correction, $P_i^{(1)\mu\nu} W_{\mu\nu}^{(0)}$
3) SCET_{II} Subleading Lagrangian (not treated here) $P_i^{(0)\mu\nu} W_{\mathcal{L}}^{(1)\mu\nu}$:
   $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1/2)} + \mathcal{L}^{(1)} + \dots$ see e.g. [Moult et al '17]

- Assumption: Glauber Lagrangian $\mathcal{L}_G^{(0)}$ doesn't spoil factorization

Factorization at leading power

Leading power current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta} C_f^{(0)}(Q)\, \bar{\chi}^\alpha_{\bar{n},\omega_b} [S^\dagger_{\bar{n}} S_n]\, \chi^\beta_{n,\omega_a}$

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat{\Phi}_f^{\beta'\beta}(x, \vec{b}_T) = \Big\langle N \Big| \bar{\chi}_n^\beta(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi_n^{\beta'}(0) \Big| N \Big\rangle$$

$$\hat{\Delta}_f^{\alpha\alpha'}(z, \vec{b}_T) = \frac{1}{2z} \sum\!\!\!\!\!\!\!\int_X \Big\langle 0 \Big| \delta(\omega_b - \overline{\mathcal{P}}_{\bar{n}})\, \chi_{\bar{n}}^\alpha(b_\perp) \Big| h, X \Big\rangle \Big\langle h, X \Big| \bar{\chi}_{\bar{n}}^{\alpha'}(0) \Big| 0 \Big\rangle$$

- Soft Wilson lines yield the TMD soft function

$$\mathcal{S}(b_T) = \frac{1}{N_c} \mathrm{tr} \Big\langle 0 \Big| \big[S_n^\dagger(b_\perp)\, S_{\bar{n}}(b_\perp)\big]\, \big[S_{\bar{n}}^\dagger(0)\, S_n(0)\big] \Big| 0 \Big\rangle .$$

- Combine into the quark correctors

$$\Phi_f^{\beta'\beta}(x, \vec{b}_T) = \hat{\Phi}_f^{\beta'\beta}(x, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}\,, \qquad \Delta_f^{\alpha'\alpha}(z, \vec{b}_T) = \hat{\Delta}_f^{\alpha'\alpha}(z, \vec{b}_T) \sqrt{\mathcal{S}(b_T)}$$

$\Rightarrow$ Factorized leading power hadronic tensor

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[\Phi_f(x, \vec{b}_T)\, \gamma_\perp^\mu\, \Delta_f(z, \vec{b}_T)\, \gamma_\perp^\nu\Big] .$$

- Hard function: $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$

Structure functions at leading power

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$\Phi_f^{\beta'\beta}(x,\vec{p}_T) = \frac{1}{4}\left\{ f_1 \not{n} + \mathrm{i} h_1^\perp \frac{[\not{p}_\perp, \not{n}]}{2M_N} \right\}^{\beta'\beta},$$

$$\Delta_f^{\alpha'\alpha}(z,\vec{k}_T) = \frac{1}{4}\left\{ D_1 \not{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\not{k}_\perp, \not{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha}$$

- $h_1^\perp$ Boer-Mulders function, $H_1^\perp$ Collins function
- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{-1}^{(0)} = \mathcal{F}\left[\mathcal{H}^{(0)}\, f_1 D_1\right],$$

$$W_3^{(0)} = \mathcal{F}\left[-\frac{2\, p_{Tx}\, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h}\mathcal{H}^{(0)}\, h_1^\perp H_1^\perp\right],$$

[Bacchetta et al '06]

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2 p_T\, \mathrm{d}^2 k_T\, \delta^2\Big(\vec{q}_T + \vec{p}_T - \vec{k}_T\Big)\, \omega(\vec{p}_T, \vec{k}_T)\, \mathcal{H}_f(Q)\, g_f(x, p_T)\, D_f(z, k_T)$$

- Contract $W^{(0)\mu\nu}$ with $P_{1,2}^{(1)\mu\nu}$, we get the kinematic corrections for $W_{1,2}$

Subleading Operators: $\mathcal{P}_\perp$ acting on the collinear fields

Unique hard operators to all orders [Feige et al '17]

$$J^{(1)\mu}_{\mathcal{P}} \sim \frac{C_f^{(0)}}{2\omega_a} \bar\chi_{\bar n,\omega_b} [S^\dagger_{\bar n} S_n] \gamma^\mu \, \not{\mathcal{P}}_\perp \not{\bar n} \chi_{n,\omega_a} \,, \quad J^{(1)\mu}_{\mathcal{P}^\dagger} \sim \frac{C_f^{(0)}}{2\omega_b} \bar\chi_{\bar n,\omega_b} \not{n} \not{\mathcal{P}}^\dagger_\perp \gamma^\mu [S^\dagger_{\bar n} S_n] \chi_{n,\omega_a}$$

- RPI relates these two operators with the leading power operator

$\Rightarrow$ The Wilson coefficients is identical to the leading power one $C_f^{(0)}(Q)$

- Plug these currents into $\left(J^{(1)\dagger\mu}_{\mathcal{P}} + J^{(1)\dagger\mu}_{\mathcal{P}^\dagger}\right) J^{(0)\nu} + J^{(0)\dagger\mu} \left(J^{(1)\nu}_{\mathcal{P}} + J^{(1)\nu}_{\mathcal{P}^\dagger}\right)$

$$\hat W^{(1)\mu\nu}_{\mathcal{P}} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2\vec b_T \, \mathcal{H}_f^{(0)}(Q)\, \mathcal{S}(b_T) \times \left\{ \mathrm{Tr}\left[\hat\Phi_{\mathcal{P}f}(x,\vec b_T)\, \gamma^\mu \, \hat\Delta_f(z,\vec b_T)\, \gamma^\nu \right] + \mathrm{Tr}\left[\hat\Phi_f(x,\vec b_T)\, \gamma^\mu \, \hat\Delta_{\mathcal{P}f}(z,\vec b_T)\, \gamma^\nu \right] \right\} .$$

$$\hat\Phi^{\beta'\beta}_{\mathcal{P}f}(x,\vec b_T)$$

$$\equiv \frac{1}{2Q} \theta(\omega_a) \left\{ \left\langle N \middle| \bar\chi^\beta_n(b^\mu_\perp) \, [\not{\mathcal{P}}_\perp \not{\bar n} \, \chi_{n,\omega_a}(0)]^{\beta'} \middle| N \right\rangle + \left\langle N \middle| \left[\bar\chi_n(b^\mu_\perp) \not{\bar n} \not{\mathcal{P}}^\dagger_\perp \right]^\beta \chi^{\beta'}_{n,\omega_a}(0) \middle| N \right\rangle \right\}$$

$$= \mathrm{i} \frac{1}{2Q} \frac{\partial}{\partial b^\rho_\perp} \left[\gamma^\rho_\perp \not{\bar n} \,, \, \hat\Phi_f(x,\vec b_T) \right]^{\beta'\beta} ,$$

Subleading Operators: $\mathcal{P}_\perp$ acting on the collinear fields

- Define $\Phi_{\mathcal{P}}$, $\Delta_{\mathcal{P}}$ and $W_{\mathcal{P}}^{(1)\mu\nu}$

$$\Phi_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T) \equiv \mathrm{i}\frac{1}{2Q}\frac{\partial}{\partial b_\perp^\rho}\Big[\gamma_\perp^\rho\,\bar{\not{n}}\,,\,\Phi_f(x,\vec{b}_T)\Big]^{\beta'\beta}\,,\ \text{where } \Phi_f^{\beta'\beta}(x,\vec{b}_T) = \hat{\Phi}_f^{\beta'\beta}(x,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)$$
$$\times\Big\{\mathrm{Tr}\Big[\Phi_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\,\Delta_f(z,\vec{b}_T)\,\gamma^\nu\Big] + \mathrm{Tr}\Big[\Phi_f(x,\vec{b}_T)\,\gamma^\mu\,\Delta_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\Big]\Big\}\,.$$

- Equivalent to $\hat{W}_{\mathcal{P}}^{(1)\mu\nu}$ (noticing that $(n_\mu - \bar{n}_\mu)P_i^{\mu\nu} = \mathcal{O}(\lambda)$)

$$W_{\mathcal{P}}^{(1)\mu\nu} - \hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)\,\frac{\mathrm{i}}{Q}\left(\frac{\partial}{\partial b_\perp^\rho}\sqrt{\mathcal{S}(b_T)}\right)\sqrt{\mathcal{S}(b_T)}$$
$$\times\Big\{(\bar{n}^\nu - n^\nu)\,\mathrm{Tr}\Big[\gamma_\perp^\rho\,\hat{\Phi}_f(x,\vec{b}_T)\,\gamma^\mu\,\hat{\Delta}_f(z,\vec{b}_T)\Big] + (n^\mu - \bar{n}^\mu)\,\mathrm{Tr}\Big[\hat{\Phi}_f(x,\vec{b}_T)\,\gamma_\perp^\rho\,\hat{\Delta}_f(z,\vec{b}_T)\,\gamma^\nu\Big]\Big\}$$

- Same leading power functions appear, in momentum space

$$\Phi_{\mathcal{P}f}(x,\vec{p}_T) = \frac{1}{2Q}\Big[\not{p}_\perp\bar{\not{n}}\,,\,\Phi_f\Big] = \frac{1}{2Q}\Big\{f_1\not{p}_\perp - ih_1^\perp\frac{p_T^2\,[\not{n},\bar{\not{n}}]}{2M_N}\Big\}$$

Subleading Operators: with $\mathcal{B}_\perp$ insertion

- Fields and currents of definite helicity [Moult et al '15]

$$\mathcal{B}^a_{n\pm} = -\varepsilon_{\mp\mu}(n,\bar{n})\,\mathcal{B}^{a\mu}_{n\perp,\omega_c}\,,\ \chi^\alpha_{n\pm} = \frac{1\pm\gamma_5}{2}\chi^\alpha_{n,\omega_a}\,,\ J^{\bar{\alpha}\beta}_{\bar{n}n\pm} = \mp\sqrt{\frac{2}{\omega_a\,\omega_b}}\,\frac{\varepsilon^\mu_\mp(\bar{n},n)}{\langle n\mp|\bar{n}\pm\rangle}\,\bar{\chi}^{\bar{\alpha}}_{\bar{n}\pm}\,\gamma_\mu\chi^\beta_{n\pm}$$

- The complete set of operators in the helicity basis [Feige et al '17]

$$O^{(1)a\,\bar{\alpha}\beta}_{1+-} = \mathcal{B}^a_{n+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,,\qquad O^{(1)a\,\bar{\alpha}\beta}_{1-+} = \mathcal{B}^a_{n-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,,$$
$$O^{(1)a\,\bar{\alpha}\beta}_{2--} = \mathcal{B}^a_{\bar{n}-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,,\qquad O^{(1)a\,\bar{\alpha}\beta}_{2++} = \mathcal{B}^a_{\bar{n}+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,.$$

- Parity and charge conjugation invariance $\Rightarrow C^{(1)}_{\lambda_3\lambda_{12}} = C^{(1)}_{-\lambda_3-\lambda_{12}}$

$\Rightarrow$ Combination of operators appear as

$$\mathcal{B}_{n+}J_{\bar{n}n-} + \mathcal{B}_{n-}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\,[S^\dagger_{\bar{n}}S_n]\,\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
$$\mathcal{B}_{\bar{n}-}J_{\bar{n}n-} + \mathcal{B}_{\bar{n}+}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\not{\mathcal{B}}_{\perp\bar{n},\omega_c}\,[S^\dagger_{\bar{n}}S_n]\,\chi_{n,\omega_a}$$

- Same soft Wilson lines as leading power

Subleading Operators: with $\mathcal{B}_\perp$ insertion

$$\frac{1}{\sqrt{\omega_a \omega_b}}\, \bar\chi_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n]\, \not{\mathcal{B}}_{\perp n,-\omega_c} \chi_{n,\omega_a}\,, \qquad \frac{1}{\sqrt{\omega_a \omega_b}}\, \bar\chi_{\bar n,\omega_b} \not{\mathcal{B}}_{\perp \bar n,\omega_c}\, [S_{\bar n}^\dagger S_n]\, \chi_{n,\omega_a}$$

- Hermiticity + Kinematics + $n \leftrightarrow \bar n$: only one $C_f^{(1)}$ which is real
- Summing over helicities gives

$$\sum_{\lambda_e,\lambda_{12},\lambda_3} C_f^{(1)}\, \mathcal{B}_{\lambda_3}\, J_{\bar n n \lambda_{12}} J_{\lambda_e} \sim J_{\mathcal{B}}^{(1)\mu}\, J_{e\mu}$$

  where J_{λ_e} is the the leptonic current with definite helicity
- This yields

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar n^\mu) \int \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c\, C_f^{(1)}(Q,\omega_c)$$
$$\times \Big[\delta(\omega_a+\omega_c-Q)\, \delta(\omega_b-Q)\, \bar\chi_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n] \not{\mathcal{B}}_{\perp n,-\omega_c} \chi_{n,\omega_a}$$
$$+ \delta(\omega_a-Q)\, \delta(\omega_b+\omega_c-Q)\, \bar\chi_{\bar n,\omega_b} \not{\mathcal{B}}_{\perp \bar n,\omega_c} [S_{\bar n}^\dagger S_n] \chi_{n,\omega_a} \Big]$$

Subleading Operators: with $\mathcal{B}_\perp$ insertion

Denoting $\xi = \omega_c/Q$, define the q-q-g correlators as

$$\hat{\tilde{\Phi}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) \equiv Q\,\langle N|\left[\bar{\chi}^{\beta}_{n,\omega_a}\,\mathcal{B}^{\rho}_{\perp n,-\omega_c}\right](b^\mu_\perp)\,\chi^{\beta'}_{n}(0)\,|N\rangle\ ,$$

$$\hat{\tilde{\Delta}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) \equiv \frac{Q}{2z}\sum\!\!\!\!\!\!\int_X \langle 0|\left[\bar{\chi}^{\beta}_{\bar{n},\omega_b}\,\mathcal{B}^{\rho}_{\perp\bar{n},\omega_c}\right](b^\mu_\perp)\,|h,X\rangle\,\langle h,X|\,\chi^{\beta'}_{\bar{n}}(0)\,|0\rangle$$

$$\tilde{\Phi}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) = \hat{\tilde{\Phi}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)},$$
$$\tilde{\Delta}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) = \hat{\tilde{\Delta}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W^{(1)\mu\nu}_{\mathcal{B}} = \frac{2z}{Q}\sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\int_0^1 \mathrm{d}\xi\,\mathcal{H}^{(1)}(Q,\xi)$$
$$\times\,\mathrm{Tr}\left[\tilde{\Phi}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\,\gamma_\rho\,\Delta_f(z,\vec{b}_T)\,\gamma^\nu_\perp + \Phi_f(x,\vec{b}_T)\,\gamma^\mu_\perp\,\tilde{\Delta}^{\rho}_{\mathcal{B}\,f}(z,\xi,\vec{b}_T)\,\gamma_\rho + \text{h.c.}\right].$$

$$\mathcal{H}^{(1)}(Q,\xi) = C^{(1)}_f(Q,\xi)\,C^{(0)}_f(Q)$$

In momentum space, $\tilde{\Phi}^{\rho\,\beta'\beta}_{\mathcal{B}f}$ can be decomposed as [Boer, Mulders, Pijlman '03]

[Bacchetta, Mulders, Pijlman '04]

$$\tilde{\Phi}^{\rho\,\beta'\beta}_{\mathcal{B}f}(x,\xi,\vec{p}_T) = \frac{xM_N}{2}\left\{\left[\left(\tilde{f}^\perp - \mathrm{i}\tilde{g}^\perp\right)\frac{p_{\perp\sigma}}{M_N}\left(g^{\rho\sigma}_\perp - \mathrm{i}\epsilon^{\rho\sigma}_\perp\gamma_5\right) + \mathrm{i}\left(\tilde{h} + \mathrm{i}\,\tilde{e}\right)\gamma^\rho_\perp\right]\frac{\not{n}}{2}\right\}^{\beta'\beta},$$

Results

$$P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu)\,, \quad P_2^{\mu\nu} = \mathrm{i}(\tilde{t}^\mu \tilde{x}^\nu - \tilde{x}^\mu \tilde{t}^\nu)$$

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\, h_1^\perp H_1^\perp \Big] \qquad \text{(Kinematic corrections)}$$

$$-\,\mathcal{H}^{(0)}\Big[\frac{p_{Tx} + k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp \Big] \qquad \text{(From the } \mathcal{P} \text{ operators)}$$

$$+\,\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} k_{Tx}\,\tilde{h}\,H_1^\perp \Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1 \tilde{D}^\perp + \frac{M_h}{M_N} p_{Tx}\,h_1^\perp \tilde{H} \Big)\Big]\Big\}$$

(From the $\mathcal{B}$ operators)

$$W_2 = \mathcal{F}\Big\{ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{g}^\perp D_1 + \frac{M_N}{M_h} k_{Tx}\,\tilde{e}\,H_1^\perp \Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1 \tilde{G}^\perp + \frac{M_h}{M_N} p_{Tx}\,h_1^\perp \tilde{E} \Big)\Big]\Big\}$$

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2 p_T\,\mathrm{d}^2 k_T\,\delta^2\big(\vec{q}_T + \vec{p}_T - \vec{k}_T\big)\,\omega(\vec{p}_T, \vec{k}_T) \times \int_0^1 \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),p_T)\,D_f(z,(\xi),k_T)$$

New in our results

- Soft function, same as leading power (as conjectured in [Bacchetta et al '19])
- Appearance of two hard functions, $\mathcal{H}^{(0)}(Q)$ and $\mathcal{H}^{(1)}(Q,\xi)$
- Dependence on ξ in $\mathcal{H}^{(1)}(Q,\xi)$ and the functions $\tilde{f}^\perp$, $\tilde{D}^\perp$, ...

Further Discussions

Implications from our results

- Rapidity anomalous dimension is the same as at leading power

$$\tilde{\Phi}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\zeta) = \hat{\tilde{\Phi}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T,\mu,\nu^2/\zeta)\sqrt{\mathcal{S}(b_T,\mu,\nu)},$$

$$\Rightarrow \frac{\mathrm{d}\log\tilde{\Phi}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}}{\mathrm{d}\log\zeta} = \frac{1}{4}\frac{\mathrm{d}\log\mathcal{S}}{\mathrm{d}\log\nu} = \frac{1}{4}\gamma_\nu(\mu,b_T)$$

Comparison with literature

- At leading order, $C_f^{(1)}$ is independent on ξ from tree level matching, ξ can be integrated in qqg correlators. $W_{1,2}$ then fully agrees with [Bacchetta et al '06] at leading order (after inclusion of the soft function, as conjectured in [Bacchetta et al '19])
- Anomalous dims of $C_f^{(1)}$ have been calculated to one loop, with single log dependence on ξ [Beneke et al, '17 '18]

$\Rightarrow$ Confirms the nontrivial ξ dependence

$\Rightarrow$ Disproves the simpler factorization theorem in [Bacchetta et al '19]

Summary & Outlook

- Derived factorization of $W^{\mu\nu}$ at subleading power, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_\perp$
- Showed the factorization formulae of subleading structure functions W_1 and W_2, including contributions from the kinematic correction and the subleading operators
- Future Directions
  - Resummation of W_1 and W_2
  - Possible contributions from the SCET_{II} subleading Lagrangian

Summary & Outlook

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- Future Directions
  - ▷ Resummation of W_1 and W_2
  - ▷ Possible contributions from the SCET_{II} subleading Lagrangian

Thanks for your attention!