Resummation of large logs in the presence of endpoint divergences in SCET

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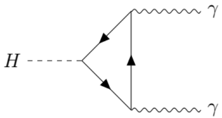
Based on work done with Z.Liu, M.Neubert and X.Wang
arXiv 2009.06779
arXiv 2009.04456

Resummation in SCET: $H \to \gamma\gamma$

Precision calculations require large log resummation.



Coefficients of the 3 loop amplitude for $H \to \gamma\gamma$ (from our analytical results):

$$C_F \left[\frac{C_F}{90} L^6 + \dots \right] = 0.01975 L^6 - 0.31111 L^5 - 8.74342 L^4 - 68.6182 L^3 + \dots$$
$$+ \left(0.02963 L^5 + 0.79012 L^4 + 3.57918 L^3 + \dots \right) n_f$$

See Xing's talk on Thursday

Confirmed with numerical results in literature.

(Czakon and M. Niggetiedt, '20)

Resummation of only leading and next to leading logs is insufficient!

SCET offers a consistent framework for RG-improved PT.

Renormalized factorization formula for $H \to \gamma\gamma$

Consistent large log resummation to higher orders requires:

- Renormalized factorization formula for the amplitude

$$
\begin{aligned}
\mathcal{M}_b =& H_1(\mu)\langle O_1(\mu)\rangle + 2\int_0^1 dz\Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!]\Big] \\
&+ \lim_{\sigma\to -1} H_3(\mu)\int_0^{M_h}\frac{d\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{d\ell_+}{\ell_+} J\left(M_h\ell_-,\mu\right) J\left(-M_h\ell_+,\mu\right) S\left(\ell_+\ell_-,\mu\right)
\end{aligned}
$$

- RGEs that hold beyond the LL and NLL

The hard coefficient H_1 has the most non-trivial RGE.
Derived from the RG invariance of the full amplitude and the re-factorization conditions.

$$
\frac{d}{d\ln\mu}\mathcal{M}_b = 0 = \left[\left(\frac{d}{d\ln\mu} - \gamma_{11}\right) H_1(\mu)\right]\langle O_1(\mu)\rangle + \frac{d}{d\ln\mu}T_2(\mu) + \frac{d}{d\ln\mu}T_3(\mu)
$$

Detailed derivation in arXiv2009.06779

Renormalization Group Equations

Operators

$$\frac{d}{d\ln\mu}\left\langle O_1(\mu)\right\rangle = -\gamma_{11}\left\langle O_1(\mu)\right\rangle$$

$$\frac{d}{d\ln\mu}\left\langle O_2(z,\mu)\right\rangle = -\int_0^1 dz'\gamma_{22}\left(z,z'\right)\left\langle O_2\left(z',\mu\right)\right\rangle - \gamma_{21}(z)\left\langle O_1(\mu)\right\rangle$$

$$\frac{d}{d\ln\mu}[\![\langle O_2(z,\mu)\rangle]\!] = -\int_0^\infty dz'[\![\gamma_{22}\left(z,z'\right)]\!][\![\langle O_2\left(z',\mu\right)\rangle]\!] - [\![\gamma_{21}(z)\langle O_1(\mu)\rangle]\!]$$

Jet function

$$\frac{d}{d\ln\mu}J\left(p^2,\mu\right) = -\int_0^\infty dx\gamma_J\left(p^2,xp^2\right)J\left(xp^2,\mu\right)$$

(Z.Liu and M.Neubert arXiv 2003.03393)

Soft function

$$\frac{d}{d\ln\mu}S(w,\mu) = -\int_0^\infty dw'\gamma_S\left(w,w';\mu\right)S\left(w',\mu\right)$$

(Z.Liu, BM, M.Neubert, X.Wang arXiv2005.03013)

Hard coefficients

$$\frac{d}{d\ln\mu}H_1(\mu) = D_{\text{cut}}(\mu) + \gamma_{11}H_1(\mu) + 2\int_0^1 dz\left[[H_2(z,\mu)\gamma_{21}(z) - [\![H_2(z,\mu)]\!]z[\![\gamma_{21}(z)]\!] - [\![H_2(\bar{z},\mu)]\!]\bar{z}[\![\gamma_{21}(\bar{z})]\!]\right]$$

$$\frac{d}{d\ln\mu}H_2(z,\mu) = \int_0^1 dz'H_2\left(z',\mu\right)\gamma_{22}\left(z',z\right)$$

$$\frac{d}{d\ln\mu}[\![H_2(z,\mu)]\!] = \int_0^1 dz'[\![H_2\left(z',\mu\right)]\!]\,[\![\gamma_{22}\left(z',z\right)]\!]\,, \qquad \frac{d}{d\ln\mu}H_3(\mu) = \gamma_{33}H_3(\mu)$$

Anomalous dimensions

In principle using RGEs is possible to obtain "arbitrary" higher order accuracy in log resummation.

$$\gamma_{33} = \Gamma_{\text{cusp}}\left(\alpha_s\right) \ln \frac{-M_h^2}{\mu^2} + 2\gamma_q\left(\alpha_s\right)$$

$$\gamma_{22}\left(z,z'\right) = -\frac{C_F \alpha_s}{\pi}\left\{\left[\ln z + \ln(1-z) + \frac{3}{2}\right]\delta\left(z-z'\right) + z(1-z)\left[\frac{1}{z'(1-z)}\frac{\theta\left(z'-z\right)}{z'-z} + \frac{1}{z\left(1-z'\right)}\frac{\theta\left(z-z'\right)}{z-z'}\right]_+\right\} + \mathcal{O}\left(\alpha_s^2\right)$$

$$[\![\gamma_{22}\left(z,z'\right)]\!] = -\frac{C_F \alpha_s}{\pi}\left\{\left(\ln z + \frac{3}{2}\right)\delta\left(z-z'\right) + z\left[\frac{\theta\left(z'-z\right)}{z'\left(z'-z\right)} + \frac{\theta\left(z-z'\right)}{z\left(z-z'\right)}\right]_+\right\} + \mathcal{O}\left(\alpha_s^2\right)$$

$$\gamma_J\left(p^2, xp^2\right) = \left[\Gamma_{\text{cusp}}\left(\alpha_s\right)\ln\frac{-p^2}{\mu^2} - \gamma'\left(\alpha_s\right)\right]\delta(1-x) + \Gamma_{\text{cusp}}\left(\alpha_s\right)\Gamma(1,x) + \mathcal{O}\left(\alpha_s^2\right)$$

$$\gamma_S\left(w,w';\mu\right) = -\left[\Gamma_{\text{cusp}}\left(\alpha_s\right)\ln\frac{w}{\mu^2} - \gamma_s\left(\alpha_s\right)\right]\delta\left(w-w'\right) - 2\Gamma_{\text{cusp}}\left(\alpha_s\right) w\Gamma\left(w,w'\right) + \mathcal{O}\left(\alpha_s^2\right)$$

$$D_{\text{cut}}\left(\mu\right) = 4\int_0^\infty dx K(x)\int_1^{1/x}\frac{dz}{z}[\![\bar{H}_2(xz,\mu)]\!]\Delta_{21}(z,\mu)$$

D_{cut} depends only on the hard scale and has single logs.

$$D_{\text{cut}}(\mu) = -\frac{N_c \alpha_b}{\pi}\frac{y_b(\mu)}{\sqrt{2}}\left[\frac{C_F\alpha_s}{4\pi}16\zeta_3 + \left(\frac{\alpha_s}{4\pi}\right)^2 d_{\text{cut},2} + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$d_{\text{cut},2} \sim \ln^2\frac{M_h^2}{\mu^2}$$

Resummation in RG improved PT at NLP in SCET

Resummation in RG improved PT

$$\mathcal{M}_b = \overbrace{H_1(\mu)\langle O_1(\mu)\rangle}^{T_1} + \overbrace{2\int_0^1 dz\Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!]\Big]}^{T_2}$$
$$+ \underbrace{\lim_{\sigma\to -1} H_3(\mu)\int_0^{M_h}\frac{d\ell_-}{\ell_-}\int_0^{\sigma M_h}\frac{d\ell_+}{\ell_+} J\left(M_h\ell_-,\mu\right) J\left(-M_h\ell_+,\mu\right) S\left(\ell_+\ell_-,\mu\right)\Big|_{LP}}_{T_3}$$

- **At LO in RG improved PT the first two series are resummed.**

$$T_i \sim \exp\left[L\cdot g_0(\alpha L) + g_1(\alpha L) + \alpha\cdot g_2(\alpha L) + ...\right]$$
$$\left.\equiv \sum_n \alpha^n\cdot L^{n+1} + \sum_n \alpha^n\cdot L^n + \mathcal{O}(\alpha)\text{ corrections}\right]$$

- **For $H \to \gamma\gamma$ there is a contribution at LO RG improved PT from the T_3 and T_2 amplitude.**
- For $\mu_h \sim M_h$ then T_1 large log contribution starts at $\mathcal{O}(\alpha_s)$

Note that resummation in the RG improved PT is much more accurate than the *naiive* log expansion of the amplitude:

$$M \sim \sum\left(\alpha\cdot L^2\right)^n + \sum\left(\alpha^4\cdot L^{2n-1}\right) + ...$$

In this case large logs from higher orders are still left unresummed!

Resummation at LO in RG improved PT
T_3 amplitude

Resummation of T_3 at LO in RG improved PT

$$\mathcal{M}_b = \overbrace{H_1(\mu)\langle O_1(\mu)\rangle}^{T_1} + 2\overbrace{\int_0^1 dz \Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!] \Big]}^{T_2}$$
$$+ \underbrace{\lim_{\sigma\to -1} H_3(\mu) \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+} J\left(M_h \ell_-,\mu\right) J\left(-M_h\ell_+,\mu\right) S\left(\ell_+\ell_-,\mu\right)\Big|_{LP}}_{T_3}$$

- Cutoffs introduce higher order power corrections that should be dropped for consistency
- $\ell_+ \,, \ell_-$ integrals generate rapidity logs (collinear anomaly)
- T_3 is enhanced by two powers of log

$$T_1 \propto -2 + \frac{C_F\alpha_s}{4\pi}\left[-\frac{\pi^2}{3}L_h^2 + \left(12 + 8\zeta_3\right)L_h + \cdots\right]$$
$$T_2 \propto \frac{C_F\alpha_s}{4\pi}\left[\frac{2\pi^2}{3}L_hL_m - \frac{\pi^2}{3}L_m^2 + \cdots\right]$$
$$T_3 \propto \frac{L^2}{2} + \frac{C_F\alpha_s}{4\pi}\left[\frac{5L^4}{12} + \left(L_m - 1\right)L^3 + \left(4 - \frac{\pi^2}{3} + \frac{L_m^2}{2} - \frac{L_h^2}{2} - 3L_m\right)L^2 + \left(\frac{2\pi^2}{3} + 8\zeta_3\right)L - 8\zeta_3 L_m + \cdots\right]$$

$$L_h = \ln\left(-M_h^2/\mu^2\right)\,, \qquad L_m = \ln\left(m_b^2/\mu^2\right)$$

T_3 amplitude contains the leading and next-to-leading large logs.

Resummation of T_3 at LO in RG improved PT

$$T_3^{\text{LO}} = \frac{\alpha}{3\pi}\frac{y_b(\mu_h)}{\sqrt{2}}\int_0^{M_h}\frac{d\ell_-}{\ell_-}\int_0^{M_h}\frac{d\ell_+}{\ell_+}m_b(\mu_s)\,e^{2\mathcal{S}_F(\mu_s,\mu_h)-2\mathcal{S}_F(\mu_-,\mu_h)-2\mathcal{S}_F(\mu_+,\mu_h)}\left(\frac{-M_h\ell_-}{\mu_-^2}\right)^{a_\Gamma^-}\left(\frac{-M_h\ell_+}{\mu_+^2}\right)^{a_\Gamma^+}\left(\frac{-\ell_+\ell_-}{\mu_s^2}\right)^{-a_\Gamma^s}$$
$$\times\left(\frac{\alpha_s(\mu_s)}{\alpha_s(\mu_h)}\right)^{-\frac{\gamma_{s,0}}{2\beta_0}} e^{-2\gamma_E a_\Gamma^+}\frac{\Gamma\left(1-a_\Gamma^+\right)}{\Gamma\left(1+a_\Gamma^+\right)}e^{-2\gamma E a_\Gamma^-}\frac{\Gamma\left(1-a_\Gamma^-\right)}{\Gamma\left(1+a_\Gamma^-\right)}e^{4\gamma E a_\Gamma^s}G_{4,4}^{2,2}\left(\begin{matrix}-a_\Gamma^s, -a_\Gamma^s, 1-a_\Gamma^s, 1-a_\Gamma^s\\ 0,\ 1,\ 0,\ 0\end{matrix}\,\middle|\,\frac{m_b^2}{-\ell_+\ell_-}\right)$$

$$\mathcal{S}_F(\mu_i,\mu_h) = \frac{C_F\gamma_0^{\text{cusp}}}{4\beta_0^2}\left[\frac{4\pi}{\alpha_s(\mu_i)}\left(1-\frac{1}{r}-\ln r\right)+\left(\frac{\gamma_1^{\text{cusp}}}{\gamma_0^{\text{cusp}}}-\frac{\beta_1}{\beta_0}\right)(1-r+\ln r)+\frac{\beta_1}{2\beta_0}\ln^2 r\right]$$
$$a_\Gamma(\mu_i,\mu_h) = \frac{C_F\gamma_0^{\text{cusp}}}{2\beta_0}\ln\frac{\alpha_s(\mu_h)}{\alpha_s(\mu_i)}\,,\qquad r=\frac{\alpha_s(\mu_h)}{\alpha_s(\mu_i)}$$

$G_{4,4}^{2,2}(\cdots\mid x)$: Meijer G function
Scale fixing: $\mu_h^2 = M_h^2$

⋆ **Dynamical scale setting:** $\mu_s^2 \sim \ell_+\ell_- \qquad \mu_\pm^2 \sim \ell_\pm M_h$
⋆ $G_{4,4}^{2,2}(\cdots\mid x)$ vanishes for $x\to\infty \Rightarrow$ the scale parameters should be kept larger than m_b^2.

Resumed amplitude

Resummed amplitude expanded in power series:

$$\mathcal{M}_b^{\rm NLL} = \frac{\alpha}{3\pi} m_b \frac{y_b\left(\hat{\mu}_h\right)}{\sqrt{2}} \frac{L^2}{2} \sum_{n=0}^{\infty} (-\rho)^n \frac{2\Gamma(n+1)}{\Gamma(2n+3)} \times \left[1 + \frac{3\rho}{2L}\frac{2n+1}{2n+3} - \frac{\beta_0}{C_F}\frac{\rho^2}{4L}\frac{(n+1)^2}{(2n+3)(2n+5)}\right]$$

$$\rho = \tfrac{C_F \alpha_s(\hat{\mu}_h)}{2\pi} L^2 \,, \qquad L = \ln\left(-M_h^2/m_b^2 - i0\right)$$

Sub-leading log term doesn't agree with literature!

(Akhoury et al, '01)

The formalism can be extended to colored final states.
A very similar formula was derived for $gg \to H$

(See Xing's talk)

Tower of LL there agrees with literature. Disagreement at NLL.

(Liu and Penin, '17; Anastasiou and Penin, '20)

Resummation at LO in RG improved PT
T_2 amplitude

T_2 resummation at LO in RG improved PT

- **Completing the resummation at LO in RG improved PT requires resummation of large logs in T_2 amplitude**

$$T_2 = 2\int_0^1 dz\Big[H_2(z,\mu)\langle O_2(z,\mu)\rangle - [\![H_2(z,\mu)]\!][\![\langle O_2(z,\mu)\rangle]\!] - [\![H_2(\bar{z},\mu)]\!][\![\langle O_2(\bar{z},\mu)\rangle]\!]\Big]$$

- **T_2 has a more complicated structure $\Rightarrow$ challenging to produce an analytical result as for T_3**
- **Endpoint divergences are are cancelled between $H_2 \otimes O_2$ and $[\![H_2]\!] \otimes [\![O_2]\!]$ terms**

T_2 resummation

- **Expand O_2 and H_2 in Gegenbauer moments**

(Lepage and Brodsky, '79)

$$H_2 \otimes O_2 \equiv \int_0^1 \mathrm{d}z H_2(z,\mu) \langle O_2(z,\mu) \rangle \propto \sum_{m=0}^{\infty} h_{2m}(\mu) a_{2m}(\mu)$$

$$\frac{\mathrm{d}a_{2m}(\mu)}{\mathrm{d}\ln\mu} = -\tilde{\gamma}_{22}\left(2m, \alpha_s(\mu)\right) a_{2m}(\mu) - 2N(2m)\tilde{\gamma}_{21}\left(2m, \alpha_s(\mu)\right) \langle O_1(\mu) \rangle$$

$$N(2m) = \frac{2(4m+3)}{3(2m+1)(2m+2)}$$

- **Solution:**

$$a_{2m}(\mu) \propto \frac{4\pi}{\alpha_s(\nu)} \frac{r^{-1+\frac{\delta\gamma_{2m}^{(0)}}{2\beta_0}} - 1}{2\beta_0 - \delta\tilde{\gamma}_{2m}^{(0)}} + \left(\tilde{\gamma}_{21}^{(0)}(2m) + \frac{\beta_1}{\beta_0}\right) \frac{r^{\frac{\delta\bar{\gamma}_{2m}^{(0)}}{2\beta_0}} - 1}{\delta\tilde{\gamma}_{2m}^{(0)}} + \left(-\frac{\delta\tilde{\gamma}_{2m}^{(0)}}{2\beta_0}\frac{\beta_1}{\beta_0} - \frac{\delta\tilde{\gamma}_{2m}^{(1)}}{2\beta_0}\right) \left[\frac{r^{\frac{\delta\gamma_{2m}^{(0)}}{2\beta_0}} - 1}{\delta\tilde{\gamma}_{2m}^{(0)}} + \frac{r^{-1+\frac{\delta\bar{\gamma}_{2m}^{(0)}}{2\beta_0}} - 1}{2\beta_0 - \delta\tilde{\gamma}_{2m}^{(0)}}\right]$$

$$r = \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}$$

Requires the two loop coefficient of the Brodsky-Lepage kernel in Gegenbauer space: $\delta\tilde{\gamma}_{2m}^{(1)}$

T_2 resummation

- **Divergent part comes from the mixing of O_2 with O_1.**
  The divergent part of the convolution

$$(H_2(\mu) \otimes O_2(\mu))_{\text{div}} \propto \sum_{m=0}^{\infty} \frac{6N(2m)}{\beta_0+2C_F(2H_{2m+1}-3)} O_1(\nu)$$

- **For the subtraction terms need to solve the RGEs and then expand the result in Gegenbauer moments:**
  The divergent part is exactly the same as for $H_2 \otimes O_2$ and they cancel!

$$\left([\![H_2(z,\mu)]\!] \otimes [\![O_2(z,\mu)]\!] + [\![H_2(\bar{z},\mu)]\!] \otimes [\![O_2(\bar{z},\mu)]\!] \right)_{\text{div}} \propto \sum_{m=0}^{\infty} \frac{6N(2m)}{\beta_0 + 2C_F\left(2H_{2m+1} - \frac{1}{(2m+1)(2m+2)} - 3\right)} O_1(\nu)$$

To perform the resummation it is easier to solve the equations numerically.

(Work in progress)

Conclusions

- **Resummation of logarithms for precision purposes should be treated in RG improved PT**
- **We have shown the first subleading power observable in SCET consistently treated in RG improved PT**
- **The framework sets the basis for other application for large log resummation; for instance with colored final states $gg \to H$**